Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Chair for
INFORMATICS I
Efficient Algorithms and
Knowledge-Based Systems

Institute for Informatics

Drawing Planar Graphs with Few Segments on the Grid

Philipp Kindermann
Universität Würzburg

joint work with

Tamara Mchedlidze | Thomas Schneck | Antonios Symvonis

Karlsruher Institut für Technologie 1825 1956 2009

EBERHARD KARLS UNIVERSITÄT TÜBINGEN

NATIONALE METSOBILE TECHNOLOGIE

Visual Complexity

of geometric entities in a drawing

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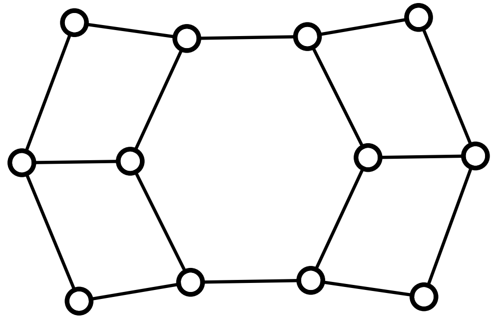

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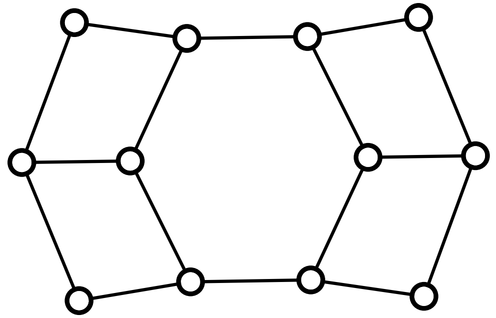

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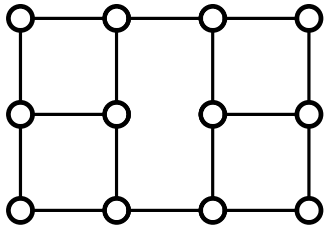

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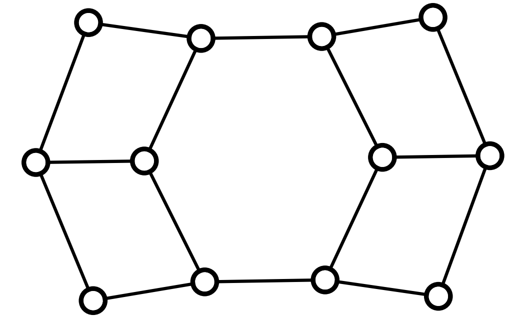

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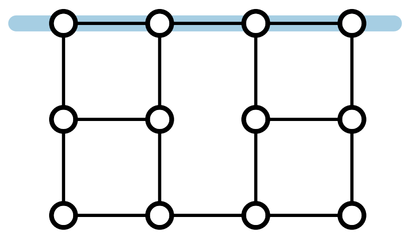

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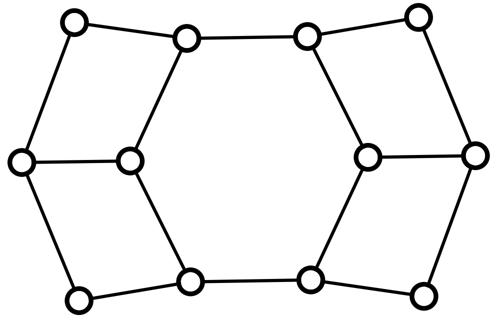

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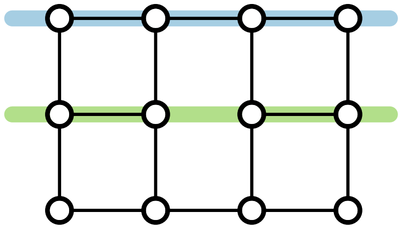

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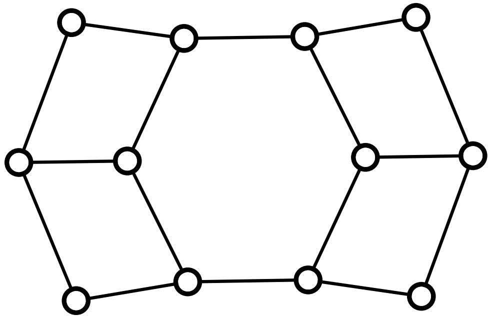

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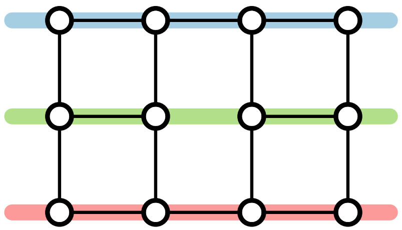

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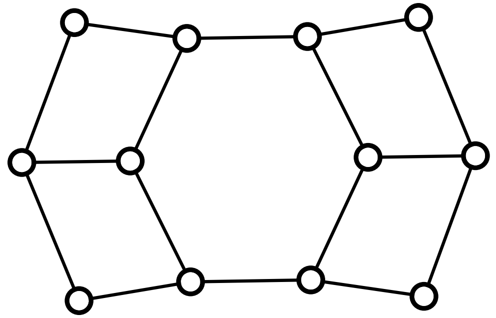

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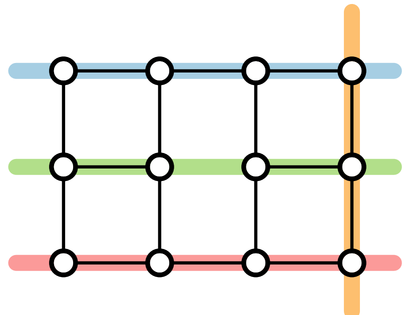

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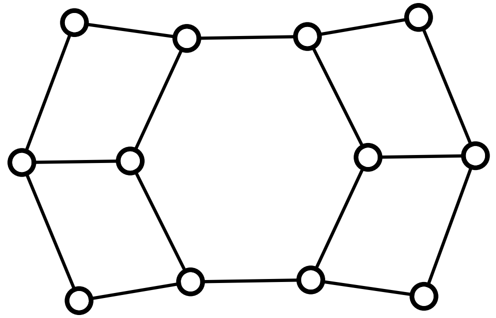

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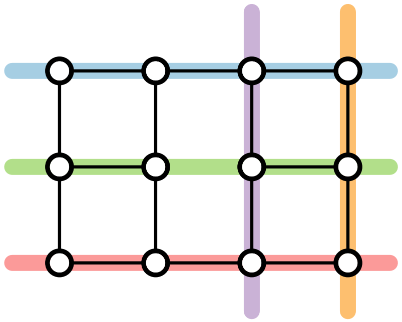

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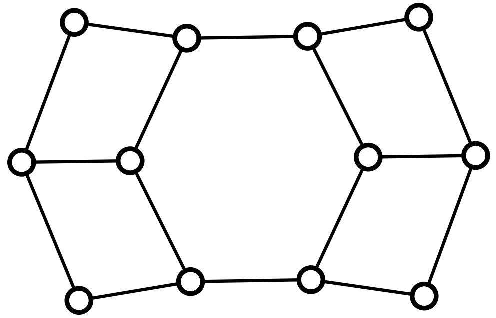

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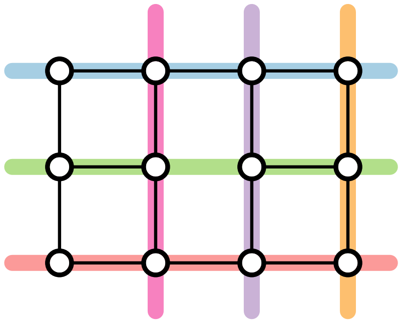

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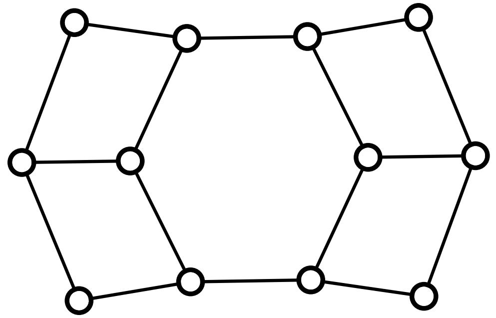

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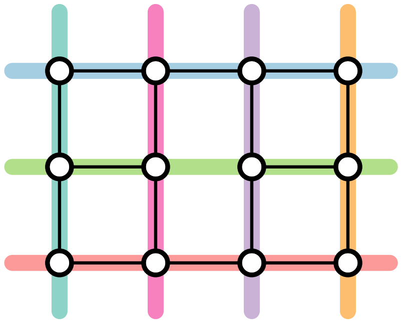

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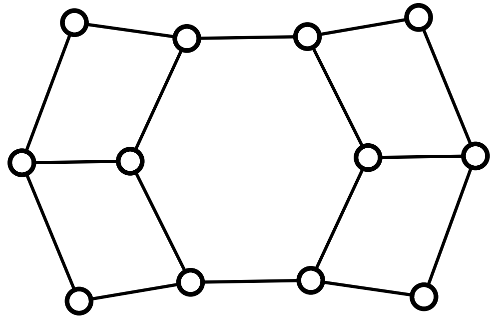

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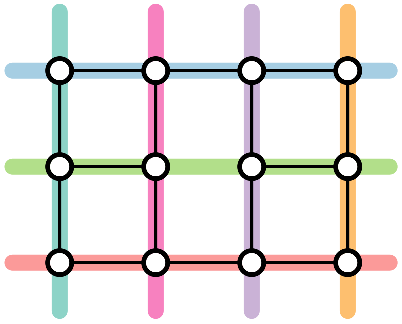

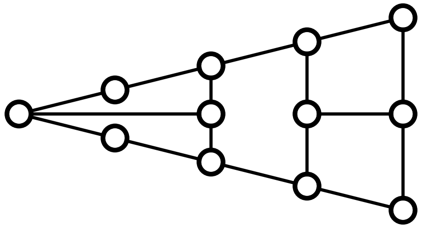

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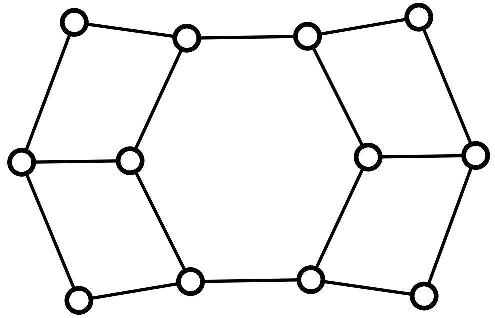

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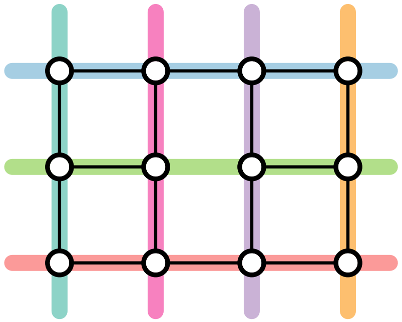

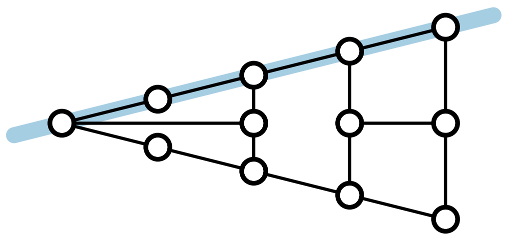

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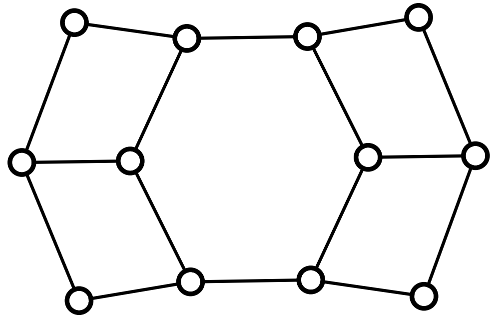

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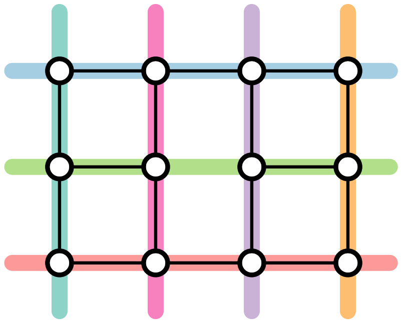

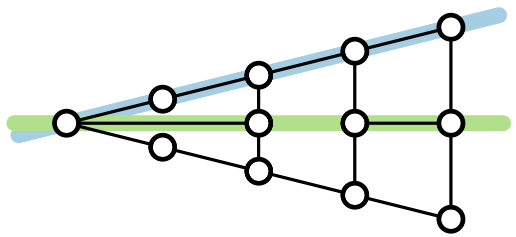

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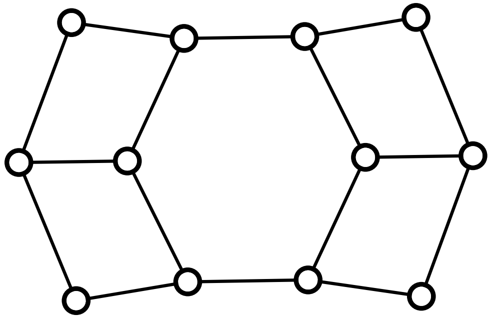

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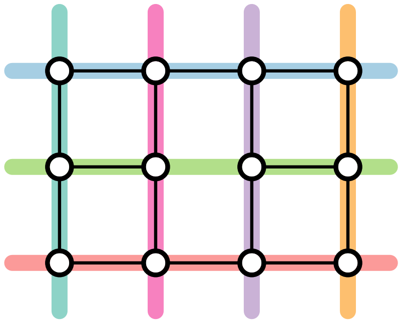

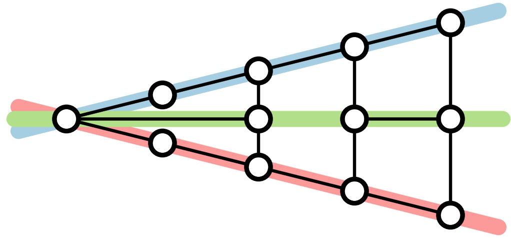

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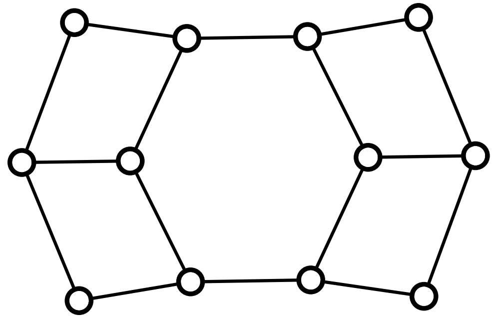

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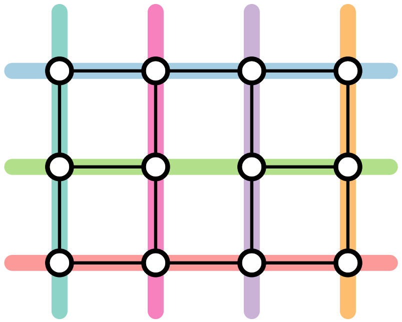

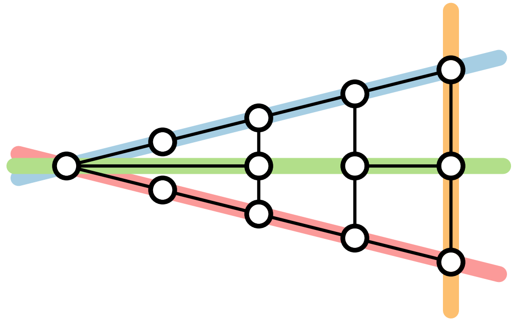

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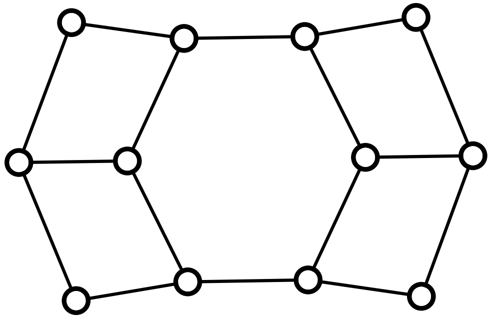

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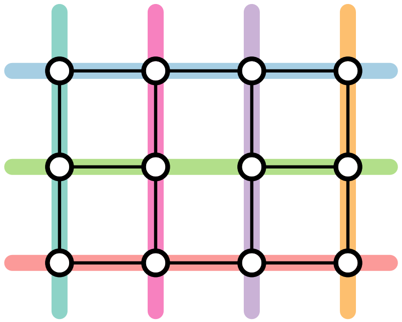

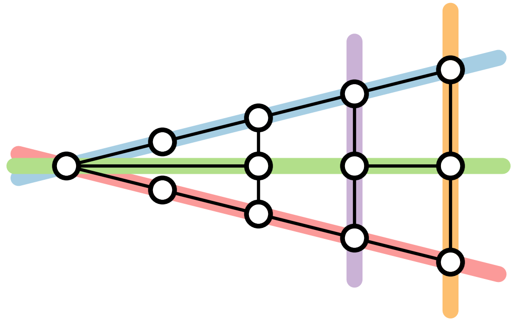

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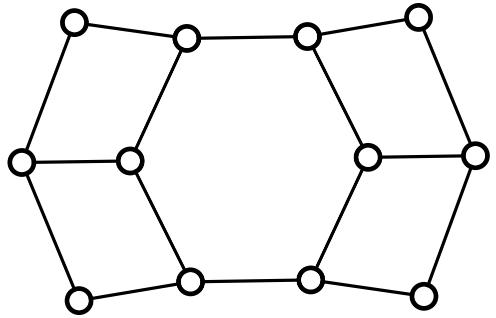

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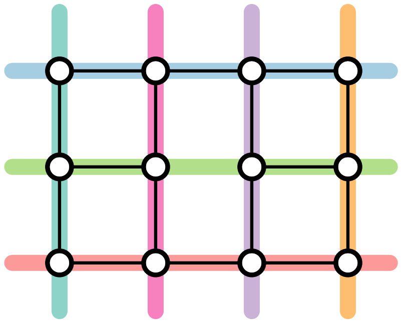

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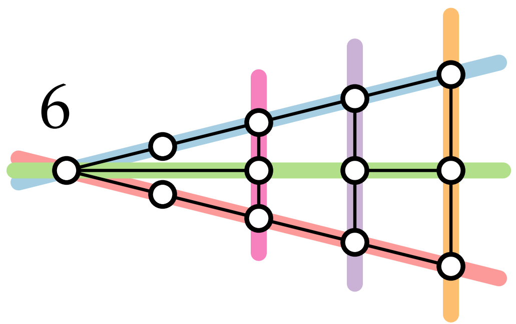

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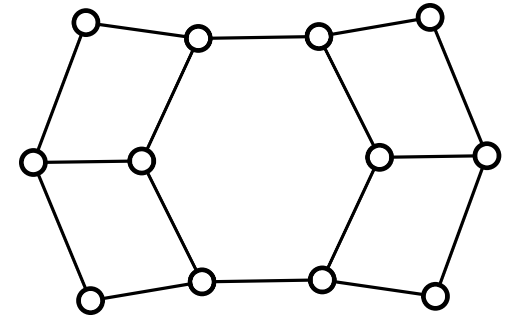

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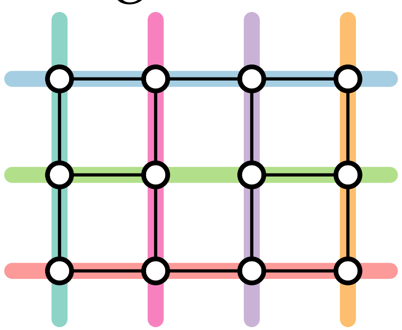

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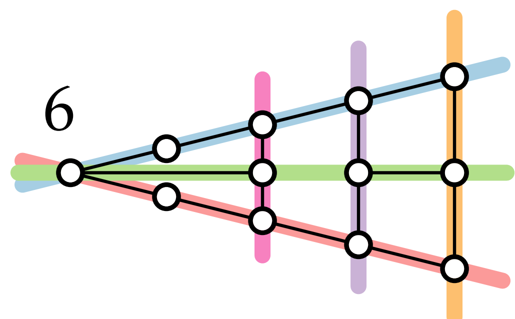

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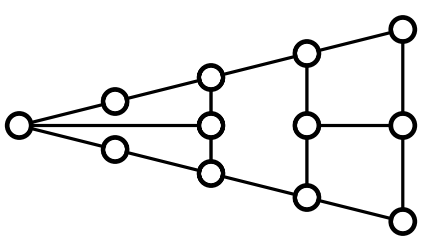

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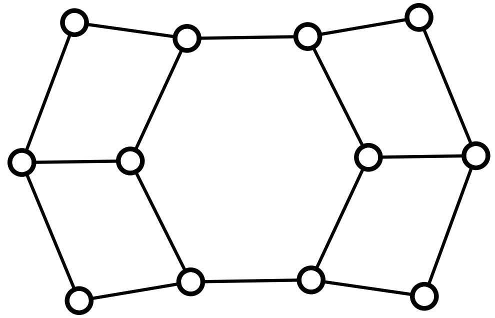

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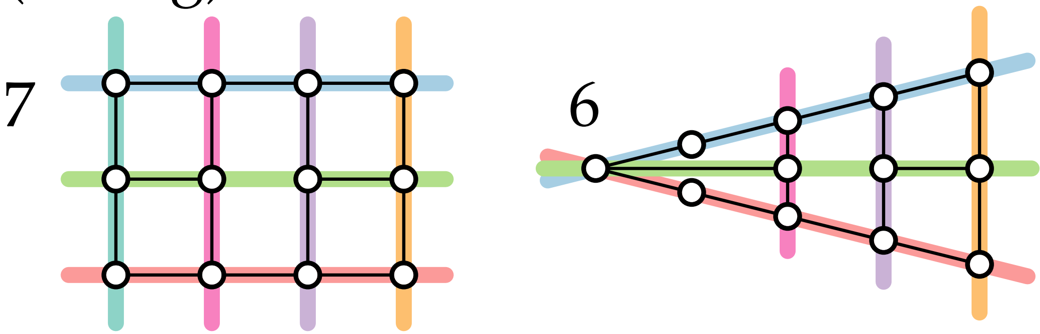

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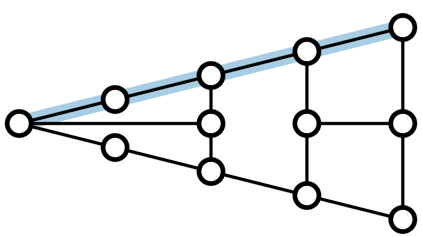

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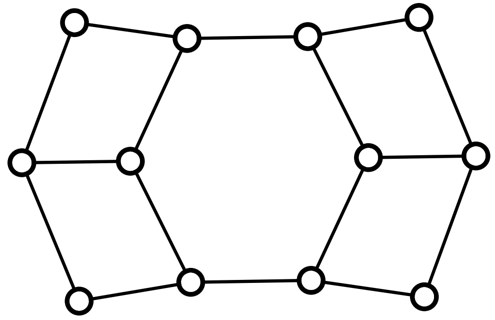

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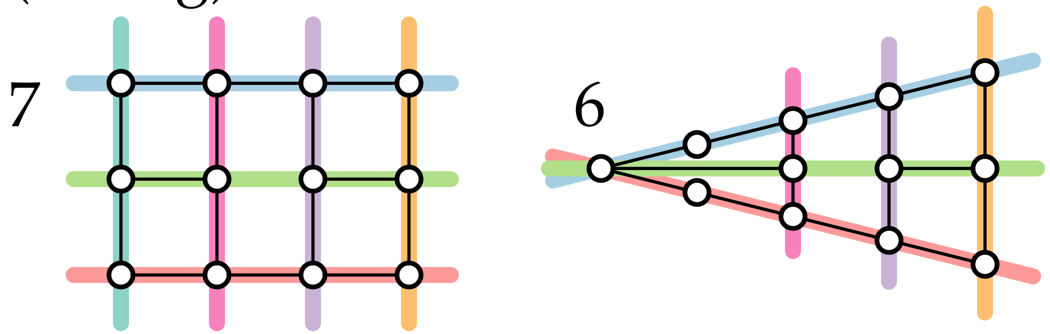

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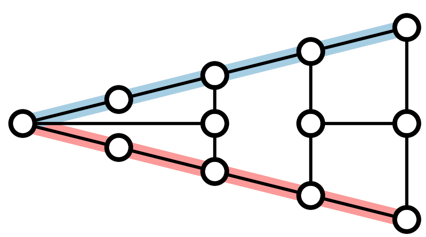

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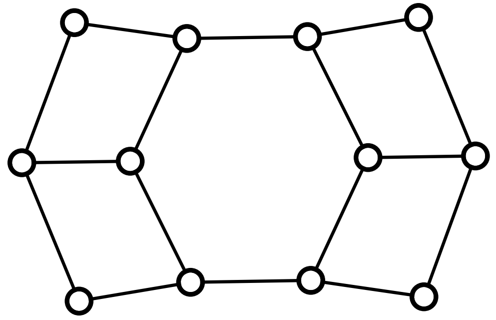

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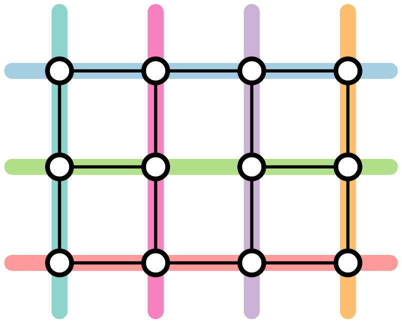

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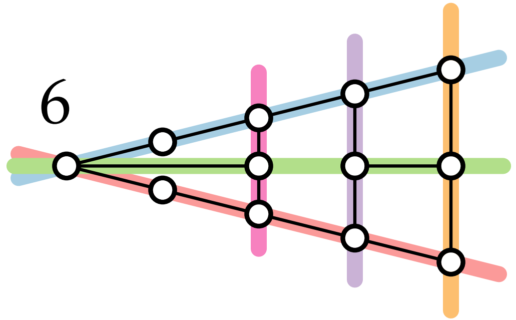

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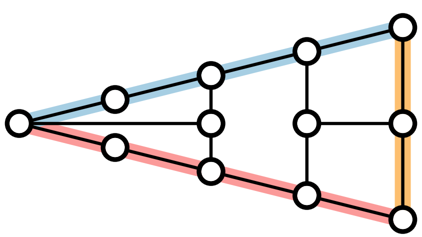

