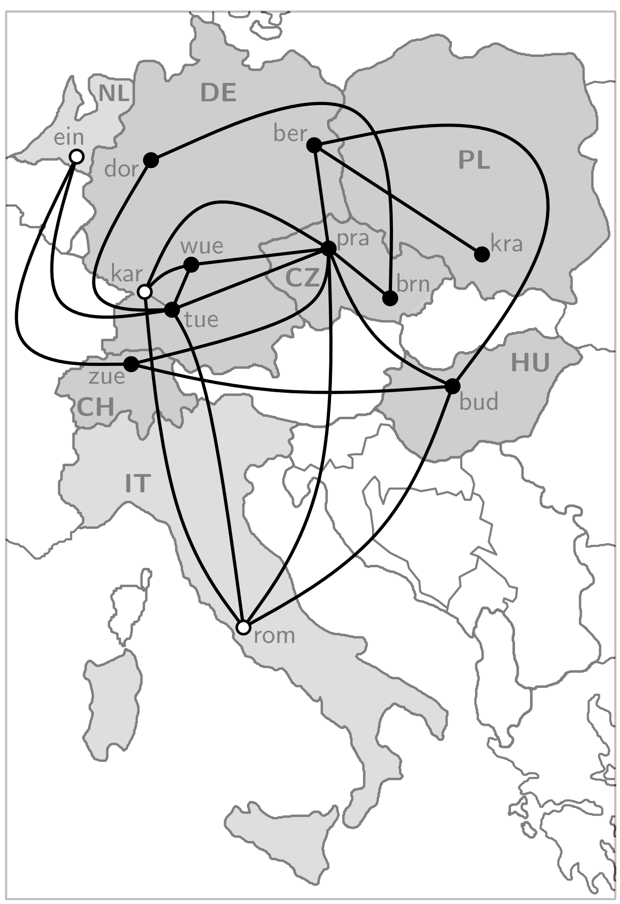
GraDR

Graph Drawings and Representations

Full proposal of EuroGIGA Project No. 10-EuroGIGA-OP-003



Section A – Collaborative Research Project

1. Description of the CRP

Visualization of graphs and networks has become crucial in many real world applications, especially nowadays when large scale networks need to be displayed in an easy-to-grasp way. Deep theoretical results in structural graph theory lead to design of fast algorithms to achieve this, while, at the same time, motivation stemming from applications strongly influences basic research in graph theory and discrete mathematics. One can hardly find a more prominent example of cross-fertilization between theory and practical applications. The historically oldest concept of graph visualization is the notion of **planarity**. The famous map colouring problem took almost 100 years to be finally resolved by a computer-aided proof [Appel and Haken 1976, Robertson et al. 1996]. Kuratowski's structural characterization of planar graphs by forbidden minors [1930] marks the beginning of modern graph theory, the first linear-time planarity testing algorithm of Hopcroft-Tarjan [1974] secured them (both planar graphs as well as Hopcroft and Tarjan) eternal placement in the hall of fame of theoretical computer science. The long line of research on planar graphs has generated a broad basis of both structural and algorithmic results. Nevertheless, interest in planarity remains vital and interesting connections and results continue to emerge: Excluded planar minors play an exceptional role in structural graph theory, recent algorithmic results treat constrained planarity problems, surprising theorems on intersection representations of planar graphs have been proved. Still, many old problems remain open, and many new ones are emerging.

The **aims** of the CRP are to attack well known hard problems both from structural and algorithmic points of view. The research will be concentrated around **planarity issues**, will go **beyond planarity** and explore **geometric representations of graphs**. Given the dynamics of the field we expect to encounter and identify new frontiers and new research directions. Since visualization of graphs is motivated by practical applications, a key ingredient of the project is cross-fertilization of theory and applications. A brief description of the current **state of the art** in these areas follows (more details can be found in the descriptions of particular work packages in the individual projects):

> **Planarity issues.** The algorithmic complexity of many constrained versions of planarity is still open. So-called SPQR trees allow us to encode all planar drawings of a 2-connected graph in linear time. This fact has been used to count the number of such drawings, and to decide whether an input graph allows a drawing satisfying certain additional constraints. Recently this technique proved useful to decide planarity of partially embedded graphs in linear time [SODA'10], but a combinatorial description of feasible instances of this problem by Kuratowski-type obstructions is still not known. Another useful set of constraints are so-called ec-constraints that can model a wide variety of constraints, such as, e.g., port constraints in electronic circuit layout. While ec-planarity testing for the case when all edges appear in some ec-constraints can be solved in polynomial time using SPQR trees, the general problem is still open. Another tantalizing question is that of clustered planarity where partial results have been achieved, but the algorithmic complexity of the general problem is still a big challenge. In many real-life situations one wants to visualize several objects at the same time. In graph drawing, this is known as simultaneous embedding, which is another area with many open questions both of structural and algorithmic nature. The planar slope number of a graph is the minimum number of different slopes of edges in a planar straight-line drawing of the graph. Only recently Jelínek et al. [GD'09] have shown that the planar slope number of outerplanar graphs of bounded maximum degree is bounded. A similar result for general planar graphs has been announced by Keszegh et al. However, in both results the upper bound depends exponentially on the maximum degree, while no matching lower bounds are known. In angular schematization the task is to draw a given graph using a given set of slopes for the edges, which is of interest, e.g., for subway maps. Surprisingly, the problem can be solved efficiently in the orthogonal case, but gets hard when allowing diagonals [TVCG'10]. A new variant

of the problem [GD'09] requires that edges are drawn as geodesics with respect to the given set of slopes. This inspires many open questions.

Beyond planarity. In real-world applications, one is often confronted with graphs that are not planar but also "not far from" planar. One then tries to optimize the graph layout in the plane, say, with respect to the number of edge crossings. It is known that the exact crossing minimization problem is NP-hard, and Cabello and Mohar recently showed that the problem remains NP-hard even on planar graphs with one added edge. On the other hand, Mutzel et al. have recently described a practical branch-and-bound approach to exact crossing minimization that performs surprisingly well on real-world graphs, and Hliněný et al. have given a series of constant-factor approximation algorithms (the latest in [SODA'10]) for crossing minimization in several restricted graph classes. These recent results of project members are an excellent basis for further research in the scope of this project. This research is also related to some interesting constrained planarity issues, namely to so called (vertex/edge/subgraph) insertion problems.

Geometric representations of graphs. In some applications it comes handy to visualize graphs by means of intersection or contact representations of geometric objects (usually in the plane). Such representations are often naturally derived from the instances under consideration. Very often, basic optimization problems (e.g., colourability, independent set, etc.) are easier (i.e., polynomial-time solvable or at least approximable) on intersection graphs of a certain type than on general graphs. We will explore the computational complexity of optimization problems restricted to intersection graphs, and also study the complexity of recognizing these classes. Unlike for many graph theoretical problems, the complexity boundary to be explored does not lie between P and NP-hardness, but rather between NP and PSPACE-completeness (e.g., intersection graphs of curves in the plane have been known to be NP-hard to recognize for a long time, but a finite recognition algorithm and NP-membership are rather recent, while for intersection graphs of straight-line segments or convex sets only PSPACE membership is known). We will also study competitiveness of optimization problems on intersection graphs in the on-line setting. The interest is in bounds for any algorithms as well as for standard heuristics (e.g., first-fit). We hope to extend our methods used for on-line colouring of intervals to other geometric objects, as well as for different types of colourings (e.g., acyclic colourings, conflict-free colourings, game colourings, etc.). We also plan to adapt other algorithmic techniques in this area based, for instance, on the recent constructive version of the Lovasz Local Lemma due to Moser and Tardos. Structural questions regarding intersection and contact representations include questions about representations of planar graphs (thus relating the areas of geometric representations and planarity issues). A classical result of Koebe from the 1930's says that every planar graph is a contact graph of disks. Recently, it has been shown that planar graphs are intersection graphs of straight-line segments and contact graphs of triangles. Natural questions about representations by segments with bounded number of directions, or homothetic triangles are open and worth close attention. An affirmative answer to the first question (with four directions) would yield a new proof of the Four Colour Theorem, the second question is related to max-tolerance graphs with applications to DNA sequencing in bioinformatics.

The main goal of this CRP is to foster collaborative research in the areas of **graph drawing** and **geometric representations** of graphs, and by coordinating and unifying the efforts of top European research groups to attack basic and notoriously difficult open problems in the area. Twenty years ago when the Graph Drawing symposium series was established by di Battista, Pach, and others the field was small and encompassed a limited number of researchers. Due to its appeal and growing importance many people from the discrete mathematics and theoretical computer science communities found their way into graph drawing. The success of the annual symposia shows how the area has flourished. The symposia also clearly show that research is scattered over different teams, fragmented over potentially interconnected topics, and often inefficiently duplicated. With our CRP we see a unique chance for unifying European forces to efficently and seriously attack pertinent difficult and important problems, open new territories and significantly push the frontiers of our knowledge. Incorporating young researchers and students in the research teams will help achieve the **second main goal** of this CRP: the transfer of our know-how to the next generation. Last but not least, we will make efforts to implement our theoretical findings in graph drawing and visualization toolkits and other applications that are being developed by member teams of the CRP.

Organization of research. In the first year, joint research will be focused mainly on problems where the team members have prior experience. At the same time new directions will be investigated, in particular directions that combine different subareas. These will be attacked in the second and third years. Research will be organized in work packages. The following table shows a list of our work packages together with the responsible sites:

	WP name	*leader*	*team*	*site*
WP01	Slope number	J. Kratochvíl	IP1-CZ-P	Prague
WP02	Angular schematization	A. Wolff	IP2-DE-W	Würzburg
WP03	Simultaneous embeddings	M. Hoffmann	IP3-CH	Zurich
WP04	Constrained embeddings	D. Wagner	AP3-DE	Karlsruhe
WP05	Clustered planarity	G. Di Battista	AP1-IT	Rome
WP06	Quasi- and near-planar graphs	J. Pach	IP4-HU	Budapest
WP07	Region constrained graph drawing	B. Speckmann	AP2-NL	Eindhoven
WP08	Crossing number	P. Hliněný	IP1-CZ-B	Brno
WP09	Coloring graphs with geometric representations	J. Grytczuk	IP5-PL	Krakow
WP10	Contact and intersection representations	S. Felsner	IP2-DE-B	Berlin
WP11	Transfer to practice	P. Mutzel	IP2-DE-D	Dortmund
WP12	Hypergraphs with applications in bioinformatics	M. Kaufmann	IP2-DE-T	Tübingen

Methodology and workplan. The main focus of this project is on basic research in discrete mathematics and theoretical computer science. As such, the main methodological approach is that of theoretical research. Our main tools come from the areas of graph theory and graph algorithms, combinatorics, combinatorial optimization, data structures, and discrete and computational geometry. Special attention will be paid to connectons with other fields, e.g., algebra and topology, and to 'non-standard' methods made possible by these connections. We will approach computational questions, depending on the context, by exact, approximation, randomized, or fixed-parameter algorithms. We will use computers to demonstrate and test results as well as for experiments that will help make conjectures and substantiate them. A key ingredient to the success of this kind of theoretical research is the effective interaction and exchange of ideas between researchers. The extra mobility as well as seminars and workshops that will be made possible by the CRP will enhance this interaction between the participating groups and lead to effective concentration of manpower.

Once a year we will organize a central meeting for all CRP members. The goal of these meetings is to review the progress and identify potential for cross-WP collaboration. Work package leaders will organize two or more specialized workshops focusing on work within their work packages. Organization and schedule of these workshops will be flexible so that the specific needs of each work package can be accounted for.

We will offer to organize the first central meeting as a bigger event where all the funded EuroGIGA CRPs can come together to learn about each other. Such a meeting would have additional impact on cross-CRP collaborations.

Apart from research workshops we will organize winter schools on particular topics targetting especially PhD students and young researchers from the participating sites. A first course in this spirit will be organized in Berlin already in the spring of 2011. This course "Methods in Graph Drawing" and the subsequent schools organized by the CRP will contribute to the transfer of expertise to the young generation, and at the same time recruit new young contributors to our teams.

The research itself will be organized in work packages specified above. Each participating site will be responsible for one work package, but will typically participate in the research on more than one work package (for details see the next section). This will enable dynamic interaction and collaboration of the research groups as well as focused leadership of each work package.

Deliverables. The table summarizes the central deliverables of the entire CRP and deliverables here that will be completed by all work package leaders. These deliverables will not be stated again in Sections B1–C3.

	description	*responsibility*	*time*
D0.1	wiki for internal information sharing	Prague	start date + 1 month
D0.2	web site for quick dissemination of GraDR results	Prague	start date + 1 month
D0.3	GraDR kick-off meeting	Prague	start date + 3 months
D0.4	GraDR midterm meeting	Krakow	2nd year
D0.5	GraDR final meeting	Berlin	3rd year
D0.6	WP workshops	WP sites	flexible
D0.7	annual progress reports	WP sites	end of each year
D0.8	winter school	Würzburg	1st year
D0.9	winter school	Zürich	2nd year

Novelty and originality. On the research level the originality of the project lies in combining expertise and experience of groups that have previously very successfully worked in the areas of graph drawing and geometric representations of graphs. We believe that interaction of ideas and expertise will be fruitful for research in both areas.

On the organizational level the project is unique in its broad spectrum in at least three dimensions. From the geographical point of view it provides substantial links between Western and (former) Eastern Europe. All teams include a wide spectrum of generations, ranging from PhD. (and sometimes undergraduate) students to senior researchers and full professors. And last but not least the workplan of most work packages guarantees direct transfer of theoretical findings to practical applications.

2. Description of the collaboration

European groups have been leading in the graph drawing community since the beginnings in the early 90s. Intensified cooperation is necessary to keep Europe in this leading position also for the next two decades. The synergy of this CRP team offers full warranty of such an intensive cooperation.

Management. The CRP will be managed on a multilevel basis depicted in Figure 1. While the Hungarian, Swiss, and Polish IP's and all AP's consist of one site each, the Czech and German IP's involve more sites. In Czechia, the participating sites are Charles University in Prague and Masaryk University in Brno. In Germany the participating sites are the universities in Berlin, Dortmund, Tübingen, and Würzburg. The management flows will be maintained along the lines of the chart. On the contrary the information flow will be more complex, including cross-level exchange of information. The CRP will set up a web page that will inform about all its activities and achievements. For the most immediate and effective way of sharing ideas and brainstorming a wiki page will be set up. Both of them will be maintained by the Czech IP.

The principal investigators and associate partners will form an executive committee of the CRP and as such will meet annually to discuss the progress and management of the project. In particular, this executive committee will evaluate the course of work of the IPs and spell recommendations

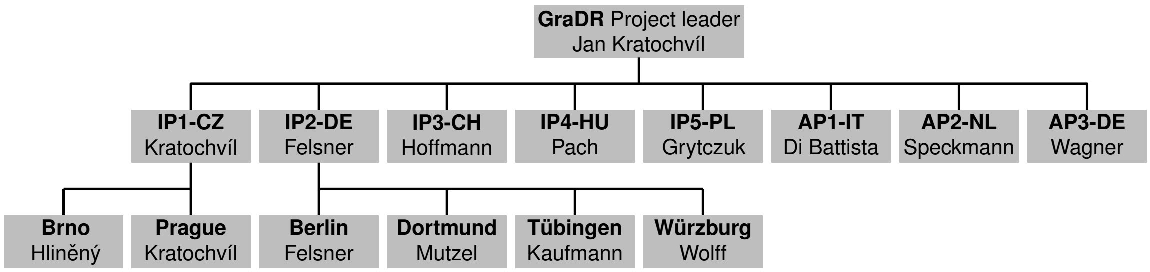


Figure 1: Management structure of GraDR.

for the future focus and exchange of expertise. Once a year we will organize a meeting for all CRP members where each work package will report on its progress. Using the mobility funds, the work package leaders will organize further specialized workshops for all researchers contributing to their work package. These specialized workshops will be organized on a flexible basis. We expect that most work packages will have one or two of these workshops between every two consecutive central CRP meetings.

Collaboration. Fostering collaboration across the teams and coordinating the research will lead to creating a critical mass needed to successfully attack difficult problems. It will also lead to efficient conduct of research, since parallel research in groups unaware of each other can be avoided. The results of previous bilateral research contacts show that the teams fit well together. This justifies strong expectations towards the performance of our project.

Most IP teams include researchers that have worked or have a potential to work in several milestone areas. Also many of the team members have collaborated before on bilateral basis. The **track record of the members of CRP** listed at the end of this section is an excerpt from more than 30 papers co-authored by researchers from more than one of the participating sites and such previous joint research is illustrated in the attached collaboration graph depicted in Fig. 2. The CRP will enable upgrading the existing bilateral collaboration to a true multilateral one by fostering joint research, organizing regular workshops and seminars and supporting frequent research visits. In terms of graph drawing, one of the goals of this CRP is to turn the collaboration graph into a **dense hypergraph**.

All IP and AP teams include senior researchers (I. Barany, J. Fiala, P. Hliněný, M. Kaufmann, J. Matoušek, P. Mutzel, M. Patrignani, E. Welzl, and A. Wolff, in addition to the team leaders) as well as young researchers and students. Research visits will be a priceless experience for students to learn and gain new expertise.

The associated partners come from countries whose national funding agencies do not participate in EuroGIGA (AP-NL and AP-IT). In the case of AP-DE, Dorothea Wagner was involved in the decision process about the acceptance of EuroGIGA as rapporteur at the Core Group of PESC and in the DFG Senate. She refrains from asking for financial support in order to prevent any suspicion of personal advantage, but her expertise is very important for our CRP.

Expertise of each IP and AP. All participating teams have a strong research potential. Yet each group has a reputation for expertise in certain areas and methodologies. These strong points of the participating groups conveniently complement each other. From the methodologies announced in the first section, the following are the strongest points of particular teams:

Discrete algorithms	IP2-DE-B, IP5-PL, AP1-IT
Combinatorial methods	IP1-CZ-P, IP1-CZ-B, IP2-DE-B, IP4-HU
Computational complexity	IP1-CZ-P, IP2-DE-W, IP3-CH, AP1-IT
Computational geometry	IP2-DE-W, IP3-CH, IP4-HU, AP2-NL
Implementations	IP2-DE-D, IP3-CH, AP3-DE
Combinatorial optimization and ILP	IP2-DE-D
Large-scale computations	IP2-DE-T

Objectives of each IP and AP. Each work package is coordinated by one participating site. The assignment of work packages to IPs and APs is shown in the following table by • signs. The × signs denote which IPs/APs envision active participation in other work packages, based either on previous results in the area or on the current research of their teams. Further collaboration is hoped for and expected.

