Drawing Planar Graphs with Few Segments on the Grid

Philipp Kindermann
Universität Würzburg

joint work with
Tamara Mchedlidze Thomas Schneck Antonios Symvonis
Visual Complexity

# of geometric entities in a drawing
Visual Complexity

# of geometric entities in a drawing
Visual Complexity

# of geometric entities in a drawing

(strong) line cover number
Visual Complexity

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segment number

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arc number

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path cover number

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arc number

slope number

path cover number
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segment number

arc number

path cover number

slope number

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5

4
Visual Complexity

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6

segment number

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arc number

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path cover number

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slope number

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all other numbers are lower bounds
### (Some) Known Results

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[2] Igamberdiev et al. 2015  
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<td>2$n$ [4]</td>
<td>9$n/3$ [4]</td>
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Tree Drawings

Tree $T$
$n$ vtcs
Tree Drawings

Tree $T$

$n$ vtcs

$\beta$ deg-2 vtcs
Tree Drawings

Tree $T$

$n$ vtcs

$\beta$ deg-2 vtcs
Tree Drawings

Tree $T$
$n$ vtcs

Remove $\beta$ deg-2 vtcs
Tree Drawings

Tree $T$
$n$ vtcs

$\Rightarrow$ Tree $T'$
$n - \beta$ vtcs

Remove $\beta$ deg-2 vtcs
Tree Drawings

Tree $T$
$n$ vtcs

⇒ Tree $T'$
$n - \beta$ vtcs

Remove $\beta$ deg-2 vtcs

$\alpha$ leaves
Tree Drawings

Tree $T$
$n$ vtcs

Remove $\beta$ deg-2 vtcs

$\Rightarrow$ Tree $T'$
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Tree Drawings