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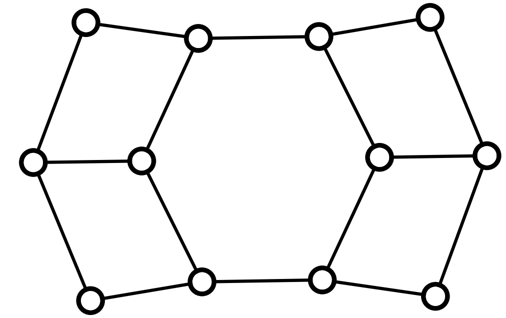

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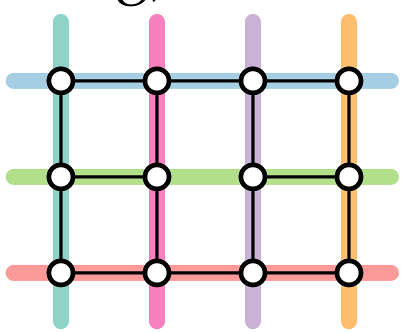

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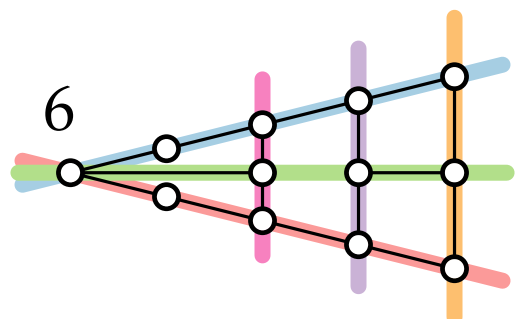

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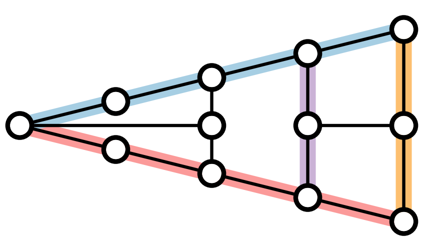

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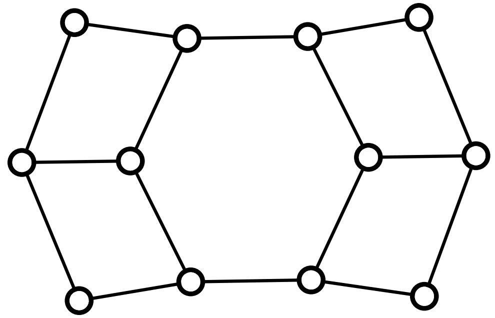

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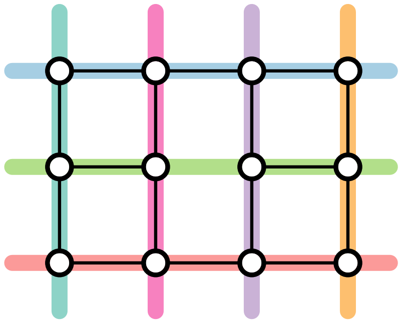

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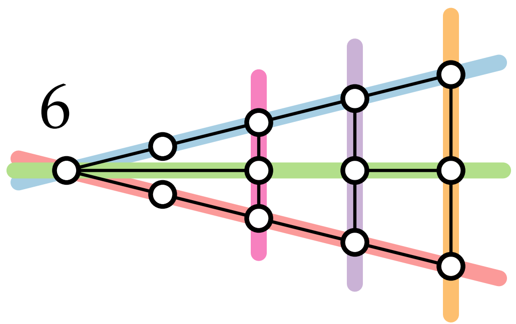

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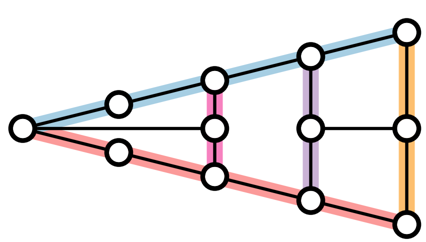

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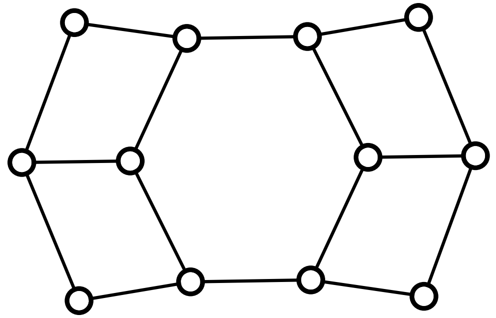

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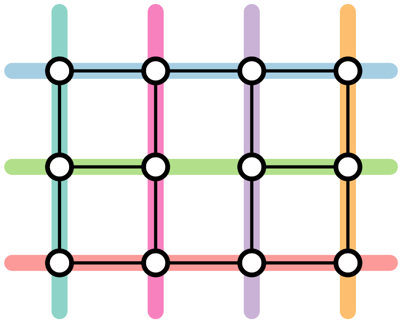

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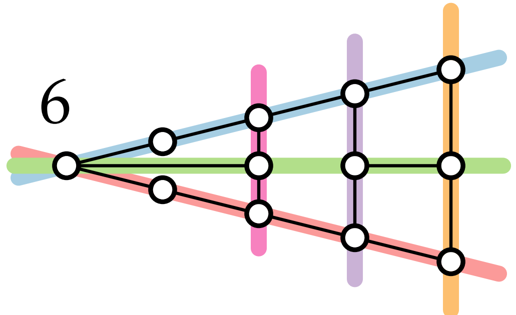

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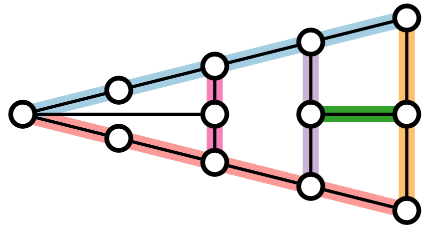

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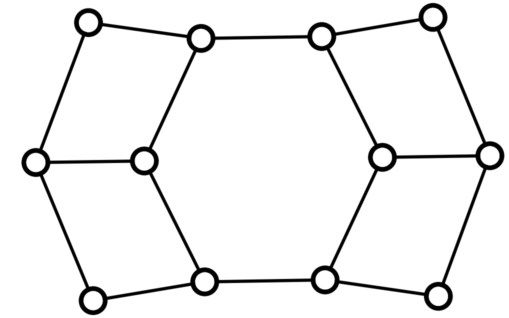

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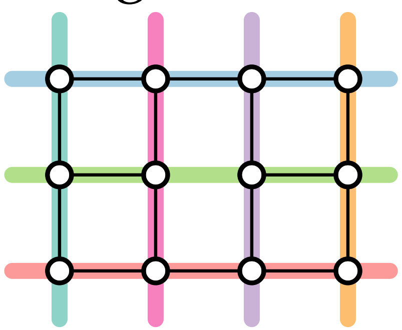

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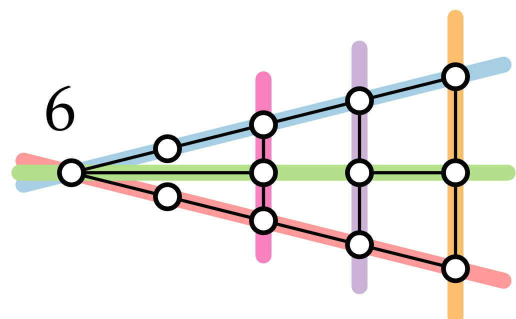

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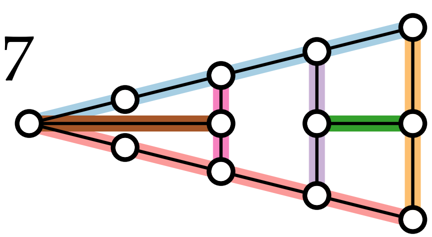

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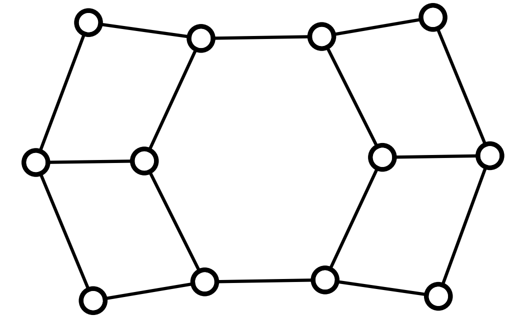

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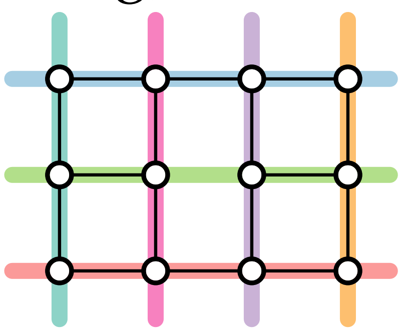

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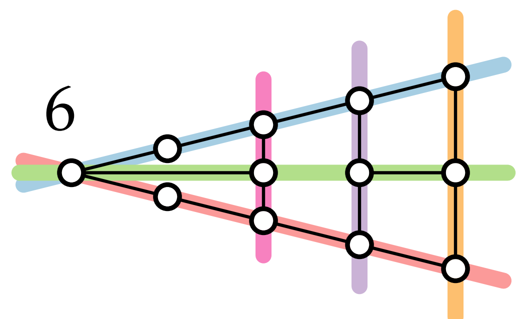

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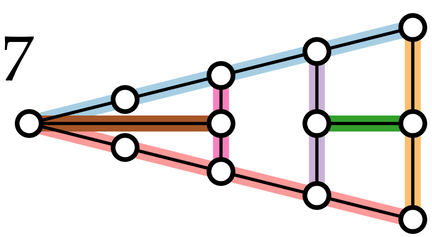

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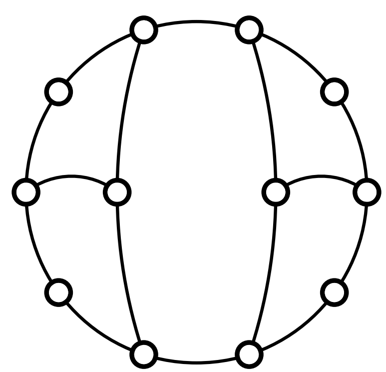

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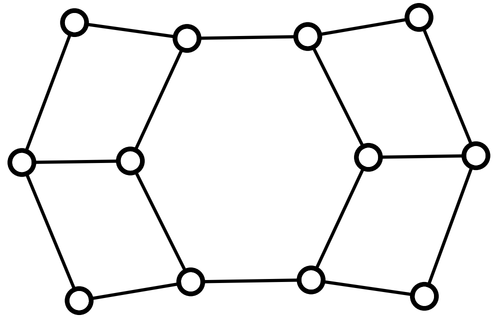

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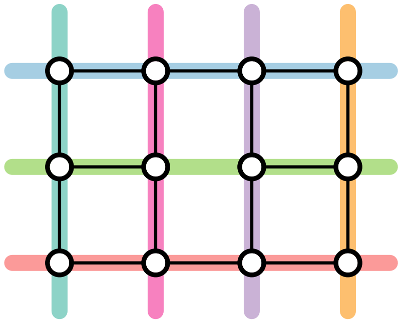

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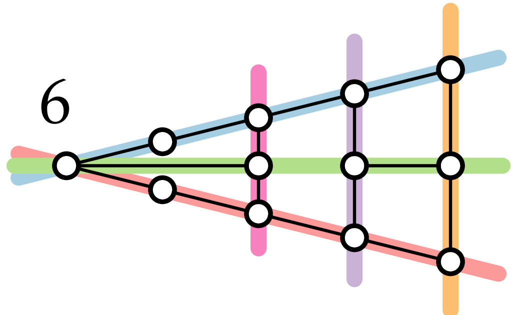

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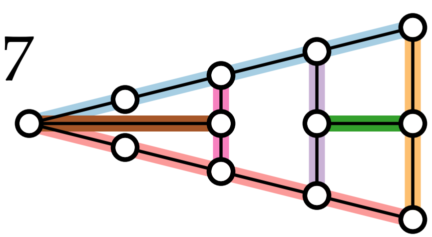

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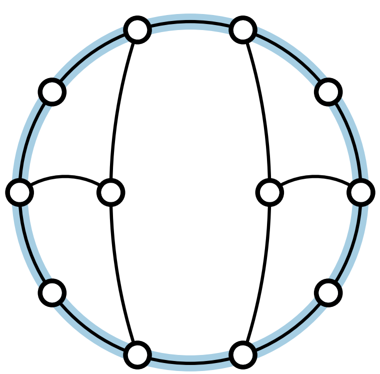

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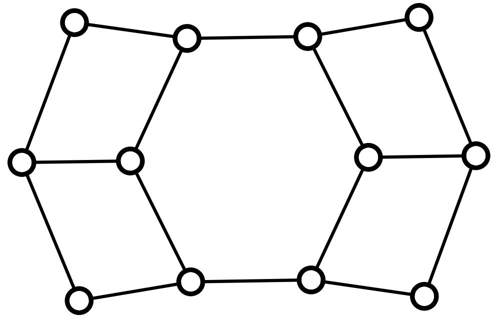

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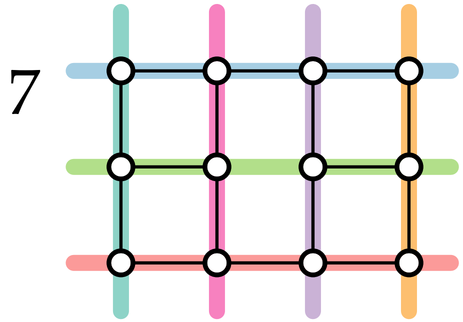

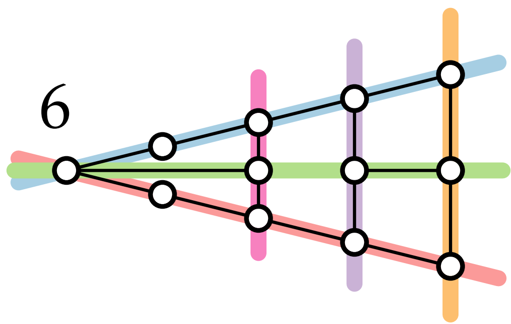

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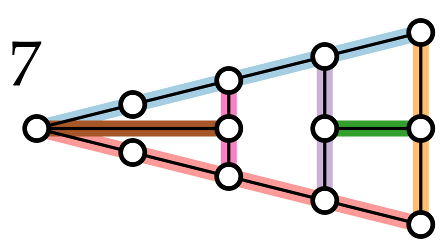

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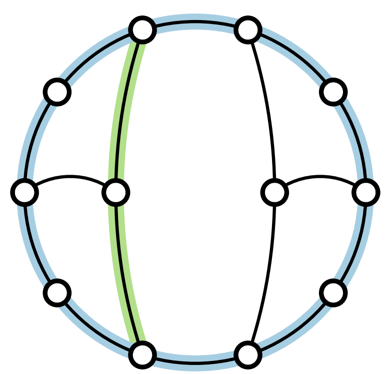

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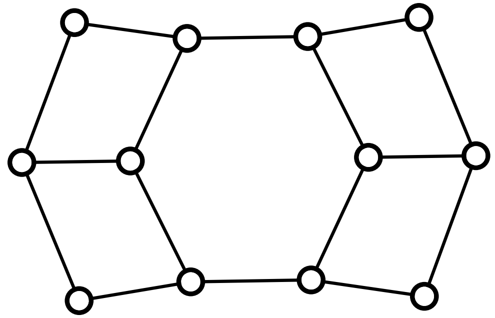

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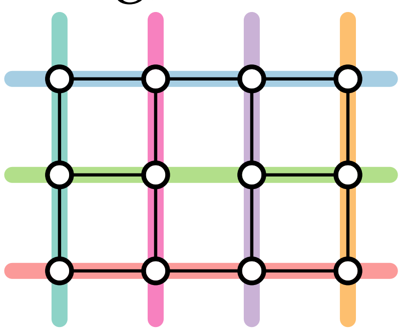

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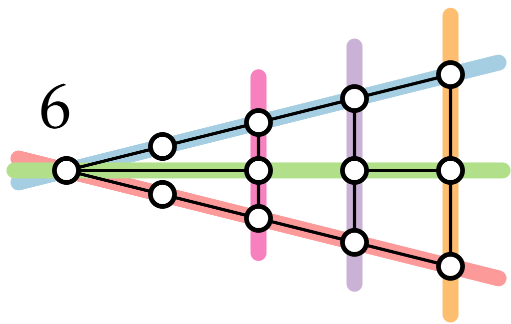

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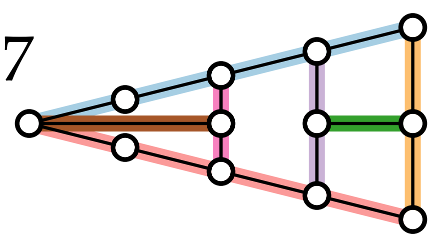

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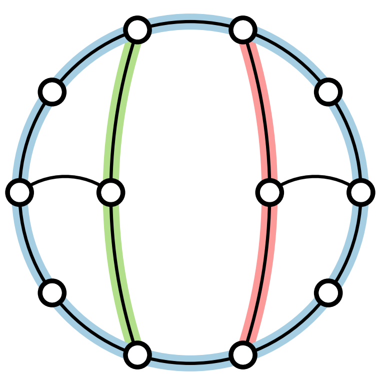

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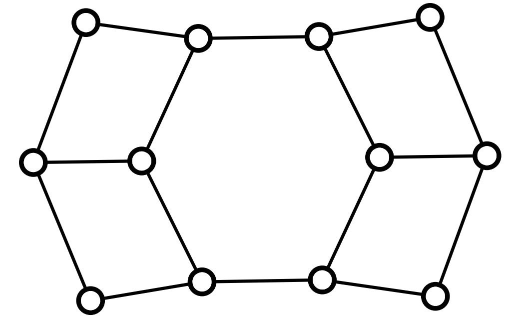

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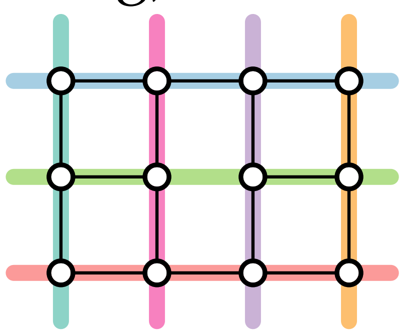

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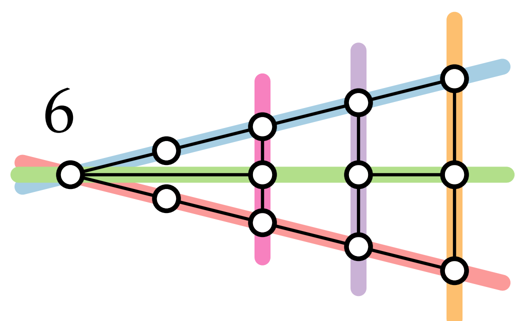

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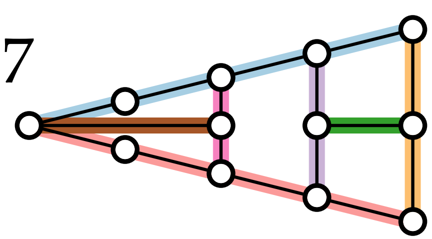

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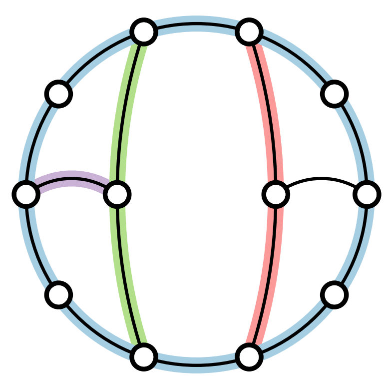

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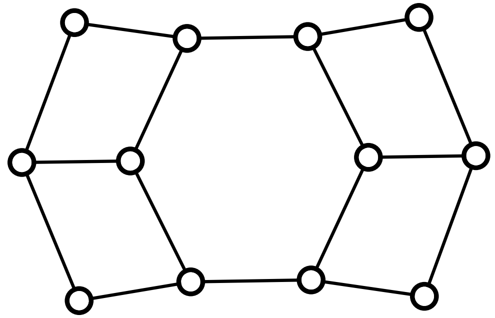

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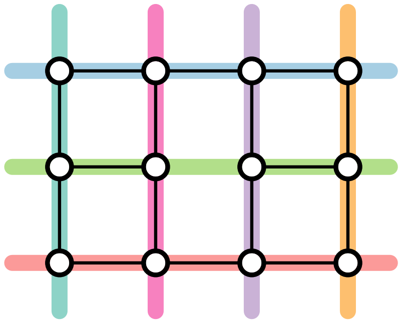

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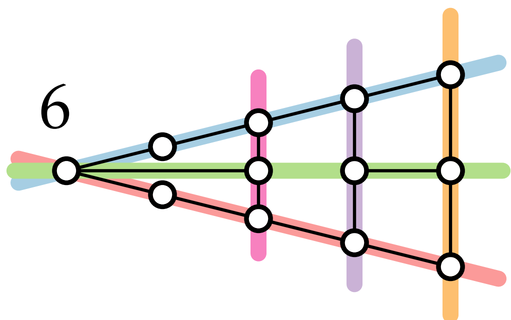

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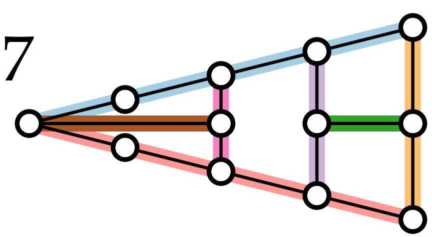

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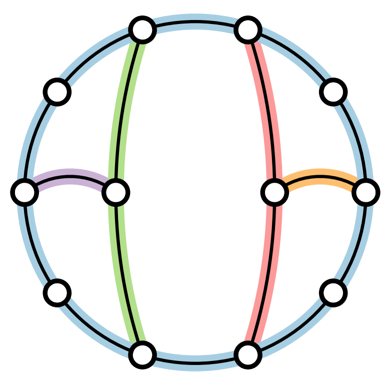

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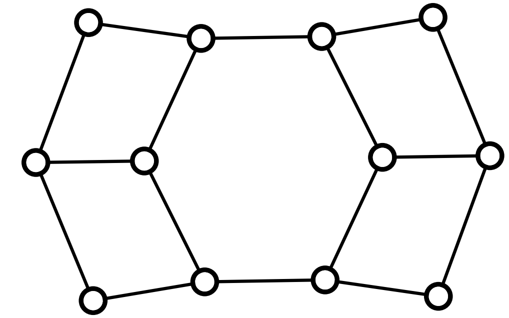

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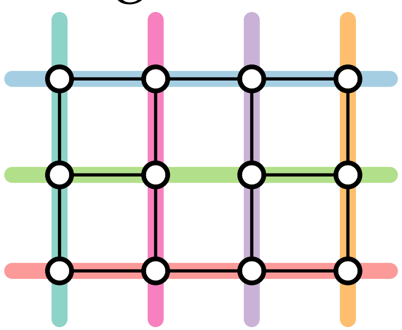

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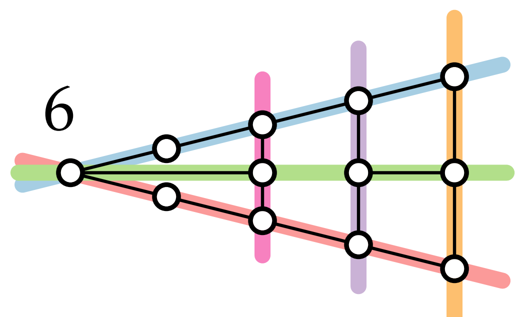

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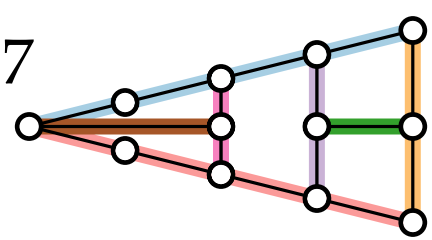

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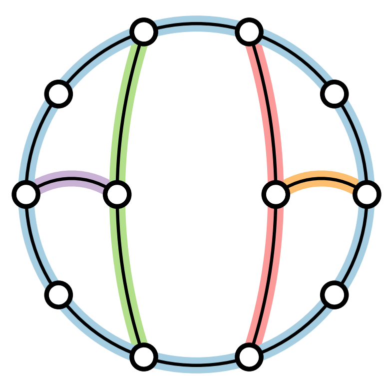

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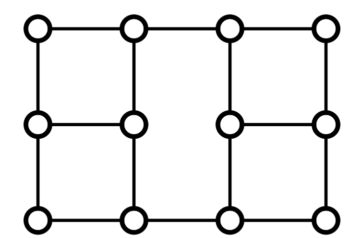

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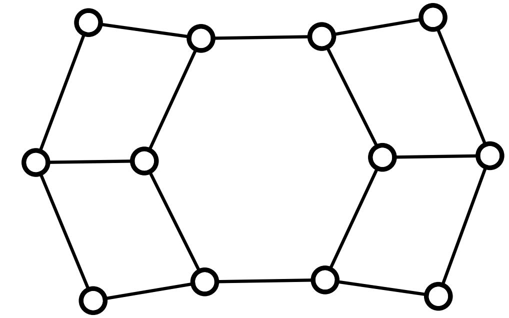

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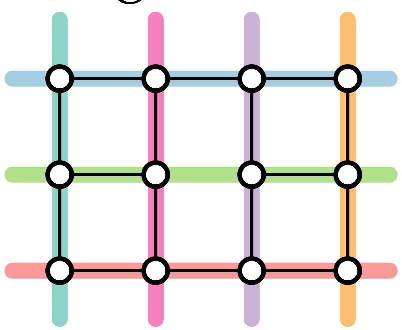

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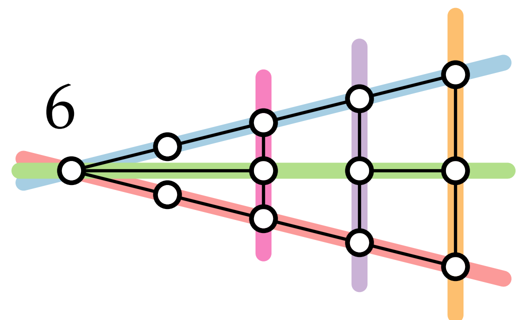

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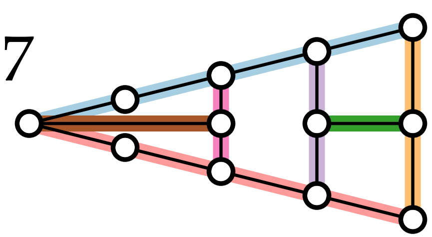