Team	WP01	WP02	WP03	WP04	WP05	WP06	WP07	WP08	WP09	WP10	WP11	WP12
IP1-CZ-P	•			×	×				×	×		×
IP1-CZ-B								•		×	×	
IP2-DE-B						×		×	×	•		
IP2-DE-D				×	×			×			•	×
IP2-DE-T		×	×				×			×		•
IP2-DE-W	×	•		×						×	×	
IP3-CH			•			×					×	
IP4-HU	×					•	×			×		
IP5-PL						×			•	×		
AP1-IT	×	×	×	×	•			×	×			
AP2-NL		×	×				•				×	×
AP3-DE		×	×	•	×							

The objectives of particular work packages, their workplans, methodologies and the current state of the art are described in the individual projects of partners that are responsible for them.

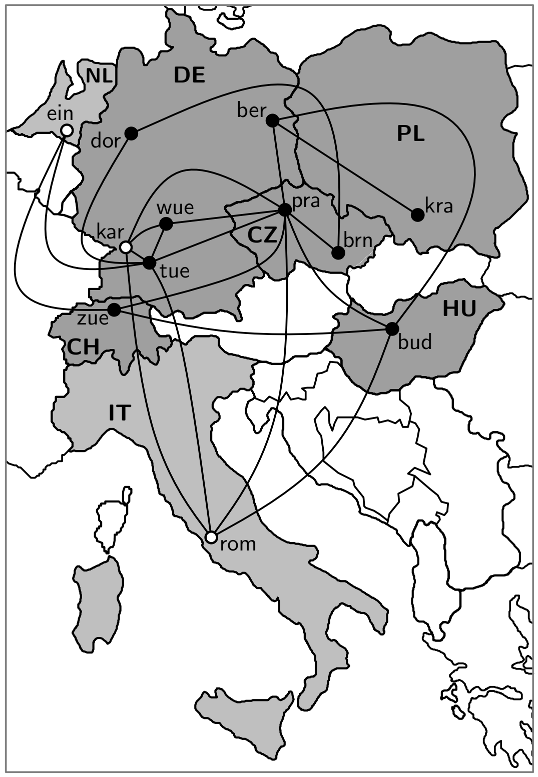


Figure 2: Past collaboration of members of the GraDR team. Sites of IPs and APs are marked by black and white vertices, respectively. Joint papers in the bibliography below are marked by edges.

Selection of cross-site papers

[1] **P. Angelini, G. Di Battista, F. Frati, V. Jelínek, J. Kratochvíl, M. Patrignani**, and **I. Rutter**. Testing planarity of partially embedded graphs. In: *Proc. ACM-SIAM Symp. Discrete Algorithms (SODA'10)*, pp. 202–221, 2010.

[2] M. Bekos, **M. Kaufmann**, A. Symvonis, and **A. Wolff**. Boundary labeling: Models and efficient algorithms for rectangular maps. *Comput. Geom. Theory Appl.*, 36(3):215-236, 2007.

[3] U. Brandes, M. Eiglsperger, **M. Kaufmann**, and **D. Wagner**. Sketch-Driven Orthogonal Graph Drawing. In: *Proc. Graph Drawing (GD'02)*, LNCS 2528:1–11. Springer, 2003.

[4] U. Brandes, C. Erten, J. Fowler, **F. Frati**, M. Geyer, **C. Gutwenger**, S.-H. Hong, **M. Kaufmann**, S. Kobourov, G. Liotta, **P. Mutzel**, and A. Symvonis. Colored Simultaneous Geometric Embeddings. *Proc. COCOON'07*, LNCS 4598:254–263. Springer, 2007.

[5] **M. Chimani, P. Hliněný**, and **P. Mutzel**. Approximating the Crossing Number of Apex Graphs. In: *Proc. Graph Drawing (GD'08)*, LNCS 5417:432–434. Springer, 2009.

[6] **S. Felsner** and **M. Pergel**. The Complexity of Sorting with Networks of Stacks and Queues. In: *Proc. Europ. Symp. Algorithms (ESA'08)*, LNCS 5193:417-429. Springer, 2008.

[7] **S. Felsner**, K. Kloch, G. Matecki, and **P. Micek**. On-line chain partitioning of up-growing orders: The case of 2-dimensional orders and semi-orders. CoRR abs/0704.1829, 2007.

[8] A. Fiat, M. Levy, **J. Matoušek**, E. Mossel, **J. Pach**, M. Sharir, Sh. Smorodinsky, U. Wagner, and **E. Welzl**. Online conflict-free coloring for intervals. In: *Proc. ACM-SIAM Symp. Discrete Algorithms (SODA'05)*, p. 545–554, 2005.

[9] X. Goaoc, **J. Kratochvíl**, Y. Okamoto, Ch.-S. Shin, A. Spillner, and **A. Wolff**. Untangling a Planar Graph. *Discrete & Comput. Geom.*, 42(4):542–569, 2009.

[10] **M. Hoffmann, B. Speckmann**, and Cs. Tóth. Pointed Binary Encompassing Trees: Simple and Optimal. *Comput. Geom. Theory & Appl.*, 43(1):35–41, 2010.

[11] **M. Kaufmann, J. Kratochvíl**, K. Lehmann, and A. Subramanian. Max-tolerance graphs as intersection graphs: cliques, cycles, and recognition. In: *Proc. ACM-SIAM Symp. Discrete Algorithms (SODA'06)*, p. 832–841, 2006.

[12] **M. Kaufmann**, M. van Kreveld, and **B. Speckmann**. Subdivision drawings of hypergraphs. *Proc. Graph Drawing (GD'08)*, LNCS 5417:396–407. Springer, 2009.

[13] **J. Kynčl, J. Pach**, and G. Tóth. Long alternating paths in bicolored point sets. *Discrete Math.*, 308(19):4315–4321, 2008.

[14] **M. Nöllenburg** and **A. Wolff**. Drawing and labeling high-quality metro maps by mixed-integer programming. *IEEE Trans. Visualization & Comput. Graphics*, to appear, 2010.

Section B1 – Individual Project IP1-CZ

Individual project's contribution to the CRP

The Czech team consists of two sites – Charles University in Prague (led by Jan Kratochvíl) and Masaryk University in Brno (led by Petr Hliněný). These teams will be responsible for work packages WP01 – Slope number and WP08 – Crossing number, respectively.

The team members will contribute to the research in other work packages as well, namely in packages WP04 – Constrained embeddings (V. Jelínek and J. Kratochvíl), WP05 – Clustered planarity (V. Jelínek and J. Kratochvíl), WP09 – Coloring graphs with geometric representations (J. Fiala and J. Matoušek), WP10 – Contact and intersection representations (J. Kratochvíl, M. Pergel, and P. Hliněný), and WP12 – Hypergraphs with applications in bioinformatics(J. Kratochvíl, M. Pergel). Contribution to these work packages is described in the corresponding individual projects. The objectives, methodology and work plans of the key work packages follow.

team	*leading*	*contributing to*
Kratochvíl	WP01 – Slope number	WP04, WP05, WP09, WP10, WP12
Hliněný	WP08 – Crossing number	WP10

WP01 – Slope number It is well known that every planar graph has a planar embedding whose edges are non-crossing straight line segments (the so called Fáry embedding). Dujmović et al. [3] pose the question of *how many different slopes* are needed for such a representation. In particular they ask if there exists a function f such that each planar graph of maximum degree Δ has a Fáry embedding that uses at most $f(\Delta)$ slopes. Jelínek et al. [4] answer this question in affirmative for outerplanar graphs. They actually prove a stronger result by showing that every planar partial 3-tree of maximum degree Δ has a Fáry embedding which uses at most $O(2^{\Delta})$ slopes. A generalization of this result to all planar graphs was most recently proved by Keszegh et al. [5]. What remains wide open is the gap between these upper bounds and the best known lower ones. To be more explicit, let $f_{planar}(\Delta)$ denote the maximum planar slope number taken over all graphs of maximum degree Δ. The following two problems are a challenge:

Is $f_{planar}(\Delta)$ bounded from above by a function polynomial in Δ?

and

$$\text{Is } \lim_{\Delta\to\infty} \frac{f(\Delta)}{\Delta} < \infty?$$

The immediate natural question is whether the upper bound c^{Δ} could be substantially improved (possibly to a polynomial one). Other questions we want to study are exact values of $f(\Delta)$ for small Δ and tight bounds for partial 2-trees (which include outerplanar graphs). Methods of discrete geometry will be exploited in attempting these questions.

We will also address related complexity questions, in particular how difficult it is to decide if a given graph has a Fáry embedding using a small number (2, 3) of slopes.

Bounding the number of slopes proves useful not only for drawing graphs, but also for their intersection representations. For instance, it is known that recognition of intersection graphs of straight lines segments in the plane that are parallel with a collection of k directions is NP-complete for every fixed $k \geq 2$ [6, 1], while NP-membership is an open problem for intersection graphs of segments (with no restriction on the number of slopes). Similarly, the complexity of the problem of finding a largest clique in an intersection graph of segments is still open [7] (while the question is polynomial time solvable if the number of slopes is bounded). These are seemingly hard problems. We hope that recently developed techniques for drawing graphs with a small number of slopes may provide inspiration for successfully attacking them.

Very recently Chalopin and Goncalves [2] proved a very surprising result that every planar graph is the intersection graph of line segments in the plane. This remarkable result revives a question that Scheinermann asked in his thesis – is every planar graph the intersection graph of straight-line segments which are parallel with at most 4 directions only? This daring conjecture implies the celebrated Four Color Theorem, and hence one should search for a counterexample first. We will perform such a search and try to combine techniques of the so-called *Order Forcing Lemma* developed by Kratochvíl and Matoušek [6] with recent methods of geometric graph theory and the computational power of our network.

Milestones of WP01

- M01.1 Improve the upper bounds for planar slope number of planar graphs and/or its subclasses (partial 3-trees, outerplanar graphs, etc.).
- M01.2 Determine tight upper bounds for the planar slope number of series-parallel graphs.
- M01.3 Determine tight upper bounds for the planar slope number of planar graphs with small maximum degree.
- M01.4 Determine the computational complexity of deciding if a planar graph has a drawing with a fixed small number of slopes.
- M01.5 Attempt to generalize results on recognition of intersection graphs of segments in bounded number of slopes to larger classes of segment intersection graphs.
- M01.6 Attempt to generalize results on the complexity of the CLIQUE problem for intersection graphs of segments in bounded number of slopes to larger classes of segment intersection graphs.
- M01.7 Try to find (by various techniques including intelligent computer search) a planar graph that does not have an intersection representation by segments in 4 directions.

References for WP01

[1] C. Ambühl and **U. Wagner**. The clique problem in intersection graphs of ellipses and triangles. *Theory Comput. Syst.* 38(3):279–292, 2005.
[2] J. Chalopin and D. Gonçalves. Every planar graph is the intersection graph of segments in the plane. In: *Proc. STOC'09*, p. 631-638, 2009.
[3] V. Dujmović, D. Eppstein, M. Suderman and D. Wood. Drawings of planar graphs with few slopes and segments. *Comput. Geom. Theory Appl.*, 38(3):194–212, 2007.
[4] **V. Jelínek**, E. Jelínková, **J. Kratochvíl**, B. Lidický, M. Tesař, and T. Vyskočil. The Planar Slope Number of Planar Partial 3-Trees of Bounded Degree. In: *Proc. Graph Drawing (GD'09)*, p. 304–315. Springer, 2010.
[5] B. Keszegh, D. Pálvőlgyi, and **J. Pach**. Drawing planar graphs of bounded degree with few slopes. Submitted.
[6] **J. Kratochvíl** and **J. Matoušek**. Intersection graphs of segments. *J. Comb. Theory, Ser. B*, 62(2):289–315, 1994.
[7] **J. Kratochvíl** and J. Nešetřil. INDEPENDENT SET and CLIQUE problems in intersection-defined classes of graphs. *Commentat. Math. Univ. Carol.*, 31(1):85–93, 1990.

WP08 – Crossing number This work package concerns the following natural problem in graph drawing: to find a drawing of a given graph that minimizes the number of pairwise edge crossings. This minimum, which obviously is a nonzero positive integer for any nonplanar graph, is called the *crossing number* of the graph. Investigation of graph crossing numbers has been initiated by Turán during WW II, and this topic has since found important applications, e.g., in VLSI design and graph visualization.

Despite decades of research, many basic questions remain unanswered – such as what are the crossing numbers of complete, complete bipartite, or toroidal grid graphs. This fact indicates the difficulty of the crossing number problem, and the problem indeed is very hard from both the theoretical and algorithmic points of view. In this respect it is worth to mention a recent involved (ILP-based) practically usable approach to exact crossing minimization by Chimani, Mutzel, and Bomze [2], cf. WP11.

One may clearly see that the theory of graph crossing number has matured especially during the last decade, evolving into a rich mathematical theory with many new results and many more open problems of a fundamental nature. Some of the recent results have been trully groundbreaking: the separation between odd and ordinary crossing numbers by Pelsmajer, Schaefer, and Stefankovič [7], hardness of computation of the crossing number even for nearly-planar graphs by Cabello and Mohar [1], or a construction of an unexpected crossing-critical family by Dvořák and Mohar [4], to mention just a few. Yet many other fundamental questions remain wide open.

The proposed work package leader, Petr Hliněný[1], has also remarkably contributed to the recent development in the area of crossing numbers. Out of his numerous related results we mention structural study of crossing-critical graphs [5] and subsequent constructions of interesting critical families, and a series of papers devoted to constant-factor approximation algorithms for the crossing number on restricted graph classes (in collaboration with Salazar [6], and with Chimani and Mutzel) such as most recent [3]. The existing collaboration of the work package leader with Chimani and Mutzel (IP2-DE) is especially noticeable with respect to our CRP. We plan to continue with intensive research in the mentioned theoretical directions of the crossing number.

We also plan to develop new exact ILP-based branch-and-cut algorithms for special graph classes for solving the crossing number problem to provable optimality (cf. [2] of Mutzel et al). For instance, we would like to develop such an algorithm able to compute all possible crossing configurations of small complete graphs. Implementations of the algorithms can be used to study the crossing number of these graph classes, hopefully leading to conjectures that will be verified afterwards. This may help us in finding new structural insights into the crossing number. We would like to put our software on a web-based server, offering everybody the possibility to compute the exact crossing number of given small instances (cf. IP2-DE and WP11).

On a more applied side, we plan to jointly explore the properties of constrained layouts of planar graphs, where constraints on the mutual placement of the vertices yields highly non-planar representations (cf. AP1-IT).

Milestones of WP08

- M08.1 Improve the lower estimates on the crossing number of surface embedded graphs, and attempt to extend the material to nonorientable surfaces.
- M08.2 Lower estimate and approximate the minor crossing number of surface graphs.
- M08.3 Try to determine the computational complexity of crossing number of almost planar graphs of bounded degree.
- M08.4 Further investigation of theoretical properties of crossing-critical graphs.
- M08.5 Attempt to give an approximative solution for multiple-edge insertion to planar graphs.
- M08.6 Studying non-planar constrained layouts of planar graphs.
- M08.7 Developing new exact ILP-based branch-and-cut algorithms for special graph classes and studying the computation results in order to get structural insights.

References for WP08

[1] S. Cabello and B. Mohar. Adding one edge to planar graphs makes crossing number hard. In: *Proc. ACM Symp. Comput. Geom. (SoCG'10)*. To appear.

[2] **M. Chimani**, **P. Mutzel**, and I. Bomze. A new approach to exact crossing minimization. In: *Proc. Europ. Sympos. Algorithms (ESA'08)*, p. 284–296, Springer 2008.

[3] **M. Chimani** and **P. Hliněný**. Approximating the crossing number of graphs embeddable in any orientable surface. In: *Proc. ACM-SIAM Symp. Discrete Algorithms (SODA'10)*, p. 918–927, 2010.

[4] Z. Dvořák and B. Mohar. Crossing-critical graphs with large maximum degree. *J. Combin. Theory Ser. B*, 100:413–417, 2010.

[5] **P. Hliněný**. Crossing-critical graphs have bounded path-width. *J. Combin. Theory Ser. B*, 88:347–367, 2003.

[6] **P. Hliněný** and G. Salazar. Approximating the Crossing Number of Toroidal Graphs. In: *Proc. Symp. Algorithms Comput. (ISAAC'07)*, LNCS 4835:148–159. Springer, 2007.

[7] M. Pelsmajer, M. Schaefer, and D. Stefankovič. Odd crossing number and crossing number are not the same. *Discrete Comput. Geom.*, 39:442–454, 2008.

Deliverables. Each work package will annually organize one to two meetings of researchers interested in its topic and prepare an annual report about the research it coordinates. Moreover the team will organize a 1 week tutorial on intersection graphs (in Year 1) and a 1 week tutorial on crossing number (in Year 2) for students from all interested sites.

[1] See `http://www.fi.muni.cz/~hlineny/work.html` for a CV of Petr Hliněný.

Justification of the budget. In the national currency (CZK), our budget is exactly the same as in the outline proposal. Due to changes in the exchange rate of the CZK against the Euro, the Euro figures have slightly changed compared to the outline proposal. (We have used the most recent rate to recalculate.)

The salary costs are planned for 6 senior researchers (J. Kratochvíl part-time 0.2, J. Matoušek 0.1, J. Fiala 0.2, M. Pergel 0.2, and V. Jelínek 0.2, P. Hliněný 0.1), 2 PhD. students (part-time 0.5 each), 1 full-time postdoc researcher acquired on the basis of an internationally announced competition, 2 staff members (secretary and technician, each 0.1) and stipends for 1-2 Master degree students.

We plan travel costs for all members of the research team to travel to conferences, workshops, tutorials, etc. (including the networking within GraDR). Further are planned consumables for both sites (Prague and Brno) and overhead (also for both Prague and Brno).

Related projects

The current research of Jan Kratochvíl is supported by institutional funding from the Czech Ministry of Education; through the project MSM0021620838 *Modern methods, structures, and systems of computer science* (leader J. Kratochvíl) and through the *Institute for Theoretical Computer Science – ITI* (grant 1M0545, led by J. Nešetřil).

Petr Hliněný's research is supported by the individual grant GA 201/08/0308 *Structural and Width Parameters in Combinatorics and Algorithmic Complexity* and by the bilateral grant 201/09/J021 *Structural Graph Theory and Parameterized Complexity* (with RWTH Aachen, P. Rossmanith) from the Czech Science Foundation. He is also partly supported by institutional funding from the Czech Ministry of Education (projects MSM0021622419 and ITI 1M0545).

Overview of other ongoing international scientific relationships

J. Kratochvíl has a long term collaboration with A. Proskurowski from University of Oregon (they mainly work on domination type problems), with F. Fomin and J.A. Telle from University of Bergen (on various topics from theoretical computer science), with P. Golovach from University of Durham and Zs. Tuza from Renyi Institute in Budapest (on variants of graph coloring problems), and with D. Kratsch from University in Metz and M. Liedloff from University Orleans (on exact exponential time algorithms).

