KAMAK 2017

Penzion Orlice, Záměl

September 17.–22.

Charles University, Prague

Organizers:

Pavel Dvořák
Robert Šámal

Brochure of open problems,
Prague, 2017.

Contents

Program

8:30 breakfast

9:30 morning session I

10:30 break

11:00 morning session II

12:30 lunch

15:00 afternoon session I

16:30 break

17:00 afternoon session II

18:30 progress reports

19:00 dinner

Wednesday is planned to be free to make an excursion in the neighborhood with everyone who would like to come.

OPEN PROBLEMS

Problem 1. *Online Ramsey number of paths on three colors (suggested by Václav Blažej)*

Source: Grytczuk et al. in 2004

Definitions.
• *An online Ramsey game with the goal graph H is a game between Builder and Painter, alternating in turns. In each round Builder draws an edge and Painter colors it either red or blue (generally k colors). Builder wins if after some round there is a monochromatic copy of the graph H, otherwise Painter is the winner.*
• *An online Ramsey number $\widetilde{r}(H)$ is the minimum number of rounds such that Builder has a winning strategy in the online Ramsey game.*

Example:
• In a c-color variant, we see that $\widetilde{r}(S_k) = c(k-1)+1$, since we can create $S_{c(k-1)+1}$ and at least k of those edges will have the same color.
• For 2-color variant, it is possible to create a monochromatic triangle in 8 moves by creating an S_5 and connecting endpoints of monochromatic S_3 with three edges.

Question: Determine bounds on $\widetilde{r}(P_n)$ for the three color variant of the online Ramsey game.

Related results:
• There is a well known linear upper bound on $\widetilde{r}(P_n)$ by Grytczuk et al. 2008 on two colors, however the same strategy does not work when Painter can use three colors.
• In 1995 Haxell et al. proved an upper bound for size-Ramsey number of induced cycles. This translates to a linear upper bound on online Ramsey number of paths for arbitrary fixed number of colors.

References:

• Jaroslaw Grytczuk, Mariusz Hałuszczak, and Hal A. Kierstead. On-line ramsey theory. *Electronic Journal of Combinatorics*, 11, 2004.
• Jaroslaw Grytczuk, Hal A. Kierstead, and Paweł Prałat. On-line ramsey numbers for paths and stars. *Discrete Mathematics & Theoretical Computer Science*, 10, 2008.
• P. E. Haxell, Y. Kohayakawa and T. Łuczak. The Induced Size-Ramsey Number of Cycles. 1995.

Problem 2. *Lower Bound for the* SPAN *Problem (suggested by Pavel Dvořák)*

Source: personal communication with M. Koucký.

Definitions.
• *A data structure for the* SPAN *problem stores a subspace* $U \subseteq \mathbb{Z}_2^n$ *and can answer a question if some vector* $x \in \mathbb{Z}_2^n$ *is in* U.
• *The* cell probe model *for a data structure* D *which can answer queries* Q *is defined as follows. The data structure* D *is mapped into* s *cells, where each cell consists of* b *bits. Each query* $q \in Q$ *is a decision tree* T_q*, where each node of the tree probes a single cell* c *and branches according to the* 2^b *different values of* c*. Each leaf of* T_q *is labeled by an answer to* q.
• *The time* t_q *used for answering the query* q *is a depth of* T_q*, i.e., the number of probes. The query time* t *of the data structure* D *is the maximum* t_q *over all queries* $q \in Q$.
• *The* bit probe model *is the cell probe model for* $b = 1$.

Example:
The data structure D can store $n - \dim(U)$ equations which define U (basically the basis of the orthogonal complement). And if we want to answer that $x \in U$ we can check if x satisfies every equation. Thus, the data structure needs n^2 bits and its query time is n.

Question: What is a lower bound of t and s for the SPAN problem in the bit probe model?