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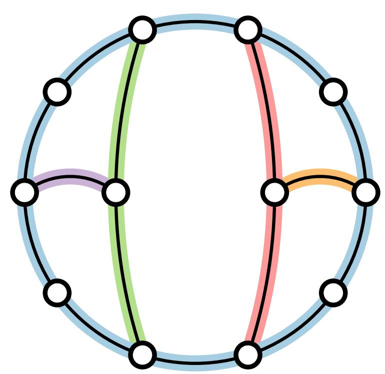

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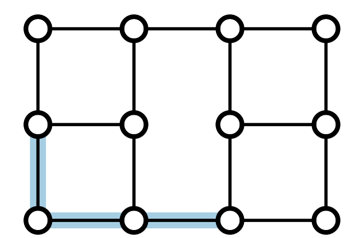

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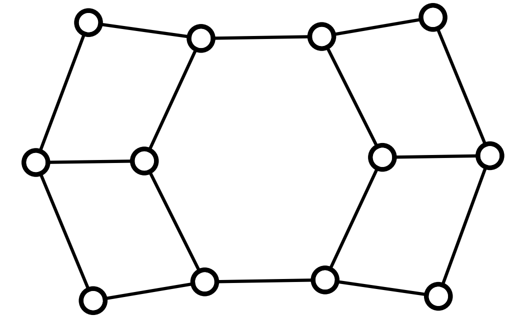

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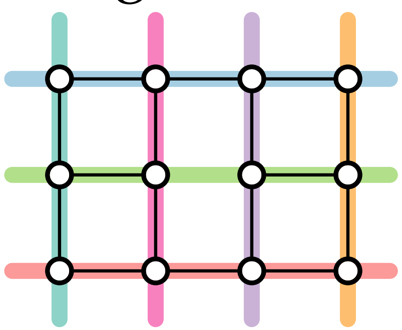

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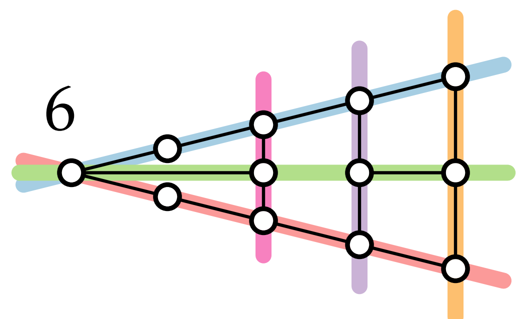

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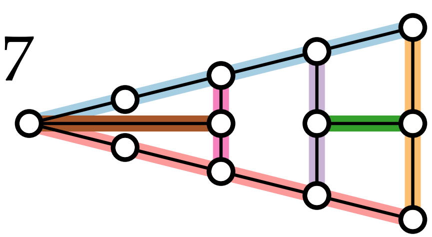

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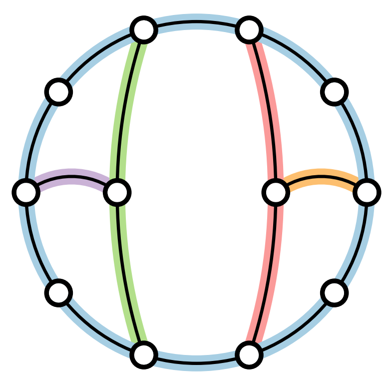

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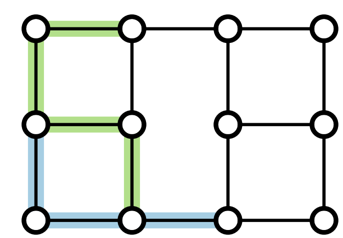
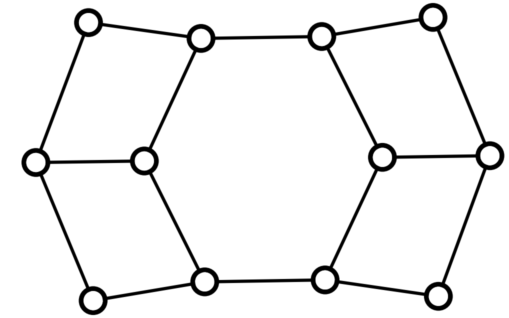


Visual Complexity

of geometric entities in a drawing



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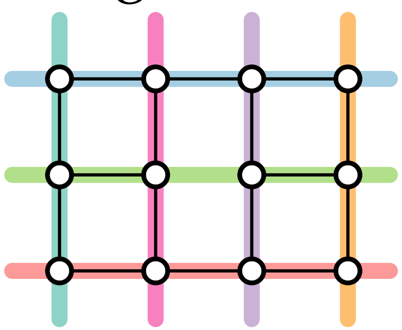

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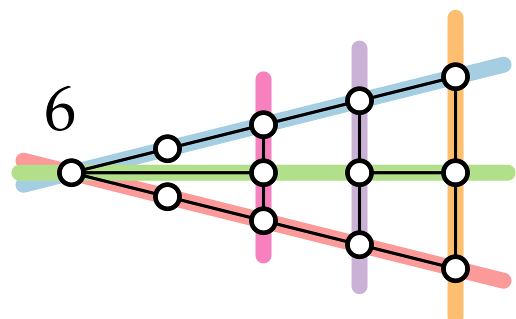

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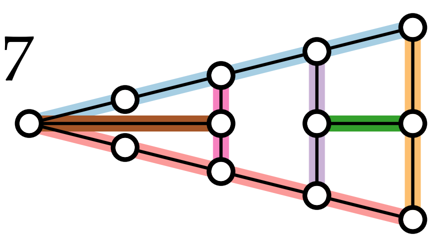

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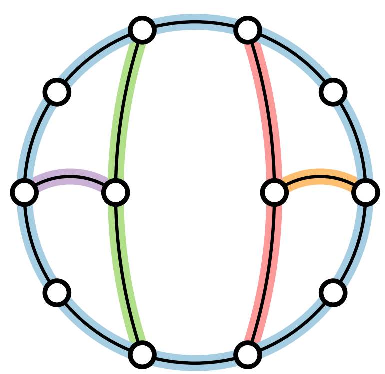

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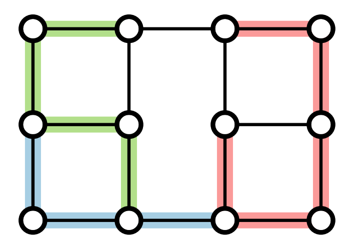

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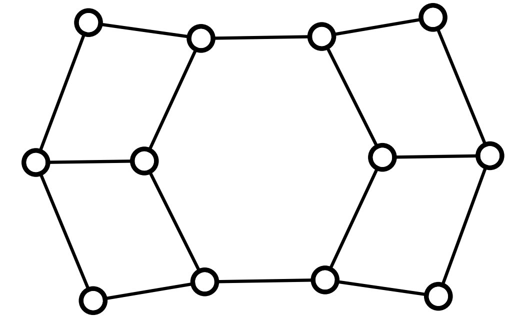

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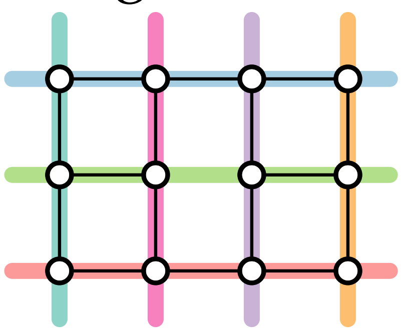

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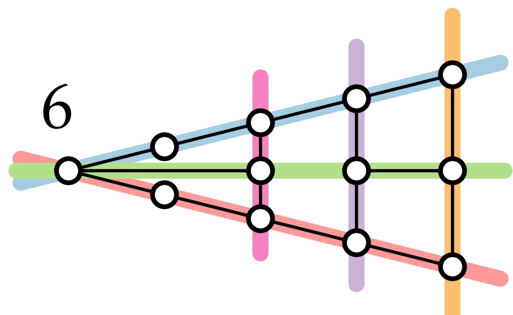

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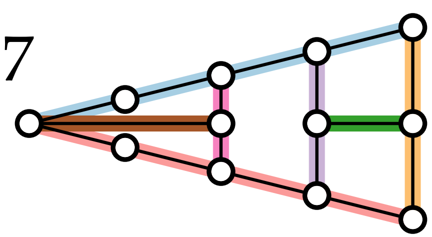

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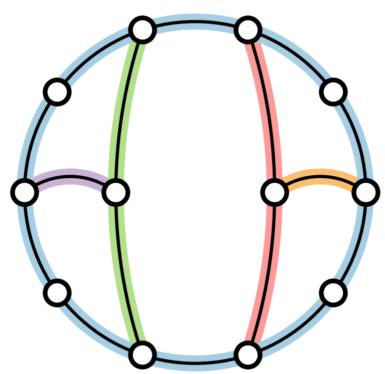

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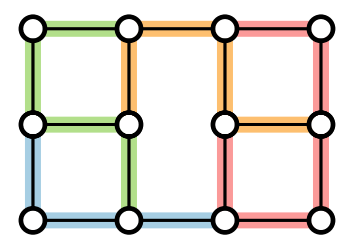
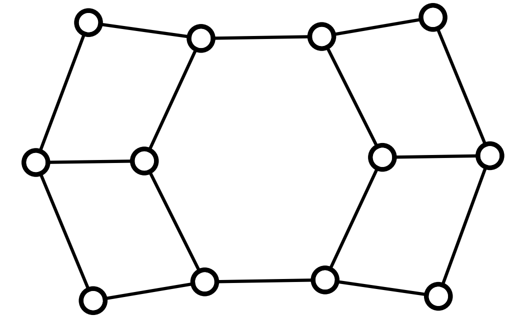


Visual Complexity



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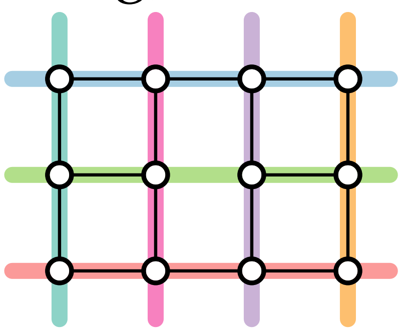

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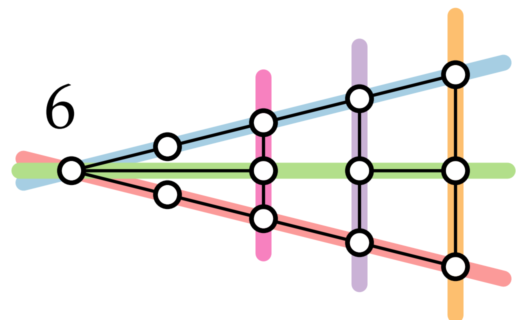

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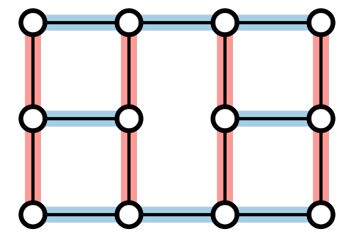

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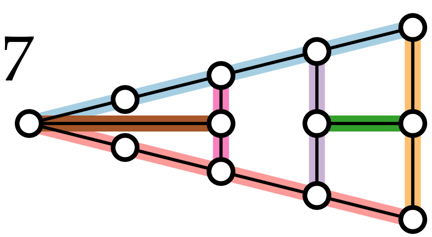

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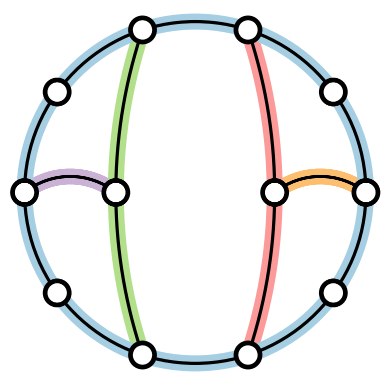

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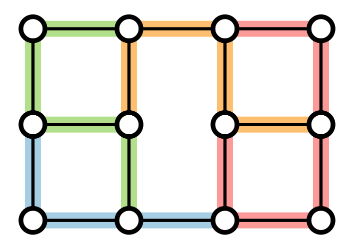
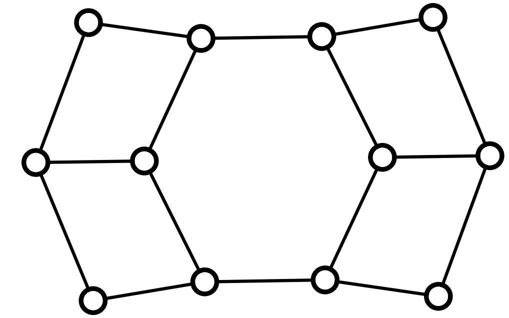


Visual Complexity



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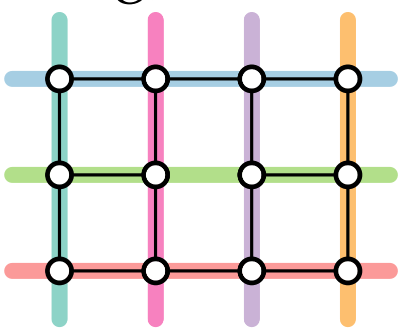

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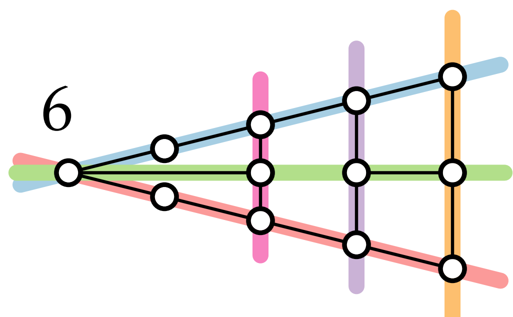

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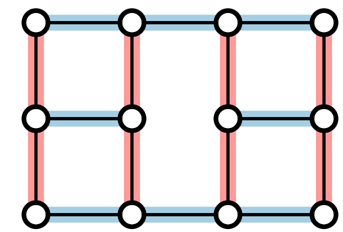

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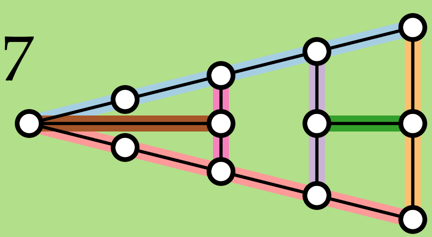

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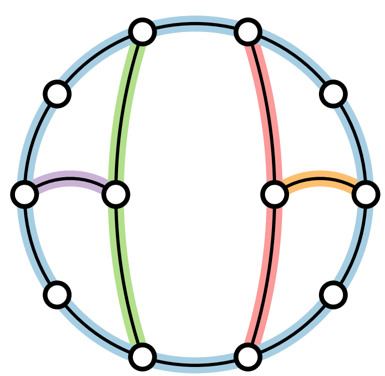

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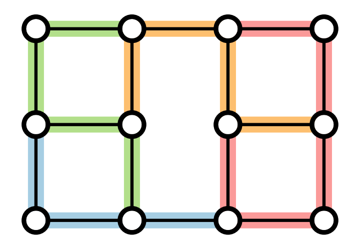
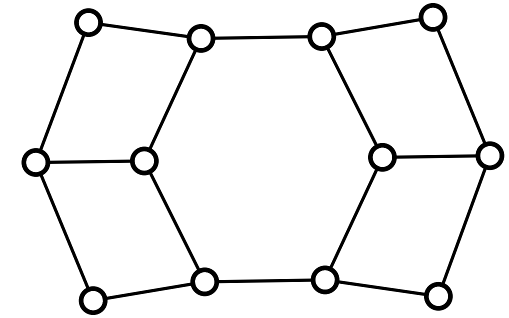


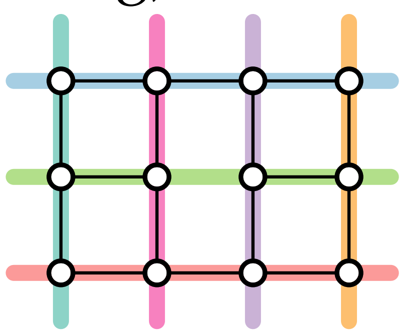
Visual Complexity

of geometric entities in a drawing

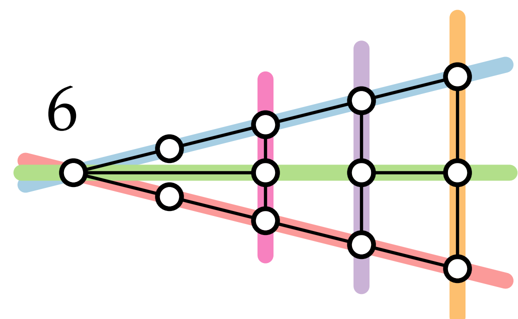


(strong) line cover number

7

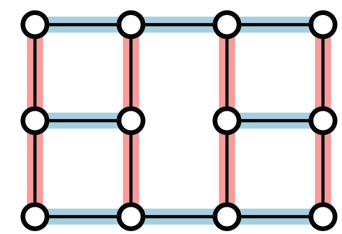


6



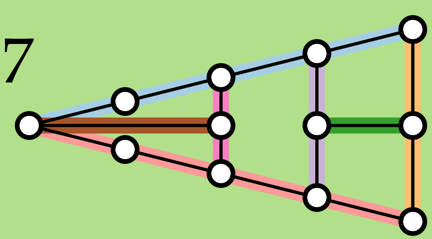
slope number

2



segment number

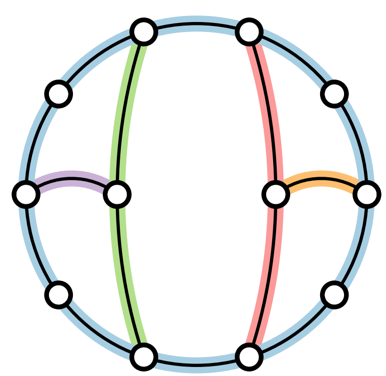
7



all other numbers
are lower bounds

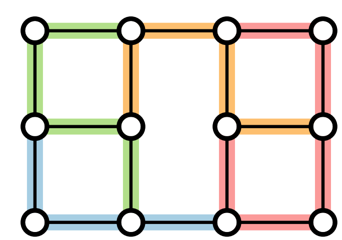
arc number

5



path cover number

4



(Some) Known Results

Class	Segments	
	Lower	Upper

(Some) Known Results

Class	Segments	
	Lower	Upper
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]

[1] Dujmović et al. 2007

(Some) Known Results

Class	Segments	
	Lower	Upper
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]
outerplanar	n [1]	

[1] Dujmović et al. 2007

(Some) Known Results

Class	Segments			
	Lower		Upper	
tree	$\vartheta/2$	[1]	$\vartheta/2$	[1]
outerplanar	n	[1]		
max. outerp.	n	[1]	n	[1]

[1] Dujmović et al. 2007

(Some) Known Results

Class	Segments			
	Lower		Upper	
tree	$\vartheta/2$	[1]	$\vartheta/2$	[1]
outerplanar	n	[1]		
max. outerp.	n	[1]	n	[1]
3-trees	$2n$	[1]	$2n$	[1]

[1] Dujmović et al. 2007

(Some) Known Results

Class	Segments			
	Lower		Upper	
tree	$\vartheta/2$	[1]	$\vartheta/2$	[1]
outerplanar	n	[1]		
max. outerp.	n	[1]	n	[1]
3-trees	$2n$	[1]	$2n$	[1]
2-connected	$2n$	[1]		

[1] Dujmović et al. 2007

(Some) Known Results

Class	Segments			
	Lower		Upper	
tree	$\vartheta/2$	[1]	$\vartheta/2$	[1]
outerplanar	n	[1]		
max. outerp.	n	[1]	n	[1]
3-trees	$2n$	[1]	$2n$	[1]
2-connected	$2n$	[1]		
3-connected	$2n$	[1]	$5n/2$	[1]

[1] Dujmović et al. 2007

(Some) Known Results

Class	Segments			
	Lower		Upper	
tree	$\vartheta/2$	[1]	$\vartheta/2$	[1]
outerplanar	n	[1]		
max. outerp.	n	[1]	n	[1]
3-trees	$2n$	[1]	$2n$	[1]
2-connected	$2n$	[1]		
3-connected	$2n$	[1]	$5n/2$	[1]
cubic 3-conn.	$n/2$	[3]	$n/2$	[2]

[1] Dujmović et al. 2007 [2] Igamberdiev et al. 2015 [3] Mondal et al. 2013

(Some) Known Results

Class	Segments			
	Lower		Upper	
tree	$\vartheta/2$	[1]	$\vartheta/2$	[1]
outerplanar	n	[1]		
max. outerp.	n	[1]	n	[1]
3-trees	$2n$	[1]	$2n$	[1]
2-connected	$2n$	[1]		
3-connected	$2n$	[1]	$5n/2$	[1]
cubic 3-conn.	$n/2$	[3]	$n/2$	[2]
triangulation	$2n$	[4]	$7n/3$	[4]

[1] Dujmović et al. 2007 [2] Igamberdiev et al. 2015 [3] Mondal et al. 2013
[4] Durocher & Mondal 2014

(Some) Known Results

Class	Segments: Lower		Segments: Upper	
tree	$\vartheta/2$	[1]	$\vartheta/2$	[1]
outerplanar	n	[1]		
max. outerp.	n	[1]	n	[1]
3-trees	$2n$	[1]	$2n$	[1]
2-connected	$2n$	[1]		
3-connected	$2n$	[1]	$5n/2$	[1]
cubic 3-conn.	$n/2$	[3]	$n/2$	[2]
triangulation	$2n$	[4]	$7n/3$	[4]
4-conn. triang.	$2n$	[4]	$9n/3$	[4]

[1] Dujmović et al. 2007 [2] Igamberdiev et al. 2015 [3] Mondal et al. 2013
[4] Durocher & Mondal 2014

(Some) Known Results

Class	Segments			
	Lower		Upper	
tree	$\vartheta/2$	[1]	$\vartheta/2$	[1]
outerplanar	n	[1]		
max. outerp.	n	[1]	n	[1]
3-trees	$2n$	[1]	$2n$	[1]
2-connected	$2n$	[1]		
3-connected	$2n$	[1]	$5n/2$	[1]
cubic 3-conn.	$n/2$	[3]	$n/2$	[2]
triangulation	$2n$	[4]	$7n/3$	[4]
4-conn. triang.	$2n$	[4]	$9n/3$	[4]
planar	$2n$	[4]	$8n/3$	[4]

[1] Dujmović et al. 2007 [2] Igamberdiev et al. 2015 [3] Mondal et al. 2013
[4] Durocher & Mondal 2014

(Some) Known Results

Class	Segments		Grid Segments	
	Lower	Upper	Segm.	Area
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]		
outerplanar	n [1]			
max. outerp.	n [1]	n [1]		
3-trees	$2n$ [1]	$2n$ [1]		
2-connected	$2n$ [1]			
3-connected	$2n$ [1]	$5n/2$ [1]		
cubic 3-conn.	$n/2$ [3]	$n/2$ [2]		
triangulation	$2n$ [4]	$7n/3$ [4]		
4-conn. triang.	$2n$ [4]	$9n/3$ [4]		
planar	$2n$ [4]	$8n/3$ [4]		

[1] Dujmović et al. 2007 [2] Igamberdiev et al. 2015 [3] Mondal et al. 2013
[4] Durocher & Mondal 2014

(Some) Known Results

Class	Segments		Grid Segments	
	Lower	Upper	Segm.	Area
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]		
outerplanar	n [1]			
max. outerp.	n [1]	n [1]		
3-trees	$2n$ [1]	$2n$ [1]		
2-connected	$2n$ [1]			
3-connected	$2n$ [1]	$5n/2$ [1]		
cubic 3-conn.	$n/2$ [3]	$n/2$ [2]	$n/2$ [2]	$O(n) \times O(n)$
triangulation	$2n$ [4]	$7n/3$ [4]		
4-conn. triang.	$2n$ [4]	$9n/3$ [4]		
planar	$2n$ [4]	$8n/3$ [4]		

[1] Dujmović et al. 2007 [2] Igamberdiev et al. 2015 [3] Mondal et al. 2013
[4] Durocher & Mondal 2014

(Some) Known Results

Class	Segments		Grid Segments	
	Lower	Upper	Segm.	Area
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]		
outerplanar	n [1]			
max. outerp.	n [1]	n [1]		
3-trees	$2n$ [1]	$2n$ [1]		
2-connected	$2n$ [1]			
3-connected	$2n$ [1]	$5n/2$ [1]		
cubic 3-conn.	$n/2$ [3]	$n/2$ [2]	$n/2$ [2]	$O(n) \times O(n)$
triangulation	$2n$ [4]	$7n/3$ [4]	$8n/3$ [5]	$2^{O}(n \log n)$
4-conn. triang.	$2n$ [4]	$9n/3$ [4]		
planar	$2n$ [4]	$8n/3$ [4]		

[1] Dujmović et al. 2007 [2] Igamberdiev et al. 2015 [3] Mondal et al. 2013
[4] Durocher & Mondal 2014 [5] Mondal 2016

(Some) Known Results

Class	Segments		Grid Segments	
	Lower	Upper	Segm.	Area
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$3n/4$ [6]	$O(n^2) \times O(n^{1.58})$
outerplanar	n [1]			
max. outerp.	n [1]	n [1]		
3-trees	$2n$ [1]	$2n$ [1]		
2-connected	$2n$ [1]			
3-connected	$2n$ [1]	$5n/2$ [1]		
cubic 3-conn.	$n/2$ [3]	$n/2$ [2]	$n/2$ [2]	$O(n) \times O(n)$
triangulation	$2n$ [4]	$7n/3$ [4]	$8n/3$ [5]	$2^{O}(n \log n)$
4-conn. triang.	$2n$ [4]	$9n/3$ [4]		
planar	$2n$ [4]	$8n/3$ [4]		

[1] Dujmović et al. 2007 [2] Igamberdiev et al. 2015 [3] Mondal et al. 2013
[4] Durocher & Mondal 2014 [5] Mondal 2016 [6] Hültenschmidt et al. 2017