P. Hliněný has active strong links to Gelasio Salazar from Mexico and Markus Chimani from Germany (the work on crossing number), and to Peter Rossmanith from Germany (on parameterized complexity).

Section B2 – Individual Project IP2 – GraDR-DE

Individual project's contribution to the CRP

Aims and Objectives. This project assembles four work packages. The coordinating researchers for these packages are the four senior scientists S. Felsner (Berlin), M. Kaufmann (Tübingen), P. Mutzel (Dortmund), and A. Wolff (Würzburg) who are acting as equitable partners in directing the research of this IP. We only include the CV of PI S. Felsner, see Annex 1. The CV data of Mutzel[2], Kaufmann[3], and Wolff[4] are available on the Web. Each site will lead one work package and contribute to one or more of the other work packages, see the table below.

team	*leading*	*contributing to*
Felsner	WP10 – Contact and intersection representations	WP06, WP08, WP09
Kaufmann	WP12 – Hypergraphs with appl. in bioinformatics	WP02, WP03, WP10
Mutzel	WP11 – Transfer to practice	WP04, WP05, WP08, WP12
Wolff	WP02 – Angular schematization	WP01, WP04, WP10, WP11

The teams are composed as follows. Felsner's team in Berlin consists of Kolja Knauer (currently Ph.D. student, probable candidate for PostDoc position) and Julia Rucker (currently master student, probable candidate for Ph.D. position). Kaufmann's team in Tübingen consists of Ph.D. students Robert Krug and Philipp Effinger, and master student Christian Zielke. Mutzel's team in Dortmund consists of Carsten Gutwenger, Karsten Klein (both Ph.D. students), and Markus Chimani (a former Ph.D. student and PostDoc, now associated). Finally, Wolff's team in Würzburg consists of PostDoc Joachim Spoerhase and Ph.D. student Martin Fink.

WP02 – Angular schematization We are interested in computing the layout of complex networks under *angular* restrictions. We refer to this problem as *angular schematization* and subsume under it also the combined effort of network construction and layout. It is striking that edge directions are being restricted in networks of very different nature and that these networks are constructed in very different communities: graph drawing, information visualization, geographic information science, computational geometry, VLSI layout, and underground mining. In some of these communities (such as graph drawing or VLSI layout), rectilinear connections have a long history, but recently octilinear connections have moved into the spotlight, bringing with them completely new problems and challenges. In other fields of application such as underground mining, it is not the number of slopes that is restricted, but there is an upper bound in the maximum slope.

As a first step, we are organizing an interdisciplinary Dagstuhl seminar (Nov. 14–19, 2010) on the topic. This meeting will ignite the discussion with researchers from application areas and will lay the foundations for further cooperation within the work package.

Workplan. Building on previous work in 2d [1], we plan to approach, in cooperation with Tübingen, the minimum Manhattan network (MMN) problem. MMNs are so called 1-spanners w.r.t. the Manhattan metric; they help to visualize so-called *split networks* in phylogenetics (cf. WP03). It is known that in 3d, constructing MMNs is APX-hard. It is an open question whether approximation algorithms exist for 3d. This is what we want to tackle. Further, we will investigate (with AP3-DE) Manhattan-geodesic drawings (which we introduced recently [2]) and (with AP1-IT) *monotone straight-line drawings* where we require for each pair of vertices that there exists a direction d such that the drawing contains a path that connects the vertices and is increasing in d. Together with Dortmund, we want to engineer methods for drawing large-scale (labeled) metro maps [3] – including rectangular stations and parallel lines. With Dortmund and Tübingen, we will attack bend minimization in the well-known Kandinsky framework [5] for which so far only a 2-approximation is known – but no hardness proof [4]. Rectilinear area-preserving schematizations of maps have recently been treated [7] – what about allowing diagonals (with AP2-NL)?

[2] `http://ls11-www.cs.tu-dortmund.de/staff/mutzel`
[3] `http://www-pr.informatik.uni-tuebingen.de/~mk`
[4] `http://www1.informatik.uni-wuerzburg.de/mitarbeiter/wolff_alexander`

Milestones of WP02

M02.1 Design approximation algorithms for minimum Manhattan networks in dimension ≥ 3.
M02.2 Attack open questions about Manhattan-geodesic drawing convention, see [2].
M02.3 Consider area-preserving schematization with diagonals.
M02.4 Investigate complexity / design algorithms for monotone straight-line drawings.
M02.5 Combine IP and heuristic methods for drawing large-scale (labeled) metro maps (including rectangular stations and parallel lines).
M02.6 Show hardness or give efficient algorithm for bend minimization in Kandinsky framework.

References for WP02

[1] M. Benkert, **A. Wolff**, F. Widmann, and T. Shirabe. The minimum Manhattan network problem: Approximations and exact solutions. *Comput. Geom. Theory Appl.*, 35(3):188–208, 2006.

[2] B. Katz, **M. Krug, I. Rutter**, and **A. Wolff**. Manhattan-geodesic embedding of planar graphs. In: *Proc. Graph Drawing (GD'09)*, LNCS 5849:207–218. Springer, 2010.

[3] **M. Nöllenburg** and **A. Wolff**. Drawing and labeling high-quality metro maps by mixed-integer programming. *IEEE Trans. Visualization & Comput. Graphics*, to appear, 2010.

[4] W. Barth, **P. Mutzel**, and C. Yildiz. A new approximation algorithm for bend minimization in the Kandinsky model. In: *Proc. Graph Drawing (GD'06)*, LNCS 4372:343–354. Springer, 2007.

[5] U. Fößmeier and **M. Kaufmann**. Drawing High Degree Graphs with Low Bend Numbers. In: *Proc. Graph Drawing (GD'95)*, LNCS:1027:254–266.

[6] M. Bekos, **M. Kaufmann, M. Nöllenburg**, and A. Symvonis. Boundary labeling with octilinear leaders. In: *Proc. Scandinavian Workshop Algorithms Theory (SWAT'08)*, LNCS 5124:234–245. Springer, 2008.

[7] W. Meulemans, A. van Renssen, and **B. Speckmann**. Area-preserving subdivision schematization. In: *Proc. Int. Conf. Geographic Inform. Sci. (GIScience'10)*, to appear 2010.

WP10 – Contact and intersection representations Intersection graphs of geometric objects have received a lot of interest from a wide range of prospects. This interest is based on the fact that geometric representations enforce strong structural properties on the graphs which can be exploited in applications, e.g. the use of interval graphs in dynamic storage allocation or the use of disk graphs in radio telecommunication. We aim at making further progress in the study of separation properties and extremal properties (see, e.g., the Pach-Sharir conjecture for the number of edges in intersection graphs excluding a bipartite subgraph [11, 10]).

Intersection graphs of curves come in many flavours, ranging from unrestricted versions (string graphs) to situations where the number of intersections between members in the family of curves is bounded by k (k-string graphs). Particularly interesting are 1-string graphs [5]. A recent breakthrough was that all planar graphs have a 1-string representation [4]. We aim at a better understanding of this class. A range of interesting graph classes is defined on the basis of curves with endpoints attached to a fixed object, this includes, e.g., outer-string graphs and interval filament graphs. If curves are restricted to follow paths on some graph from a base class we obtain graph classes ranging from interval graphs to the class of all graphs. In [8] it was pointed out that some intersection defined graph classes become recognizable in polynomial time if the girth is bounded from below. This phenomenon is worth further exploration.

Recently the notion of edge-intersection graphs has gained increasing attention. Particulary interesting is the class $\mathcal{G}_k$ of graphs admitting an edge-intersection representation by paths on a grid with the restriction that the paths have at most k bends [1, 3]. The class $\mathcal{G}_0$ coincides with the class of interval graphs and every graph is in $\mathcal{G}_k$ for some k. For a given graph G we want to determine bounds for the minimum k such that $G \in \mathcal{G}_k$. Is it true that $\mathcal{G}_4$ contains all planar graphs?

Graphs representable by contacts of noncrossing curves are planar. If contacts have to involve an end of a curve, we deal with complicated classes [7]. Without the restriction it follows from Koebe's Coin-Graph Theorem that every planar graph is representable. We aim at contact representations of planar graphs by homothetic triangles and squares. In fact, we hope to prove that every 4-connected triangulation map has a unique contact representation with homothetic triangles. First steps towards a solution of the question [2, 6] show that this relates to Schnyder woods, flip graphs, positivity of solutions of systems of linear equations and discrete harmonic functions.

Contact and intersection graphs of convex polygons homothetic to a fixed polygonal shape P form a family of interesting graph classes. For every convex polygon P, the intersection graphs of Hom_P are NP-complete to recognize [9]. For contact graphs, the recognition complexity is unknown.

Milestones of WP10

M10.1 Progress in extremal properties for box intersection graphs.
M10.2 Identification of classes of comparability graphs that are 1-string.
M10.3 Determination of the minimal k such that $G \in \mathcal{G}_k$ for bipartite graphs.
M10.5 Resolving the question wether $\mathcal{G}_4$ contains all planar graphs?
M10.6 Development of techniques to compute contact representation with homothetic triangles.
M10.7 Decision of the recognition complexity of contact graphs from Hom_P.

References for WP10

[1] A. Asinowski and A. Suk. Edge intersection graphs of systems of paths on a grid with a bounded number of bends. *Discrete Appl. Math.*, 157(14):3174–3180, 2009.
[2] M. Badent, C. Binucci, E. Di Giacomo, W. Didimo, **S. Felsner**, F. Giordano, **J. Kratochvíl**. P. Palladino, **M. Patrignani**, and F. Trotta. Homothetic triangle contact representations of planar graphs. In *Proc. Canad. Conf. Comput. Geom. (CCCG'07)*, p. 233–236, 2007.
[3] T. Biedl and M. Stern. On edge-intersection graphs of k-bend paths in grids. *Discrete Math. & Theoretical Comput. Sci.*, 12(1), 2010.
[4] J. Chalopin, D. Gonçalves, and P. Ochem. Planar graphs have 1-string representations. *Discrete Comput. Geom.*, 43(3):626–647, 2010.
[5] C. Dangelmayr, **S. Felsner**, and W. Trotter. Intersection graphs of pseudosegments: Chordal graphs. *J. Graph Algorithms Appl.*, 14:199–220, 2010.
[6] **S. Felsner**. Triangle contact representations, 2009. In: *Proc. MCW'09*.
[7] **P. Hliněný**. Classes and recognition of curve contact graphs. *J. Comb. Theory, Ser. B*, 74(1):87–103, 1998.
[8] **J. Kratochvíl** and **M. Pergel**. Geometric intersection graphs: Do short cycles help? *Proc. COCOON'07*, LNCS 4598:118–128. Springer 2007.
[9] **J. Kratochvíl** and **M. Pergel**. Intersection graphs of homothetic polygons. *Electronic Notes Discrete Math.*, 31:277–280, 2008.
[10] **J. Pach** and M. Sharir. On planar intersection graphs with forbidden subgraphs. *J. Graph Theory*, 59:205–214, 2008.
[11] R. Radoičić and G. Tóth. The discharging method in combinatorial geometry and the Pach-Sharir conjecture. In: Surveys on discrete and computational geometry. Twenty years later. *Contemp. Math.*, 453:319–342, 2008.

WP11 – Transfer to practice The aim of this work package is to give users easy access to state-of-the-art graph drawing methods. Typically, this is achieved by providing algorithm and data structure libraries, thus allowing to integrate such methods into user applications. Teams in this project already offer graph drawing libraries like the open-source C++ library OGDF [1], BN++ [2] or CGAL [3], and we want to use these libraries both for implementing and disseminating the algorithmic results of this project.

We can, however, reach but a small fraction of graph drawing users with such libraries since most users refrain from programming and simply want to use familiar, domain-specific tools. Hence, it is required to extend these tools by integrating sophisticated graph drawing methods that take application- or domain-specific restrictions into account. A very good example is Cytoscape [4], a bioinformatics software platform for visualizing molecular interaction networks. This tool has a huge user community and also provides a flexible plugin mechanism for integrating new features, e.g., new graph layout methods. Within this work package, we want to integrate tailor-made graph layout methods for networks arising in bioinfomatics applications into Cytoscape.

Furthermore, we want to give easy access to our methods for solving NP-hard optimization problems like crossing number [5]. Since these algorithms are often based on ILP formulations, their implementations utilize additional LP solver software. This complicates installation and usage of these implementations. To overcome these difficulties, we want to install a web application for the crossing number problem which allows users to send their problem instances (a graph for which the crossing number shall be computed) via a web interface, and the optimization is then run on the server. Hence, this requires no installation at all by the user, and problems can be solved without any programming involved. We think this approach can help a lot to reach a broader audience for such sophisticated methods.

Naturally, this work package is closely related to other work packages, where algorithmic results will be implemented. This includes WP02 (where ILP methods are used to draw subway maps), WP03 (which contributes code for simultaneous embeddings to CGAL), WP07 (implementing promising algorithms for flow maps), and WP12 (implementing layout algorithms for biochemical networks).

Milestones of WP11

- M11.1 Integration of the main algorithmic results of WP02, WP03, WP07, WP08, and WP12 into existing (open-source) algorithm and data structure libraries.
- M11.2 Implementation of Cytoscape plugins providing domain-specific graph layout functionality within Cytoscape (also results of WP12).
- M11.3 Setting up a server with a web interface for solving crossing number problems to provable optimality for instances of moderate size (compare WP08).

References for WP11

[1] **M. Chimani**, **C. Gutwenger**, M. Jünger, K. Klein, **P. Mutzel**, and M. Schulz. The Open Graph Drawing Framework. Poster abstract, *Proc. Graph Drawing (GD'07)*. LNCS 4875. Springer, 2008. See also `http://www.ogdf.net`.

[2] J. Küntzer, T. Blum, A. Gerasch, Ch. Backes, A. Hildebrandt, **M. Kaufmann**, O. Kohlbacher, and H.-P. Lenhof. BN++ – A biological information system. *J. Integrative Bioinformatics*, 3(2):34, 2006.

[3] CGAL, Computational Geometry Algorithms Library. See `http://www.cgal.org`.

[4] P. Shannon, A. Markiel, O. Ozier et al.: Cytoscape: A Software Environment for Integrated Models of Biomolecular Interaction Networks. *Genome Research* 13(11):2498–2504, 2003.

[5] **M. Chimani**, **P. Mutzel**, and I. Bomze. A New Approach to Exact Crossing Minimization. In: *Proc. Europ. Symp. Algorithms (ESA'08)*, LNCS 5193:284–296. Springer, 2008.

WP12 – Hypergraphs with applications in bioinformatics Hypergraphs are essential tools to model complex data relations as they arise for example in data bases. We concentrate our methodological research on applications in bioinformatics. Two research directions related to different representations of hypergraphs will be addressed.

The first direction concerns subdivision drawings [1, 2], where each vertex is represented by a face of a planar subdivision while a hyperedge is represented implicitely by the connectedness of the corresponding faces. The *planar support* is the graph structure that represents the connectedness.

The second direction addresses the more direct representation of hyperedges as trees. Each hyperedge is realized by a tree connecting the corresponding vertices. A user-friendly layout convention is the so-called *bus layout*, where each tree has only one or two main horizontal or vertical segments which interconnect the legs of the different pins, cf. [4] for a restricted model.

From the various fields where hyperedges are being used, we chose network applications in bioinformatics, e.g., in metabolic and phylogenetic networks to be the playground for our models and methods.

Workplan. In [2], several classes of planar supports such as paths and cycles have been considered. In cooperation with AP2-NL, we will investigate extensions to more general classes, especially to trees with special properties such as degree restrictions or distinguished elements as they arise in the modeling of biochemical reactions (cf. educts / products).

For the traditional tree representation of hyperedges, several practical issues have been addressed using adhoc heuristics [3] but strong models and efficient algorithms or approximations are missing even for simple models with triple or quadruple hyperedges. We will address using the bus representations together with Dortmund (previous work: [5]) and Würzburg. There are also methodological connections to WP03 - simultaneous drawings.

In Tübingen, we have developed a platform [6] to visualize and analyze biochemical network structures. It provides biological data using a powerful database, and also some basic layout features. BN++, which is free under the GPL, will serve as the interconnection with other projects concerning different geometric representations of graphs and hypergraphs and their applications to biological data. This idea has been tried out already in a successful way with the advice of collegues from the Bioinformatics department in Tübingen [7] and will be extended along this line of research. We will be in a close cooperation and coordination with Dortmund's WP11.

Milestones of WP12

M12.1 Design of algorithms of restricted classes of planar supports as they arise in applications
M12.2 Development of suitable models for one- and two-dimensional buses for hyperstructures
M12.3 Layout algorithms for bus structures using the models from above respecting issues like crossing minimization, port restrictions
M12.4 Implementation of the layout algorithms in BN++ and applications to metabolic data
M12.5 Extension and application of the geometric representation of graphs hypergraphs in the areas of metabolic pathways and phylogenetic split networks

References for WP12

[1] **M. Kaufmann**, M. van Kreveld, and **B. Speckmann**. Subdivision drawings of hypergraphs. *Proc. Graph Drawing (GD'08)*, LNCS 5417:396–407. Springer, 2009.
[2] K. Buchin, M. van Kreveld, H. Meijer, **B. Speckmann**, and K. Verbeek. On planar supports for hypergraphs. *Proc. Graph Drawing (GD'09)*, LNCS 5417:345–356. Springer, 2010.
[3] G. Sander. Layout of directed hypergraphs with orthogonal hyperedges. *Proc. Graph Drawing (GD'04)*, LNCS 3393:381–386. Springer, 2005.
[4] A. Ada, M. Coggan, P. Di Marco, A. Doyon, L. Flookes, S. Whitesides et al. On bus graph realizability. *Proc. Canad. Conf. Comput. Geom. (CCCG'07)*, p. 229–232, 2007.
[5] M. Chimani and **C. Gutwenger**. Algorithms for the Hypergraph and the Minor Crossing Number Problems *Proc. Intern. Sympos. Algor. Computat. (ISAAC'07)*, LNCS 4835:184–195. Springer, 2007.
[6] J. Küntzer, T. Blum, A. Gerasch, Ch. Backes, A. Hildebrandt, **M. Kaufmann**, O. Kohlbacher, and H.-P. Lenhof. BN++ – A biological information system. *J. Integrative Bioinformatics*, 3(2):34, 2006.
[7] K.A. Lehmann, **M. Kaufmann**, St. Steigele, and K. Nieselt. On the maximal cliques in c-max-tolerance graphs and their application in clustering molecular sequences. *Algorithms Molecular Biol.*, 1, 2006.

Justification for Budget Items. For the budget we have updated the numbers given in the outline proposal according to the specification of DFG. We are applying for a postdoc position for WP10, which is – in terms of abstraction level – probably the most demanding work package of IP2. For each of the three other work packages, we are applying for a 75% PhD student position for research plus a student assistant for implementation work. In addition, we ask for travel support to cover all work package and project meetings plus at least one trip to an international conference for each senior researcher and each PhD student / postdoc.