Related results:
• In the cell probe model for $b = n$ it can be proved that if $t = o(n)$ then $s \geq 2^{\Omega(n/t)}$ [1], which meets the upper bound if we want polynomial s. However, if we use the same proof for $b = 1$ we get the same lower bound, but the upper bound is $t = O(n^2), s = O(n^2)$ – by the data structure from the example.

References:

[1] E. Kushilevitz, N. Nisan, Communication Complexity, Cambridge University Press New York, NY, USA ,1997.

Problem 3. *Improve the lower bound related to Chvátal's toughness conjecture (suggested by Adam Kabela)*

Source: In 1973, Chvátal conjectured that there is a constant t such that every t-tough graph has a Hamilton cycle. The conjecture is still open. The progress in the study of Chvátal's conjecture is well-documented by a series of survey papers; see, for instance, Bauer, Broersma and Schmeichel (2006).

Definitions. *We recall that the* toughness *of a graph G is the minimum of $\frac{|S|}{c(G-S)}$ where $c(G-S)$ denotes the number of components of the graph $G-S$ and the minimum is taken over all sets of vertices S such that $c(G-S) \geq 2$. The toughness of a complete graph is defined as ∞. We say that a graph is* t-tough *if its toughness is at least t.*

Best known lower bound: In 2000, Bauer, Broersma and Veldman constructed $(\frac{9}{4}-\varepsilon)$-tough non-Hamiltonian graphs for every $\varepsilon > 0$ (in fact, the graphs have no Hamilton path). The construction is outlined in Figure 1. The previously best known bound is due to Enomoto et al. (1985) who presented $(2-\varepsilon)$-tough graphs which have no 2-factor (and thus no Hamilton cycle).

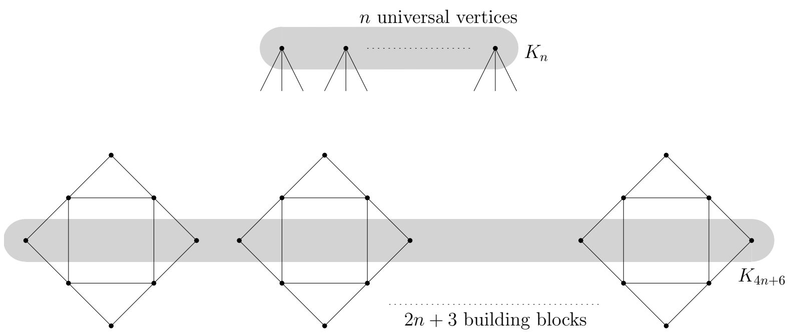

Figure 1: A sketch of the construction of $(\frac{9}{4}-\varepsilon)$-tough non-Hamiltonian graphs.

Question: Can we improve this lower bound? In other words, are there $\frac{9}{4}$-tough non-Hamiltonian graphs?

References:

• D. Bauer, H. J. Broersma, E. Schmeichel: Toughness in graphs — A survey, Graphs and Combinatorics 22 (2006), 1–35.

• D. Bauer, H. J. Broersma, H. J. Veldman: Not every 2-tough graph is Hamiltonian, Discrete Applied Mathematics 99 (2000), 317–321.

• V. Chvátal: Tough graphs and hamiltonian circuits, Discrete Mathematics 5 (1973), 215–228.
• H. Enomoto, B. Jackson, P. Katerinis, A. Saito: Toughness and the existence of k-factors, Journal of Graph Theory 9 (1985), 87–95.

Problem 4. *Induced rainbow paths (suggested by Tereza Klimošová)*

Source: Attributed to Aravind (2013), fist appeared in [1].

Definition. *We call a subgraph of a properly (vertex) colored graph* rainbow*, if no two vertices of the subgraph have the same color.*

Conjecture 1. *Let G be a triangle-free graph. Then for every colouring (not necessarily optimal) of G, there is a rainbow induced subgraph isomorphic to a $\chi(G)$-vertex path.*

The *girth* of a graph G is the length of a shortest cycle contained in the graph, we denote it by $g(G)$.