(Some) Known Results

Class	Segments		Grid Segments	
	Lower	Upper	Segm.	Area
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$3n/4$ [6] $\vartheta/2$ [6]	$O(n^2) \times O(n^{1.58})$ quasipolynomial
outerplanar	n [1]			
max. outerp.	n [1]	n [1]		
3-trees	$2n$ [1]	$2n$ [1]		
2-connected	$2n$ [1]			
3-connected	$2n$ [1]	$5n/2$ [1]		
cubic 3-conn.	$n/2$ [3]	$n/2$ [2]	$n/2$ [2]	$O(n) \times O(n)$
triangulation	$2n$ [4]	$7n/3$ [4]	$8n/3$ [5]	$2^{O}(n \log n)$
4-conn. triang.	$2n$ [4]	$9n/3$ [4]		
planar	$2n$ [4]	$8n/3$ [4]		

[1] Dujmović et al. 2007 [2] Igamberdiev et al. 2015 [3] Mondal et al. 2013
[4] Durocher & Mondal 2014 [5] Mondal 2016 [6] Hültenschmidt et al. 2017

(Some) Known Results

Class	Segments		Grid Segments	
	Lower	Upper	Segm.	Area
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$3n/4$ [6] $\vartheta/2$ [6]	$O(n^2) \times O(n^{1.58})$ quasipolynomial
outerplanar	n [1]			
max. outerp.	n [1]	n [1]	$3n/2$ [6]	$O(n) \times O(n^2)$
3-trees	$2n$ [1]	$2n$ [1]		
2-connected	$2n$ [1]			
3-connected	$2n$ [1]	$5n/2$ [1]		
cubic 3-conn.	$n/2$ [3]	$n/2$ [2]	$n/2$ [2]	$O(n) \times O(n)$
triangulation	$2n$ [4]	$7n/3$ [4]	$8n/3$ [5]	$2^{O}(n \log n)$
4-conn. triang.	$2n$ [4]	$9n/3$ [4]		
planar	$2n$ [4]	$8n/3$ [4]		

[1] Dujmović et al. 2007 [2] Igamberdiev et al. 2015 [3] Mondal et al. 2013
[4] Durocher & Mondal 2014 [5] Mondal 2016 [6] Hültenschmidt et al. 2017

(Some) Known Results

Class	Segments		Grid Segments	
	Lower	Upper	Segm.	Area
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$3n/4$ [6] $\vartheta/2$ [6]	$O(n^2) \times O(n^{1.58})$ quasipolynomial
outerplanar	n [1]			
max. outerp.	n [1]	n [1]	$3n/2$ [6]	$O(n) \times O(n^2)$
3-trees	$2n$ [1]	$2n$ [1]	$8n/3$ [6]	$O(n) \times O(n^2)$
2-connected	$2n$ [1]			
3-connected	$2n$ [1]	$5n/2$ [1]		
cubic 3-conn.	$n/2$ [3]	$n/2$ [2]	$n/2$ [2]	$O(n) \times O(n)$
triangulation	$2n$ [4]	$7n/3$ [4]	$8n/3$ [5]	$2^{O}(n \log n)$
4-conn. triang.	$2n$ [4]	$9n/3$ [4]		
planar	$2n$ [4]	$8n/3$ [4]		

[1] Dujmović et al. 2007 [2] Igamberdiev et al. 2015 [3] Mondal et al. 2013
[4] Durocher & Mondal 2014 [5] Mondal 2016 [6] Hültenschmidt et al. 2017

(Some) Known Results

Class	Segments: Lower		Segments: Upper		Grid Segments: Segm.		Grid Segments: Area
tree	$\vartheta/2$	[1]	$\vartheta/2$	[1]	$3n/4$	[6]	$O(n^2) \times O(n^{1.58})$
					$\vartheta/2$	[6]	quasipolynomial
outerplanar	n	[1]					
max. outerp.	n	[1]	n	[1]	$3n/2$	[6]	$O(n) \times O(n^2)$
3-trees	$2n$	[1]	$2n$	[1]	$8n/3$	[6]	$O(n) \times O(n^2)$
2-connected	$2n$	[1]					
3-connected	$2n$	[1]	$5n/2$	[1]			
cubic 3-conn.	$n/2$	[3]	$n/2$	[2]	$n/2$	[2]	$O(n) \times O(n)$
triangulation	$2n$	[4]	$7n/3$	[4]	$8n/3$	[5]	$2^{O}(n \log n)$
4-conn. triang.	$2n$	[4]	$9n/3$	[4]			
planar	$2n$	[4]	$8n/3$	[4]			

[1] Dujmović et al. 2007 [2] Igamberdiev et al. 2015 [3] Mondal et al. 2013

[4] Durocher & Mondal 2014 [5] Mondal 2016 [6] Hültenschmidt et al. 2017

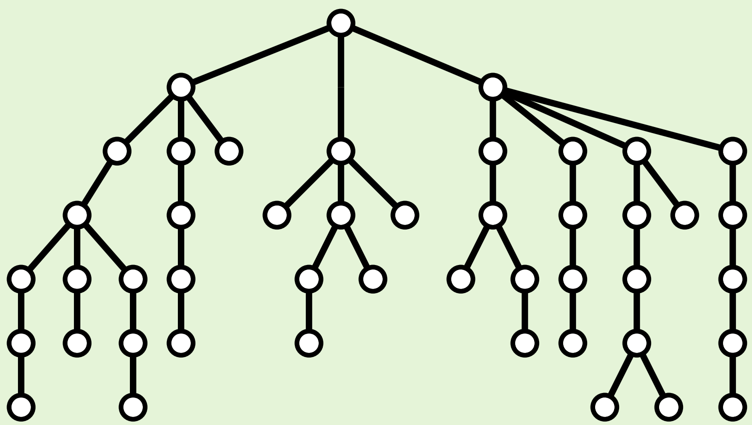
(Some) Known Results

Class	Segments: Lower	Segments: Upper	Grid Segments: Segm.	Grid Segments: Area
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$3n/4$ [6]	$O(n^2) \times O(n^{1.58})$
			$\vartheta/2$ [6]	quasipolynomial
outerplanar	n [1]			
max. outerp.	n [1]	n [1]	$3n/2$ [6]	$O(n) \times O(n^2)$
3-trees	$2n$ [1]	$2n$ [1]	$8n/3$ [6]	$O(n) \times O(n^2)$
2-connected	$2n$ [1]			
3-connected	$2n$ [1]	$5n/2$ [1]		
cubic 3-conn.	$n/2$ [3]	$n/2$ [2]	$n/2$ [2]	$O(n) \times O(n)$
triangulation	$2n$ [4]	$7n/3$ [4]	$8n/3$ [5]	$2^{O}(n \log n)$
4-conn. triang.	$2n$ [4]	$9n/3$ [4]		
planar	$2n$ [4]	$8n/3$ [4]		

[1] Dujmović et al. 2007 [2] Igamberdiev et al. 2015 [3] Mondal et al. 2013
[4] Durocher & Mondal 2014 [5] Mondal 2016 [6] Hültenschmidt et al. 2017