Related projects

In the framework of EuroGIGA Stefan Felsner is applying for funds within the project Combinatorial Problems of Point Sets and Arrangements of Objects (ComPoSe), PI Oswin Aichholzer. The emphasis of the subproject there is on arrangements of pseudolines. There is no overlap with this IP.

Petra Mutzel has a joint project with Michael Jünger (Universität zu Köln) in the priority program SPP 1307 *Algorithm Engineering* titled *Planarization Methods in Automatic Graph Drawing* of the German Science Foundation. The project focusses on algorithm engineering methods for the planarization approach with applications to software engineering and bioinformatics. Our contributions mentioned in the EuroGiga project are disjunct with our DFG application (funded since 2007).

Section B3 – Individual Project IP3-CH

Individual project's contribution to the CRP

We will lead WP03 and contribute to at least WP06 and WP11. Apart from PI Michael Hoffmann, the team at ETH Zürich includes the following senior researchers whose expertise and contacts are relevant to the project, although they do not promise particular deliverables within this project: Bernd Gärtner (combinatorial and geometric optimization $\rightarrow$ WP11), Uli Wagner (combinatorial methods, in particular related to crossing numbers and intersection graphs $\rightarrow$ WP01,08,10, strong links to Prague), and Emo Welzl (combinatorial methods, in particular related to non-crossing configurations $\rightarrow$ WP06,08,09, strong links to Budapest).

WP03 – Simultaneous embeddings

a) Aims and objectives. The challenge to effectively visualize several structures on the same ground set simultaneously has emerged as an active research area in graph drawing over the past five years. Originally, the term *simultaneous (geometric) embedding/drawing* was coined by Brass et al. [3] for a very specific setting. But we will use it here more generally to refer to any kind of problem in which two or more graphs are to be drawn simultaneously, usually subject to certain constraints.

One reason for the strong interest in simultaneous embeddings lies in a wealth of applications in areas such as bio-informatics, network analysis, and software engineering. For instance, in order to compare two phylogenetic trees one would like to have a "nice" simultaneous drawing of two trees sharing the same set of leaves [16]. More generally, simultaneous drawings come into play whenever large or dynamically changing graphs are to be visualized. Given a large graph, only part of it can be shown at a time with a sufficient level of detail. When browsing from one such part to a neighboring one, it is desirable to have a certain overlap that is common to both parts and drawn the same way for both parts. In this way, the user can maintain her mental map [17] and more easily keep track of her current location in the graph. In the context of dynamic graphs, a simultaneous drawing for several instances of the graph at different timesteps allows to visually examine the differences in order to, for instance, draw conclusions regarding the capacity utilization of a network.

Beyond these motivating applications the study of simultaneous embeddings revealed a rich collection of interesting and challenging combinatorial and algorithmic problems, many of them still far from being solved. The goal of this project/WP is, on one hand, to gain more insights into some of these problems and, on the other hand, to develop new models in order to address some shortcomings of the existing settings studied so far.

Although some related questions have been studied earlier, the term *simultaneous embedding* goes back to Brass et al. [3], who studied the following problem: Given two[5] (planar) graphs G_1 and G_2 on the same vertex set V, find an embedding of V into R^2 such that for both G_1 and G_2 the induced straight line drawing is plane. Note that the drawings of G_1 and G_2 are considered separately and so an edge of one graph may freely cross an edge of the other. Another way to think of this model is to imagine G_1 and G_2 being drawn on two parallel planes in R^3—one plane contains the drawing of G_1, the other contains the drawing of G_2—such that under orthogonal projection the two copies of each vertex end up at the same point.

Brass et al. [3] showed that a simultaneous embedding exists for any pairs of path, cycles or caterpillars, but it is not always possible for three paths or two general planar graphs. This left open the question for trees, which was answered negatively by Geyer, Kaufmann, and Vrt'o [13] who described two trees that cannot be embedded simultaneously. Most recently, Angelini et al. [1] even claimed an example of a path and a tree that do not admit a simultaneous embedding. On top of this it is NP-hard to decide whether two given graphs on the same vertex set admit a simultaneous embedding [8].

[5] For brevity, we talk about two graphs only, the generalization of these questions to a larger number of graphs is obvious.

Altogether it seems that this model of simultaneous embedding is too strong to be generally useful. Therefore, one main focus of this WP is to develop and study relaxed—or even different—notions of simultaneous embeddings that—hopefully—are applicable to larger classes of graphs. Some possible candidates for such notions along with specific questions that we would like to address are highlighted below.

A possible relaxation is to allow edges to bend [7]. From a theoretical point of view, one bend per edge is always sufficient: a recent construction of Everett et al. [9] yields a family of sets of n points in R^2 on which every planar graph on n vertices can be drawn using at most one bend per edge. However, such a *universal* point set is unlikely to be the best option for any given instance and the point coordinates grow exponentially in n.

Alternatively, edges can be drawn as higher (but still small) degree algebraic curves, such as circular or parabolic arcs. Very little is known for this setting, other than how to combine an outerplanar graph with a path using circular arcs [5].

Instead of allowing more freedom to draw edges only one could also give up on the requirement that each vertex be at the very same location in both drawings. A natural option is to allow vertices to reside in different locations but try to keep them "close" to each other. Unfortunately, at least with respect to metric distance this is not possible in general [11]. But it may still work for certain classes of graphs or in case that the graphs under consideration are sufficiently *similar*, for an appropriate notion of similarity.

A different notion of proximity is employed in *matched drawings*; here the correspondence between two images of a vertex is expressed by demanding them to be placed at the same y-coordinate, that is, height in both drawings (and no other vertex be placed at this height). Not any two planar graphs allow a matched drawing [14], but at least for some special classes of graphs [15], in particular, for two trees [14] this is always possible. Despite these encouraging results many questions in the context of matched drawings are still open. For instance, are there other classes of graphs for which a matched drawing is always possible? Also, the algorithm by Di Giacomo et al. [14] uses exponential area, and the complexity of deciding whether or not two given graphs admit a matched drawing is still unresolved.

In all models discussed so far, both input graphs shared the same set of vertices. What if only a subset of the vertex set is common to both graphs? This question of whether the correspondence/mapping between the vertices of the input graphs is total or partial is in some sense orthogonal to the model for simultaneous embedding under consideration. From an application point of view, an interesting special case is when the input consists of two trees on the same set of leaves, a so-called *tanglegram* [4].

Closely related though fundamentally different and not so relevant from an application point of view is the scenario in which no vertex mapping is given. In this case the input consists of two graphs on n vertices and the goal is to find a set of n points in R^2 on which both graphs admit a plane straight line drawing (as before, considered separately). This can always be done for any number of outerplanar graphs [2] and for an outerplanar and a planar graph [3], but the general case of two planar graphs is still open.

We conclude by mentioning the packing problem, which is a problem on topological graphs, that is, the drawings need not be straight line. Here the drawings are not considered in isolation but instead their union must be plane and each (combinatorial) edge may be used by at most one of the graphs (also called a *tight packing* sometimes). It is a long-standing open problem whether or not any pair of non-star trees admits a tight packing. García et al. [12] showed that any non-star tree can be packed with a copy of itself and any non-star tree can be packed with any path. A few other special cases have been resolved as well [10], but the general problem appears still widely open.

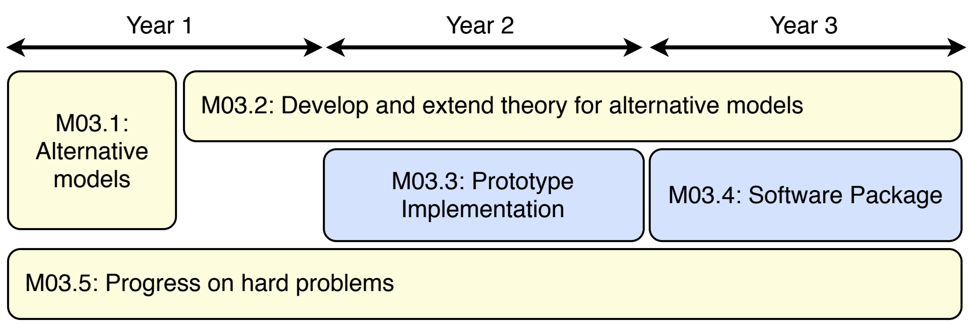
b) Methodologies and Experiments. We are interested both in theoretical results and in practical implementations. The theoretical side encompasses existential statements that ensure the existence of a certain type of embedding, algorithms that can construct it—ideally efficiently—, or negative results that show—typically by example—that specific families of graphs do not admit such an embedding, or hardness reductions hinting that it is unlikely that the embedding can be constructed efficiently. In a search for counterexamples we will employ computer experiments, for instance, by enumerating certain families of possible input graphs, where in order. Other than that, our tools are pencil and paper.

On the practical side, we will implement our algorithms—where promising—in order to examine their real-world performance on various data, both in terms of efficiency and quality of the drawing. The data sets may be synthetic or based on real-world data, depending on the problem at hand. Our ultimate goal are methods that provide both theoretical guarantees *and* good results in practice.

c/d) Work plan, milestones and deliverables. Milestones are listed in the table below.

M03.1 Develop alternative notions of simultaneous embeddings and analyze their potential, all from a combinatorial (Which graph classes can be handled?), algorithmic (Can such drawings be constructed efficiently?), and a practical (Do our methods produce "nice" drawings for reasonable instances?) point of view.
M03.2 Develop and extend the theory for the most promising models obtained in M03.1.
M03.3 Develop a prototype implementation for the most promising models obtained in M03.1 and M03.2.
M03.4 Polish and document the prototype obtained in M03.3, such that it can be published and—if successful—be submitted for possible inclusion into an algorithms library such as, e.g., CGAL.
M03.5 Try to make some progress on the long standing and probably hard problems within the area.

M03.1 and M03.2 will be tackled in collaboration with Eindhoven (AP2-NL), regarding M03.5 we will also collaborate with Karlsruhe (AP3-DE), Rome (AP1-IT), and Tübingen (IP2-DE). We expect M03.1 to be reached after the first six months of the project. Then work on M03.2 will go on till the end of the project. In parallel, work on M03.3 will kick in at begin of Year 2 and be superseded by M03.4 at begin of Year 3. The work on M03.5 will be an ongoing effort over the duration of the whole project. Our timetable is summarized in the figure shown below.



Deliverables beyond the annual progress reports and WP meetings (see CRP description), as well as publications at workshops, conferences, (and, eventually, journals) are listed in the table below.

D03.1 for M03.4: a software package, to be published under an open source license (Year 3);
D03.2 for M03.5: a dedicated workshop for project members and selected external researchers (within the first two years);
D03.3 for M03.5: a report summarizing these problems and the state-of-the-art, where possible, highlighting the particular difficulties to overcome (Year 2);
D03.4 for M03.5: a webpage/repository of open problem descriptions in the spirit of TOPP [6], that hopefully can be maintained and extended over time in order to disseminate information and so serve as a basis for further research activities (Year 3).

e) Justification for Budget Items. SNF grants 50% PhD positions only, which translate to a 60% position at ETH. Salary is based on ETH accounting rules for 2010 with a 2% inflation compensation added and include 13.5% social insurance (also fixed by ETH regulations). EUR figures are based on an exchange rate of 1 CHF = 0.726 EUR.

In order to foster collaboration among the partners within the project, we plan to go for several extended research visits. Therefore, a substantial amount of travel money is essential to run the project successfully.

References for WP03

[1] P. Angelini, M. Geyer, **M. Kaufmann**, and D. Neuwirth. On a tree and a path with no geometric simultaneous embedding. arXiv.org, 2010.

[2] P.K. Bose. On embedding an outer-planar graph in a point set. *Comput. Geom. Theory Appl.*, 23(3):303–312, 2002.

[3] P. Brass, E. Cenek, C.A. Duncan, A. Efrat, C. Erten, D.P. Ismailescu, S. Kobourov, A. Lubiw, and J.S. Mitchell. On simultaneous planar graph embeddings. *Comput. Geom. Theory Appl.*, 36(2):117–130, 2007.

[4] K. Buchin, M. Buchin, J. Byrka, **M. Nöllenburg**, Y. Okamoto, R. I. Silveira, and **A. Wolff**. Drawing (complete) binary tanglegrams: Hardness, approximation, fixed-parameter tractability. In *Graph Drawing (Proc. GD '08)*, volume 5417 of *Lecture Notes Comput. Sci.*, pages 324–335. Springer-Verlag, 2009.

[5] J. Cappos, A. Estrella-Balderrama, J. Fowler, and S. Kobourov. Simultaneous graph embedding with bends and circular arcs. *Comput. Geom. Theory Appl.*, 42(2):173–182, 2009.

[6] E. Demaine, J.S. Mitchell, and J. O'Rourke. The Open Problems Project. `http://maven.smith.edu/~orourke/TOPP/`.

[7] C. Erten and S. Kobourov. Simultaneous embedding of planar graphs with few bends. *J. Graph Algorithms Appl.*, 9(3):347–364, 2005.

[8] A. Estrella-Balderrama, E. Gassner, M. Jünger, M. Percan, M. Schaefer, and M. Schulz. Simultaneous geometric graph embeddings. In *Graph Drawing (Proc. GD '07)*, volume 4875 of *Lecture Notes Comput. Sci.*, pages 280–290. Springer-Verlag, 2008.

[9] H. Everett, S. Lazard, G. Liotta, and S. Wismath. Universal sets of n points for one-bend drawings of planar graphs with n vertices. *Discrete Comput. Geom.*, 43(2):272–288, 2010.

[10] **F. Frati**, M. Geyer, and **M. Kaufmann**. Planar packing of trees and spider trees. *Inf. Proc. Lett.*, 109(6):301–307, 2009.

[11] **F. Frati**, **M. Kaufmann**, and S. Kobourov. Constrained simultaneous and near-simultaneous embeddings. *J. Graph Algorithms Appl.*, 13(3):447–465, 2009.

[12] A. García, C. Hernando, F. Hurtado, M. Noy, and J. Tejel. Packing trees into planar graphs. *J. Graph Theory*, 40(3):172–181, 2002.

[13] M. Geyer, **M. Kaufmann**, and I. Vrt'o. Two trees which are self-intersecting when drawn simultaneously. *Discrete Math.*, 309(7):1909–1916, 2009.

[14] E. D. Giacomo, W. Didimo, M. van Kreveld, G. Liotta, and **B. Speckmann**. Matched drawings of planar graphs. *J. Graph Algorithms Appl.*, 13(3):423–445, 2009.

[15] L. Grilli, S.-H. Hong, G. Liotta, H. Meijer, and S. Wismath. Matched drawability of graph pairs and of graph triples. *Comput. Geom. Theory Appl.*, 43(6–7):611–634, 2010.

[16] A. Lozano, R. Y. Pinter, O. Rokhlenko, G. Valiente, and M. Ziv-Ukelson. Seeded tree alignment and planar tanglegram layout. In *Proc. 7th Internat. Workshop on Algorithms in Bioinformatics*, volume 4645 of *Lecture Notes Comput. Sci.*, pages 98–110. Springer-Verlag, 2007.

[17] K. Misue, P. Eades, W. Lai, and K. Sugiyama. Layout adjustment and the mental map. *J. Visual Lang. Comput.*, 6(2):183–210, 1995.

Related projects

None, as far as dedicated funding is concerned.

Overview of other ongoing international scientific relationships

Michael Hoffmann is involved in the CGAL project (cf. `http://www.cgal.org`), among others with the groups of Jean-Daniel Boissonat (INRIA Sophia), Kurt Mehlhorn (MPI), and Dan Halperin (Tel Aviv). Other contacts outside of GraDR include a long-term collaboration with Csaba D. Tóth (Univ. of Calgary) on various aspects of geometric graphs and planar subdivisions and strong links to the group of Oswin Aichholzer (TU Graz) and Yoshio Okamoto (Tokyo Tech).

Section B4 - Individual Project IP4-HU

Individual project's contribution to the CRP

a) Aims and Objectives. The Hungarian team is led by János Pach at the Alfréd Rényi Institute. János Pach also holds a chair at EPFL Lausanne, Switzerland. The other senior team members are Imre Bárány, Balázs Keszegh, Gábor Tardos, and Géza Tóth (all at Rényi Institute). The team is responsible for work package WP06 – Quasi- and near-planar graphs, which is described below. The team members will also contribute to work packages WP01, WP07, and WP10.

WP06 – Quasi- and near-planar graphs A *geometric graph* is a graph drawn in the plane so that its vertices are represented by points in general position (i.e., no three are collinear) and its edges by straight-line segments connecting the corresponding points. *Topological graphs* are defined similarly, except that now each edge can be represented by any simple Jordan arc passing through no vertices other than its endpoints. If any two edges of G have at most one point in common (including their endpoints), then G is said to be a *simple* topological graph. Clearly, every geometric graph is simple.

It follows from Euler's Polyhedral Formula that every topological graph with n vertices and more than $3n-6$ edges has a pair of crossing edges. According to Kuratowski's theorem, it also contains a K_5 minor or a $K_{3,3}$ minor. What happens if, instead of these substructures, we want to guarantee the existence of some larger configurations involving several crossings? What kind of *unavoidable* substructures must occur in every geometric (or topological) graph G having n vertices and more than Cn edges, for an appropriately large constant $C > 0$? To investigate this problem is one of the main objectives of the present proposal.

The problem is related to some classical topics in graph theory. A theorem of Kostochka [7] and Thomason [15] states that for any positive integer r, every graph of n vertices and at least $cr\sqrt{\log r}n$ edges has a K_r minor. This immediately implies that if the chromatic number $\chi(G)$ of G is at least $2cr\sqrt{\log r}+1$, then G has a K_r minor. According to Hadwiger's notorious conjecture, for the same conclusion it is enough to assume that $\chi(G) \geq r$. This is known to be true for $r \leq 6$ (see [13]).

A (k,l)*-grid* in a topological graph is a pair of edge subsets E_1, E_2 such that $|E_1| = k$, $|E_2| = l$, and every edge in E_1 crosses every edge in E_2. It was shown in [10] that for any $k, l \geq 1$, there exists a constant $c_{k,l}$, such that any topological graph on n vertices and more than $c_{k,l}n$ edges contains a (k,l)-grid. Tardos and Tóth [14] showed that we can also find a (k,l)-grid such that all k edges in E_1 are incident to the same vertex and all l edges in E_2 are incident to another vertex. However, it appears to be much harder to find a *"natural"* (k,l)*-grid*, i.e., a grid in which the edges in E_1 are pairwise disjoint and the edges in E_2 are pairwise disjoint. We plan to attack the conjecture [2] that $C_{k,l}n$ edges also suffice in this case, for a suitable constant $C_{k,l}n$. Note that we do not even know whether there exists a constant C_k such that every simple topological graph on n vertices and more that C_kn edges has k disjoint edges! We know, however, that the last statement is true for geometric graphs [12], so that the first goal ("milestone") may be to address this special case. The initial results are promising. We know that the natural grid conjecture is true for *convex* geometric graphs (i.e., when the vertices form the vertex set of a convex n-gon) and in the special case $k+2, l = 1$. We also know that the maximum number of edges that a simple topological graph of n vertices and no natural (k,l)-grid cannot have more than n times a polylogarithmic number of edges [2].