It is known that every properly colored graph (not necessarily by minimum number of colors) has a rainbow path on $\chi(G)$ vertices but this path might not be induced (corollary of Galai-Roy theorem). It follows the conjecture for graphs with girth larger than chromatic number. Moreover, it is easy to observe that the conjecture holds for $\chi(G) \leq 3$.

In [1], it was shown that the conjecture holds for graphs where girth equals chromatic number.

Theorem 1. *Let G be a graph with $g(G) = \chi(G)$ whose vertices have been properly coloured. Then there exists an induced rainbow path on $\chi(G)$ vertices in G.*

This implies that the conjecture holds also for $\chi(G) = 4$.

A relaxed version of the conjecture was proven in [2] and later strengthened in [3].

Theorem 2. *For all $s \geq 1$ there exists c such that for every properly colored graph G with girth at least 5 and $\chi(G) > c$ there is a rainbow induced subgraph isomorphic to an s-vertex path.*

Theorem 3. *For all $\kappa, s \geq 1$ there exists c such that for every properly colored graph G with $\omega(G) \leq \kappa$ and $\chi(G) > c$, there is a rainbow induced subgraph of G isomorphic to an s-vertex path.*

In [3], the authors remark that they do not believe the conjecture (but do not have a counterexample and seem to have tried to find one only manually) and that analogous statement is false for graphs that are not paths. They also ask a related question: is it true that for all fixed s, κ, if G is a graph with $\omega(G) \leq \kappa$ and $\chi(G)$ sufficiently large then in every colouring of G there is a hole in which some set of s consecutive vertices is rainbow?

References:

[1] J. Babu, M. Basavaraju, L. Chandran and M. Francis, “On induced colorful graphs in trianglefree graphs”, arXiv:1604.06070.

[2] A. Gyárfás and G. Sárközy, “Induced colorful trees and paths in large chromatic graphs”, Electronic J. Combinatorics 23 (2016), #P4.46.

[3] A. Scott and P. Seymour: “Induced subgraphs of graphs with large chromatic number. IX. Rainbow paths.” arXiv preprint arXiv:1702.01094.

Problem 5. *Minimizing trace of even power (suggested by Tereza Klimošová)*

Source: Dan Král', personal communication, motivated by graphons.

Definition. *For a $n \times n$ matrix $A = (a_{ij})$ and a partition $\mathcal{P}$ of $[n]$ we define a* partition averaged *matrix $B = (b_{ij})$ to be a $n \times n$ matrix such that for every $P_k, P_\ell \in \mathcal{P}$ every b_{ij} satisfying $(i,j) \in P_k \times P_\ell$ is equal to the average of entries a_{ij} satisfying $(i,j) \in P_k \times P_\ell$.*

Example: For a matrix

$$A = \begin{pmatrix} 0 & 2 & 1 \\ 2 & 0 & 3 \\ 1 & 3 & 4 \end{pmatrix}$$

and a partition $\mathcal{P} = \{\{1,2\},\{3\}\}$, $a(A,\mathcal{P}) =$

$$a(A,\mathcal{P}) = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 4 \end{pmatrix}$$

Problem: For a symmetric matrix with entries from $[0,1]$, show that trace of sixth (or, more generally $2m$-th) power of the matrix is at least trace of sixth (or $2m$-th) power of its partition averaged matrix for any partition.

Known for the trivial partition $\mathcal{P} = \{[n]\}$. Also known for fourth power [1].

References:

[1] J.W. Cooper, D. Král', T.L. Martins: Finitely forcible graph limits are universal. Preprint available as arXiv:1701.03846.

Problem 6. *Maker Breaker Games (suggested by Karel Král)*

Source: MCW 2017.

Definitions.
• *Two players – maker and breaker are playing a game on a given connected multigraph. Breaker starts by claiming k previously unclaimed edges then maker claims k previously unclaimed edges and then they alternate in turns. Maker's goal is to claim all edges of some spanning tree of the given graph, breaker's goal is to prevent maker from doing that.*

Question: Study the case when $k = 2$ and find a characterization of graphs where maker wins. Find the complexity of deciding if maker wins.