Tree Drawings

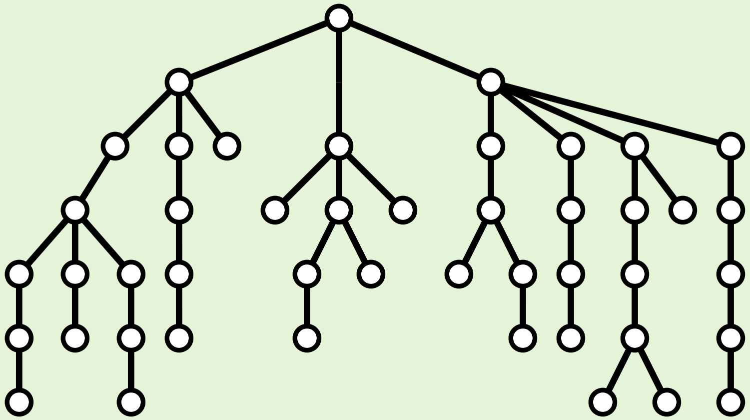
Tree T
n vtcs



Tree Drawings

Tree T

n vtcs

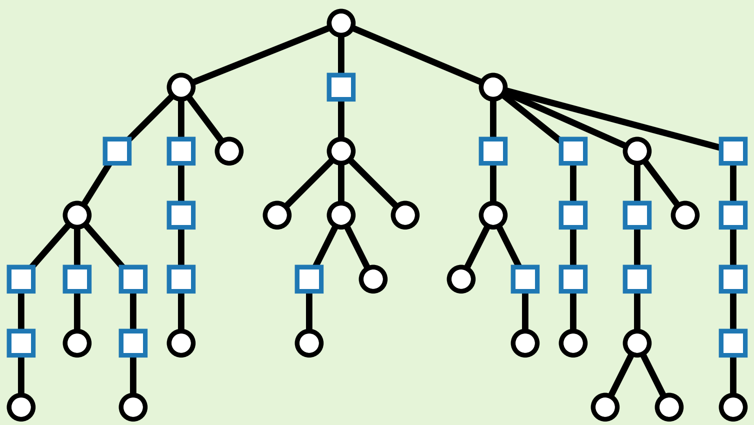


β deg-2 vtcs

Tree Drawings

Tree T

n vtcs

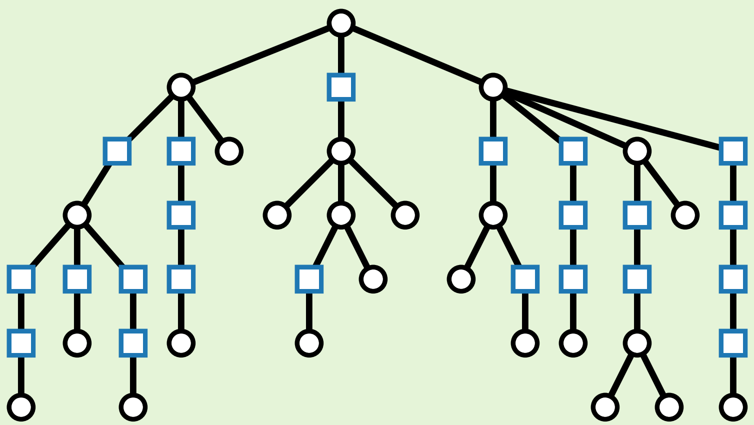


β deg-2 vtcs

Tree Drawings

Tree T

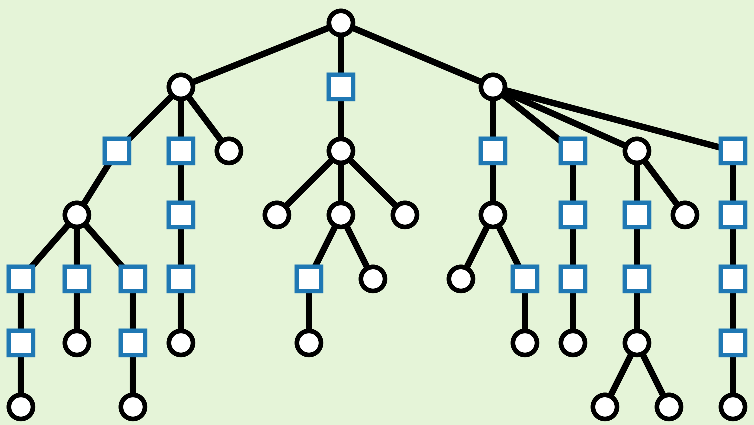
n vtcs



Remove β deg-2 vtcs

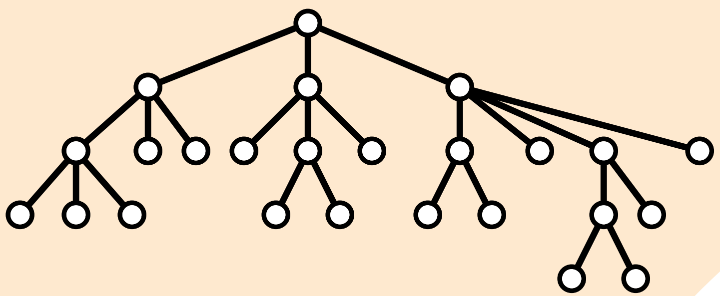
Tree Drawings

Tree T
n vtcs



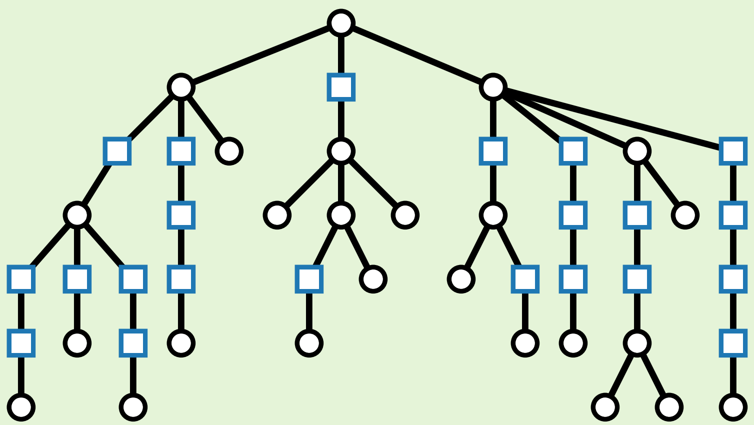
Remove β deg-2 vtcs

$\Rightarrow$ Tree T'
$n-\beta$ vtcs



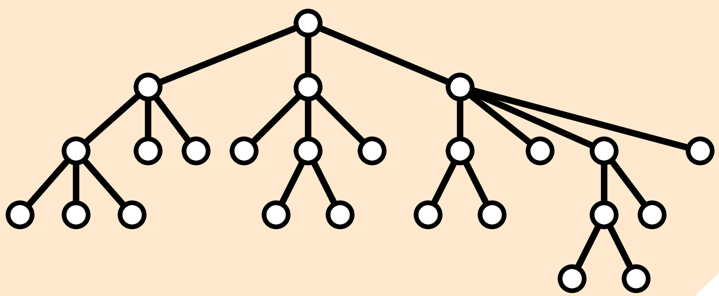
Tree Drawings

Tree T
n vtcs



Remove β deg-2 vtcs

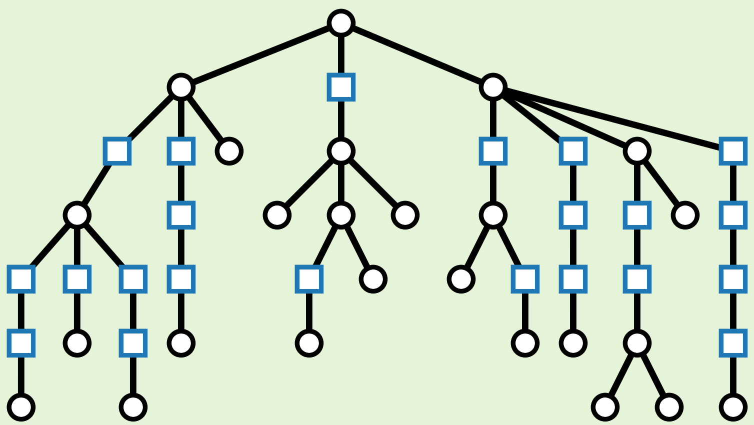
$\Rightarrow$ Tree T'
$n - \beta$ vtcs



α leaves

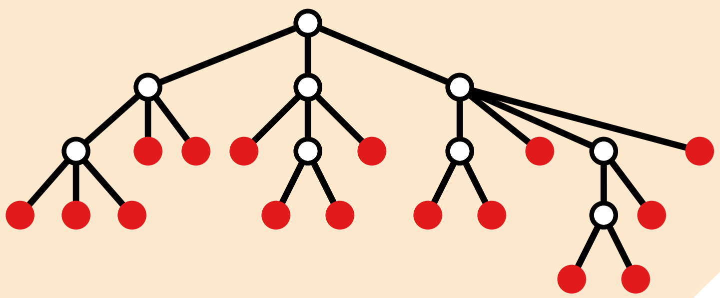
Tree Drawings

Tree T
n vtcs



Remove β deg-2 vtcs

$\Rightarrow$ Tree T'
$n-\beta$ vtcs

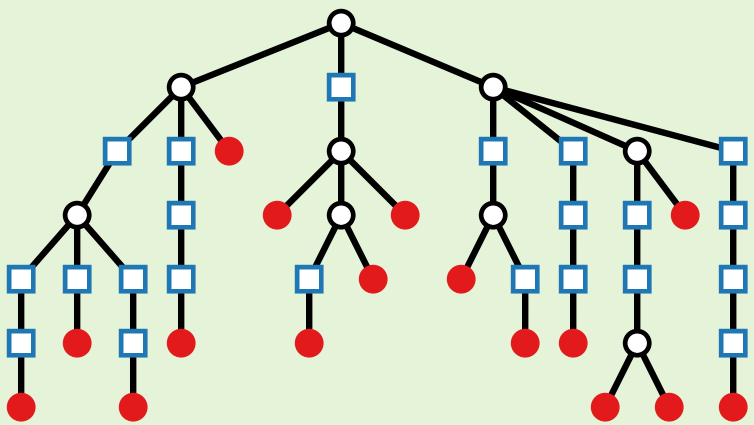


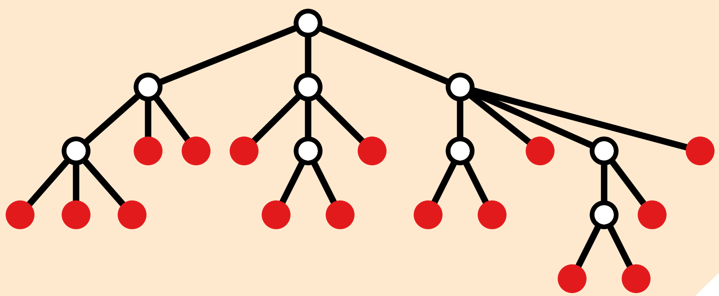
α leaves

Tree Drawings

Tree T
n vtcs

$\Rightarrow$ Tree T'
$n - \beta$ vtcs





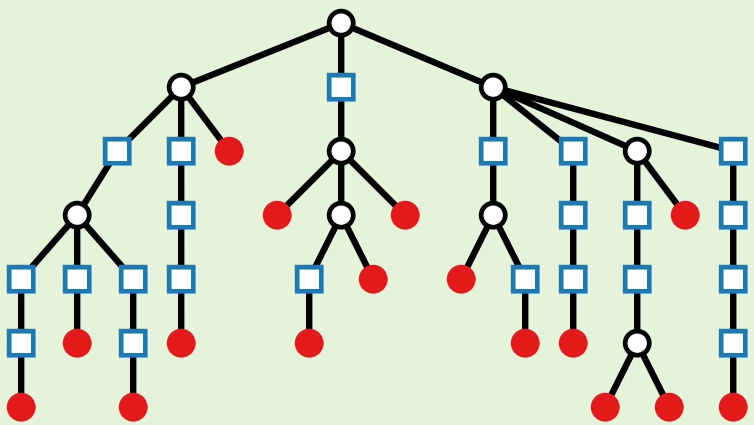
Remove β deg-2 vtcs

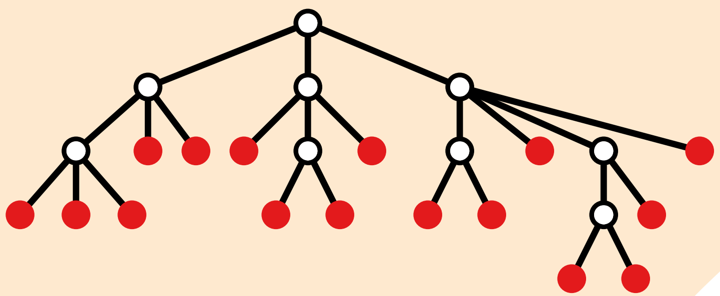
α leaves

Tree Drawings

Tree T
n vtcs

$\Rightarrow$ Tree T'
$n - \beta$ vtcs



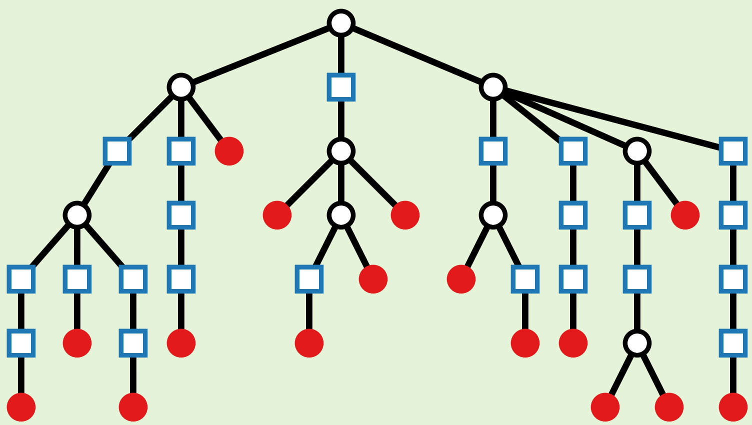


Remove β deg-2 vtcs

Remove α leaves

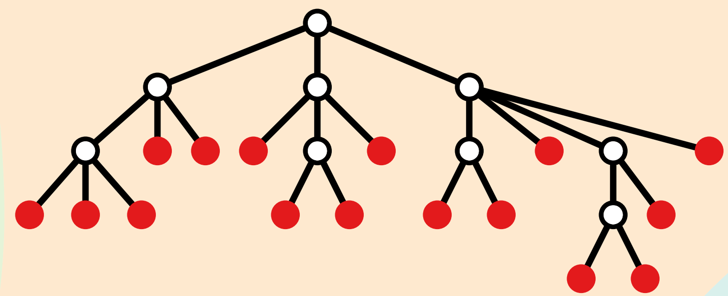
Tree Drawings

Tree T
n vtcs



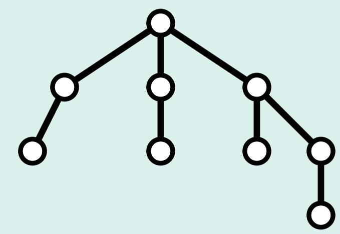
Remove β deg-2 vtcs

$\Rightarrow$ Tree T'
$n-\beta$ vtcs



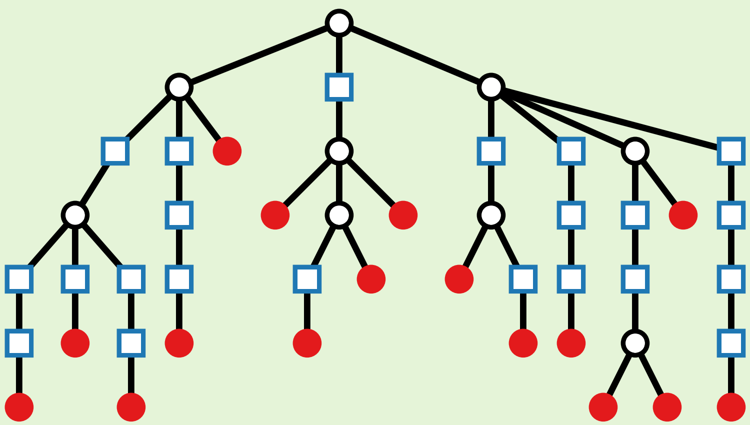
Remove α leaves

$\Rightarrow$ Tree T''
$n-\alpha-\beta$ vtcs



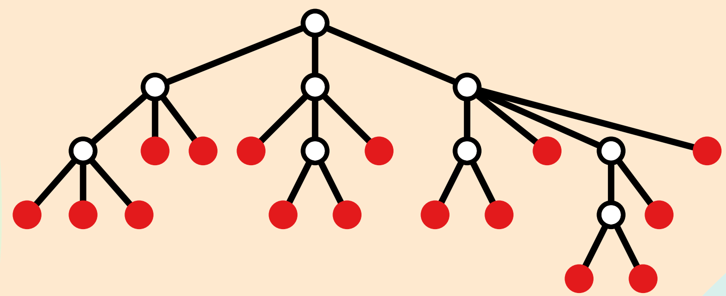
Tree Drawings

Tree T
n vtcs



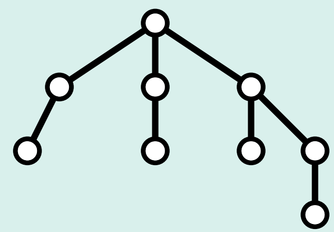
Remove β deg-2 vtcs

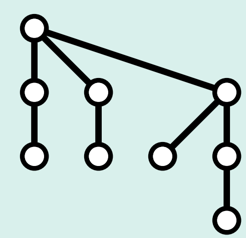
$\Rightarrow$ Tree T'
$n-\beta$ vtcs



Remove α leaves

$\Rightarrow$ Tree T''
$n-\alpha-\beta$ vtcs

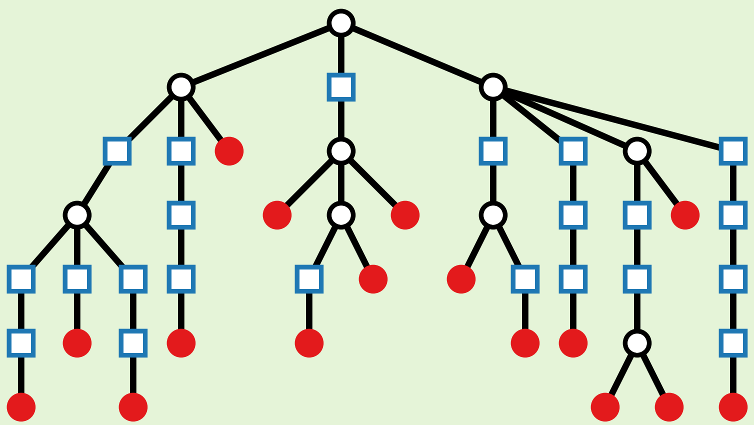




$n-\alpha-\beta$
segment

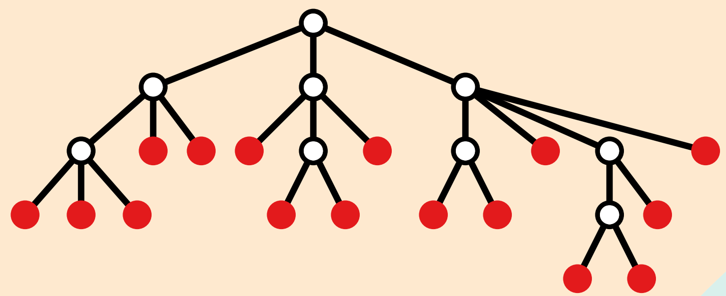
Tree Drawings

Tree T
n vtcs



Remove β deg-2 vtcs

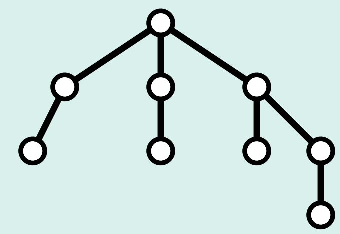
$\Rightarrow$ Tree T'
$n-\beta$ vtcs

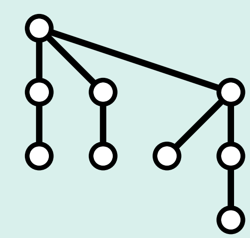


Remove α leaves

+ $\alpha/2$ segments

$\Rightarrow$ Tree T''
$n-\alpha-\beta$ vtcs

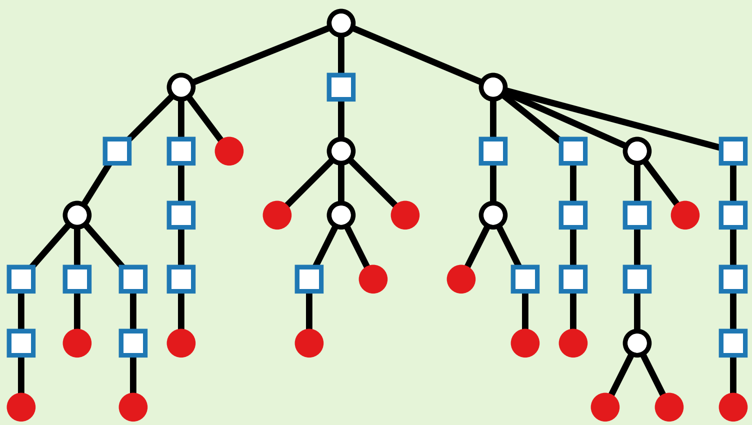




$n-\alpha-\beta$
segment

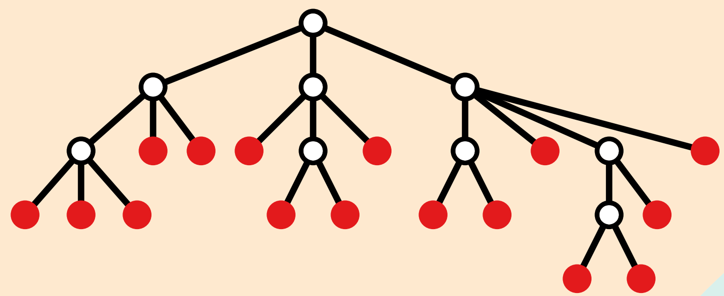
Tree Drawings

Tree T
n vtcs



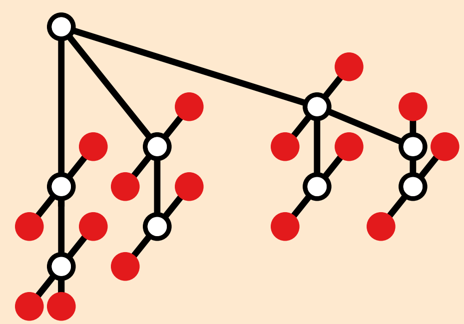