Some of the above questions grew out of an attempt to strengthen the famous *Crossing Lemma* of Ajtai, Chvátal, Newborn, Szemerédi [4] and Leighton [8], which is one of the most efficient tools in combinatorial geometry and graph drawing. It states that every topological graph with n vertices and $e > 4n$ edges has an edge that crosses at least a positive constant times e^2/n^2 edges. It was shown in [6] that in simple topological graphs, or if the number of crossings between a pair of edges is bounded by a constant, one can also find a (k,k)-grid with k at least a constant times e^2/n^2. We will try to further strengthen the lemma, which may have interesting consequences in its applications to Erdős-type distance problems.

One of the major objectives of this project is to make progress in the study of another notoriously difficult problem in geometric graph theory. A topological graph is called r*-quasi-planar* if it has no r pairwise crossing edges. Is it true that for any fixed r, the maximum number of edges that an r-quasi-planar topological graph of n vertices can have is at most linear in n? It is known to be true for $r = 3, 4$ [3], [1], and for convex geometric graphs. For geometric graphs, the best known bound is due to Valtr [16].

A topological graph G is called r*-locally planar* if G has no selfintersecting path of length at most r. Roughly speaking, this means that the embedding of the graph is planar in a neighborhood of radius $r/2$ around each vertex. In [11], we showed that there exist 3-locally planar geometric graphs with n vertices and with at least constant times $n \log n$ edges. Somewhat surprisingly, Tardos managed to extend this result to any fixed $r \geq 3$. He constructed a sequence of r-locally planar geometric graphs with n vertices and a slightly superlinear number of edges. We know that the maximum number of edges of a 3-locally planar geometric graph is $O(n \log n)$, and that this estimate is tight. However, for topological graphs nothing better than an upper bound of $O(n^{3/2})$ is known. Our aim is to improve the last bound.

A topological graph is called a *thrackle (generalized thrackle)* if any pair of its edges meet exactly once (an odd number of times), counting their common endpoints if applicable. These graphs appear to be the exact opposites of plane graphs, in which no nonadjacent edges have a point in common. However, the difference between the two notions is not that large: a bipartite graph can be drawn as a generalized thrackle if and only if it is planar [9]. According to Conway's thrackle conjecture, every thrackle has at most as many edges as vertices. We plan to improve on the best known upper bound, $|E(G)| < 1.5|V(G)|$, established by Cairns and Nikolayevsky [5]. We hope that in the process we will uncover some hidden structural properties of intersection patterns of edges, in the spirit of Tutte [17].

b) Work Plan. At the first stage, we plan to intensify our (long existing) collaboration with the other teams involved in the project, concentrating on the key themes outlined in the present proposal. We will work together with the teams led by Kratochvíl (IP1-CZ), Mutzel (IP2-Dortmund), Hoffmann (IP4-CH), and Speckmann (AP2-NL) on problems related to slope numbers and crossing numbers of graphs, and with the group of Di Battista (AP1-IT) on queue numbers and related parameters of planar graphs.

References for WP06

[1] E. Ackerman. On the maximum number of edges in topological graphs with no four pairwise crossing edges. *Discrete Comput. Geom.*, 41:365–375, 2009.

[2] E. Ackerman, J. Fox, **J. Pach**, and A. Suk. On grids in topological graphs. In: *Proc. ACM Symp. Comput. Geometry (SoCG'09)*, p. 403–412, 2009.

[3] P.K. Agarwal, B. Aronov, **J. Pach**, R. Pollack, and M. Sharir. Quasi-planar graphs have a linear number of edges. *Combinatorica*, 17:1–9, 1997.

[4] M. Ajtai, V. Chvátal, M. Newborn, and E. Szemerédi. Crossing-free subgraphs. In: *Theory and Practice of Combinatorics, North-Holland Math. Stud.*, 60:9–12. North-Holland, 1982.

[5] G. Cairns and Y. Nikolayevsky. Bounds for generalized thrackles. *Discrete Comput. Geom.*, 23:191–206, 2000.

[6] J. Fox, **J. Pach**, and C.D. Tóth. A bipartite strengthening of the crossing lemma. *J. Combin. Theory Ser. B*, 100:23–35, 2010.

[7] A.V. Kostochka. Lower bound of the Hadwiger number of graphs by their average degree, *Combinatorica*, 4:307–316, 1984.

[8] T. Leighton. New lower bound techniques for VLSI. *Math. Systems Theory*, 17:47–70, 1984.

[9] L. Lovász, **J. Pach**, and M. Szegedy. On Conway's thrackle conjecture. *Discrete Comput. Geom.*, 18:369–376, 1997.

[10] **J. Pach**, R. Pinchasi, M. Sharir, and **G. Tóth**. Topological graphs with no large grids, *Graphs Combin.*, 21:355–364, 2005.

[11] **J. Pach**, R. Pinchasi, **G. Tardos**, and **G. Tóth**. Geometric graphs with no self-intersecting path of length three. *European J. Combin.*, 25:793–811, 2004.

[12] **J. Pach** and J. Törőcsik. Some geometric applications of Dilworth's theorem. *Discrete Comput. Geom.*, 12:1–7, 1994.

[13] N. Robertson, P. Seymour, and R. Thomas. Hadwiger's conjecture for K_6-free graphs. *Combinatorica*, 13:279–361, 1993.

[14] **G. Tardos** and **G. Tóth**. Crossing stars in topological graphs. *SIAM J. Discrete Math.*, 21:737–749, 2007.

[15] A. Thomason. An extremal function for contractions of graphs. *Math. Proc. Cambridge Philos. Soc.*, 95:261–265, 1984.

[16] P. Valtr. On geometric graphs with no k pairwise parallel edges. *Discrete Comput. Geom.*, 19:461–469, 1998.
[17] W. T. Tutte. Toward a theory of crossing numbers. *J. Comb. Theory*, 8:45–53, 1970.

c) Deliverables and/or Milestones. As we proceed, first we plan to publish technical reports (mainly for the participating teams), and then research and survey papers summarizing our results. At the last stage, we plan to disseminate our findings and the outstanding open problems on larger conferences.

Some of milestones were mentioned in the main text of the proposal. For instance, we hope to report at an early stage an improved upper bound on Conway's thrackle conjecture and some progress on the maximum number of edges an r-quasi-planar geometric graph with n vertices can have.

d) Additional description/justification for Budget Items. We plan to hire and involve two Ph.D. students (or one Ph.D. student and a postdoctoral researcher) in the project. The budgeted salaries are those that have been communicated to us by the Human Resources department of Rényi Institute.

In addition, according to the above work plan, we need to spend a substantial part of our budget on travel (visiting the participating teams, inviting and consulting other experts, participating in conferences). We need to put aside some money for 5 PC workstations, for the new Ph.D. students as well as for the visitors.

Related Projects

In the framework of EuroGIGA János Pach is applying for funds within the project Combinatorial Problems of Point Sets and Arrangements of Objects (ComPoSe), PI Oswin Aichholzer. The emphasis of the subproject there is on structural properties of arrangements of convex bodies. The questions addressed in ComPoSe are purely combinatorial and focus on properties of *geometric set systems* and *geometric order types* so that there is no overlap with problems addressed in this IP.

János Pach is currently involved in the OTKA project entitled *"Discrete Geometry"* (2007–2011). This is a joint project with nine co-PIs sharing an overall budget of 10,000 euros per year.

János Pach is also involved in two three-years grants (2008–2011) in the US, from NSF and from NSA, entitled *"Geometric Arrangements and Their Algorithmic Applications"* (with Richard Pollack and Micha Sharir as co-PIs) and *"Geometric Graph Theory"*. The budgets of these grants are $ 550,000 and $ 130,000, respectively, and they fund several Ph.D. students and postdoctoral researchers. All three of the above grants run out at the end of the summer 2011.

In addition, János Pach is the recipient of a three years grant from the Swiss SNF (2009-2012), entitled *"Intersection Patterns of Geometric Objects"*. The budget supports a Ph.D. student and a postdoctoral researcher working at EPFL for 120,000 Sfr/year.

Overview of Other Ongoing International Scientific Relationships

Apart from several members of the current project, we are collaborating with a number of other individuals and teams across the world. Our most frequent collaborators are Micha Sharir (Tel Aviv), Rom Pinchasi (Haifa), Jacob Fox (Cambridge), Adrian Dumitrescu (Milwaukee), Radoš Radoičić (New York), and József Solymosi (Vancouver). The covered topics include problems in combinatorial and computational geometry, as well as graph and hypergraph theory.

Section B5 – Individual Project IP5-PL

Individual project's contribution to the CRP

The Polish team is led by Jarosław Grytczuk at the Jagiellonian University in Kraków. The other members are Tomasz Krawczyk and Piotr Micek. The group is completed by three PhD students and three Master students. The team is responsible for work package WP09 – Coloring graphs with geometric representations, which is described below. The team members will also contribute to work packages WP06 and WP10.

WP09 – Coloring graphs with geometric representations Coloring problems constitute the core of combinatorics from the very beginning. They are fascinating for many reasons, ranging from purely aesthetic to practical ones. Coloring problems for geometric structures are most exciting in both respects. Indeed, on one hand we have grand challenges (like determining the chromatic number of the plane), on the other, a variety of topics inspired by real-world applications (like dynamic storage allocation, frequency assignment, VLSI design, etc.).

In this project we are mainly interested in coloring intersection graphs of intervals and their two-dimensional relatives (rectangles, disks, etc.). In a typical problem we are given a family of objects whose members are to be colored so that the intersecting ones get different colors. The task is to find an effective coloring procedure using as few colors as possible.

Basically we distinguish two settings: static (off-line), when the entire structure is known in advance, and dynamic (on-line), when the objects are coming one by one and should be colored immediately and irrevocably. The latter model corresponds to many real-world applications in which the input is gradually disclosed over time. As an intermediate variant we study coloring games in which the structure is given in advance, but only one of the players cares about minimizing the number of colors. We also plan to investigate other types of colorings in this geometrical vein (e.g. game colorings, nonrepetitive colorings, etc.).

One direction of our research focuses on a general question: how much beneficial it is to use geometric representation, rather than just an abstract graph, in a coloring algorithm? For instance, it is known that in the on-line coloring of unit interval graphs one needs at least $2\omega - 1$ colors in the abstract case (which is best possible [6]), while currently best lower bound in the geometric case is $\frac{3}{2}\omega$ (where ω denotes the clique number). We aim at closing this gap. Similar dissonance is expected for bounded tolerance graphs. Since these graphs are complements of comparability graphs [7], we may use here on-line chain partitioning algorithms for posets [4]. By the recent progress in this area made by Bosek and Krawczyk [5] we know that $O(\omega^{16 \log \omega})$ colors are sufficient, while the current lower bound is $\binom{\omega+1}{2}$. We feel however that making use of geometric representation one can get here an on-line algorithm using the number of colors polynomial in ω. Recently it was proved that the first-fit algorithm uses $\Theta(\omega)$ colors on p-tolerance graphs [8], which are a subclass of bounded tolerance graphs.

There are many ways of generalizing intervals from the real line to higher dimensions. For most of them the coloring problems become much harder even in the off-line setting. Consider for instance the class of intersection graphs of line segments in the plane. It is not known if the chromatic number of these graphs is bounded in terms of the clique number (cf. [10]). The situation is not so hopeless for box graphs (intersection graphs of axis-parallel rectangles), though there is no progress on the half-century-old upper bound of $O(\omega^2)$ due to Asplund and Grünbaum [2] (and nothing better than $\Omega(\omega)$ is known from below). Restricting to translated or homothetic copies of one particular figure allows for linear upper bounds on the chromatic number ($3\omega - 2$ in the former case, and $6\omega - 6$ in the later), as proved in [9]. Moreover, these bounds are valid in a stronger sense, where the chromatic number is replaced by the coloring number (the smallest k for which a graph is $(k-1)$-degenerate). These bounds are known to be sharp only for unit disks. We aim at improving them for other classes, like general disk graphs, square graphs, etc. We also hope for obtaining (close to) optimal on-line algorithms for some of these classes, including general box graphs.

Aside from attacking classical problems, we would like to introduce some new topics into the area. One of them is the graph coloring game [3], where two players color a given graph (or its geometric representation) together, but only one of them is interested in minimizing the number of colors used (the other one tries to maximize it). The value of the game when both players play perfectly is known as the game chromatic number. The game was originally proposed for planar graphs, and, despite extensive efforts, the exact value remains a mystery (for some time it was not even clear if it is finite). We hope to achieve some progress here, perhaps by using appropriate geometric representation of planar graphs.

A related concept that recently gained considerable attention is nonrepetitive graph coloring [1]. In such coloring repetitive paths (of any length) are forbidden (a path is repetitive if its first half looks exactly like the second half). A major challenge in this topic is to decide whether every planar graph is nonrepetitively k-colorable for some absolute constant k.

Milestones of WP09

- M09.1 Improve the bounds for the on-line approximability of the chromatic number for unit interval graphs.
- M09.2 Try to provide a polynomial on-line approximation of the chromatic number for bounded tolerance graphs; analyze the possibilities of on-line approximation for general tolerance graphs.
- M09.3 Provide/improve upper bounds for the chromatic/coloring number in terms of the clique number for various classes of intersection graphs.
- M09.4 Provide a reasonable on-line coloring algorithm for box graphs.
- M09.5 Improve the bounds for the game chromatic number of planar graphs.
- M09.6 Verify whether the nonrepetitive chromatic number of planar graphs is finite.

References for WP09

[1] N. Alon, **J. Grytczuk**, M. Hałuszczak, and O. Riordan. Nonrepetitive colorings of graphs. *Random Struct. Algorithms*, 21(3–4):336–346, 2002.

[2] E. Asplund and B. Grünbaum. On a coloring problem. *Math. Scand.*, 8:181–188, 1960.

[3] T. Bartnicki, J. Grytczuk, H. A. Kierstead, and X. Zhu. The map coloring game. *Amer. Math. Monthly*, 114:793–803, 2007.

[4] B. Bosek, **S. Felsner**, K. Kloch, **T. Krawczyk**, G. Matecki, and **P. Micek**. On-line chain partitions of orders: a survey. Submitted.

[5] B. Bosek and **T. Krawczyk**. A subexponential upper bound for on-line chain partitioning problem. Manuscript.

[6] **S. Felsner**, K. Kloch, G. Matecki, and **P. Micek**. On-line chain partitions of up-growing semi-orders. Submitted.

[7] M. C. Golumbic and C. L. Monma. A generalization of interval graphs with tolerances. In: *Proc. Southeastern Conf. Combinatorics, Graph Theory & Comput.*, vol. 35, p. 321–331, 1982.

[8] H. A. Kierstead and K. R. Saoub. First-fit coloring of bounded tolerance graphs. Manuscript.

[9] S.-J. Kim, A. Kostochka, and K. Nakprasit. On the chromatic number of intersection graphs of convex sets in the plane. *Electr. J. Comb.*, 11(1):paper 52, 2004.

[10] A. Kostochka and J. Nešetřil. Coloring relatives of intervals on the plane, I: chromatic number versus girth. *Europ. J. Combin.*, 19:103–110, 1998.

Deliverables

We will have a weekly seminar on coloring problems for geometric structures (twice per week in the first semester). In the first year of the project, we plan to organize a workshop on on-line algorithms for younger researchers from all interested sites. We will also organize annual meetings devoted to the topic of our work package, gathering all contributors.

Justification of the budget

The budget is planned in the same amounts as it was in the outline proposal. The salary costs are planed for 1 senior researcher (J. Grytczuk part-time 0.5), 2 postdocs (Tomasz Krawczyk, Piotr Micek, each 0.5), 3 PhD students (part-time 0.75 each), and stipends for 3 students. We plan travel costs for all members of the research team to travel to conferences, meetings, tutorials, visits, etc. (including the networking within GraDR). Some consumables and necessary overhead are also complied.

Related projects

The current research of Jarosław Grytczuk is supported by institutional funding from the Polish Ministry of Science and Higher Education through the project *Algorithmic problems in combinatorics on words* (grant N N206 257035; individual project). He is also conducting two PhD projects sponsored by the same institution, namely *Coloring distance graphs on the integers* (grant N N201 271335) and *Game coloring of graphs* (grant N201 2128 33).

The Polish Ministry of Science and Higher Education also supports the team project *On-line algorithms and combinatorial games* (grant N N206 492338) led by Piotr Micek. Team members Tomasz Krawczyk and Bartosz Walczak are contributing.

Overview of other ongoing international scientific relationships

Jarosław Grytczuk collaborates frequently with Noga Alon (Tel Aviv University), Boštjan Brešar (University of Maribor), Hal Kierstead (Arizona State University), Sandi Klavžar (University of Ljubljana), and Xuding Zhu (Natinal Sun Yat-sen University). The covered topics include nonrepetitive colorings of graphs, game chromatic number, and on-line Ramsey theory.

Section C1 – Associated Project AP1-IT

Associated project's contribution to the CRP

a) Aims and Objectives. The Rome team is led by Prof. Giuseppe Di Battista and includes Patrizio Angelini (postdoc), Fabrizio Frati (postdoc), and Maurizio Patrignani (Assistant Prof.) of the Roma Tre University, and Anna Galluccio (Senior Researcher) of the IASI Institute of the Italian National Research Council (CNR). The CV data of Angelini [6], Frati [7], Galluccio [8], and Patrignani [9], are available on the Web. We will lead WP05 and contribute to WP01, WP02, WP03, WP08, and WP09.

WP05 – Clustered planarity A *clustered graph* (G, T) is a graph G together with a set T of *clusters*, that are recursively nested sets of vertices of G. Clustered graphs are widely used in applications where it is needed at the same time to represent relationships between entities and to group entities with semantic affinities. For example, in the Internet network, links among routers give rise to a graph; geographically close routers are grouped into areas, which in turn are grouped into Autonomous Systems. Clustered graphs can also be effectively applied to represent graphs at different levels of abstractions and to make easy the navigation of large graphs.