Related results:
• (Lehman and Edmonds) In the case when $k = 1$ maker wins iff the graph contains two edge disjoint spanning trees (thus deciding if maker wins is poly-time).
• (Chvátal and Erdős) if k is sufficiently large breaker wins on a clique of size n (where the breaking point of large is around $k = n/\log n$).

References:

Biased positional games, Chvátal, Vašek and Erdös, Paul, Annals of Discrete Mathematics, 2, 221–229, 1978, Elsevier.

Problem 7. *Open perfect neighborhood sets on trees (suggested by Josef Malík)*

Source: Proposed by Stephen T. Hedetniemi in 2006 ([1]).

Definitions.
• *Let* $G = (V, E)$ *be a graph and let* $S \subset V$ *be an arbitrary subset of vertices.*
• *A vertex* $v \in V$ *is open perfect if* $|N(v) \cap S| = 1$.
• *We call* S *an open perfect neighborhood set, if every vertex* $v \in V$ *is either open perfect or is adjacent to an open perfect vertex with respect to* S.
• *The open perfect neighborhood number* $\theta_o(G)$ *is the minimum size of an open perfect neighborhood set in* G.
• *The upper open perfect neighborhood number* $\Theta_o(G)$ *is the maximum size of an open perfect neighborhood set in* G.

Question:

Design a polynomial-time algorithm that computes the value $\theta_o(T)$ for any tree T or show that the problem is NP-complete on trees.

Design a polynomial-time algorithm that computes the value $\Theta_o(T)$ for any tree T or show that the problem is NP-complete on trees.

Related results:
• The problem of deciding whether a graph has an open perfect neighborhood set of size at most k is NP-complete for bipartite and chordal graphs ([2]).

References:

[1] Hedetniemi, S.T. Unsolved algorithmic problems on trees. *AKCE International Journal of Graphs and Combinatorics*, 3 (2006), 1-37.

[2] Hedetniemi, S.T., Jacobs, D.P., Laskar, R., and Pillone D. Open perfect neighborhood sets in graphs. *Unpublished manuscript*, dated August 23, 1996.

Problem 8. *Local dimension of Partial Ordered Set (suggested by Tomáš Masařík)*

Definitions.
• Dimension of a Poset P dim(P) *is a minimum number of linear extensions that represents a given poset.*
• Local Dimension of a Poset ldim(P) *is a minimum number of partial linear extensions that represents a given poset.*

Question:
• Two element removal lemma. At least for posets of height 3. (For each poset P there exists two elements x, y such that

$$\mathrm{ldim}(P \setminus \{x, y\}) + 1 \leq \mathrm{ldim}(P))$$

• Determine the local dimension of some specific well-known posets. e.g. Subset poset.

Related results:
• Two element removal lemma for posets of height 2.
• Four element removal lemma.

References: Jinha Kim, Ryan R. Martin, Tomáš Masařík, Warren Shull, Heather Smith, Andrew Uzzell, Zhiyu Wang: Local Dimension 2017+

Problem 9. *Complexity of some problems on P_ℓ-free graphs (suggested by Jana Novotná)*

Definitions.
• *A graph is P_ℓ-*free *if it does not contain a path on ℓ vertices as an induced subgraph.*
• Independent Odd Cycle Transversal *is an independent set of vertices such that the rest of the graph is bipartite.*
• Independent Feedback Vertex Set *is an independent set of vertices such that the rest of the graph is a forest.*
• Dominated Induced Matching *is an independent set of vertices such that the rest of the graph is an induced matching.*