Remove β deg-2 vtcs

$\Rightarrow$ Tree T'
$n-\beta$ vtcs



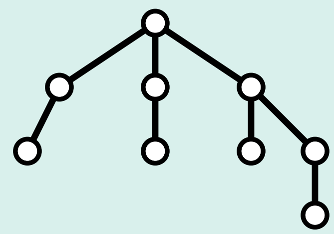
Remove α leaves

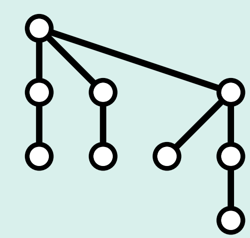
$+\ \alpha/2$ segments



$n-\alpha/2-\beta$
segments

$\Rightarrow$ Tree T''
$n-\alpha-\beta$ vtcs

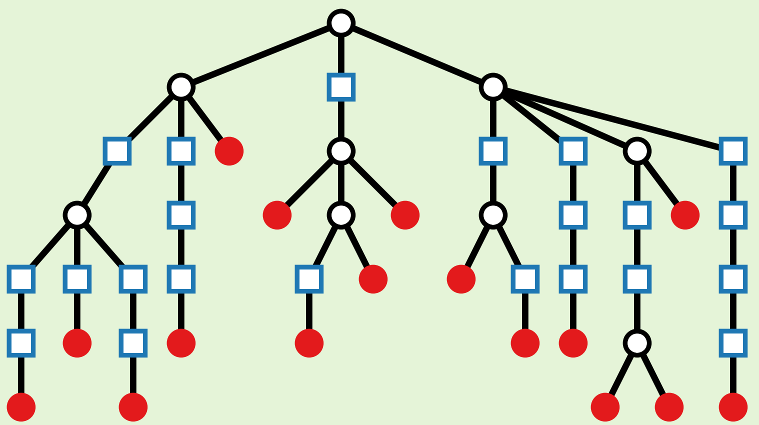




$n-\alpha-\beta$
segment

Tree Drawings

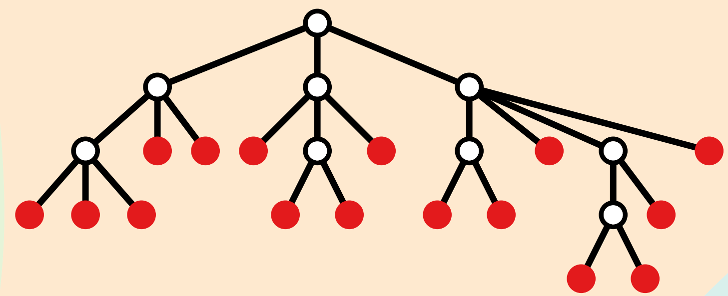
Tree T
n vtcs



Remove β deg-2 vtcs

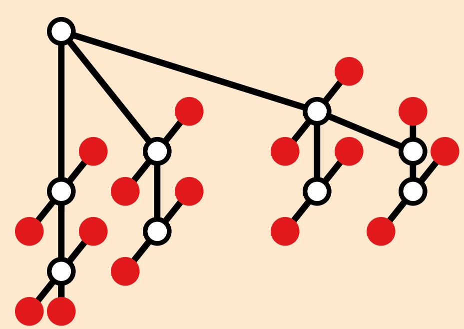
+ 0 segments

$\Rightarrow$ Tree T'
$n-\beta$ vtcs



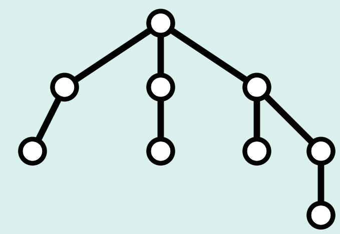
Remove α leaves

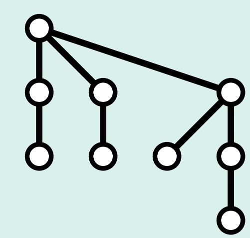
+ $\alpha/2$ segments



$n-\alpha/2-\beta$
segments

$\Rightarrow$ Tree T''
$n-\alpha-\beta$ vtcs

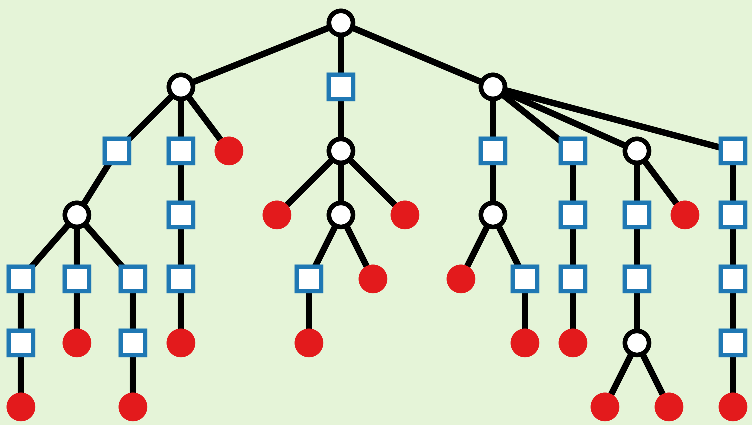




$n-\alpha-\beta$
segment

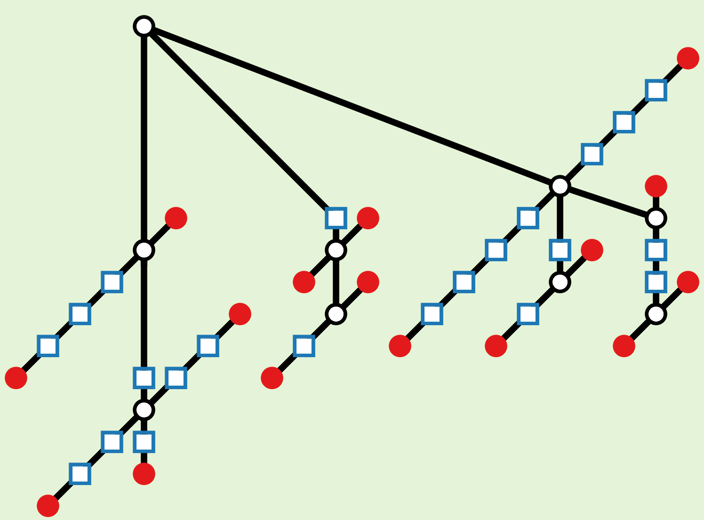
Tree Drawings

Tree T
n vtcs

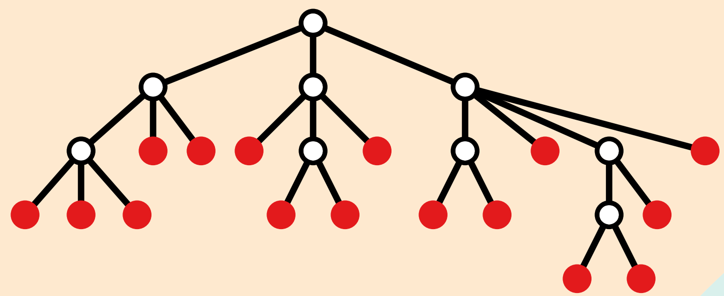


Remove β deg-2 vtcs

+ 0 segments

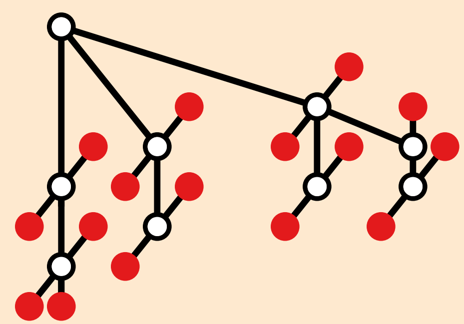


$\Rightarrow$ Tree T'
$n-\beta$ vtcs



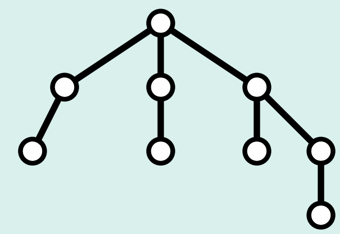
Remove α leaves

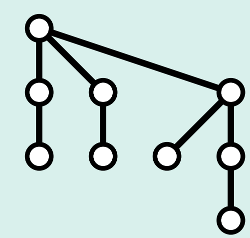
+ $\alpha/2$ segments



$n-\alpha/2-\beta$
segments

$\Rightarrow$ Tree T''
$n-\alpha-\beta$ vtcs

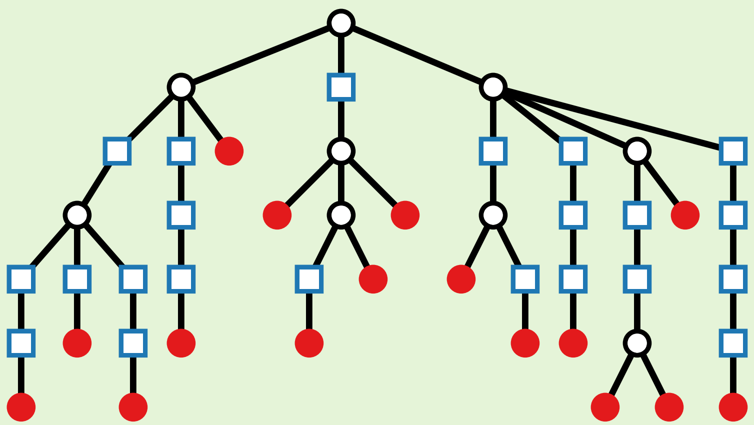




$n-\alpha-\beta$
segment

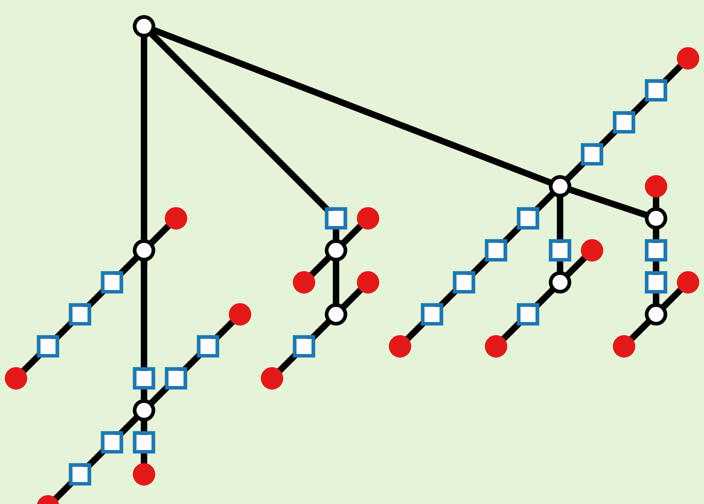
Tree Drawings

Tree T
n vtcs

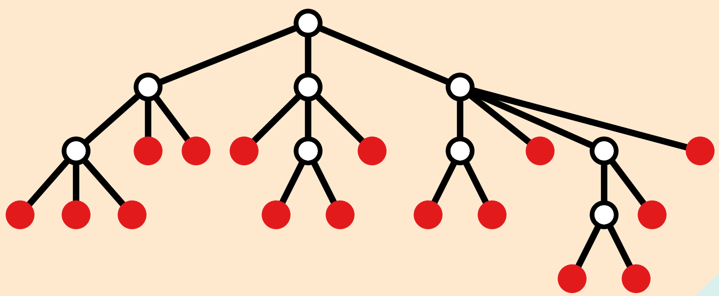


Remove β deg-2 vtcs

+ 0 segments

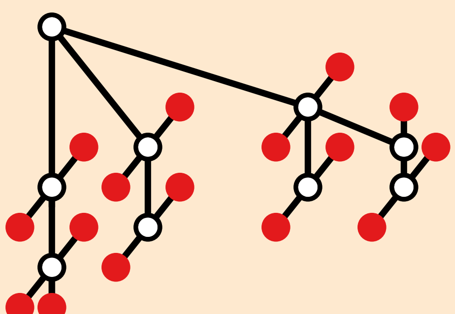


$\Rightarrow$ Tree T'
$n-\beta$ vtcs



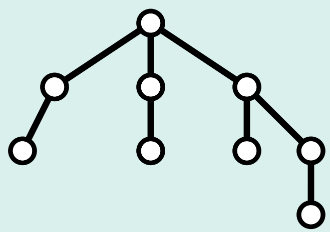
Remove α leaves

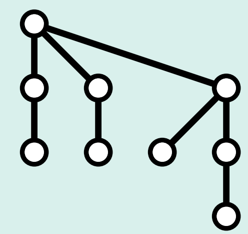
+ $\alpha/2$ segments



$n-\alpha/2-\beta$
segments

$\Rightarrow$ Tree T''
$n-\alpha-\beta$ vtcs

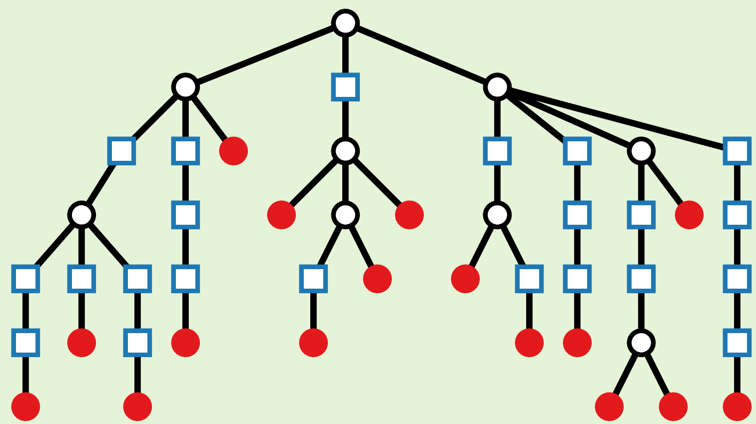




$n-\alpha-\beta$
segment

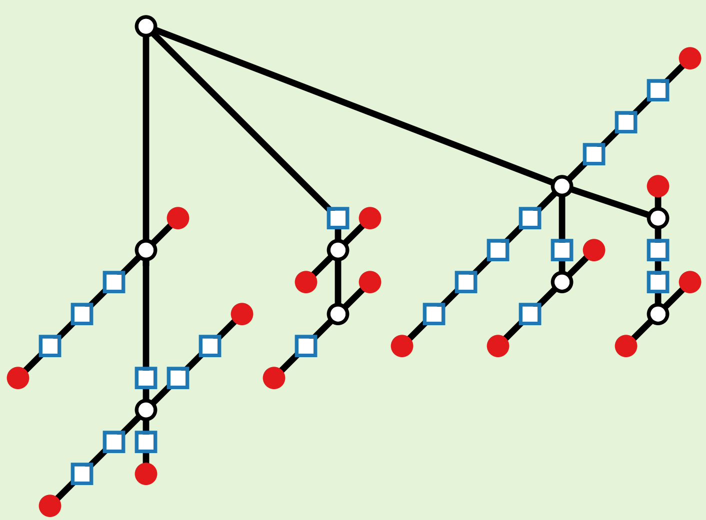
Tree Drawings

Tree T
n vtcs

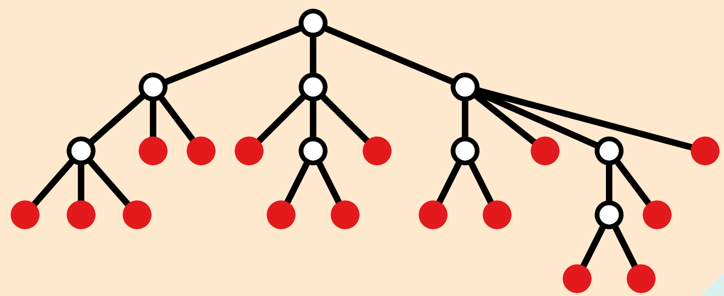


Remove β deg-2 vtcs

+ 0 segments

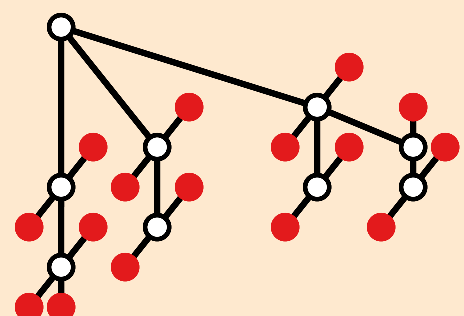


$\Rightarrow$ Tree T'
$n-\beta$ vtcs



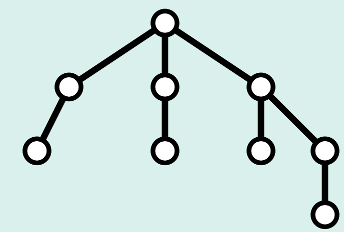
Remove α leaves

+ $\alpha/2$ segments

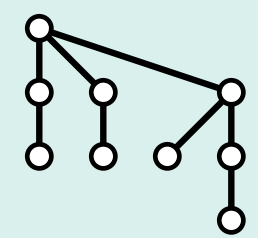


$n-\alpha/2-\beta$
segments

$\Rightarrow$ Tree T''
$n-\alpha-\beta$ vtcs



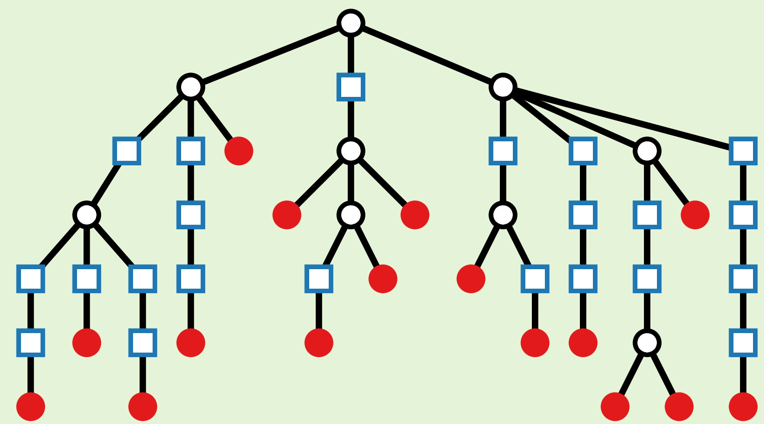
$\alpha > (n-\beta)/2$



$n-\alpha-\beta$
segment

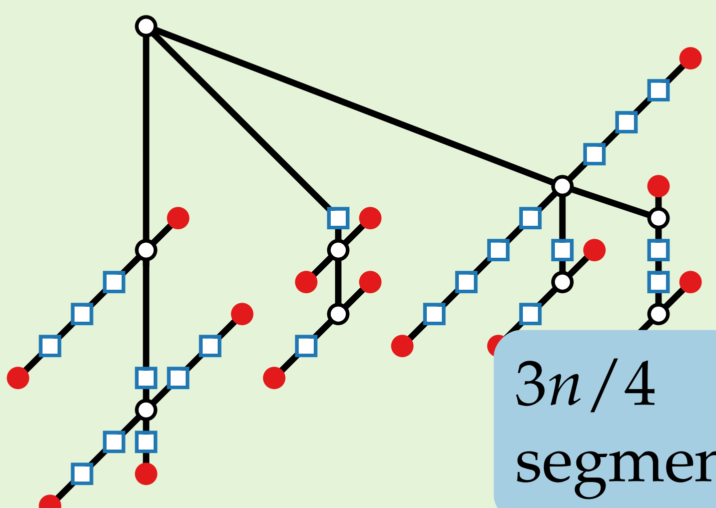
Tree Drawings

Tree T
n vtcs