Visualizing clustered graphs turns out to be a difficult problem, due to the simultaneous need for a readable drawing of the underlying structure and for a good rendering of the recursive clustering relationship. As for the visualization of graphs, the most important aesthetic criterion for a drawing of a clustered graph to be "nice" is commonly regarded to be the *planarity*. A drawing of a clustered graph (G, T) is c-*planar* if the drawing of G is planar and each cluster is represented by a simple closed region that contains all and only the vertices of the cluster, that does not intersect other clusters, and that intersects each edge at most once.

We are interested in the complexity of the c-*planarity problem*, that is, given a clustered graph, does it admit a c-planar drawing? The problem has unknown complexity and is one of the most studied problems in the graph drawing community during the last ten years, see, e.g. [1–8].

Although the problem is known to be linear when each cluster induces a connected subgraph (c-*connected clustered graphs*) [3], no efficient algorithm is known for drawing non-c-connected clustered graphs. On one side, for several special families of non-c-connected clustered graphs the problem turned out to be polynomial [8, 4, 7, 6]. On the other side, the problem of efficiently computing a c-planar drawing seems to be elusive even in the case of clustered graphs where the underlying graph has a fixed embedding and only clusters' regions need to be computed.

Milestones of WP05

- M05.1 Identify classes of clustered graphs for which the problem can be efficiently solved.
- M05.2 Establish relationships between the problem and other problems known to be difficult.
- M05.3 Determine geometric representations of c-planar clustered graphs. That is, assuming that a clustered graph admits a c-planar drawing, find one of such drawings with straight-line edges, with good resolution, with clusters represented as convex polygons, etc.

References for WP05

[1] M. Chimani, **C. Gutwenger**, M. Jansen, **K. Klein**, and **P. Mutzel**. Computing maximum c-planar subgraphs. In: *Proc. Graph Drawing (GD'08)*, LNCS 5417:114–120. Springer, 2009.

[2] S. Cornelsen and **D. Wagner**. Completely connected clustered graphs. *J. Discrete Algorithms*, 4(2):313–323, 2006.

[3] P. F. Cortese, **G. Di Battista**, **F. Frati**, **M. Patrignani**, and M. Pizzonia. C-planarity of c-connected clustered graphs. *J. Graph Algorithms Appl.*, 12(2):225–262, 2008.

[4] P.F. Cortese, **G. Di Battista**, **M. Patrignani**, and M. Pizzonia. Clustering cycles into cycles of clusters. *J. Graph Algorithms Appl.*, 9(3):391–413, 2005.

[6] http://www.dia.uniroma3.it/~compunet/www/view/person.php?id=angelini

[7] http://www.dia.uniroma3.it/~compunet/www/view/person.php?id=frati

[8] http://www.iasi.cnr.it/new/people.php/id_subject/13

[9] http://www.dia.uniroma3.it/~compunet/www/view/person.php?id=titto

[5] P. Eades, Q. Feng, and H. Nagamochi. Drawing clustered graphs on an orthogonal grid. *J. Graph Algorithms Appl.*, 3(4):3–29, 1999.

[6] E. Jelínková, J. Kára, **J. Kratochvíl**, **M. Pergel**, O. Suchý, and T. Vyskocil. Clustered planarity: Small clusters in cycles and Eulerian graphs. *J. Graph Algorithms Appl.*, 13(3):379–422, 2009.

[7] M.T. Goodrich, G.S. Lueker, and J.Z. Sun. C-Planarity of Extrovert Clustered Graphs. In: *Proc. Graph Drawing (GD'05)*, LNCS 3843:211–222. Springer, 2006.

[8] **C. Gutwenger**, M. Jünger, S. Leipert, **P. Mutzel**, M. Percan, and R. Weiskircher. Advances in c-planarity testing of clustered graphs. In: *Proc. Graph Drawing (GD'02)*, LNCS 2528:220–235. Springer, 2002.

b) Methodologies/Experiments. Possible lines of attack to the clustered planarity problem include determining significant classes of clustered graphs for which the problem can be efficiently solved or establishing relationships between the problem and other problems known to be difficult.

A promising perspective is the one of exploring the relationship between clustered planarity and *greedy drawings*, where between any two vertices a path exists such that the Euclidean distance from an intermediate vertex to the destination decreases at every step. Such a relationship is expecially evident for Hamiltonian graphs. This creates interesting connections with WP09, that studies combinatorial aspects of geometric representations of graphs.

Another problem that is related to clustered planarity is the *simultaneous embedding with fixed edges* problem, where two or more planar graphs on the same set of vertices are given and a planar drawing of each graph is requested such that a vertex has the same coordinates in all the drawings, edges are Jordan curves, and common edges are represented by the same curve. Determining the computational complexity of the simultaneous embedding with fixed edges problem could result in better understanding the computational complexity of clustered planarity.

Contribution to WP01. Graphs are commonly used to represent intersection of prescribed objects on the plane. These graphs are called intersection graphs and among them a special role is played by interval graphs and chordal graphs. Both families of graphs are widely studied and a number of optimization problems have been solved efficiently on these classes. We intend to generalize the class of interval graphs using new concepts of intervals such as fuzzy intervals, 2-intervals and balanced 2-intervals. We study the structural properties of these new classes of intersection graphs and attempt to decompose them into smaller pieces in analogy with the recent decomposition for claw-free graphs of Chudnovsky and Seymour. Once we obtain a treatable decomposition for these graphs we could consider the computational complexity of classical graph optimization problems such as the maximum weight stable set or the dominating set on the building blocks of the decomposition and aim to develop efficient algorithms for these problems.

Contribution to WP02. In this WP we plan to investigate a drawing convention called *monotone straight-line*, where we require for each pair of vertices that there exists a direction d such that the drawing contains a path that connects the vertices and is increasing in d. The target is to identify classes of graphs admitting monotone straight-line drawings and investigate their resource requirements (e.g. area requirements).

Contribution to WP03 We will study the *simultaneous embedding* problem, where two or more planar graphs on the same set of vertices are given and a planar straight-line drawing of each graph is requested such that a vertex has the same coordinates in all the drawings. The target is to identify classes of graphs admitting a simultaneous embedding.

Contribution to WP04 We will further improve our knowledge on constrained embeddings of planar graphs when part of the embedding is provided in advance. We also plan to relate this problem to other constrained planarity problems.

Contribution to WP08 We will study representations of planar graphs that are highly non-planar, like the queue and the stack layouts. A *queue* (*stack*) layout on k queues (stacks) of a graph $G(V, E)$ is a total ordering of V and a partition of E into k sets $E_1, E_2, \ldots, E_k$, such that no two edges in the same queue (stack) nest (cross). Edges (u, v) and (w, z) *nest* (*cross*) if $u < w < z < v$ or $w < u < v < z$ ($u < w < v < z$ or $w < u < z < v$). We will try to establish which is the amount of queues (stacks) that are needed to construct a queue (stack) layout of a planar graph. The target is to determine the number of queues (stacks) that are needed to construct a queue (stack) layout of specific families of planar graphs.

Contribution to WP09 We will study combinatorial aspects of several geometric representations of graphs. Namely, we will study *minimum spanning tree* representations, where the drawing of a tree coincides with the Euclidean minimum spanning tree of its vertices. The objective is to determine the area requirements of minimum spanning tree representations.

c) Work Plan. Regarding WP05, we will first outline constrained planarity problems that could be formulated as special cases of clustered planarity. This classification will allow to identify further classes of clustered graphs on which we will focus the search for polynomial planarity algorithms (or a hardness proof). With respect to WP02 we will device algorithms to build monotone straight-line drawings of classes of graphs with increasing complexity (such as trees, planar biconnected, planar simply-connected, etc). This same approach will be applied to computing simultaneous embeddings of graphs (WP03) where we will first focus on the instances featuring strong connectivity properties. We will continue the collaboration on partially embedded graphs (WP04) with the teams IP1-CZ and AP3-DE led by Kratochvíl and Wagner, respectively.

d) Deliverables

In addition to contributing to the central deliverables of the entire CRP, we will organize the Bertinoro seminar “Visualization of Large Graphs” (WP05) during 2011.

Information on funding

To participate in the CRP, we plan to use three sources of funding: (a) annual funding provided by the Università Roma Tre for research groups (confirmed for 2010); (b) funding provided by the IASI CNR (confirmed for 2010); and (c) part of the funding provided by the AlgoDEEP Project no. 2008TFBWL4 of the MIUR (confirmed).

Section C2 – Associated Project AP2-NL

Associated project's contribution to the CRP

a) Aims and Objectives This project coordinates work package WP07 – Region constrained graph drawing and is lead by Bettina Speckmann. It further includes two PhD students, Kevin Verbeek and Wouter Meulemans. We plan to also contribute to work packages WP02 – Angular schematization, WP03 – Simultaneous embeddings, WP11 – Transfer to practice, and WP12 – Hypergraphs with applications in bioinformatics.

WP07 – Region constrained graph drawing Highway, train, and river networks, airline and VLSI routing maps, information flow over the internet, and the flow of goods and people between different regions all have one thing in common: they can be effectively visualized as a region constrained graph – a graph, whose embedding is fixed, but not completely. The vertices of a region constrained graph are restricted to lie in a prescribed region while the edges might or might not be required to follow a particular channel. The edges of a region constrained graph can also carry weights – for example, the flow along that edge – which can be visualized by drawing the graph with thick edges.

Consider, for example, a subway map. The location of the stations and the lines is determined by their physical locations – but not completely. Stations can be moved but preferably not too much so that they will still be correctly identified by the user. The lines do not need to exactly follow the laid tracks, since the user cares mostly about the connectivity of the network. So we can draw the subway network in a simplified way that aligns stations and draws lines with only a few orientations. The input geometry of the network is disturbed, but the clarity of the drawing is greatly improved.

Consider, as a second example, a flow map. Such a map is a drawing of a region constrained graph where the vertices are the regions of a geographic map. They are connected by a directed edge if there is a flow between them. Each edge has a weight, which corresponds to the flow on this edge. Each node is restricted to be drawn somewhere in its corresponding region. There is no direct routing prescription for the edges, but they need to be drawn thickly – proportional to their weight – and with few bends. Furthermore, they should preserve the readability of the underlying map, that is, they should not obscure important features. Finding efficient drawing algorithms for these types of region constrained graphs is a challenging open problem. The geometric restrictions on the location of the nodes and edges of a region constrained graph usually stem from an actual physical map, such as a geographic map. A good drawing of a region constrained graph might therefore also have to avoid obstacles, such as important features of a map or pins on a VLSI chip.

Classical graph drawing algorithms pay attention only to the connectivity of the input graph. They strive to create a clear drawing of the given graph and determining the location of the vertices often constitutes the bulk of the problem. By contrast, algorithms that draw region constrained graphs know – up to a region – the location of all vertices and need to work under this restriction to create as good a drawing as possible, also taking additional constraints on the edges into account. Drawing region constrained graphs is a fairly new and relatively unexplored area at the intersection of graph drawing, computational geometry, and automated cartography. We plan to explore fundamental theoretical questions and, at the same time, develop efficient tools that generate high-quality flow maps.

Maximizing angular resolution. The angular resolution of a drawing is the minimum angle formed by any pair of adjacent incident edges. If the locations of the vertices are known and the edges are drawn as straight line segments, then the angular resolution is fixed. If we can place the vertices at will, then a placement that maximizes the resolution can be found for many classes of graphs (see [3] and [5]). Gutwenger and Mutzel [4] show how to maximize angular resolution for planar graphs when drawing on a grid and with a bounded number of bends. However, nothing is known for the case that each vertex is restricted to lie in a particular region, say a unit square.

Avoiding obstacles. This question is motivated by the specific drawings of region constrained graphs that form the overlay of a flow map. A good drawing should avoid important features of the underlying map. However, these constraints are somewhat softer than similar questions that arise in VLSI design: we cannot place a wire on an obstacle, but we can (and often have to) cross region boundaries or bodies of water.

Drawing crossings clearly. Many region constrained graphs do not have planar drawings. Hence it is important to draw the crossings clearly. There is some very recent work on drawing graphs with right angle crossings (see for example [1]). It would be very interesting to see how to extend these results to regions constrained graphs (this will most likely require bends in the edges).

Drawing with fat edges. This problem is also motivated by flow maps. Duncan et al. [2] describe algorithms to draw planar graphs with edges of prescribed thickness while maximizing the distance between edges. Speckmann and Verbeek [7] study the NP-hard problem of finding non-crossing thick minimum-link rectilinear paths which are homotopic to a set of input paths in an environment with rectangular obstacles. Can these results be extended to arbitrary graphs? Can one incorporate obstacle avoidance of large obstacles (coast lines)? How about drawing with curved arcs as it is often seen in atlases? Phan et al. [6] show how to bundle flows that follow the same direction. Unfortunately their algorithm often introduces large local distortions, ignores the geometry of the underlying map, and does not deal with crossings.

Milestones of WP07

- M07.1 Study the complexity of crossing minimization on simple region constrained graphs such as matchings. If possible extend these results to test for planarity of general region constrained graphs.
- M07.2 Try to extend the concept of right angle crossing drawings to region constrained graphs.
- M07.3 Develop an algorithm that intuitively groups and joins flows into a single flow tree without introducing intersections.
- M07.4 Consider multiple flow trees and minimize intersections while maximizing the angular resolution at crossings.
- M07.5 Extend the above algorithm to incorporate obstacles and flow channels (for example along coast lines).
- M07.6 Implement and test the algorithms on real-world data.

References for WP07

[1] W. Didimo, P. Eades, and G. Liotta. Drawing Graphs with Right Angle Crossings. In: *Proc. 11th Algorithms and Data Structures Symposium (WADS)*, pp. 206–217, LNCS 5664, 2009.

[2] C. A. Duncan, A. Efrat, S. G. Kobourov, and C. Wenk. Drawing with fat edges. *Intern. Journal of Foundations of Computer Science* 17(5):1143–1163, 2006.

[3] M. Formann, T. Hagerup. J. Haralambides, **M. Kaufmann**, F.T. Leighton, A. Simvonis, **E. Welzl**, G. Woeginger. Drawing Graphs in the Plane with High Resolution. In: *Proc. 31st Symposium on Foundations of Computer Science*, pp. 86–95, 1990.

[4] **C. Gutwenger** and **P. Mutzel**. Planar Polyline Drawings with Good Angular Resolution. In: *Proc. 6th International Symposium on Graph Drawing (GD)*, pp. 167–182, LNCS 1547, 1998.

[5] S. Malitz and A. Papakostas. On the Angular Resolution of Planar Graphs *SIAM Journal on Discrete Mathematics* 7(2):172–183, 1994.

[6] D. Phan, L. Xiao, R. Yeh, P. Hanrahan, and T. Winograd. Flow Map Layout. In: *Proc. 2005 IEEE Symposium on Information Visualization (INFOVIS)*, pp. 219–224, 2005.

[7] **B. Speckmann** and **K. Verbeek**. Homotopic Rectilinear Routing with Few Links. In: *Proc. 9th Latin American Theoretical Informatics Symposium (LATIN)*, pp. 468–479, LNCS 6034, 2010.

b) Methodologies/Experiments We plan to study algorithmic questions related to the drawing of region constrained graphs. For each of these questions we would either like to find polynomial time algorithms – at least for interesting special cases – or we would like to establish that they are NP-hard. We will also investigate approximate or fixed-parameter tractable solutions.

c) Work Plan In the beginning, we plan to intensify our collaboration with the team IP4-HU on the more fundamental theoretical questions such as crossing numbers of region constrained graphs. The immense expertise of this team should help us to quickly obtain results. Metro maps and other schematic maps are in essence region constrained graphs. Hence some of our questions are related to the topics considered in WP02 – Angular schematization and we are planning a close collaboration with the team of A. Wolff (IP2-DE) as well. Our first two milestones are independent from the others and we hence plan to simultaneously attack both the more theoretical and the more applied questions.

Information on funding

All members of WP07 – Region constrained graph drawing hold positions at TU Eindhoven. The PhD students are funded by the Netherlands Organisation for Scientific Research (NWO) under project no. 639.022.707 “Drawing Geometric Networks”. Kevin Verbeek is funded until Sept. 2012, Wouter Meulemans is funded until Sept. 2014.

Section C3 – Associated Project AP3-DE

Associated project's contribution to the CRP

a) Aims and Objectives This project coordinates the research efforts for work package WP04 – Constrained embeddings. The main researcher is Dorothea Wagner[10]; her CV is included in the appendix. The other members of this project will be the two PhD students Marcus Krug and Ignaz Rutter as well as the postdoc Martin Nöllenburg. We aim to additionally contribute to work packages WP02 – Angular schematization, WP03 – Simultaneous embeddings, and WP05 – Clustered planarity.

WP04 – Constrained embeddings In order to obtain drawings that are intuitively readable, it is desirable to meet the user's expectation concerning the arrangement of objects in a drawing. Choosing relative positions of objects according to the user's expectation, for instance, facilitates localization of objects and their relations. We model this kind of expectation as a set of constraints that restrict the valid embeddings of a given graph and we study the problem of computing feasible embeddings for different kinds of constraints. In the presence of several feasible embeddings we wish to additionally optimize æsthetic criteria to further enhance readability.

We focus on two different types of constraints; *ec-constraints* and *partial embedding constraints*. Ec-constraints restrict the cyclic orderings of edges around vertices in an embedding and thus restrict the relative positions of subgraphs in the drawing. Partial embedding constraints require that a given subgraph is drawn in a prescribed fashion. This facilitates recognition of familiar substructures. The æsthetic criteria we would like to optimize include the depth of the embedding, the size of the outer face and the number of bends in orthogonal drawings.

Ec-constraints have recently been considered by Gutwenger et al. [3]. They show that there is an efficient algorithm for inserting an edge into a planar graph with the minimum number of crossings in the presence of ec-constraints. Angelini et al. [1] present an efficient planarity test for graphs with partial embedding constraints. In particular, this solves the problem of augmenting a given topological planar drawing with additional edges and vertices. In contrast to this, it is NP-hard to decide whether a given planar straight-line drawing can be augmented in this way [6]. Embeddings with minimum block depth and maximum external face have been investigated by Gutwenger and Mutzel [4]. Angelini et al. [2] present an $O(n^4)$ time algorithm for computing a minimum face-depth embedding of a planar graph.

Along this line of research, we plan to investigate various embedding problems subject to ec-constraints or partial embedding constraints for different drawing styles, such as orthogonal drawings and geodesic drawings, which were recently introduced by Katz et al. [5]. Our aim is to determine the complexity status of these problems and, if possible, to devise efficient algorithms for them. Additionally, we will study combinatorial characterizations of solvable instances, in order to provide efficient certifying algorithms that either compute a valid embedding or some kind of forbidden substructure to prove that no valid embedding exists.

Further we will consider the so-called orientation problem for orthogonal graph drawing. Given that we draw vertices as boxes, the orientation problem asks for an orthogonal drawing in which the sides of attachment of edges to vertices are restricted. For instance, this can be used to prescribe the angles between adjacent edges.