ℓ :	≤ 4	5	6	7	8	≥ 9
3-Coloring	P	P	P	P	?	?
Vertex Cover	P	P	P	?	?	?
Independent Vertex Cover	P	P	P	P	P	P
Connected Vertex Cover	P	?	?	?	?	?
Feedback Vertex Set	P	?	?	?	?	?
Independent Feedback Vertex Set	P	P	?	?	?	?
Connected Feedback Vertex Set	P	?	?	?	?	?
Near-Bipartiteness	P	P	?	?	?	?
Odd Cycle Transversal	P	?	?	?	?	?
Independent Odd Cycle Transversal	P	P	?	?	?	?
Connected Odd Cycle Transversal	P	?	?	?	?	?
Dominating Induced Matching	P	P	P	P	P	?

red row: NP-complete on H-free graphs if H is not a linear forest
black row: not true (e.g. H is claw $\rightarrow$ polynomial time)
green row: not known (H contains cycle is open case)

Figure 2: Overview of complexity of problems on P_ℓ free graphs.

Question:
• Determine computational complexity of some problems on P_ℓ-free graphs. See Figure 1.
• or possibly determine the complexity of 4-coloring for P_6-free graphs.
• or possibly determine the complexity of 3-coloring for P_ℓ-free graphs, for all $\ell \geq 8$.

Related results:

• There are various results concerning several well studied problem, see Figure 1.

References:

• Cardoso, Korpelainen and Lozin: On the complexity of the dominating induced matching problem in hereditary classes of graphs, 2011.
• Brandstädt and Mosca: Weighted Efficient Domination for P_6-Free Graphs in Polynomial Time, 2017.
• Bonamy, Dabrowski, Feghali, Johnson, Paulusma:Independent Feedback Vertex Sets for Graphs of Bounded Diameter, 2017.
• Chiarelli, Hartinger, Johnson, Milanič, and Paulusma: Minimum Connected Transversals in Graphs: New Hardness Results and Tractable Cases Using the Price of Connectivity, 2017+.
• Grzesik, Klimošová, Pilipczuk, and Pilipczuk, 2017+.

Problem 10. *Pattern-avoiding permutations and number of inversions (suggested by Michal Opler)*

Source: Proposed by Claesson, Jelínek and Steingrímsson in 2011

Definitions.
• *Let* $\mathcal{S}_n$ *be the set of permutations of the letters* $\{1, 2, \ldots, n\} = [n]$.
• *We say that two sequences of distinct numbers* $a_1, \ldots, a_n$ *and* $b_1, \ldots, b_n$ *are* order-isomorphic *if for every two indices* $i < j$ *we have* $a_i < a_j$ *if and only if* $b_i < b_j$.
• *Given two permutations* $\pi \in S_n$ *and* $\sigma \in S_k$, *we say that* π contains σ *if there is a* k*-tuple* $1 \leq i_1 < i_2 < \cdots < i_k \leq n$ *such that the sequence* $\pi_{i_1}, \pi_{i_2}, \ldots, \pi_{i_k}$ *is order-isomorphic to* σ
• *For a pattern* σ *let* $\mathcal{S}_n(\sigma)$ *be the set of all* σ*-avoiding permutations of length* n, *and* $S_n(\sigma)$ *its cardinality.*
• *An inversion in a permutation* $\pi = \pi_1\pi_2\cdots\pi_n$ *is a pair* (i, j) *such that* $1 \leq i < j \leq n$ *and* $\pi_i > \pi_j$. *Let* $S_n^k(\sigma)$ *denote the number of* σ*-avoiding permutations of length* n *with* k *inversions.*

Example:
Permutation 51234 has 4 inversions each of the form $(1, i)$ for $i \in \{2, 3, 4, 5\}$.

Questions:
• Is it true that for all nonnegative integers n and k, we have $S_n^k(1324) \leq S_{n+1}^k(1324)$?
• Let σ be any pattern that is not an identity, i.e. $12\cdots n$. Is it true that for all nonnegative integers n and k, we have $S_n^k(\sigma) \leq S_{n+1}^k(\sigma)$?