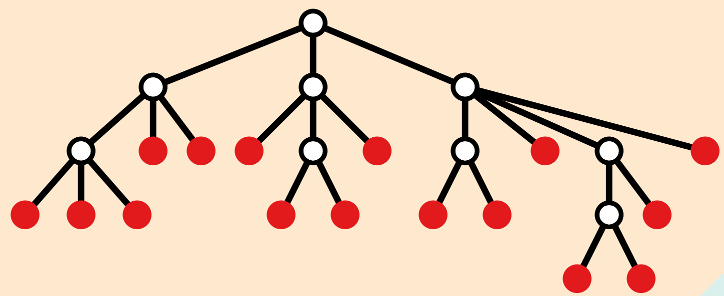
Remove β deg-2 vtcs

+ 0 segments



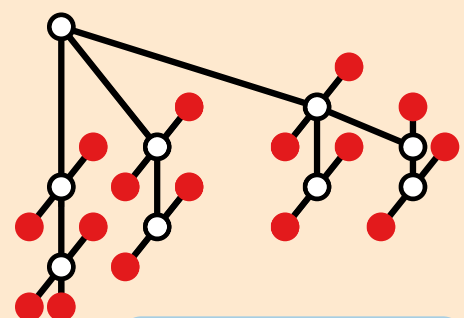
$3n/4$
segments

$\Rightarrow$ Tree T'
$n-\beta$ vtcs



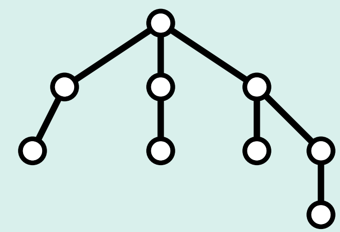
Remove α leaves

+ $\alpha/2$ segments

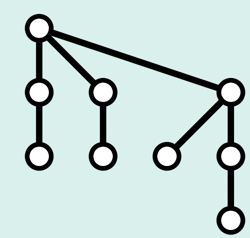


$n-\alpha/2-\beta$
segments

$\Rightarrow$ Tree T''
$n-\alpha-\beta$ vtcs

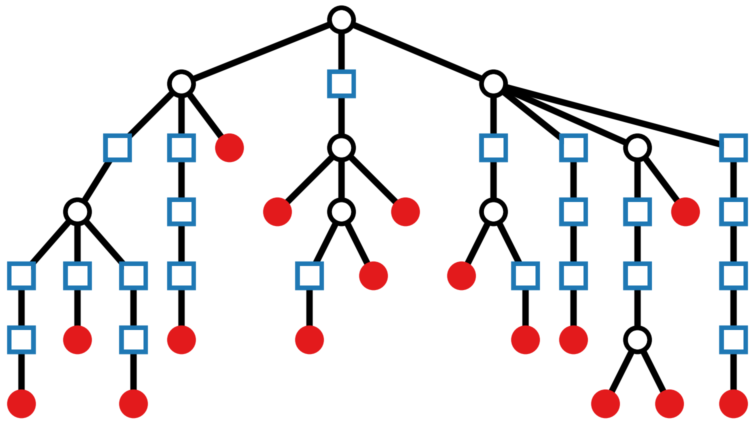


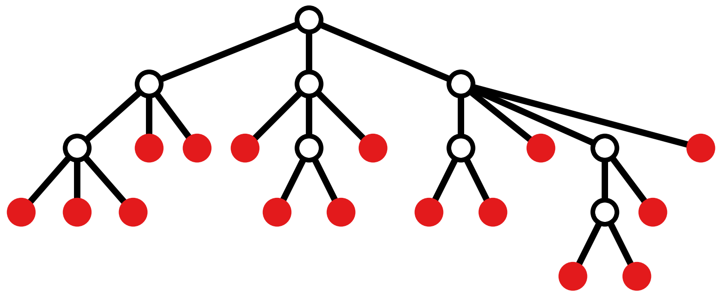
$\alpha > (n-\beta)/2$

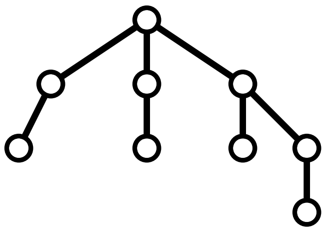


$n-\alpha-\beta$
segment

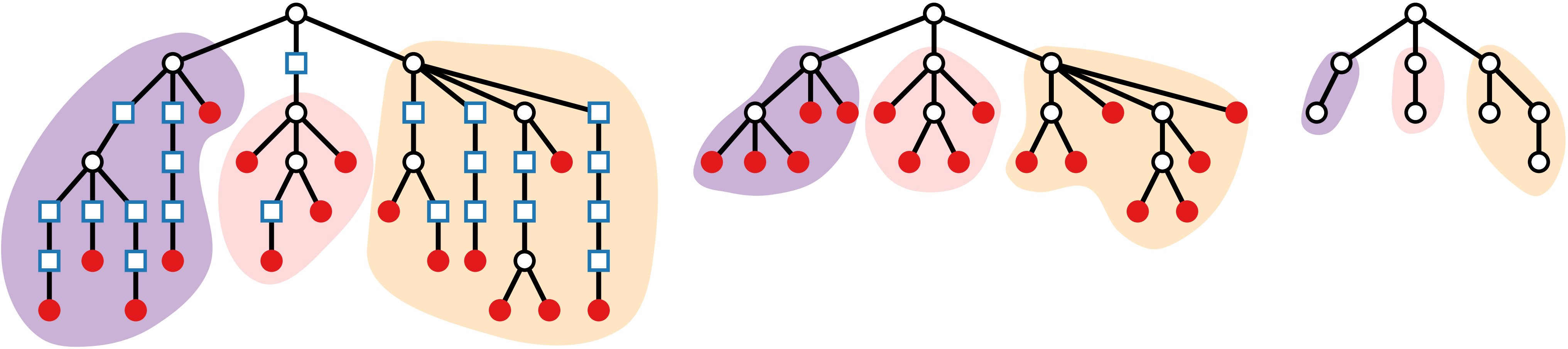
Tree Drawings





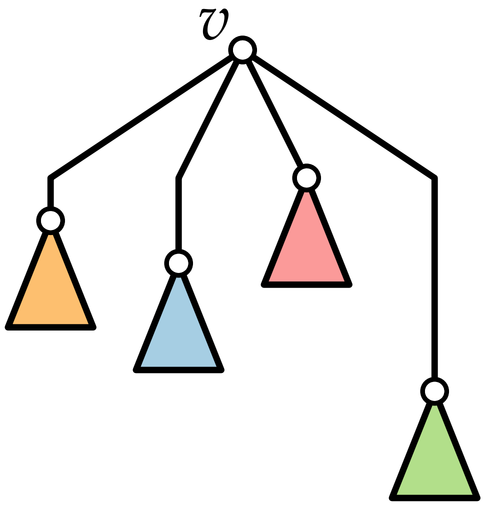


Tree Drawings

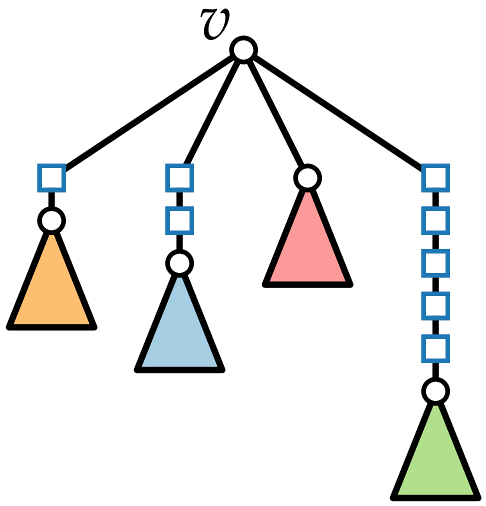


Tree Drawings

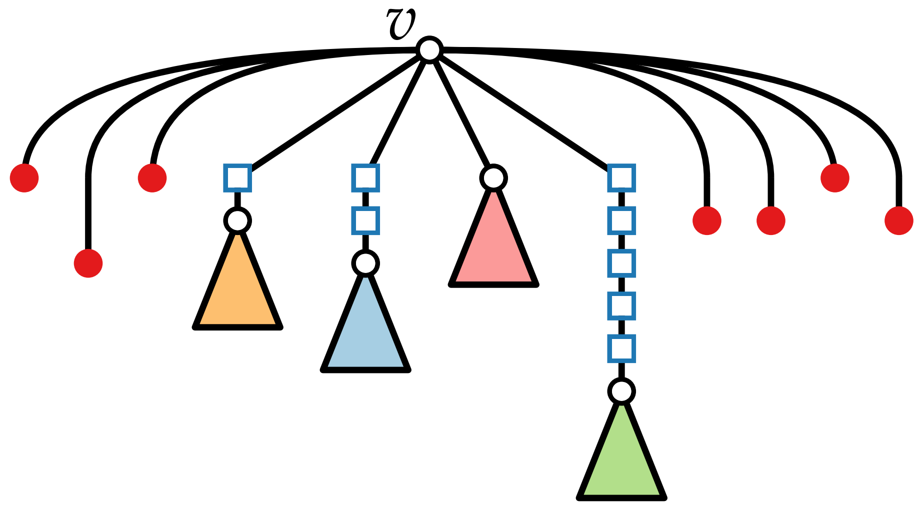
Tree Drawings



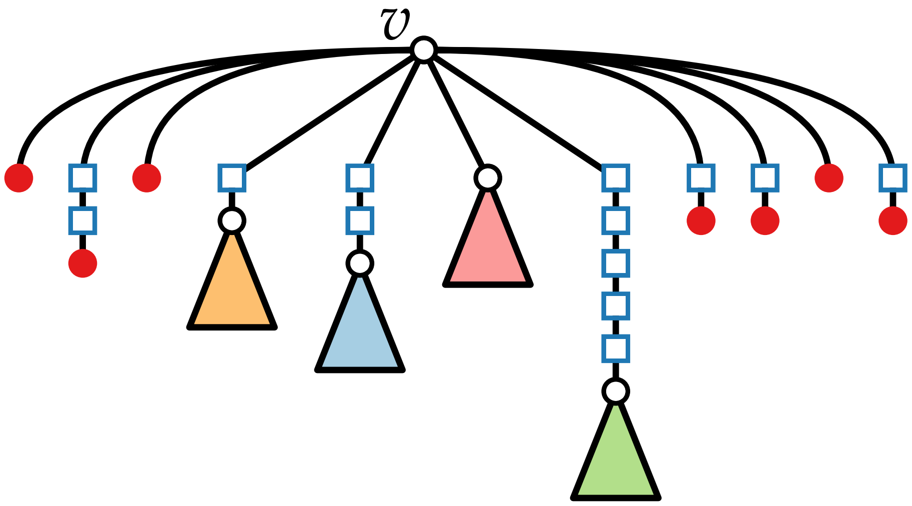
Tree Drawings



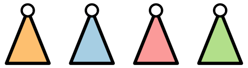
Tree Drawings

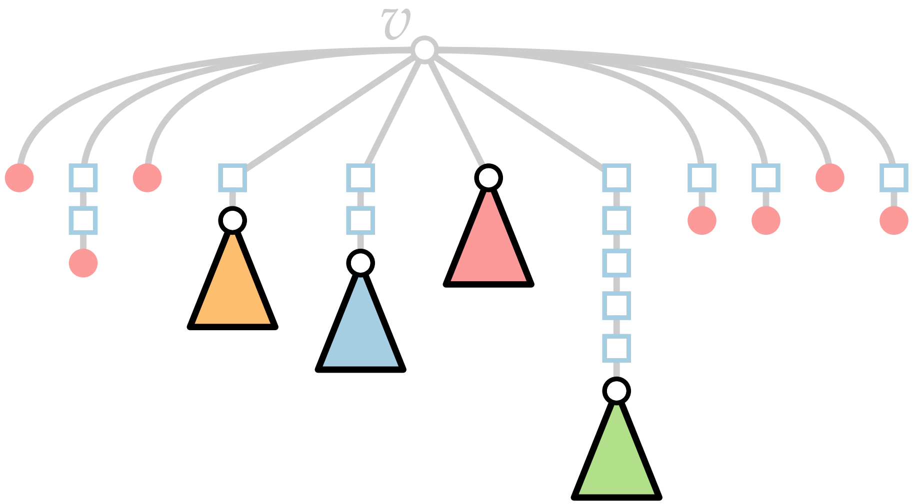


Tree Drawings

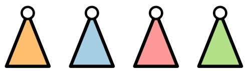


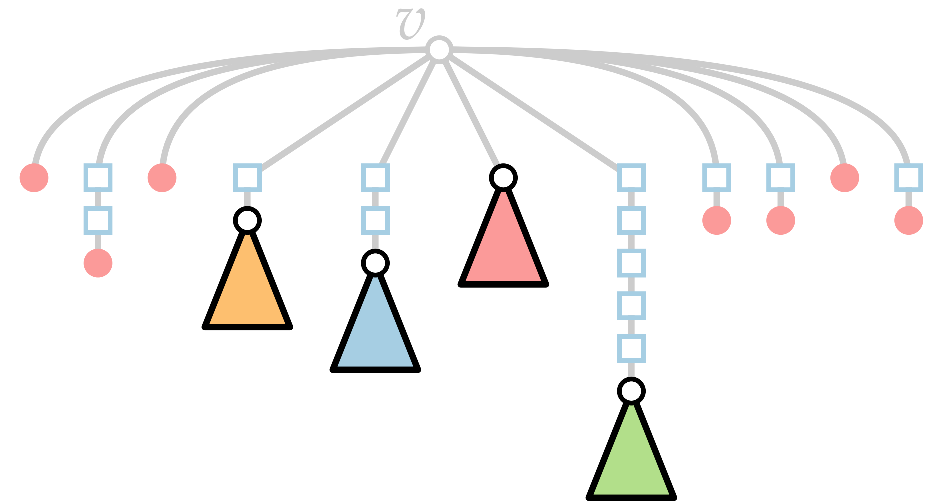
Tree Drawings

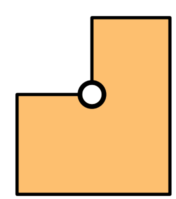
(1) Draw 



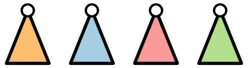
Tree Drawings

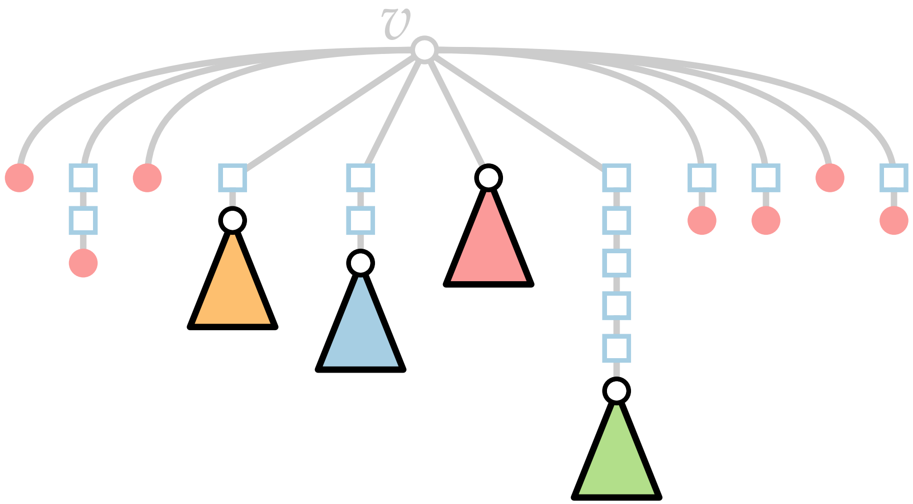
(1) Draw 

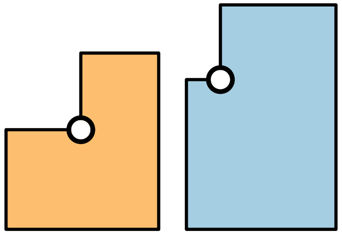




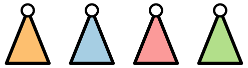
Tree Drawings

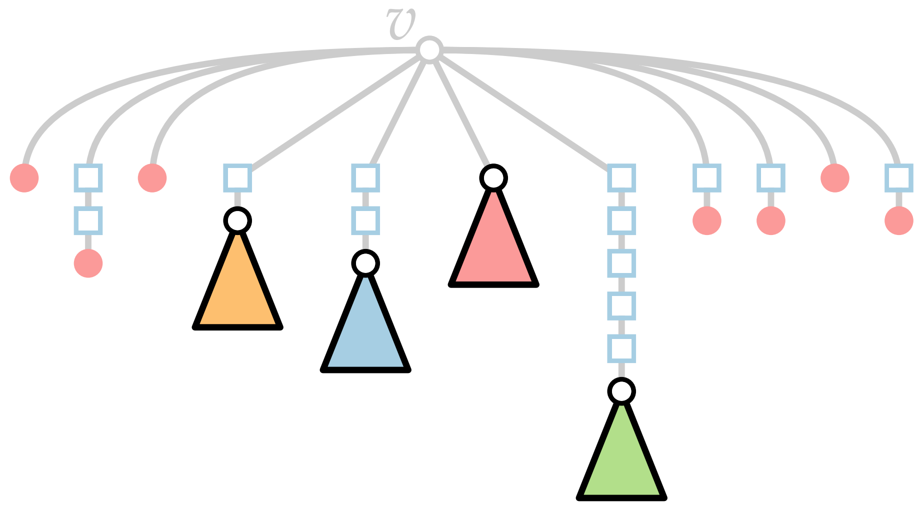
(1) Draw 

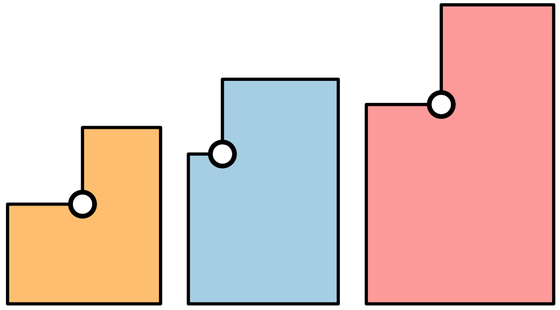




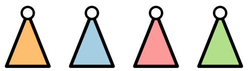
Tree Drawings

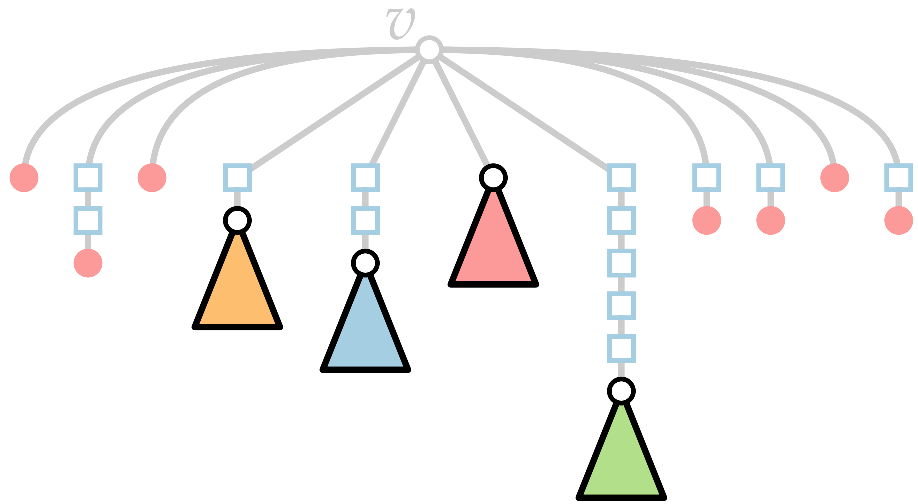
(1) Draw 

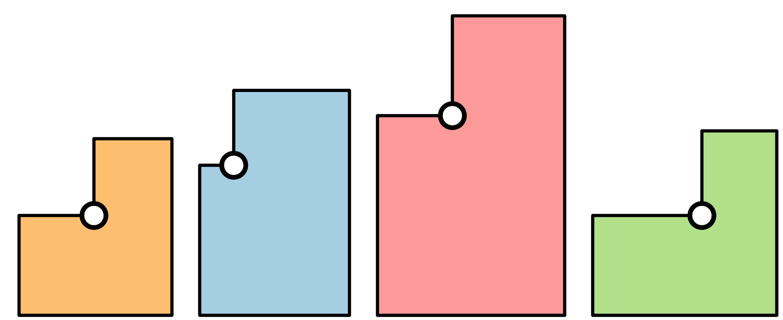




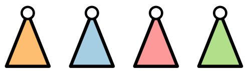
Tree Drawings

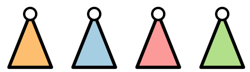
(1) Draw 

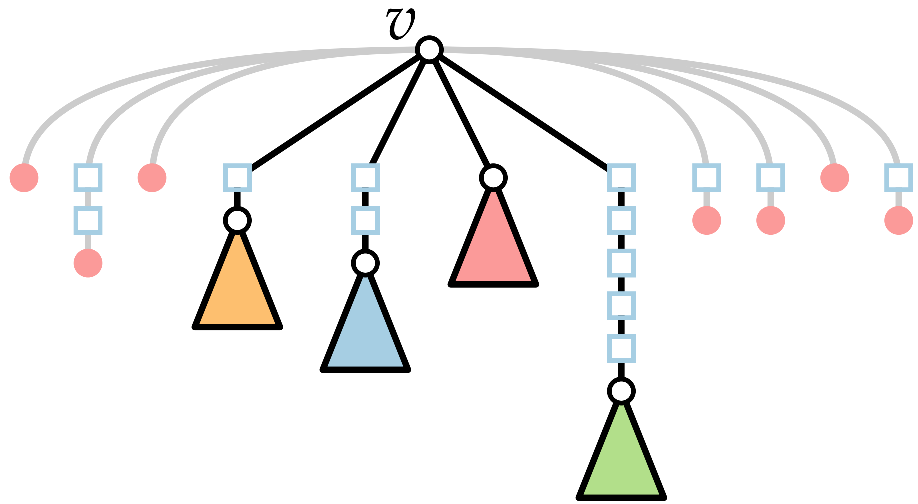


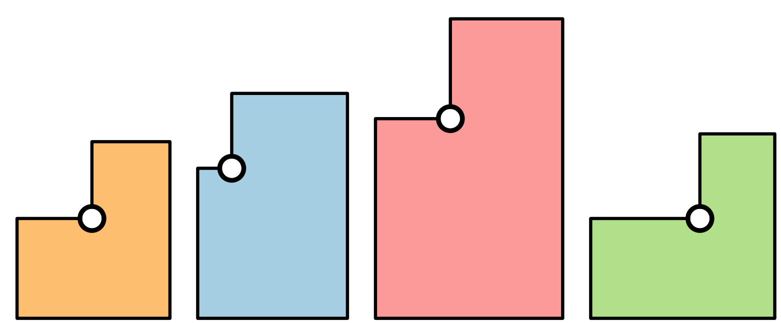


Tree Drawings

(1) Draw 

(2) Layout v + 

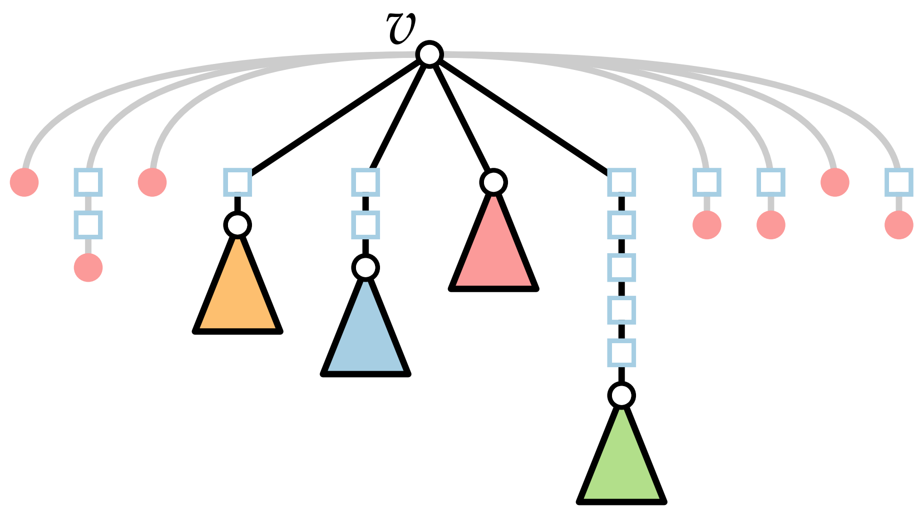


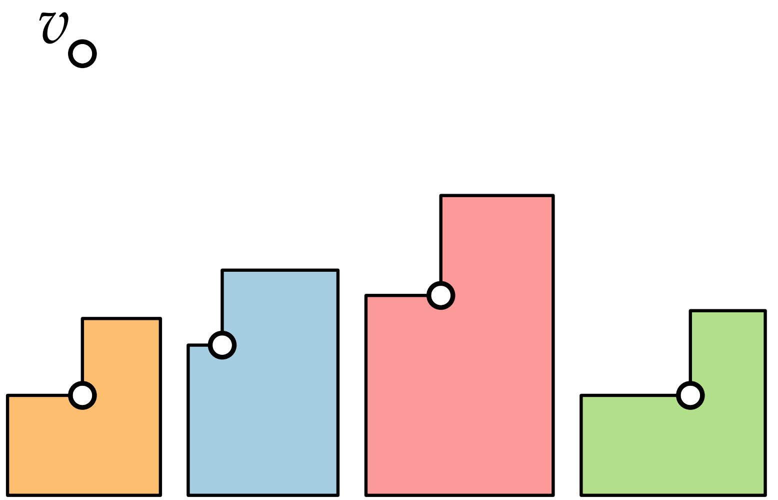


Tree Drawings

(1) Draw

(2) Layout v +

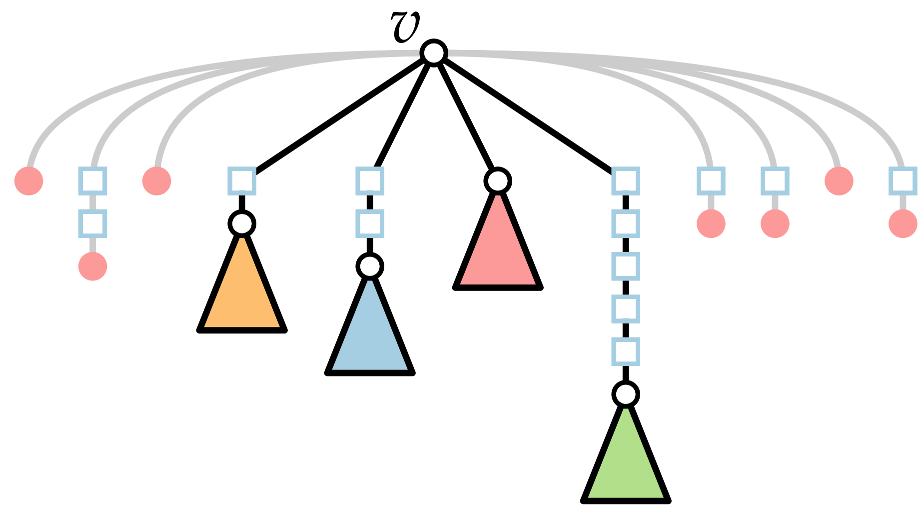


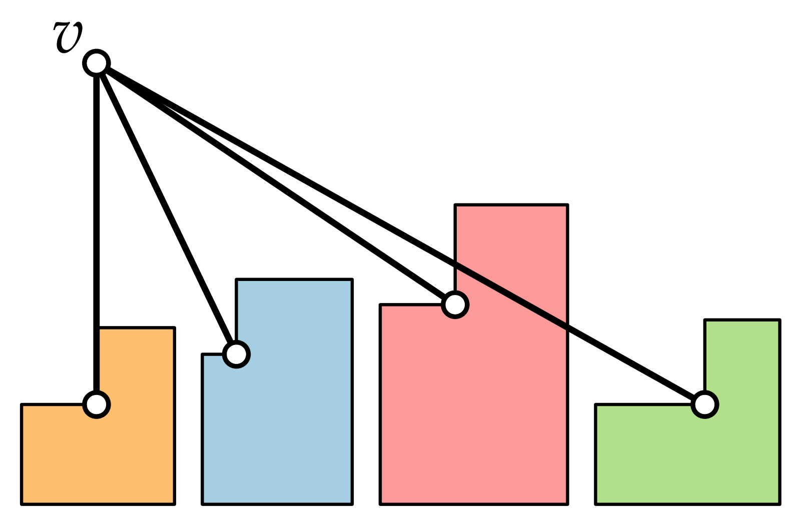


Tree Drawings

(1) Draw

(2) Layout v +

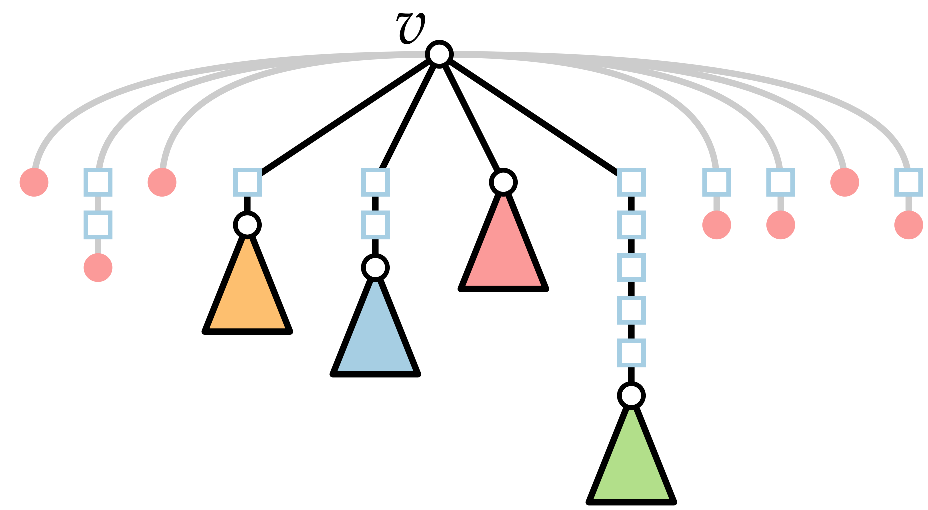


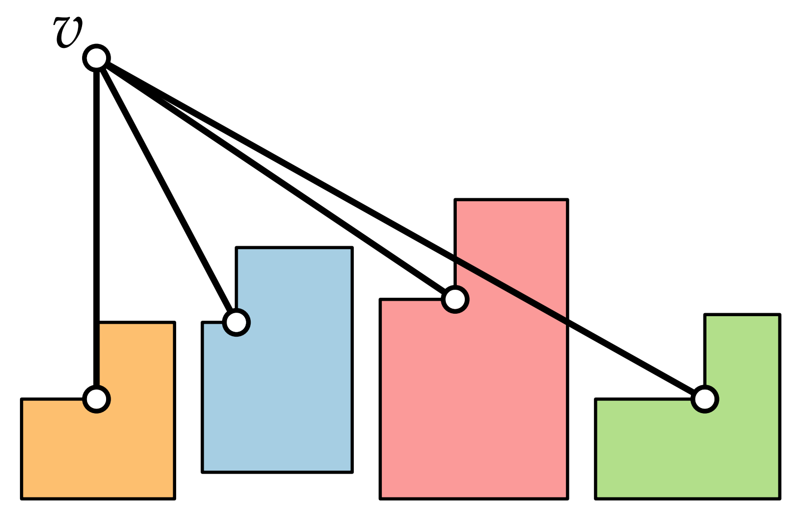


Tree Drawings

(1) Draw

(2) Layout v +

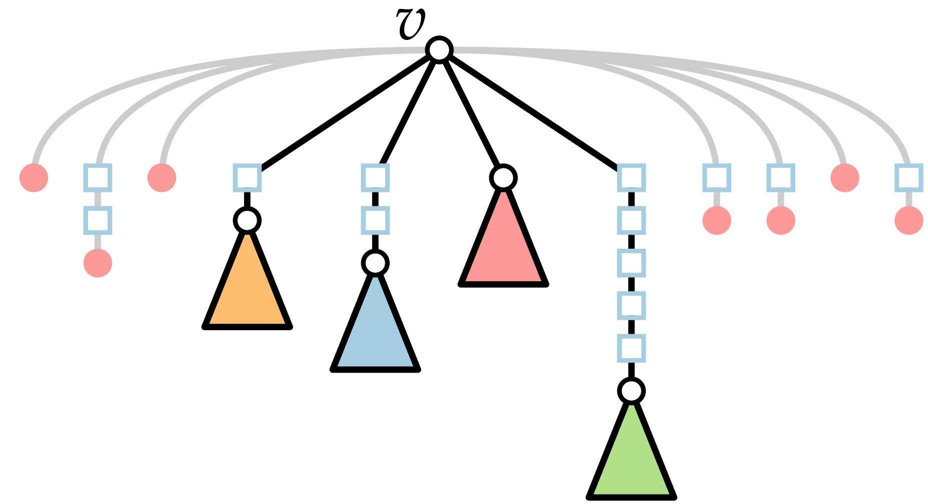


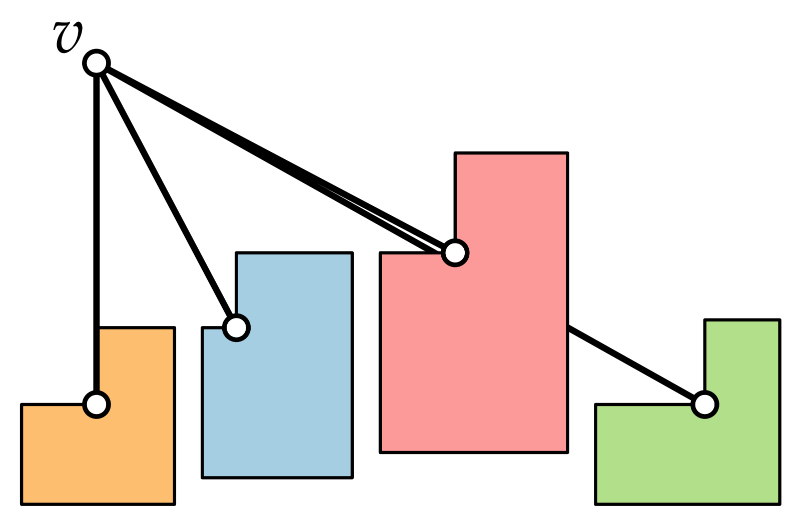


Tree Drawings

(1) Draw

(2) Layout v +

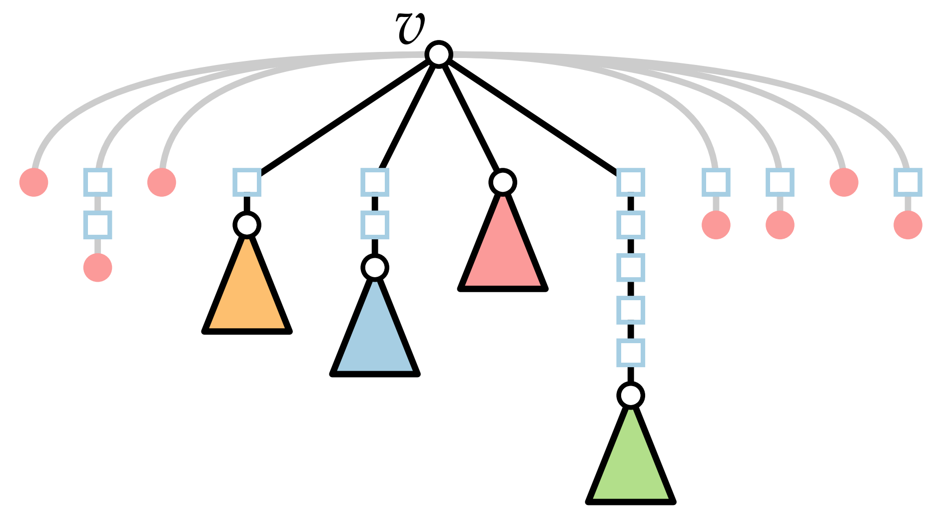


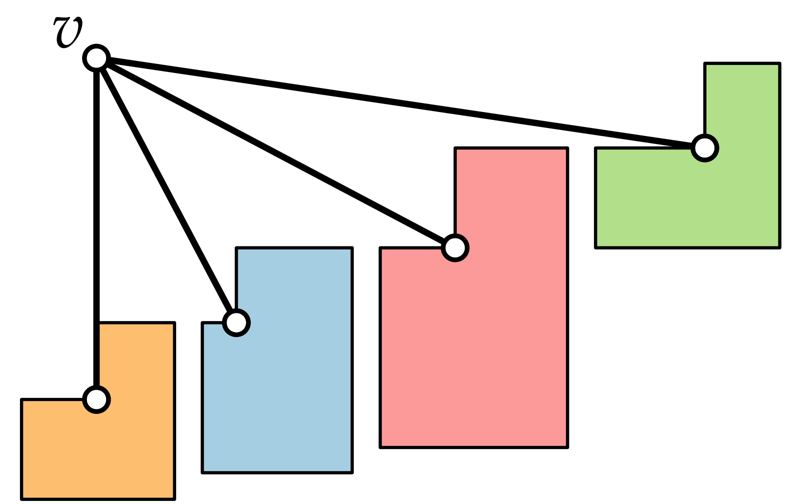