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Tree Drawings

Tree $T$
$n$ vtcs

⇒ Tree $T'$
$n - \beta$ vtcs

Remove $\beta$ deg-2 vtcs

Remove $\alpha$ leaves
Tree Drawings

Tree $T$
$n$ vtcs

$\Rightarrow$ Tree $T'$
$n - \beta$ vtcs

Remove $\beta$ deg-2 vtcs

Remove $\alpha$ leaves

$\Rightarrow$ Tree $T''$
$n - \alpha - \beta$ vtcs
Tree Drawings

Tree $T$
$n$ vtcs

$\Rightarrow$ Tree $T'$
$n - \beta$ vtcs

Remove $\beta$ deg-2 vtcs

Remove $\alpha$ leaves

$\Rightarrow$ Tree $T''$
$n - \alpha - \beta$ vtcs

$n - \alpha - \beta$ segments
Tree Drawings

Tree $T$
$n$ vtcs

⇒ Tree $T'$
$n - \beta$ vtcs

Remove $\beta$ deg-2 vtcs

⇒ Tree $T''$
$n - \alpha - \beta$ vtcs

$n - \alpha - \beta$ segments

Remove $\alpha$ leaves
$+ \frac{\alpha}{2}$ segments
Tree Drawings

Tree $T$
$n$ vtcs

⇒ Tree $T'$
$n - \beta$ vtcs

Remove $\beta$ deg-2 vtcs

⇒ Tree $T''$
$n - \alpha - \beta$ vtcs

Remove $\alpha$ leaves
$+ \alpha/2$ segments

$n - \alpha/2 - \beta$ segments

$n - \alpha - \beta$ segment
Tree Drawings

Tree $T$
$n$ vtcs

$\Rightarrow$ Tree $T'$
$n - \beta$ vtcs

Remove $\beta$ deg-2 vtcs
$+ 0$ segments

Remove $\alpha$ leaves
$+ \alpha/2$ segments

$n - \alpha/2 - \beta$ segments

$\Rightarrow$ Tree $T''$
$n - \alpha - \beta$ vtcs

$n - \alpha - \beta$ segment
Tree Drawings

Tree $T$

$n$ vtcs

⇒ Tree $T'$

$n - \beta$ vtcs

Remove $\beta$ deg-2 vtcs

$+ 0$ segments

⇒ Tree $T''$

$n - \alpha - \beta$ vtcs

Remove $\alpha$ leaves

$+ \alpha/2$ segments

$n - \alpha/2 - \beta$ segments

$n - \alpha - \beta$ segment
Tree Drawings

Tree $T$
$n$ vtcs

$\Rightarrow$ Tree $T'$
$n - \beta$ vtcs

$\Rightarrow$ Tree $T''$
$n - \alpha - \beta$ vtcs

Remove $\beta$ deg-2 vtcs
+ 0 segments

Remove $\alpha$ leaves
+ $\alpha/2$ segments

$n - \alpha/2 - \beta$ segments

$n - \alpha - \beta$ segment
Tree Drawings

Tree $T$
$n$ vtcs

$\Rightarrow$ Tree $T'$
$n - \beta$ vtcs

$\Rightarrow$ Tree $T''$
$n - \alpha - \beta$ vtcs

Remove $\beta$ deg-2 vtcs
$+ 0$ segments

Remove $\alpha$ leaves
$+ \alpha/2$ segments

$n - \alpha/2 - \beta$ segments

$\alpha > (n - \beta)/2$
Tree Drawings

Tree $T$
$n$ vts

$\Rightarrow$ Tree $T'$
$n - $vts

$\Rightarrow$ Tree $T''$
$n - $vts

Remove $\beta$ deg-2 vts
$+ 0$ segments

$3n/4$ segments

Remove $\alpha$ leaves
$+ \alpha/2$ segments

$\alpha > (n - \beta)/2$

$n - \alpha/2 - \beta$ segments

$n - \alpha - \beta$ segment
Tree Drawings
Tree Drawings
Tree Drawings
Tree Drawings
Tree Drawings
Tree Drawings
Tree Drawings

(1) Draw △ □ □ ▢ ▢
Tree Drawings

(1) Draw ▲ ▲ ▼ ▼
Tree Drawings

(1) Draw  

\[ \text{Diagram of trees} \]
Tree Drawings

(1) Draw △ □ □ ▢ ▤
Tree Drawings

(1) Draw ▲ ▲ ▲ ▲
Tree Drawings

(1) Draw Δ Δ Δ Δ

(2) Layout \( \nu + \) Δ Δ Δ Δ
Tree Drawings

(1) Draw

(2) Layout $v +$
Tree Drawings

(1) Draw ▲ ▼ ▼ ▼

(2) Layout $v +$ ▲ ▼ ▼ ▼ ▼
Tree Drawings

(1) Draw ▲ ▲ ▲ ▲
(2) Layout $v + ▲ ▲ ▲ ▲ ▲$
Tree Drawings

(1) Draw

(2) Layout $v +$
Tree Drawings

(1) Draw △ △ △ △

(2) Layout \( v + \) △ △ △ △ △
Tree Drawings

(1) Draw △ △ △ △
(2) Layout v + △ △ △ △
(3) Add □
Tree Drawings

(1) Draw

(2) Layout $v +$

(3) Add
Tree Drawings

(1) Draw △ △ △ △

(2) Layout v + △ △ △ △

(3) Add □
Tree Drawings

(1) Draw △ △ △ △
(2) Layout $v + △ △ △ △$
(3) Add □
Tree Drawings

(1) Draw

(2) Layout $v +$

(3) Add

(4) Sort by #
Tree Drawings

(1) Draw

(2) Layout $v +$

(3) Add

(4) Sort by #
Tree Drawings

(1) Draw

(2) Layout

(3) Add

(4) Sort by#

(5) Place + on common segments in order
Tree Drawings

(1) Draw ▲ ▼ ▲ ▼ ▲
(2) Layout $v + ▲ ▼ ▲ ▼ ▲$
(3) Add □
(4) Sort ● by #
(5) Place ● + □ on common segments in order
Tree Drawings