Related results:
• Answering the question positively would yield an improved upper bound on the Stanley-Wilf limit of 1324. More specifically, it would imply that the limit is at most 13.002 while the current best bounds are 10.27 from below and 13.5 from above. See Jelínek, Claesson and Steingrímsson.
• It was shown to be true for major index which is a related permutation statistic.

References:
A. Claesson, V. Jelínek, E. Steingrímsson, Upper bounds for the Stanley-Wilf limit of 1324 and other layered patterns, J. Combin. Theory Ser. A 119 (8) (2012) 1680–1691.
M. Opler, Major index distribution over permutation classes, Advances in Applied Mathematics 80 (2016) 131–150.

Problem 11. *Hardness of train problem - revisited (suggested by Veronika Slívová)*

Source: Proposed by Karthik C. S. in 2017.

Definitions.
• *A* switch graph *is a directed graph G in which every vertex v has exactly two outgoing edges $s_0(v), s_1(v)$ (loops and multi-edges are allowed).*
• *A train travels from a given vertex o (origin) along the switch graph. Whenever a train arrives to a vertex it continues along the active edge (starting with an edge s_0) and switches the active edge to the another one. The train stops only at a given vertex d (destination). The problem* Arrival *asks whether the train stops.*
• *Problem* S-Arrival *asks for a certificate that the train reaches the destination d or a certificate for the fact that it will travel infinitely long.*
• Local-OPT *is a problem where given two circuits $S\colon \{0,1\}^n \to \{0,1\}^n$ and $V\colon \{0,1\}^n \to [2^n]$ we want to find a string x satisfying $V(x) \geq V(S(x))$.*
• Local-Max-Cut *Given a weighted graph find a partitioning of its vertices into two disjoint parts such that the sum of weights of cut edges cannot be increased by moving a vertex from one part to another one.*
• Sink-of-path *is a problem where given two circuits $S\colon \{0,1\}^n \to \{0,1\}^n$ and $V\colon \{0,1\}^n \to [2^n]$ and a string $s \in \{0,1\}^n$ we want to find a string x such that $x = S^r(s)$ for some r and $V(x) \geq V(S(x))$.*

Question 1: Is S-Arrival PLS-hard? In other words can we reduce Local-OPT or Local-Max-Cut to S-Arrival.

Question 2: Is the problem of determining how many times is each edge used (finding running profile) FPSPACE-complete (an equivalent of PSPACE but we ask for a whole solution except of asking only for an existence of a solution)? Equivalently is the problem Sink-of-path reducible to the train problem?

Note that we are interested in finding a solution of those problems. It means that A is reducible to B only if we have algorithms X, Y such that X given an instance of problem A returns an instance of problem B and Y given a solution of problem B returns a solution of problem A. Moreover if a solution x of problem B is locally optimal, then also the solution $Y(x)$ has to be locally optimal.

Related results:
• Problem Arrival is in $NP \cap co - NP$. (Dohrau et al.)
• Problem S-Arrival is in the class PLS. (Karthik C. S.)

References:

Dohrau, Jérôme, et al. “ARRIVAL: A zero-player graph game in NP $\cap$ coNP.” arXiv preprint arXiv:1605.03546 (2016).

Karthik, C. S. “Did the train reach its destination: The complexity of finding a witness.” Information Processing Letters 121 (2017): 17-21.

Schäffer, Alejandro A. “Simple local search problems that are hard to solve.” SIAM journal on Computing 20.1 (1991): 56-87.

Problem 12. *Improve bounds on dispersion of the unit cube (suggested by Jakub Sosnovec)*

Source: Proposed by Aicke Hinrichs [1].