[10] http://i11www.iti.uni-karlsruhe.de/Dorothea_Wagner

Milestones of WP04

M04.1 Provide a combinatorial characterization of solvable instances of partially embedded planar graphs.
M04.2 Development of an algorithm for the case that a subgraph of a graph is restricted by ec-constraints.
M04.3 Investigate complexity of orthogonal and Manhattan-geodesic drawing with partial embedding constraints.
M04.4 Is it possible to bound the number of bends per edge in an orthogonal or Manhattan-geodesic drawing in the presence of embedding constraints?
M04.5 What is the complexity of augmenting a partial drawing with few crossings? For instance, is it possible to optimally insert an edge into a partially embedded graph?
M04.6 Solve the orientation problem for orthogonal graph drawing.

References for WP04

[1] P. Angelini, **G. Di Battista**, **F. Frati**, **V. Jelínek**, **J. Kratochvíl**, **M. Patrignani**, and **I. Rutter**. Testing planarity of partially embedded graphs. In: *Proc. ACM-SIAM Symp. Discrete Algorithms (SODA'10)*, p. 202–221, 2010.
[2] P. Angelini, **G. Di Battista**, and **M. Patrignani**. Finding a minimum-depth embedding of a planar graph in $O(n^4)$ time. *Algorithmica*, 2010. doi: 10.1007/s00453-009-9380-6
[3] **C. Gutwenger**, **K. Klein**, and **P. Mutzel**. Planarity testing and optimal edge insertion with embedding constraints. *J. Graph Alg. Appl.*, 12(1):73–95, 2008.
[4] **C. Gutwenger** and **P. Mutzel**. Graph embedding with minimum depth and maximum external face. In: *Proc. Graph Drawing (GD'03)*, LNCS 2912:259–272. Springer, 2004.
[5] B. Katz, **M. Krug**, **I. Rutter**, and **A. Wolff**. Manhattan-geodesic embedding of planar graphs. In: *Proc. Graph Drawing (GD'09)*, LNCS 5894:207–218. Springer, 2010.
[6] **M. Patrignani**. On extending a partial straight-line drawing. *Found. Comput. Sci.*, 17(5):1061–1069, 2006.

b) Methodologies/Experiments We aim to study algorithmic aspects of fundamental constrained embedding problems. We are interested in determining the complexity status of these problems as well as devising efficient exact or approximate algorithms with provable guarantees.

c) Work Plan We will continue the collaboration on partially embedded graphs with the teams IP1-CZ and AP1-IT led by Kratochvíl and Di Battista, respectively. As a first step, we aim to find a combinatorial characterization of solvable instances. Further, we will investigate problems involving ec-constraints in collaboration with P. Mutzel (IP2-DE). Since most of our milestones are not incremental, we will pursue them independently. As the problems we consider are related to topics treated in WP02 – Angular schematization, we plan a close collaboration with the team of A. Wolff (IP2-DE).

Information on funding

All members of AP3-DE hold positions at the Karlsruhe Institute of Technology, which are funded by the federal state of Baden-Württemberg, Germany.

Annex 1

Short CVs of the principal investigators of all IPs and APs.

Jan Kratochvíl, PI of IP1-CZ

Jan Kratochvíl[11] is a full professor of theoretical computer science at Charles University in Prague since 2003. He received his Ph.D. in mathematics also at Charles University in 1987. He was employed as assistant professor and associate professor at Charles University from 1987. He also lectured as a visiting associate professor at University of Oregon in Eugene in 1995 and 1999. He is a deputy director of DIMATIA (Center for Discrete Mathematics, Theoretical Computer Science, and Applications) at Charles University in Prague (since 1996) and the head of Department of Applied Mathematics (from 2003). He is the president of the Czech Mathmetical Society (2002–2010) and chair of the Committee for Support of East European mathematicians of the European Mathematical Society.

The main research area of Jan Kratochvíl is algorithmic graph theory and computational complexity of graph theoretical problems. He has published 2 monographs (one of them about string graphs, i.e., intersection graphs of curves in the plane) and 120 papers in international journals and refereed conference proceedings (his 90 entries in the ISI Web of Science show over 450 citations recorded in this database). He has delivered invited talks at a number of conferences (among others Eurocomb, Sevilla 2007; 6th Slovenian Graph Theory Conference, Bled, 2007; IWOCA 2007, Newcastle; 2nd International Polish Graph Theory Conference, Bedlewo, 2008; Graph Theory at Sandbjerg Manor, C. Thomassen's 60th birthday conference, 2008; New Directions in Algorithms, Combinatorics, and Optimization, Tom Trotter's 65th birthday conference, Atlanta, 2008).

Jan Kratochvíl is a member of editorial boards of the journals *SIAM Journal on Discrete Mathematics*, *Discrete Mathematics and Theoretical Computer Science* (electronic journal), and *Theoretical Computer Science* (Elsevier). He has guest editted nine special volumes of international journals (4 *Discrete Mathematics*, 4 *Discrete Applied Mathematics*, 1 *Theoretical Computer Science*) and was a (co-)editor of four volumes from the Lecture Notes in Computer Science series. He has served in programme committees of many theoretical computer science conferences and (co-)chaired four of them (Graph Drawing 1999, MFCS 2004, IWOCA 2009, TAMC 2010).

Jan Kratochvíl has supervised 5 Ph.D. students (among them P. Hliněný, J. Fiala, D. Král, and M. Pergel).

Stefan Felsner, PI of IP2-DE

Stefan Felsner[12] is professor for Discrete Mathematics at Technische Universität Berlin. After his mathematics studies in Vienna he arrived in Berlin in 1988 to persue a Ph.D. which he received in 1992. The thesis 'Interval Orders: Combinatorial Structure and Algorithms' was awarded a Tiburtius price for outstanding dissertations. With a scholarship funded by the German science foundation (DFG) he went to a PostDoc stay at Bellcore and DIMACS. 1993 he became Wissenschaftlicher Assistent at the Department of Mathematics and Computer Science of Freie Universität Berlin. After the habilitation he became Oberassistent (Associate Professor) and later in 2003 full professor.

He has published a monograph on geometric graphs and arrangements and 60 papers in international journals and refereed conference proceedings, of these 54 are listed in the ISI Web of Science. He has delivered invited talks at several conferences. Since 2002 he is co-chair of the annual conference Kolloquium über Kombinatorik, the main German combinatorics meeting. He organized the 2005 edition of EuroComb (European Conference of Combinatorics, Graph Theory and Applications). He is a member of editorial boards of the international journals Order and DMTCS – Discrete Mathematics and Theoretical Computer Science. He is a member of Research

[11] `http://kam.mff.cuni.cz/~honza/`

[12] `http://www.math.tu-berlin.de/~felsner`

Training Group “Methods for Discrete Structures” and of “Berlin Mathematical School” (a Graduate School in the framework of the German “Excellence Initiative”). He supervised 4 Ph.D. students.

Michael Hoffmann, PI of IP3-CH

Michael Hoffmann[13] is senior researcher and lecturer at ETH Zürich, Switzerland, since 2005. His research interests are in algorithms and data structures, in particular, for graphs, discrete and computational geometry, software design and combinatorial games.

Born May 6th, 1970, in Berlin, Germany; 1989–1996 Diploma in Mathematics at FU Berlin, Germany (Thesis: “Line-sweep auf einem Gitter”, advisors: Emo Welzl and Helmut Alt (co-referee)); 2000–2005 Ph.D. at ETH Zürich, Switzerland (Thesis: “On the existence of paths and cycles”, advisors: Emo Welzl and Erik Demaine (co-referee)).

Some 25 papers in journals and refereed conference proceedings; program committee chair of the 27th European Workshop on Computational Geometry (EuroCG) 2011; program committee member of the 16th European Symposium on Algorithms (ESA 2008—Engineering and Applications Track); member and current Board Manager of the CGAL[14] editorial board.

János Pach, PI of IP4-HU

János Pach[15] is with the Rényi Institute of Mathematics of the Hungarian Academy of Science, where he became Research Associate in 1977, Senior Research Fellow in 1986, and Research Adviser in 1995. He has been Chair of Combinatorial Geometry at the Ecole Polytechnique Fédérale de Lausanne (EPFL) since 2008. Before, he was with the City College, New York (CUNY), where he became a full professor in 1992 and a Distinguished Professor of Computer Science in 2004. He had visiting positions at the University College London, McGill University Montreal, at the DIMACS Center for Discrete Mathematics, at Tel Aviv University, at the Hebrew University, at the Ecole des Hautes Etudes en Sciences Sociales, Paris and at the Mathematical Sciences Research Institute, Berkeley.

Pach received the Grünwald Medal of the J. Bolyai Mathematical Society in 1982, the Young Researchers’ Award of the Hungarian Academy of Sciences in 1984, the Lester R. Ford Award of the Mathematical Association of America, in 1990, the Rényi Prize of the Mathematical Institute of the Hungarian Academy of Sciences in 1993 and the Academy Award of the Hungarian Academy of Sciences in 1998. He has given numerous invited talks and lectures.

Pach is co-editor-in-chief of the journal *Discrete and Computational Geometry*. He is in the editorial boards of *Combinatorica*, *Computational Geometry: Theory and Applications*, *Geombinatorics Quarterly*, *Graphs and Combinatorics*, *SIAM Journal of Discrete Mathematics*, *Applied Mathematics Research*, *International Journal of Computer Mathematics*, and *Central European Journal of Mathematics*. He is in the advisory board of the *Handbook of Discrete and Computational Geometry* (CRC Press).

Pach has co-edited seven and co-authored three books.

Jarosław Grytczuk, PI of IP5-PL

Jarosław Grytczuk[16] is a full professor at the Faculty of Mathematics and Computer Science of Jagiellonian University in Kraków, since 2007. He received his Ph.D. and habilitation degrees in mathematics from Adam Mickiewicz University, Poznań, in 1996 and 2006, respectively. His research interests lie in algorithmic graph theory, combinatorics on words, discrete geometry, and number theory. He presented invited talks at several international conferences and colloquia (e.g., at Cycles and Colourings, Stara Lesna, Slovakia 2008; 2nd Polish Combinatorial Conference, Będlewo, Poland 2008; Workshop on Graph Theory, Kaohsiung, Taiwan 2009; Google tech talk,

[13] `http://www.inf.ethz.ch/~hoffmann`
[14] Computational Geometry Algorithms Library, cf. `http://www.cgal.org/`
[15] `http://www.renyi.hu/~pach/`
[16] `http://tcs.uj.edu.pl/Grytczuk`

Google, Kraków, Poland, 2009 (invited lecture, available on YouTube)). He successfully managed four research grants of the Polish Ministry of Science and Higher Education and an international bilateral Polish-Slovenian project on *Graph invariants and graph products* in 2004/2005.

Guiseppe di Battista, PI of AP1-IT

Giuseppe Di Battista[17] is a Professor of Computer Science at Universita Roma 3. He received a Ph.D. in Computer Science from the University of Rome "La Sapienza". His current research interests include Graph Drawing and Information Visualization. He has published more than 100 papers in the above areas and has given several invited lectures worldwide. He is one of the authors of a book[18] by Prentice Hall on Graph Drawing. He served and chaired program committees of international symposia and is editor and guest editor of international journals. He is a founding member of the steering committee for the Graph Drawing Symposium. His research has been funded by the Italian National Research Council, by the EU, and by several industrial sponsors like Cabletron Systems, Enterasys, CM Sistemi, Finsiel, Integra Sistemi, and Sysdata. He has been national and/or local coordinator of many Projects of Relevant Italian National Interest (PRIN) of the MIUR.

Bettina Speckmann, PI of AP2-NL

Bettina Speckmann[19] is an associate professor at the department of mathematics and computer science of the Technical University of Eindhoven (the Netherlands). She received her diploma degree in mathematics from WWU Münster (Germany) in 1996 and her PhD in computer science from the University of British Columbia (Canada) in 2001. She spent two years as a postdoc at the Institute for Theoretical Computer Science of ETH Zurich (Switzerland) and was an assistant professor at TU Eindhoven from 2003 until 2008.

Bettina Speckmann's research interests include the design and analysis of algorithms and data structures, discrete and computational geometry, applications of computational geometry to geographic information systems, and graph drawing. She has published more than 60 papers in international journals and refereed conference proceedings. She has served on numerous program committees, was co-organizer of EuroCG 2005, and will be PC co-chair and co-organizer of Graph Drawing 2011. She currently holds a prestigious VIDI grant of the Netherlands Organization for Scientific Research and is member of the steering and management committee of a European COST action. In March 2010 she became a member of The Young Academy of the Royal Netherlands Academy of Arts and Sciences.

Dorothea Wagner, PI of AP3-DE

Dorothea Wagner[20] is a full professor for Informatics at the Karlsruhe Institute of Technology (KIT). Her research interests include design and analysis of algorithms and algorithm engineering, graph algorithms, computational geometry and discrete optimization, particularly applied to transportation systems, network analysis, data mining and visualization.

Among other activities she is vice president of the DFG (Deutsche Forschungsgemeinschaft – German Research Foundation) and speaker of the scientific advisory board of Dagstuhl – Leibniz Center for Informatics, member of the Standing Committee for Physical and Engineering Sciences (PESC) of the European Science Foundation (ESF) and Fellow of the Gesellschaft für Informatik (GI). She was coordinator of the DFG priority program "Algorithmics of Large and Complex Networks", member of the DFG research training group "Self-organizing Sensor-Actuator-Networks" and one of the initiators of the DFG priority program "Algorithm Engineering", coordinator of the EU RTN AMORE, and participant of the EU projects COSIN, DELIS, CREEN, and

[17] http://www.dia.uniroma3.it/people/gdb
[18] http://www.cs.brown.edu/people/rt/gdbook.html
[19] http://www.win.tue.nl/~speckman
[20] http://i11www.iti.uni-karlsruhe.de/en/members/Dorothea_Wagner/

ARRIVAL. She is editor-in-chief of the *Journal on Discrete Algorithms* and member of the editorial boards of *Journal of Graph Algorithms and Applications*, *Computational Geometry: Theory and Applications*, *Electronic Proceedings in Theoretical Computer Science*, and *Leitfäden der Informatik* (Teubner-Verlag).

Dorothea Wagner obtained her diploma and Ph.D. degrees in mathematics from the RWTH Aachen in 1983 and 1986, respectively, and her habilitation degree in 1992 from the TU Berlin. She was full professor for Computer Science at Universität Konstanz during the years 1994–2003.

Annex 2

A list of the ten most relevant publications of each PI and AP during the last five years.

List of selected publications for IP1-CZ

[1] V. Jelínek, E. Jelínková, J. Kratochvíl, B. Lidický, M. Tesař, and T. Vyskočil. The Planar Slope Number of Planar Partial 3-Trees of Bounded Degree. In: *Proc. Graph Drawing (GD'09)*, LNCS 5849:304–315. Springer, 2010.

[2] M.D. Coury, P. Hell, J. Kratochvíl, and T. Vyskočil. Faithful Representations of Graphs by Islands in the Extended Grid. In: *Proc. LATIN'10*, LNCS 6034:131–142. Springer 2010.

[3] P. Angelini, G. Di Battista, F. Frati, V. Jelínek, J. Kratochvíl, M. Patrignani, and I. Rutter. Testing planarity of partially embedded graphs. In: *Proc. ACM-SIAM Symp. Discrete Algorithms (SODA'10)*, p. 202–221. 2010.

[4] X. Goaoc, J. Kratochvíl, Y. Okamoto. Ch.-S. Shin, A. Spillner, and A. Wolff. Untangling a Planar Graph. *Discr. Comput. Geometry*, 42:542–569, 2009.

[5] V. Jelínek, E. Jelínková, J. Kratochvíl, and B. Lidický. Clustered planarity: Embedded clustered graphs with two-component clusters. In: *Proc. Graph Drawing (GD'08)*, LNCS 5417:121–132. Springer 2009.

[6] M.C. Doirado, J.G. Gimbel, J. Kratochvíl, F. Protti, and J.L. Szwarcfiter. On the computation of the hull number of a graph. *Discr. Math.*, 309:5668–5674, 2009.

[7] J. Fiala and J. Kratochvíl. Locally constrained graph homomorphisms – structure, complexity, and applications. *Comput. Sci. Review*, 2(2):97–111, 2008.

[8] E. Jelínková, J. Kára, J. Kratochvíl, M. Pergel, O. Suchý, and T. Vyskočil. Clustered planarity: Small clusters in Eulerian graphs. In: *Proc. Graph Drawing (GD'07)*, LNCS 4875:303–314. Springer, 2008.

[9] J. Kratochvíl and M. Pergel. Geometric intersection graphs: Do short cycles help? In: *Proc. Conf. Comput. Combinatorics (COCOON'07)*, LNCS 4598:118-128. Springer, 2007.

[10] H. Broersma, F.V. Fomin, J. Kratochvíl, and G. Woeginger. Planar graph coloring avoiding monochromatic subgraphs: Trees and paths make it difficult. *Algorithmica*, 44:343–361, 2006.

List of selected publications for IP2-DE

[1] Melanie Badent, Carla Binucci, Emilio Di Giacomo, Walter Didimo, Stefan Felsner, Francesco Giordano, Jan Kratochvíl, Pietro Palladino, Maurizio Patrignani, and Francesco Trotta. Homothetic triangle contact representations of planar graphs. In *Proc 19th An. Can. Conf. on Comp. Geom., CCCG 2007*, pages 233–236, 2007.

[2] Nicolas Bonichon, Stefan Felsner, and Mohamed Mosbah. Convex drawings of 3-connected planar graphs. *Algorithmica*, 47:399–420, 2007.

[3] Stefan Felsner. Convex drawings of planar graphs and the order dimension of 3-polytopes. *Order*, 18:19–37, 2001.

[4] Stefan Felsner. Geodesic embeddings and planar graphs. *Order*, 20:135–150, 2003.

[5] Stefan Felsner. Geometric Graphs and Arrangements. Advanced Lectures in Mathematics. Vieweg Verlag, 2004.

[6] Stefan Felsner. Lattice structures from planar graphs. *Electronic Journal of Combinatorics*, 11(R15):24p., 2004.

[7] Stefan Felsner, Bartołomiej Bosek, Kamil Kloch, Tomasz Krawczyk, Grzegorz Matecki, and Piotr Micek. On-line chain partitions of orders: A survey, 2010. submitted to Order.

[8] Stefan Felsner, Guiseppe Liotta, and Stephen Wismath. Straight-line drawings on restricted integer grids in two and three dimensions. *Journal of Graph Algorithms and Applications*, 7:363–398, 2003.

[9] Stefan Felsner and Mareike Massow. Parameters of bar k-visibility graphs. *Journal of Graph Algorithms and Applications*, 12:5–27, 2008.

[10] Stefan Felsner and Florian Zickfeld. Schnyder woods and orthogonal surfaces. *Discrete & Computational Geometry*, 40:103–126, 2008.