(1) Draw

(2) Layout $v +$

(3) Add

(4) Sort ● by #

(5) Place ● + ■ on common segments in order
Tree Drawings

(1) Draw

(2) Layout $v +$

(3) Add

(4) Sort $\bullet$ by #

(5) Place $\bullet + \square$ on common segments in order
Tree Drawings

(1) Draw

(2) Layout $v +$

(3) Add $\square$

(4) Sort $\bullet$ by #

(5) Place $\bullet + \square$ on common segments in order
Tree Drawings

(1) Draw △ △ △ △

(2) Layout $v +$ △ △ △ △

(3) Add □

(4) Sort ☆ by #

(5) Place ☆ + □ on common segments in order
Tree Drawings

(1) Draw △ □ □ □ □

(2) Layout \( v + \) △ □ □ □ □

(3) Add □

(4) Sort ● by #

(5) Place ● + □ on common segments in order
Tree Drawings

(1) Draw △ △ △ △

(2) Layout v + △ △ △ △

(3) Add □

(4) Sort ● by #

(5) Place ● + □ on common segments in order
Tree Drawings

(1) Draw triangle, square, arrowhead, diamond

(2) Layout $v +$ triangle, square, arrowhead, diamond

(3) Add square

(4) Sort circle by #

(5) Place circle, plus, square on common segments in order

$3n/4$ segments
Tree Drawings

1. Draw ▲ △ □ □
2. Layout $v + ▲ △ □ □$
3. Add □
4. Sort ● by #
5. Place ● + □ on common segments in order

$3n/4$ segments
$n \times n$ grid
Tree Drawings

1. Draw △ □ ▲ △
2. Layout v + △ □ ▲ △
3. Add □
4. Sort • by #
5. Place • + □ on common segments in order

$3n/4$ segments

$n \times n$ grid
Tree Drawings

(1) Draw △ △ △ △

(2) Layout v + △ △ △ △

(3) Add □

(4) Sort ● by #

(5) Place ● + □ on common segments in order

3n/4 segments
n × n grid
Tree Drawings

(1) Draw ▲ ▲ ▲ ▲
(2) Layout ◆ + ▲ ▲ ▲ ▲
(3) Add □
(4) Sort ◆ by #
(5) Place ◆ + □ on common segments in order

$3n/4$ segments
$n \times n$ grid
Tree Drawings

(1) Draw ▲ ▲ ▲ ▲

(2) Layout $v + \ ▲ ▲ ▲ ▲$

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$3n/4$ segments

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Tree Drawings

(1) Draw △ △ △ △

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3n/4 segments

\( n \times n \) grid
Tree Drawings

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$3n/4$ segments

$n \times n$ grid

height ✓
Tree Drawings

(1) Draw △ △ △ △

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(3) Add □

(4) Sort ● by #

(5) Place ● + □ on common segments in order

3n/4 segments

n × n grid

height ✓

width
Tree Drawings

(1) Draw ▲▲▲▲
(2) Layout v + ▲▲▲▲
(3) Add □
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3n/4 segments
n × n grid

height ✓
width
Tree Drawings

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$3n/4$ segments

$n \times n$ grid

height ✓

width

$n \times n$ grid
Tree Drawings

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(2) Layout ▼ + ▲ ▼ ▼ ▼ ▼
(3) Add □
(4) Sort ● by #
(5) Place ● + □ on common segments in order

$3n/4$ segments
$n \times n$ grid

height ✓
width
Tree Drawings

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(3) Add

(4) Sort by #

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$3n/4$ segments

$n \times n$ grid

height ✓

width
Tree Drawings

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(2) Layout \( v + \) △ △ △ △
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3n/4 segments
\( n \times n \) grid

height ✓
width ✓
## Improved Results

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Orderly Spanning Trees

[Chiang, Lin, Lu '05]
Orderly Spanning Trees

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[Chiang, Lin, Lu ’05]
Orderly Spanning Trees

[Chiang, Lin, Lu ’05]

neighbors of $v$ in circ. order:
Orderly Spanning Trees

neighbors of $v$ in circ. order:

(1) parent

[Chiang, Lin, Lu ’05]
neighbors of $v$ in circ. order:

(1) parent
(2) $N^+(v)$: diff. subtree (left)
neighbors of $v$ in circ. order:

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2. $N^+(v)$: diff. subtree (left)
neighbors of $v$ in circ. order:

(1) parent

(2) $N^+(v)$: diff. subtree (left)
Orderly Spanning Trees

neighbors of $v$ in circ. order:

1. parent
2. $N^+(v)$: diff. subtree (left)
3. children
neighbors of $v$ in circ. order:

1. parent
2. $N^+(v)$: diff. subtree (left)
3. children
4. $N^-(v)$: diff. subtree (right)
neighbors of $v$ in circ. order:

1. parent
2. $N^+(v)$: diff. subtree (left)
3. children
4. $N^-(v)$: diff. subtree (right)
neighbors of \( v \) in circ. order:

1. parent
2. \( N^+(v) \): diff. subtree (left)
3. children
4. \( N^-(v) \): diff. subtree (right)
Slope-Disjoint Drawing of a Tree

Assign angle interval to each vtx

Angelini et al. ’12
Slope-Disjoint Drawing of a Tree

Assign angle interval to each vtx

[Angelini et al. ’12]
Slope-Disjoint Drawing of a Tree

Assign angle interval to each vtx
All segments in $T[v]$ in interval

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Assign angle interval to each vtx
All segments in $T[v]$ in interval
Intervals of children: disjoint subintervals that contain parent edge

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[Hossain & Rahman ’15]
Slope-disjoint drawing of orderly spanning tree on $O(n) \times O(n^2)$ grid
⇒ planar (monotone) drawing on $O(n) \times O(n^2)$ grid

[Angelini et al. ’12]
Slope-Disjoint Drawing of a Tree

Assign angle interval to each vtx
All segments in $T[v]$ in interval
Intervals of children: disjoint subintervals that contain parent edge

[Angelini et al. ’12]

Slope-disjoint drawing of orderly spanning tree on $O(n) \times O(n^2)$ grid
⇒ planar (monotone) drawing on $O(n) \times O(n^2)$ grid
doesn’t change the slopes!

[Angelini et al. ’12]

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⇒ planar (monotone) drawing on $O(n) \times O(n^2)$ grid
doesn’t change the slopes!
Obtaining a slope-disjoint drawing
Obtaining a slope-disjoint drawing

ccw pre-order traversal
Obtaining a slope-disjoint drawing

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Obtaining a slope-disjoint drawing

ccw pre-order traversal
Obtaining a slope-disjoint drawing

ccw pre-order traversal
Obtaining a slope-disjoint drawing

ccw pre-order traversal
Obtaining a slope-disjoint drawing

ccw pre-order traversal
reuse slope whenever possible
Obtaining a slope-disjoint drawing

ccw pre-order traversal
reuse slope whenever possible
Obtaining a slope-disjoint drawing

ccw pre-order traversal
reuse slope whenever possible
Obtaining a slope-disjoint drawing

ccw pre-order traversal
reuse slope whenever possible
otherwise use highest slope +1
Obtaining a slope-disjoint drawing

ccw pre-order traversal
reuse slope whenever possible
otherwise use highest slope +1
Obtaining a slope-disjoint drawing

ccw pre-order traversal
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Obtaining a slope-disjoint drawing

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Obtaining a slope-disjoint drawing

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reuse slope whenever possible
otherwise use highest slope +1
Obtaining a slope-disjoint drawing

ccw pre-order traversal
reuse slope whenever possible
otherwise use highest slope $+1$
Obtaining a slope-disjoint drawing

ccw pre-order traversal
reuse slope whenever possible
otherwise use highest slope +1
Obtaining a slope-disjoint drawing

cCW pre-order traversal
reuse slope whenever possible
otherwise use highest slope +1
Obtaining a slope-disjoint drawing

ccw pre-order traversal
reuse slope whenever possible
otherwise use highest slope +1

highest slope: $n$
Obtaining a slope-disjoint drawing

ccw pre-order traversal
reuse slope whenever possible
otherwise use highest slope +1

highest slope: n
max. width: n
Obtaining a slope-disjoint drawing

ccw pre-order traversal
reuse slope whenever possible
otherwise use highest slope +1

highest slope: $n$
max. width: $n$

$\Rightarrow n \times n^2$ grid
Obtaining a slope-disjoint drawing

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3-conn. planar graph
\[ \Rightarrow (8n - 14)/3 \text{ segments, } O(n) \times O(n^2) \text{ grid} \]
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<tr>
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<tbody>
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<td>$\theta/2$ [1]</td>
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