Definitions.
• *Let $\mathcal{B}_d$ be the set of all axis-parallel open boxes inside $[0,1]^d$, i.e.,*

$$\mathcal{B}_d = \{I_1 \times \cdots \times I_d : I_1, \ldots, I_d \subset [0,1] \text{ are open intervals}\}.$$

For a set T of n points in $[0,1]^d$, the volume of the largest open box avoiding all points from T is called the dispersion *of T and is defined as*

$$\mathrm{disp}(T) = \sup_{B \in \mathcal{B}_d,\, B \cap T = \emptyset} \mathrm{vol}(B),$$

where $\mathrm{vol}(I_1 \times \cdots \times I_d) = |I_1| \cdots |I_d|$*. Note that the supremum is attained, since there are only finitely many inclusion-maximal boxes $B \in \mathcal{B}_d$ avoiding T.*
• *The* minimal dispersion *for any point set is defined as*

$$\mathrm{disp}^*(n,d) = \inf_{T \subset [0,1]^d,\, |T|=n} \mathrm{disp}(T).$$

Again, observe that the infimum is actually attained, since any sequence of n-element point sets inside $[0,1]^d$ has a convergent subsequence.
• *Sometimes it is more convenient to work with the inversion of minimal dispersion, defined as*

$$N(r,d) = \min\{n \in \mathbb{N} : \mathrm{disp}^*(n,d) \le r\}.$$

Questions:

1. Improve known bounds on minimal dispersion. Especially interesting would be to prove (or disprove) that $\mathrm{disp}^*(n,d) = O(d/n)$.

2. Give explicit constructions of (small) points sets that achieve small dispersion. Note that both results in [3] and [4] use probabilistic arguments to find such sets.

Related results:

- Trivial lower bound: $\mathrm{disp}^*(n,d) \ge 1/(n+1)$

- Improved lower bound [2]: $\mathrm{disp}^*(n,d) \geq \frac{\log_2 d}{4(n+\log_2 d)}$, also can be reformulated as $N(r,d) \geq \frac{1-4r}{4r}\log d$.
- Upper bound [2]: $\mathrm{disp}^*(n,d) \leq \frac{2^{7d+1}}{n}$
- Very recent upper bound [3]: $\mathrm{disp}^*(n,d) \leq 4\frac{d}{n}\log(\frac{9n}{d})$ – note that this is almost $O(d/n)$, but the dependence on n is slightly worse than the previous bound, so this bound is not strictly better.
- Upper bound asymptotically tight in d [4]: $N(r,d) \leq c_r \log d$, where c_r is a (large) constant depending only on r.

References:

1. Report from Oberwolfach Mini-Workshop: Perspectives in High-Dimensional Probability and Convexity, chapter Discrepancy & dispersion of point distributions, `https://www.mfo.de/document/1706c/preliminary_OWR_2017_10.pdf`
2. C. Aistleitner, A. Hinrichs and D. Rudolf, On the size of the largest empty box amidst a point set, `https://arxiv.org/abs/1507.02067`
3. D. Rudolf, An upper bound of the minimal dispersion via delta covers, `https://arxiv.org/pdf/1701.06430`
4. J. Sosnovec, A note on minimal dispersion of point sets in the unit cube, `https://arxiv.org/pdf/1707.08794`

Problem 13. *A matching inequality (suggested by Robert Šámal)*

Source: Proposed by Eberhard Triesch at MCW 1997. Also by Jan Volec at MCW 2017.
Question: Suppose that $G = (V, E)$ and $G' = (V, E')$ are bipartite graphs on the same vertex set V and with the same 2-colouring $V = U \cup W$ where both colour classes U and W contain n elements. Assume further that for all $A \subset U$ the number of neighbours in G is at least as large as in G':

$$|N_G(A)| \geq |N_{G'}(A)| \quad \text{for all } A \subset U.$$

Then the number of perfect matchings in G is at least as large as the number of perfect matchings in G'.

Related results:
- The case where G' is 1-regular is just the Hall's "marriage theorem".
- The result would follow from a positive solution to Aharoni–Keich conjecture on a certain generalization of determinants.

References:

- `kam.mff.cuni.cz/~kamserie/serie/clanky/1997/s339.ps`
- and references therein.
- R. Aharoni, U. Keich, *A generalization of the Ahlswede–Daykin inequality*, Discrete Math. 152 (1996), pp. 1–12.