List of selected publications for IP3-CH

[1] M. Al-Jubeh, M. Hoffmann, M. Ishaque, D. L. Souvaine, and C. D. Tóth. Convex partitions with 2-edge connected dual graphs. *J. Combin. Optimization*, 2010, to appear.

[2] T. Christ, M. Hoffmann, Y. Okamoto, and T. Uno. Improved bounds for wireless localization. *Algorithmica*, 57(3):499–516, 2010.

[3] A. Francke and M. Hoffmann. The Euclidean degree-4 minimum spanning tree problem is NP-hard. In *Proc. 25th Annu. Sympos. Comput. Geom.*, pages 179–188, 2009.

[4] M. Hoffmann, B. Speckmann, and C. D. Tóth. Pointed binary encompassing trees: Simple and optimal. *Comput. Geom. Theory Appl.*, 43(1):35–41, 2010.

[5] O. Aichholzer, T. Hackl, M. Hoffmann, C. Huemer, A. Pór, F. Santos, B. Speckmann, and B. Vogtenhuber. Maximizing maximal angles for plane straight line graphs. In *Proc. 10th Workshop Algorithms Data Struct.*, volume 4619 of *Lecture Notes Comput. Sci.*, pages 458–469. Springer-Verlag, 2007.

[6] O. Aichholzer, T. Hackl, M. Hoffmann, A. Pilz, G. Rote, B. Speckmann, and B. Vogtenhuber. Plane graphs with parity constraints. In *Proc. 11th Algorithms and Data Struct. Sympos.*, volume 5664 of *Lecture Notes Comput. Sci.*, pages 13–24. Springer-Verlag, 2009.

[7] S. Hert, M. Hoffmann, L. Kettner, S. Pion, and M. Seel. An adaptable and extensible geometry kernel. *Comput. Geom. Theory Appl.*, 38(1–2):16–36, 2007.
[8] U. Adamy, M. Hoffmann, J. Solymosi, and M. Stojaković. Coloring octrees. *Theoret. Comput. Sci.*, 363(1):11–17, 2006.
[9] R. Haas and M. Hoffmann. Chordless paths through three vertices. *Theoret. Comput. Sci.*, 351(3):360–371, 2006.
[10] M. Hoffmann and Y. Okamoto. The minimum weight triangulation problem with few inner points. *Comput. Geom. Theory Appl.*, 34(3):149–158, 2006.

List of selected publications for IP4-HU

[1] E. Ackerman, J. Fox, J. Pach, and A. Suk. On grids in topological graphs. In: *ACM Symp. Comput. Geom. (SoCG'09)*, p. 403–412, 2009.
[2] J. Fox and J. Pach. Coloring k_k-free intersection graphs of geometric objects in the plane. In: *ACM Symp. Comput. Geom. (SoCG'08)*, p. 346–354, 2008.
[3] J. Fox, J. Pach, and C.D. Tóth. A bipartite strengthening of the crossing lemma. *J. Combin. Theory Ser. B*, 100:23–35, 2010.
[4] A. Holmsen, J. Pach, and H. Tverberg. Points surrounding the origin. *Combinatorica*, 28:633–644, 2008.
[5] B. Keszegh, J. Pach, D. Pálvölgyi, and G. Tóth. Drawing cubic graphs with at most five slopes. *Comput. Geom. Theory Appl.*, 40:138–147, 2008.
[6] J. Pach, R. Pinchasi, M. Sharir. Solution of Scott's problem on the number of directions determined by a point set in 3-space. *Discrete & Computational Geometry*, 38:399–441, 2007.
[7] J. Pach, J. Solymosi, and G. Tardos. Crossing numbers of imbalanced graphs. *Journal of Graph Theory*, 64:12–21, 2010.
[8] J. Pach, A. Suk, and M. Treml. Tangencies between families of disjoint regions in the plane. In: *ACM Symp. Comput. Geom. (SoCG'10)*, p. 423–428, 2010.
[9] J. Pach and G. Tardos. Conflict-free colourings of graphs and hypergraphs. *Combinatorics, Probability & Computing*, 18:819–834, 2009.
[10] J. Pach and G. Tóth. How many ways can one draw a graph? *Combinatorica*, 26:559–576, 2006.

List of selected publications for IP5-PL

[1] A. Noga, J. Grytczuk, M. Lasoń, and M. Michałek. Splitting necklaces and measurable colorings of the real line. *Proc. Amer. Math. Soc.*, 137:1593–1599, 2009.
[2] T. Bartnicki, J. Grytczuk, and S. Niwczyk. Weight choosability of graphs. *J. Graph Theory*, 60(3):242–256, 2009.
[3] J. Grytczuk, H. A. Kierstead, and P. Prałat. On-line Ramsey numbers for paths and stars. *Discrete Math. Theor. Comput. Sci.*, 10(3):63–74, 2008.
[4] S. Czerwiński and J. Grytczuk. Invisible runners in finite fields. *Inform. Process. Lett.*, 108(2):64–67, 2008.
[5] J. Grytczuk. Thue type problems for graphs, points, and numbers. *Discrete Math.*, 308(19):4419–4429, 2008.
[6] T. Bartnicki, B. Brešar, J. Grytczuk, M. Kovše, Z. Miechowicz, and I. Peterin. Game chromatic number of Cartesian product graphs. *Electron. J. Combin.*, 15(1):paper 72, 13 pages, 2008.
[7] T. Bartnicki and J. Grytczuk. A note on the game chromatic index of graphs. *Graphs Combin.*, 24(2):67–70, 2008.
[8] T. Bartnicki, J. Grytczuk, and H. A. Kierstead. The game of arboricity. *Discrete Math.*, 308(8):1388–1393, 2008.
[9] N. Alon and J. Grytczuk. Breaking the rhythm on graphs. *Discrete Math.*, 308(8):1375–1380, 2008.
[10] T. Bartnicki, J. Grytczuk, H. A. Kierstead, and X. Zhu. The map-coloring game. *Amer. Math. Monthly*, 114(9):793—803, 2007.

List of selected publications for AP1-IT

[1] P. Angelini, G. Di Battista, F. Frati, V. Jelinek, J. Kratochvíl, M. Patrignani, and I. Rutter. Testing planarity of partially embedded graphs. In: *ACM-SIAM Symp. Discrete Algorithms (SODA '10)*, p. 202–221, 2010.
[2] P. Angelini, G. Di Battista, and M. Patrignani. Finding a minimum-depth embedding of a planar graph in $o(n^4)$ time. *Algorithmica*, 2010. (DOI: 10.1007/s00453-009-9380-6).
[3] P.F. Cortese, G. Di Battista, F. Frati, M. Patrignani, and M. Pizzonia. C-planarity of c-connected clustered graphs. *J. Graph Algorithms Appl.*, 12(2):225–262, 2008.
[4] P.F. Cortese, G. Di Battista, A. Moneta, M. Patrignani, and M. Pizzonia. Topographic visualization of prefix propagation in the Internet. *IEEE Trans. Visual. Comput. Graphics*, 12(5):725–732, 2006.
[5] P.F. Cortese, G. Di Battista, M. Patrignani, and M. Pizzonia. On embedding a cycle in a plane graph. *Discrete Math.*, 309(7):1856–1869, 2009.
[6] G. Di Battista, G. Drovandi, and F. Frati. How to draw a clustered tree. *J. Discrete Algorithms*, 7(4):479–499, 2009.
[7] G. Di Battista, Th. Erlebach, A. Hall, M. Patrignani, M. Pizzonia, and Th. Schank. Computing the types of the relationships between autonomous systems. *IEEE/ACM Trans. Networking*, 15(2):267–280, 2007.
[8] G. Di Battista and F. Frati. Efficient c-planarity testing for embedded flat clustered graphs with small faces. *J. Graph Algorithms Appl.*, 13(3):349–378, 2009. Special issue on selected papers from GD'07.
[9] G. Di Battista, F. Frati, and M. Patrignani. On embedding a graph on the grid with the maximum number of bends and other bad features. *Theory Comput. Syst.*, 44(2):143–149, 2009. Special issue on selected papers from FUN'07.
[10] G. Di Battista, G. Liotta, and S. Whitesides. The strength of weak proximity. *J. Discrete Algorithms*, 4(3):384–400, 2006.

List of selected publications for AP2-NL

[1] M. van Kreveld and B. Speckmann. On Rectangular Cartograms. *Computational Geometry: Theory and Applications* 37(3):175–187, 2007. (Special issue of invited papers from the 20th European Workshop on Computational Geometry.)

[2] D. Eppstein, E. Mumford, B. Speckmann, and K. Verbeek. Area-Universal Rectangular Layouts. In: *Proc. 25th ACM Symposium on Computational Geometry (SoCG)*, pp. 267–276, 2009.

[3] O. Aichholzer, T. Hackl, M. Hoffmann, A. Pilz, G. Rote, B. Speckmann, and B. Vogtenhuber. Plane Graphs with Parity Constraints. In: *Proc. 11th Algorithms and Data Structures Symposium (WADS)*, pp. 13 –24, LNCS 5664, 2009.

[4] M. Kaufmann, M. van Kreveld, and B. Speckmann. Subdivision Drawings of Hypergraphs. In: *Proc. 16th International Symposium on Graph Drawing (GD 08)*, pp. 396–407, LNCS 5417, 2009.

[5] M. de Berg, E. Mumford, and B. Speckmann. On Rectilinear Duals for Vertex-Weighted Plane Graphs. *Discrete Mathematics* 309(7):1794–1812, 2009. (Special issue of invited papers from the 13th International Symposium on Graph Drawing.)

[6] E. Di Giacomo, W. Didimo, M. van Kreveld, G. Liotta, and B. Speckmann. Matched Drawings of Planar Graphs *Journal of Graph Algorithms and Applications* 13(3):423–445, 2009. (Special issue of invited papers from the 15th International Symposium on Graph Drawing.)

[7] S. Cabello, M. van Kreveld, G. Liotta, H. Meijer, B. Speckmann, and K. Verbeek. Geometric Simultaneous Embeddings of a Graph and a Matching. In: *Proc. 17th International Symposium on Graph Drawing (GD 09)*, pp. 183–194, LNCS 5849, 2010.

[8] K. Buchin, M. van Kreveld, H. Meijer, B. Speckmann, and K. Verbeek. On Planar Supports for Hypergraphs. In: *Proc. 17th International Symposium on Graph Drawing (GD 09)*, pp. 345–356, LNCS 5849, 2010.

[9] M. Hoffmann, B. Speckmann, and C.D. Tóth. Pointed Binary Encompassing Trees: Simple and Optimal. *Computational Geometry: Theory and Applications* 43(1):35–41, 2010. (Special issue of invited papers from the 14th Annual Fall Workshop on Computational Geometry.)

[10] W. Meulemans, A. van Renssen, and B. Speckmann. Area-Preserving Subdivision Schematization. In: *Proc. 6th International Conference on Geographic Information Science (GIScience)*, 2010 (to appear).

List of selected publications for AP3-DE

[1] V. Aggarwal, A. Feldmann, M. Gaertler, R. Görke, and D. Wagner. Modelling overlay-underlay correlations using visualization. *Telektronikk*, 104(1):114–125, 2008

[2] P. Angelini, G. Di Battista, F. Frati, V. Jelínek, J. Kratochvíl, M. Patrignani, and I. Rutter. Testing planarity of partially embedded graphs. In: *Proc. ACM-SIAM Symp. Discrete Algorithms (SODA'10)*, p. 202–221, 2010.

[3] M. Bekos, M. Kaufmann, M. Nöllenburg, and A. Symvonis. Boundary labeling with octilinear leaders. *Algorithmica*, doi: 10.1007/s00453-009-9283-6, 2009

[4] R. Bauer, D. Delling, P. Sanders, D. Schieferdecker, D. Schultes, and D. Wagner. Combining hierarchical and goal-directed speed-up techniques for Dijkstra's algorithm. In: *Proc. Workshop on Experimental Algorithms (WEA'07)*, LNCS 5038:303–318. Springer, 2008.

[5] D. Delling, A. Gemsa, M. Nöllenburg, and T. Pajor. Path schematization for route sketches. In: *Proc. Scandinavian Workshop Algorithm Theory (SWAT'10)*, LNCS 6139:285–296. Springer, 2010

[6] D. Delling, R. Görke, C. Schulz, and D. Wagner. ORCA reduction and contraction graph clustering, In: *Proc. Algorithmic Aspects in Information and Management (AAIM'09)*, LNCS 5564:152–165. Springer, 2009.

[7] R. Görke, M. Gaertler, and D. Wagner. LunarVis - Analytic visualizations of large graphs. In: *Proc. Graph Drawing (GD'07)*, LNCS 4875:352–364. Springer, 2008

[8] B. Katz, M. Krug, I. Rutter, and A. Wolff. Manhattan-geodesic embedding of planar graphs. In: *Proc. Graph Drawing (GD'09)*, LNCS 5894:207–218. Springer, 2010.

[9] M. Krug and D. Wagner. Minimizing the area for planar straight-line grid drawings. In: *Proc. Graph Drawing (GD'07)*, LNCS 4875:207–212. Springer, 2008

[10] M. Nöllenburg and A. Wolff. Drawing and labeling high-quality metro maps by mixed-integer programming. *IEEE Transactions on Visualization and Computer Graphics*, preprint accepted for publication, 2010

Annex 3

List of all research grants received in the past five years (optional for APs)

List of grants for IP1-CZ

1. Jan Kratochvíl is the coordinator (PI) of institutional grant project *Modern methods, structures, and systems in computer science* at Charles University (2005–2011, annual budget 1.5 million Euros).
2. Jan Kratochvíl is the deputy director of the national research centre *Institute for Theoretical Computer Science* (1M0545; PI J. Nešetřil).
3. Petr Hliněný leads two Czech Science Foundation grants: GA 201/08/0308 *Structural and Width Parameters in Combinatorics and Algorithmic Complexity*, and bilateral GA 201/09/J021 *Structural Graph Theory and Parameterized Complexity* (with RWTH Aachen, P. Rossmanith). Both the grants end in 2010.
4. In 2006–2008 Jan Kratochvíl was co-PI of the bilateral Czech-U.S. project ME 885 *Graph structures, graph operators and computational complexity*. Participating sites were Charles University in Prague, University of West Bohemia in Pilsen, University of Oregon in Eugene, and University of Washington in Seattle-Tacoma. (PI from the Czech side was Z. Ryjáček from Pilsen.)

List of grants for IP2-DE

1. In the DFG priority program SPP 1307: *Algorithm Engineering*, *Planarisation methods in automatic graph drawing*, since 2007, joint project of Petra Mutzel (TU Dortmund) with Michael Jünger (Cologne University). The specific problems considered in that project are disjunct with the milestones (work plan) for the Eurogiga project.
2. In the BMWi research focus program: *Intelligent logistics in freight and economic traffic*, project: *Dynamic and integrative disposition in piece goods haulage facilities (DISS)*, 2007–2010, Petra Mutzel (TU Dortmund) with Uwe Clausen (TU Dortmund).
3. In the NRW and EU-funded *Centre for Applied Proteomics (CAP)* which was part of the *Life Sciences InnovationPlatform Dortmund*, project *Bioinformatics*, 2006–2008, Petra Mutzel (TU Dortmund) jointly with the Medical Proteom Centre Bochum.
4. In the DFG coordinated program SFB 531: *Design and management of complex technical processes and systems with Computational Intelligence methods*, project B10: *Application of Computational Intelligence in Combinatorial Optimization*, 2006–2008, joint project of Petra Mutzel (TU Dortmund) with Martin Skutella.
5. Michael Kaufmann is supported by the DFG grant KA-812-15 *Graph Drawing for Business Processes*. This project is application-oriented and has no overlap with the proposal.
6. In the DFG SPP 1335 *Scalable Visual Analytics* Michael Kaufmann is involved in the development of the BiNA platform. BiNA will serve as a platform for implementations and applications of our layout algorithms for hypergraphs
7. Stefan Felsner is funded by the DFG individual grant FE-340/7 *Graph orientations in space and in the plane*. The proposal to this project also mentions triangle contact representations. The focus of research conducted in the project is on generation, enumeration, and structural properties of orientations.
8. Stefan Felsner is a member of the Research Training Network "Methods for Discrete Structures" (MDS), which has started in 2006 and is currently under review for an extension until 2015. It involves researchers from the three big universities in Berlin. The topics of IP2 (and of the whole CRP) complement the research in MDS, which has a much broader scope.

List of grants for IP3-CH

M. Hoffmann is currently not being supported by any grants, as far as dedicated funding is concerned. The Zurich team as a whole, however, has a lot of experience with running externally-funded projects. For example, Uli Wagner currently has two projects funded by the Swiss National Science Foundation (NSF); *Combinatorial and Computational Aspects of Embeddings* and *k-Sets and Geometric Graphs*. Topicwise, these projects are disjoint from our CRP, but close enough to allow for cooperation. Bernd Gärtner also has a project with the Swiss NSF; namely about *Support Vector Machines: Geometry, Combinatorics and Algorithms*. This project has nothing to do with the content of our CRP.

List of grants for IP4-HU

1. János Pach is currently involved in the OTKA project entitled *"Discrete Geometry"* (2007–2011). This is a joint project with nine co-PIs sharing an overall budget of 10,000 euros per year.

2. János Pach is involved in two three-years grants (2008–2011) in the US, from NSF and from NSA, entitled *"Geometric Arrangements and Their Algorithmic Applications"* (with Richard Pollack and Micha Sharir as co-PIs) and *"Geometric Graph Theory"*. The budgets of these grants are $ 550,000 and $ 130,000, respectively, and they fund several Ph.D. students and postdoctoral researchers. All three of the above grants run out at the end of the summer 2011.

3. János Pach is the recipient of a three years grant from the Swiss SNF (2009-2012), entitled *"Intersection Patterns of Geometric Objects"*. The budget supports a Ph.D. student and a postdoctoral researcher working at EPFL for 120,000 Sfr/year.

List of grants for IP5-PL

1. Polish–Slovenian Bilateral Cooperation, *Graph invariants and graph products*, 2004–2005 (principal investigator on Polish side).

2. Polish Ministry of Science and Higher Education, *Thue problems for graphs*, 2004–2006, Grant 1 P03A 01727 (individual).

3. Polish Ministry of Science and Higher Education, *Game coloring of graphs*, 2007–2009, Grant N201 212833 (supervisor).

4. Polish Ministry of Science and Higher Education, *Coloring Distance Graphs on the Integers*, 2008–2010, Grant N201 271335 (supervisor).

5. Polish Ministry of Science and Higher Education, *Algorithmic problems in Combinatoics on Words*, 2008–2011, Grant N206 257035 (individual).

6. Pending: Polish Ministry of Science and Higher Education, *Additive colorings of graphs* (supervisor).