Problem 14. *The binary paint shop problem (suggested by Robert Šámal)*

Source: Proposed by Meunier&Neveu in 2012.

Definitions.
• *A* double occurrence word *w is a word in which every letter occurs exactly twice.*
• Legal 2-coloring *of w is a coloring of individual letters such that each letter occurs once red and once blue.*
• *The goal is to find a legal 2-coloring of w with minimal number of color changes. Let us call this quantity $cc(w)$.*
• *Also let cc_n be the expectation of $cc(w)$ when w is a random double occurrence word of length $2n$.*

Example:
ADEB*AFCB***C***DE***F** – 4 changes, but 2 changes possible with a different coloring.

Question: Is $cc_n = o(n)$?

Related results:
• The problem is also called necklace splitting by Alon and others.
• Computing $cc(w)$ is NP-complete, even APX-hard to approximate.
• It is easy to show that $cc_n \leq \frac{n}{2} + o(n)$.
• It is known that $cc_n \leq \frac{2}{5}n + \frac{7}{10}$.
• Other related problems about complexity/algorithms.

References:
• `http://gt-alea.math.cnrs.fr/alea2015/transp/Meunier.pdf`
• `http://kam.mff.cuni.cz/workshops/mcw/work18/mcw12andres.pdf`
• and references therein.

Problem 15. *Home-Away-Pattern Set Feasibility (suggested by Pavel Veselý)*

Source: Proposed by Dirk Briskorn at MAPSP 2017.

Definitions.
• *We consider a sport league with an even number n of teams.*
• *In a* Single Round Robin Tournament *(SRRT) there are $n-1$ rounds of matches such that each team plays once per round and each pair of teams plays exactly one match. In each match, one of the two teams plays at home.*
• *A* Home-Away-Pattern *(HAP) is a table h with rows corresponding to teams and columns corresponding to rounds such that $h_{t,i} = \mathrm{H}$ iff team t plays at home in round i; otherwise $h_{t,i} = \mathrm{A}$.*
• *In the* Home-Away-Pattern Set Feasibility *problem the goal is to determine whether for a given HAP h there exists an SRRT such that for each team t and round i we have $h_{t,i} = \mathrm{H}$ iff team t plays at home in round i in the SRRT. We call such h* feasible.

Examples:

A feasible HAP:

	1	2	3	4	5
1	H	H	H	H	H
2	A	H	H	H	A
3	H	A	H	A	A
4	A	A	A	H	H
5	H	H	A	A	H
6	A	A	A	A	A

An infeasible HAP:

	1	2	3	4	5
1	A	H	H	A	A
2	A	H	A	H	A
3	A	H	H	H	A
4	H	A	A	H	H
5	H	A	H	A	H
6	H	A	A	A	H

Question: Are there some nice (combinatorial?) necessary or sufficient conditions for feasible HAPs? Or can we even find a characterization?

Previous results:
There are some obvious necessary conditions for feasible HAPs:
• In each round the number of away-teams must equal the number of home-teams.
• Any two teams must have different rows.
There is an integer programming formulation and the corresponding linear programming relaxation provides a necessary condition [Briskorn D., 2008] which is the strongest one known so far. The author conjectured that this condition is also sufficient.

References:

• Briskorn D. (2008): Feasibility of home-away-pattern sets for round robin tournaments, Operations Research Letters, Vol. 36, No. 3, pp 283-284.
• Kashiwabara, K. (2009). Scheduling partial round robin tournaments subject to home away pattern sets. the electronic journal of combinatorics, 16(1), R55.
• Horbach, A. (2010). A combinatorial property of the maximum round robin tournament problem. Operations Research Letters, 38(2), 121-122.
• Miyashiro R., Iwasaki H., Matsui T. (2003) Characterizing Feasible Pattern Sets with a Minimum Number of Breaks. In: Practice and Theory of Automated Timetabling IV. PATAT 2002. LNCS 2740.