Tree Drawings

(1) Draw

(2) Layout v +



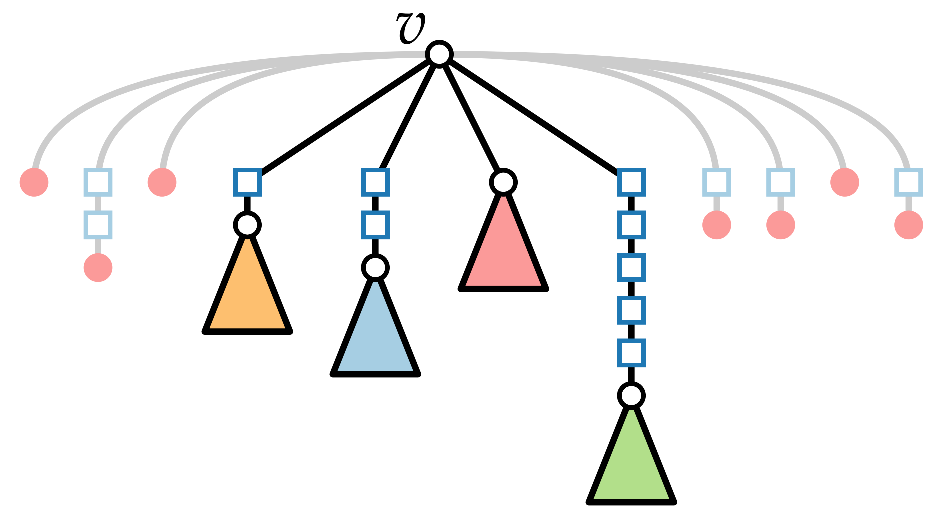


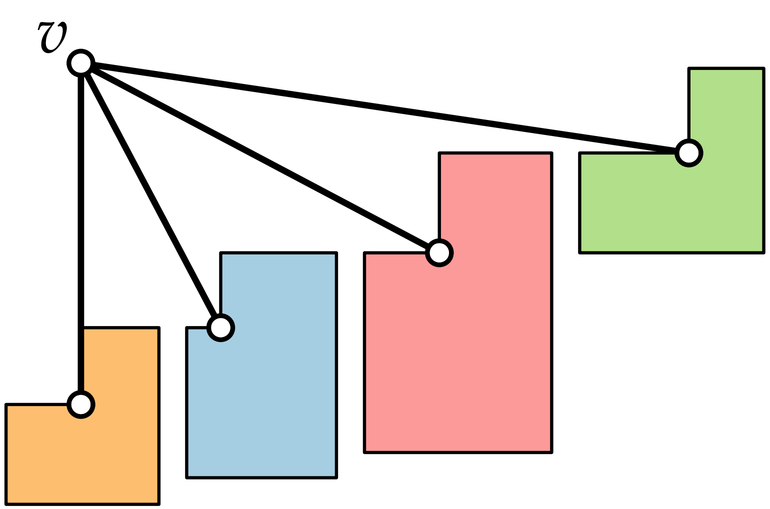
Tree Drawings

(1) Draw

(2) Layout v +

(3) Add



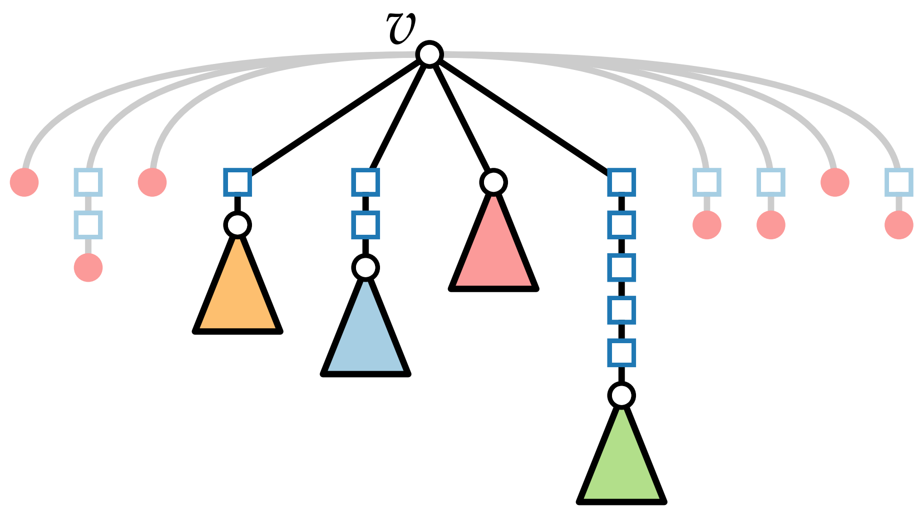


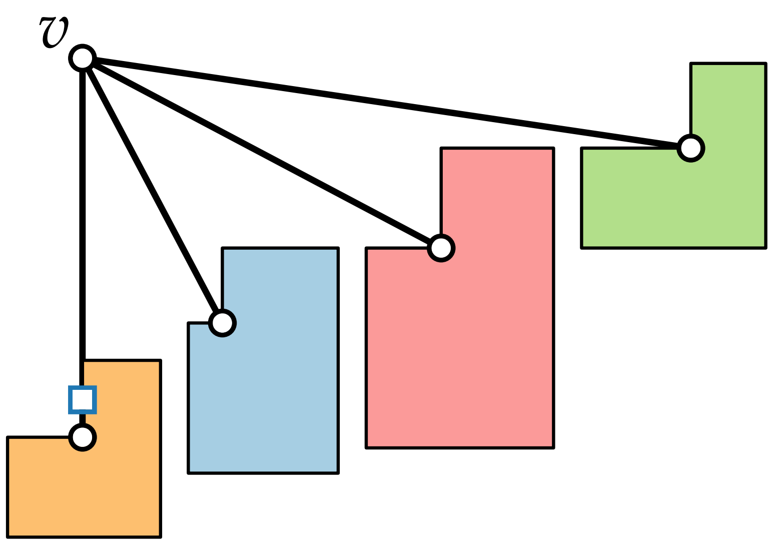
Tree Drawings

(1) Draw

(2) Layout v +

(3) Add



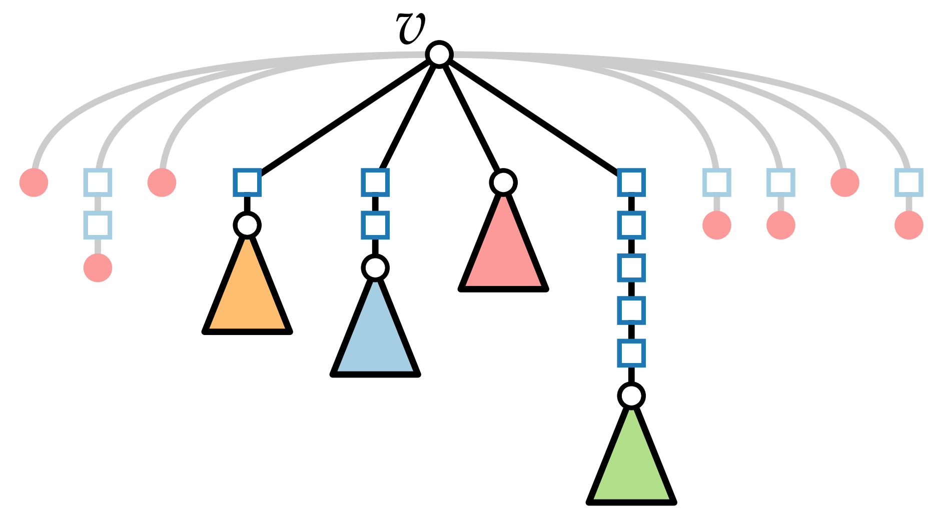


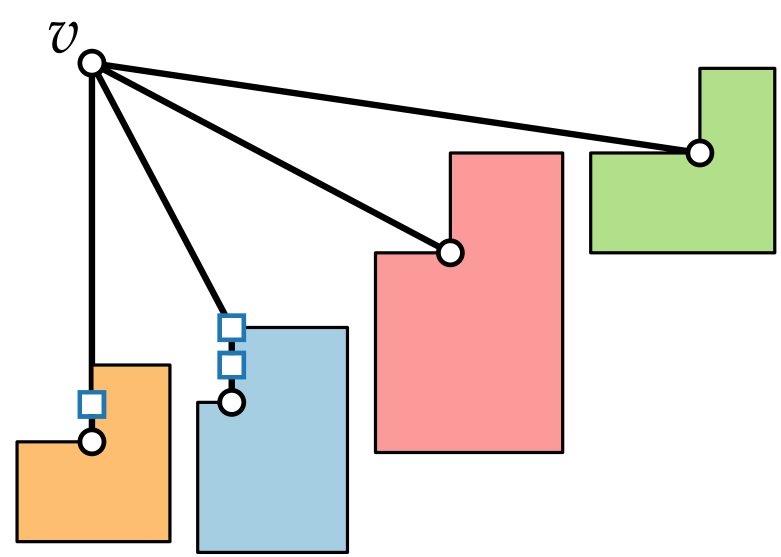
Tree Drawings

(1) Draw

(2) Layout v +

(3) Add



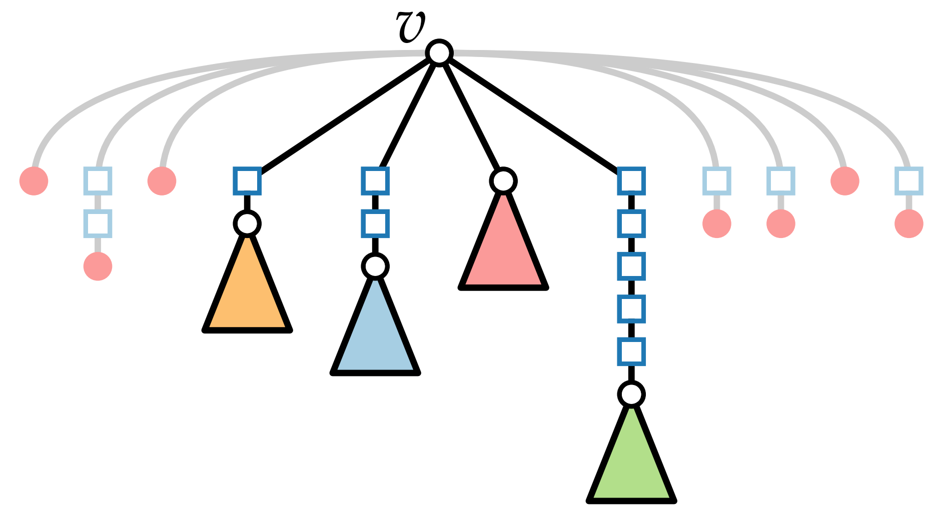


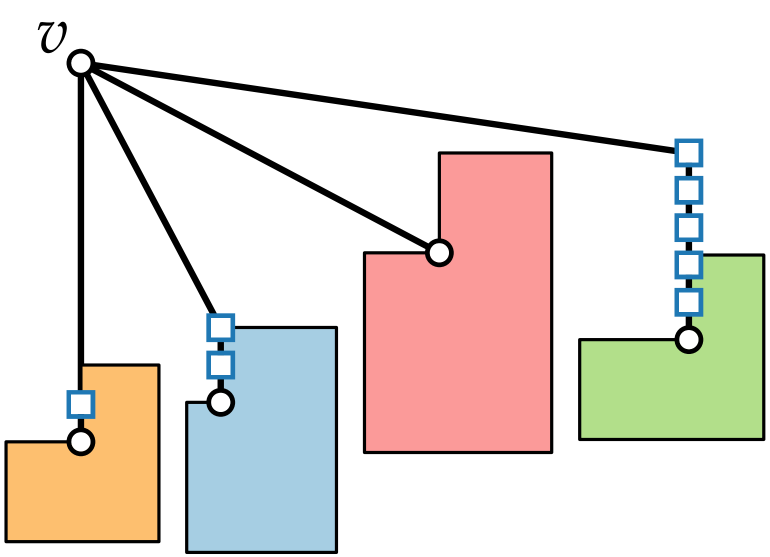
Tree Drawings

(1) Draw

(2) Layout v +

(3) Add





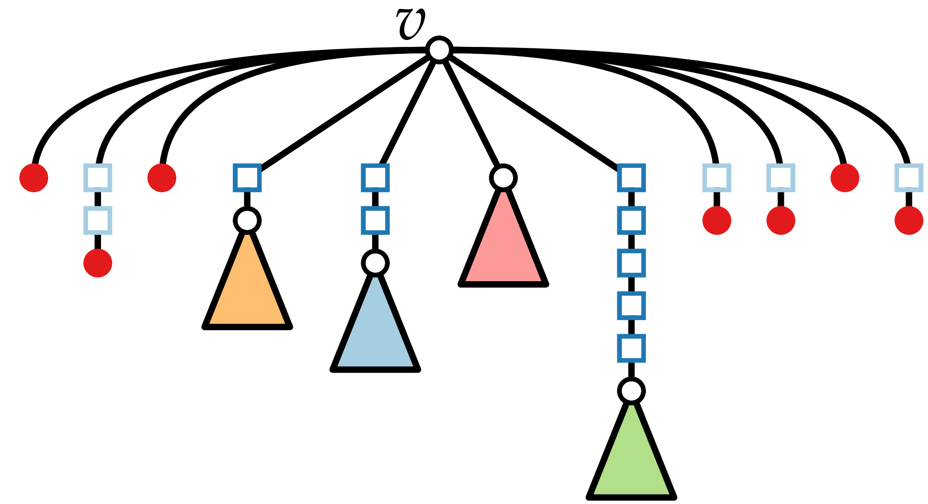
Tree Drawings

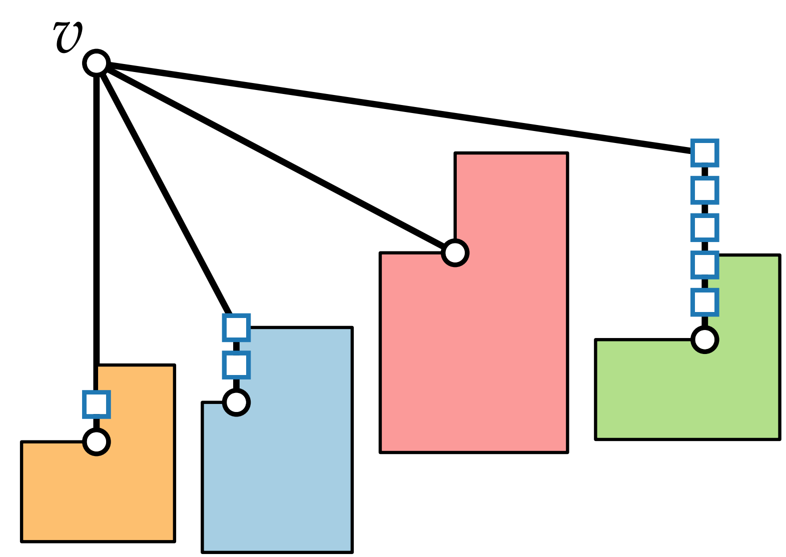
(1) Draw

(2) Layout v +

(3) Add

(4) Sort by #





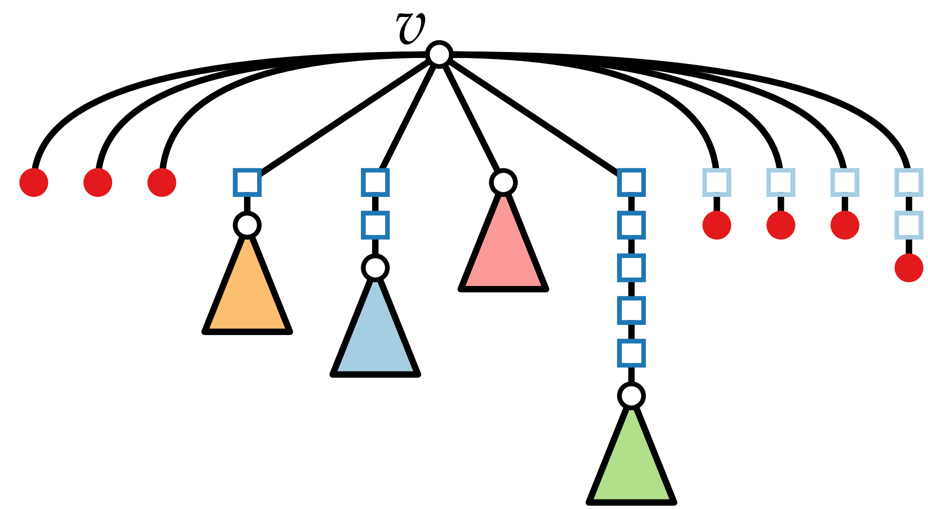
Tree Drawings

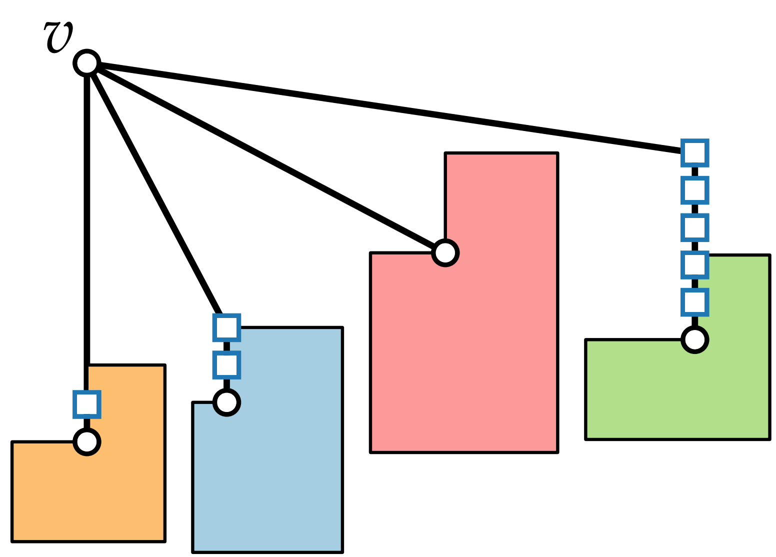
(1) Draw

(2) Layout v +

(3) Add

(4) Sort by #





Tree Drawings

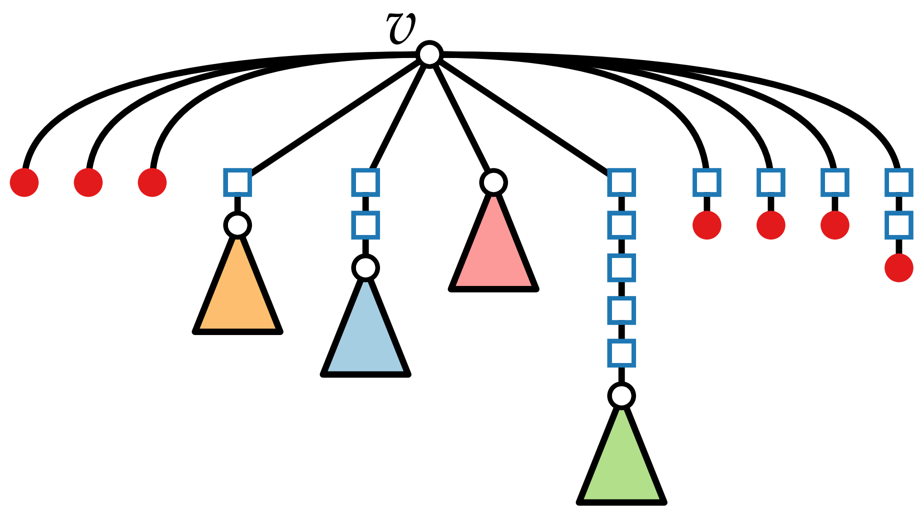
(1) Draw

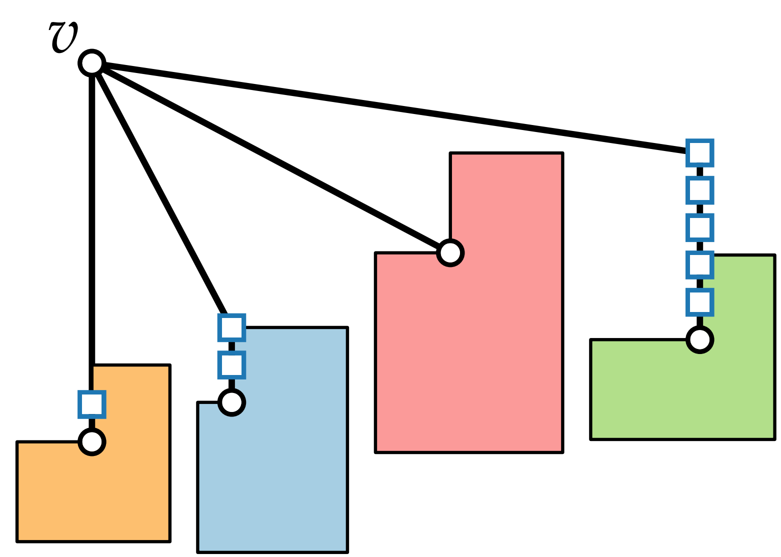
(2) Layout v +

(3) Add

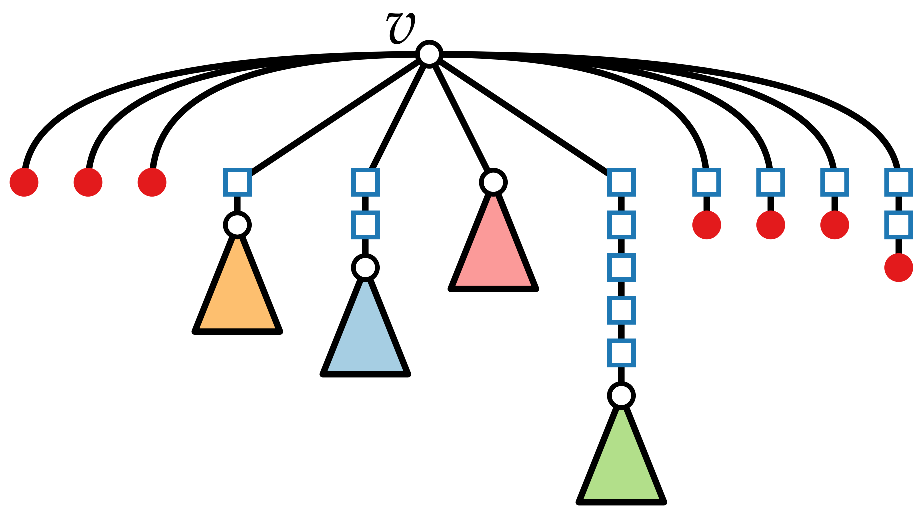
(4) Sort by #

(5) Place + on common segments in order





Tree Drawings



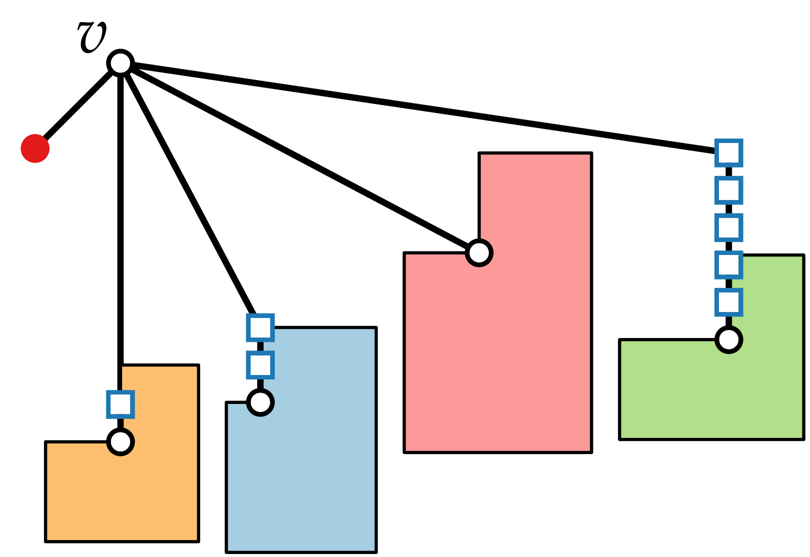
(1) Draw

(2) Layout v +

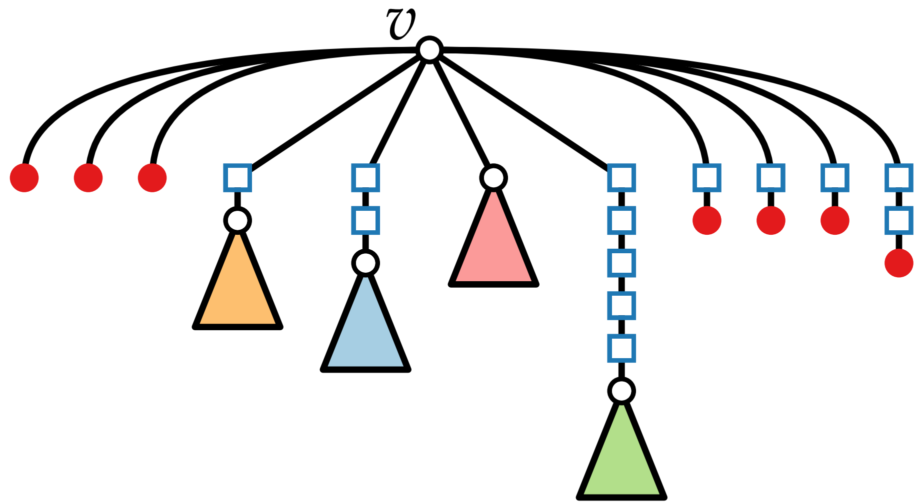
(3) Add

(4) Sort by #

(5) Place + on common segments in order



Tree Drawings



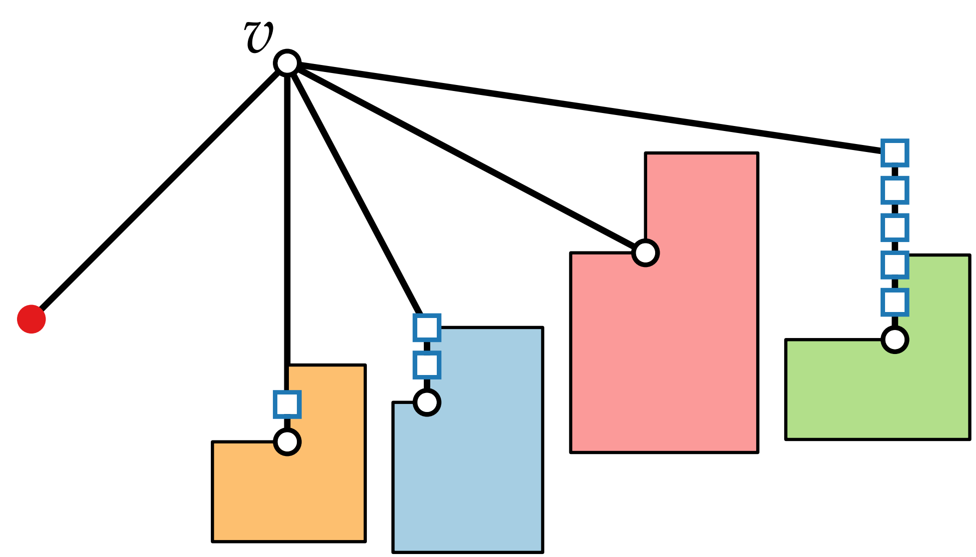
(1) Draw

(2) Layout v +

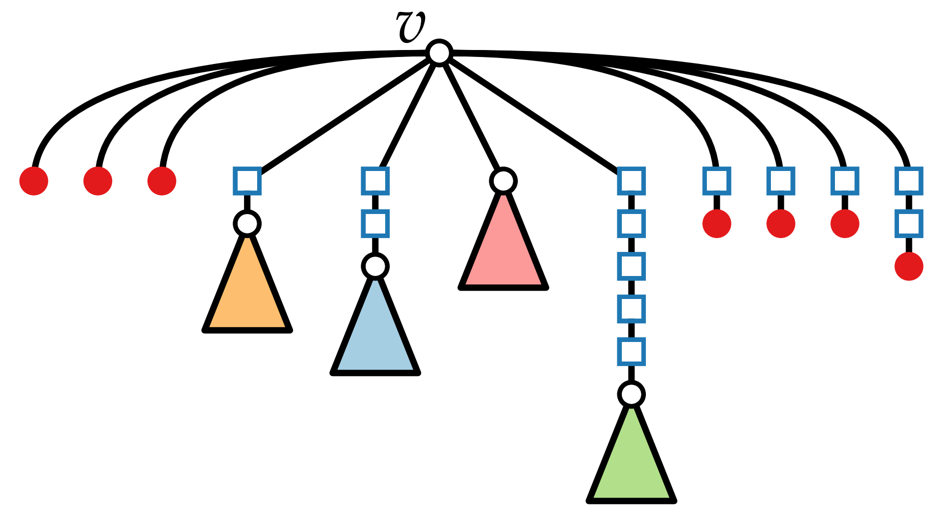
(3) Add

(4) Sort by #

(5) Place + on common segments in order



Tree Drawings



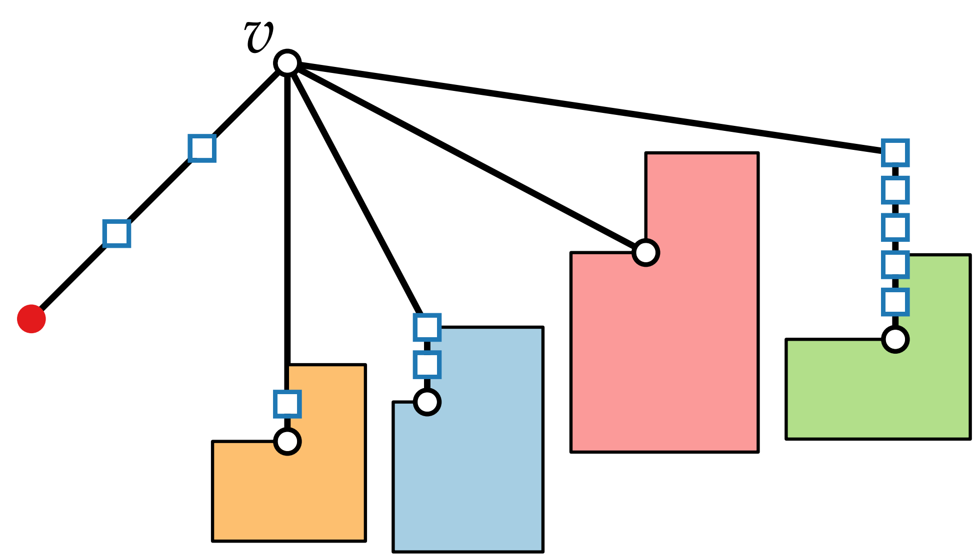
(1) Draw

(2) Layout v +

(3) Add

(4) Sort by #

(5) Place + on common segments in order



Tree Drawings

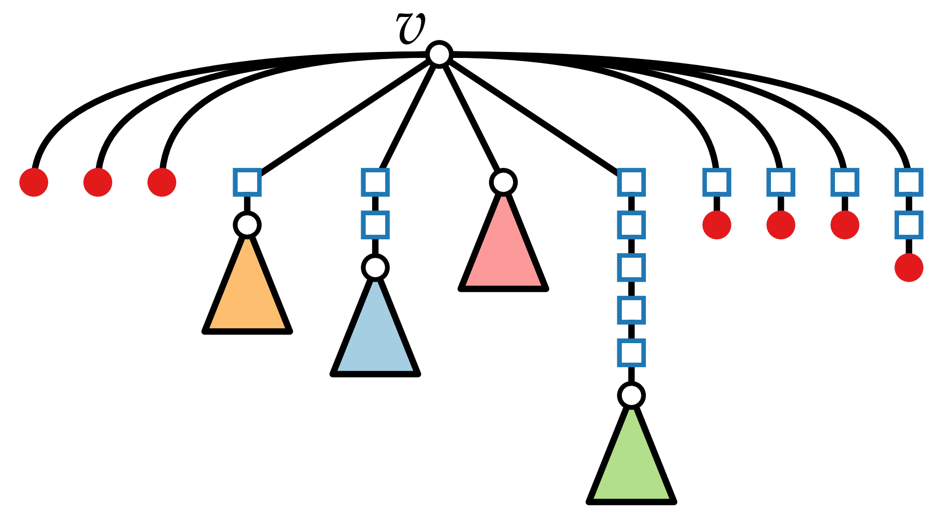
(1) Draw

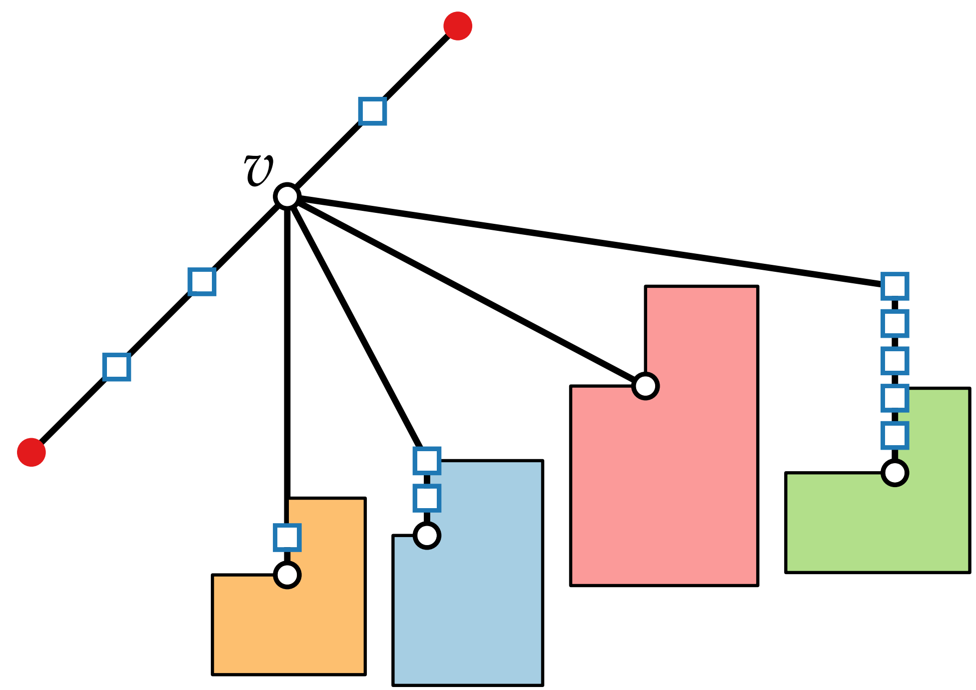
(2) Layout v +

(3) Add □

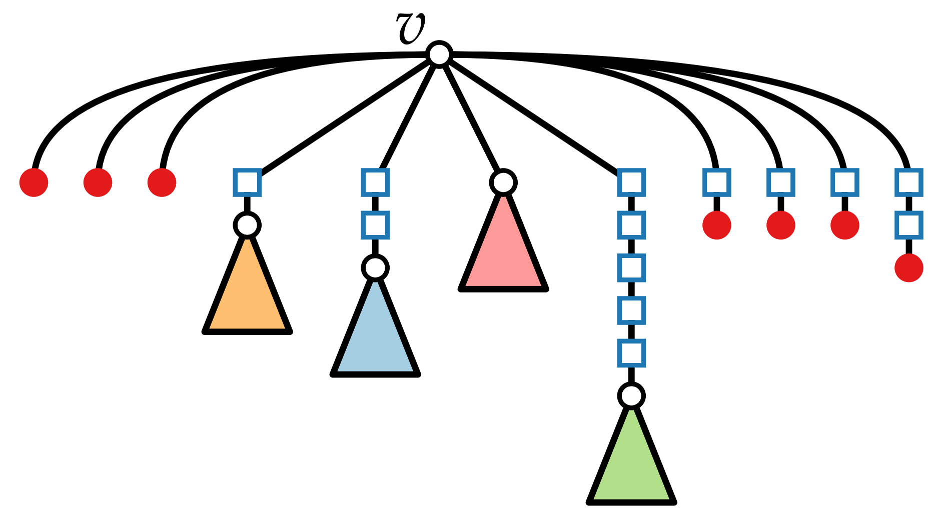
(4) Sort ● by #

(5) Place ● + □ on common segments in order





Tree Drawings



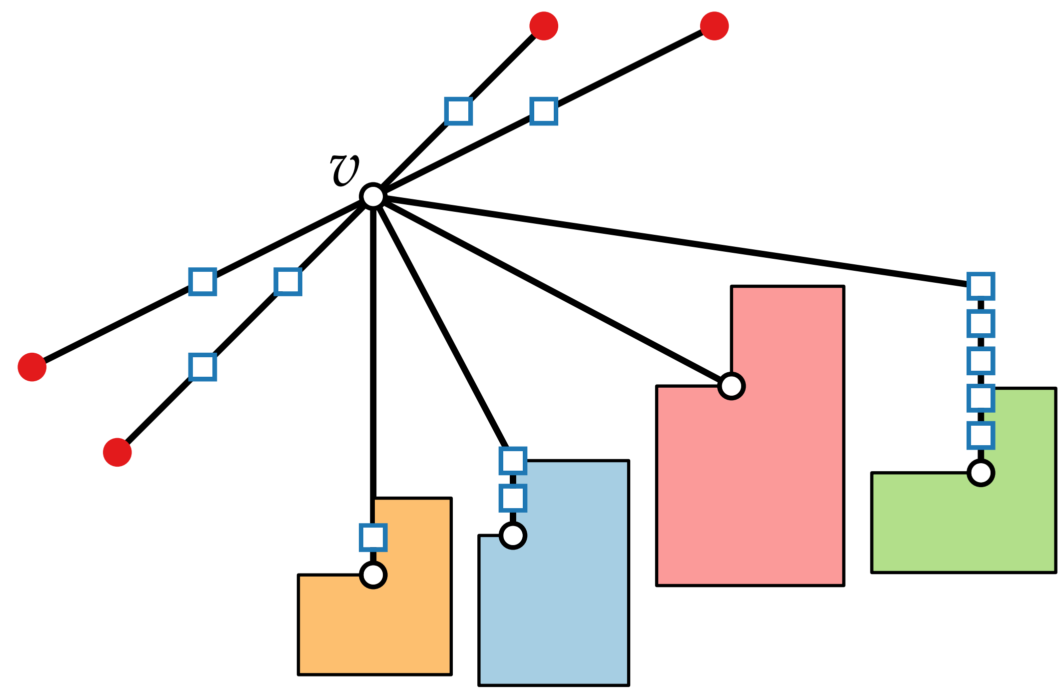
(1) Draw

(2) Layout v +

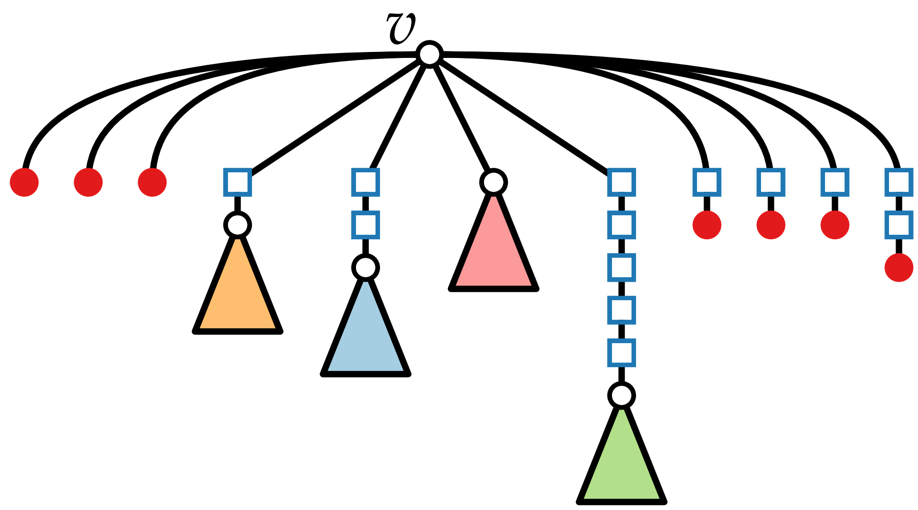
(3) Add

(4) Sort by #

(5) Place + on common segments in order



Tree Drawings



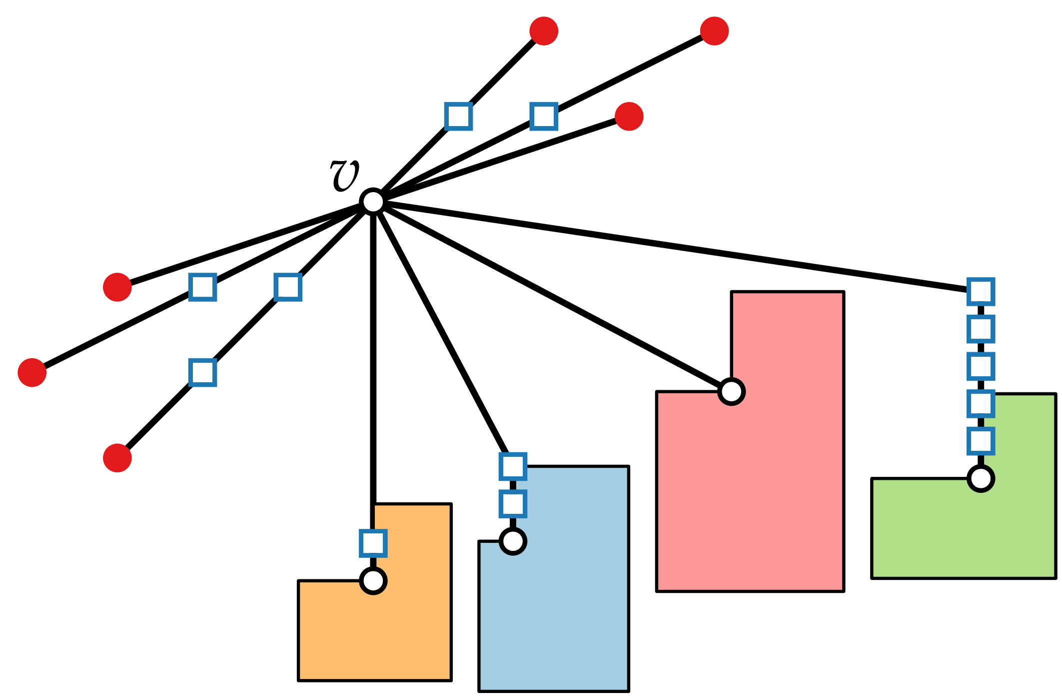
(1) Draw △ △ △ △

(2) Layout v + △ △ △ △

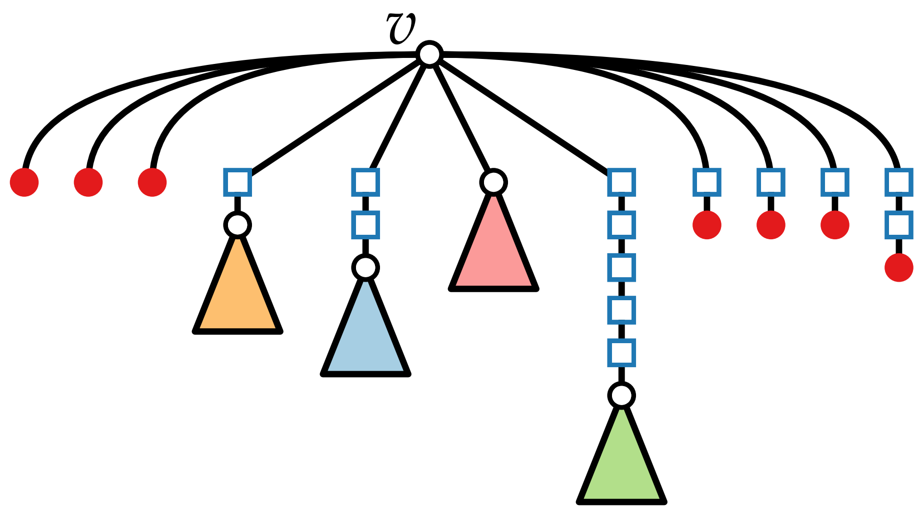
(3) Add □

(4) Sort ● by #

(5) Place ● + □ on common segments in order



Tree Drawings



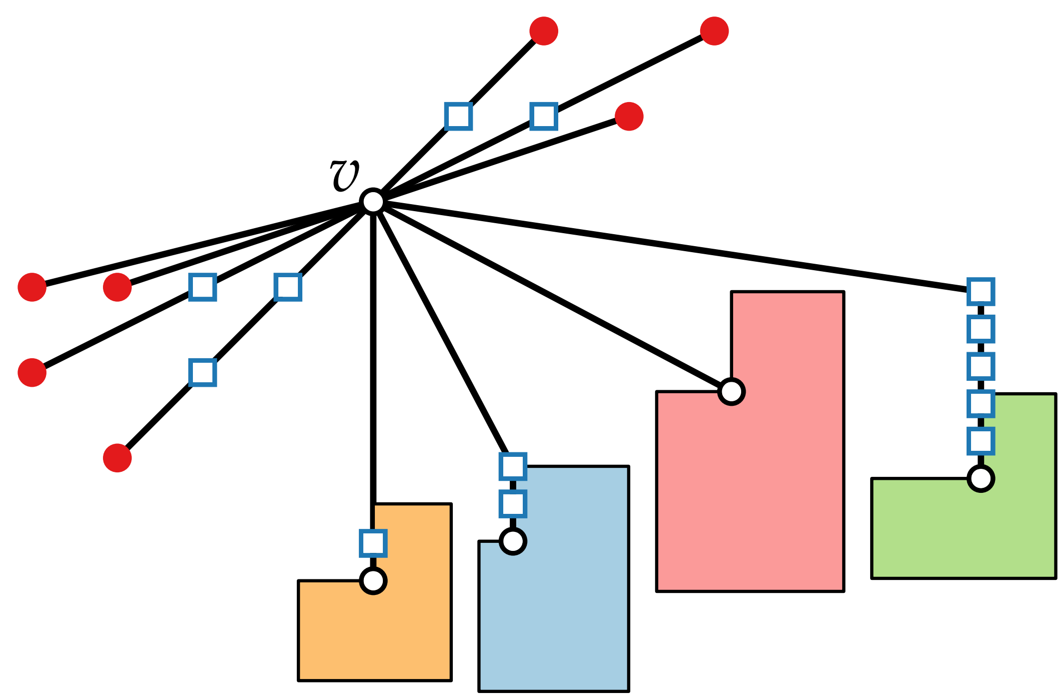
(1) Draw

(2) Layout v +

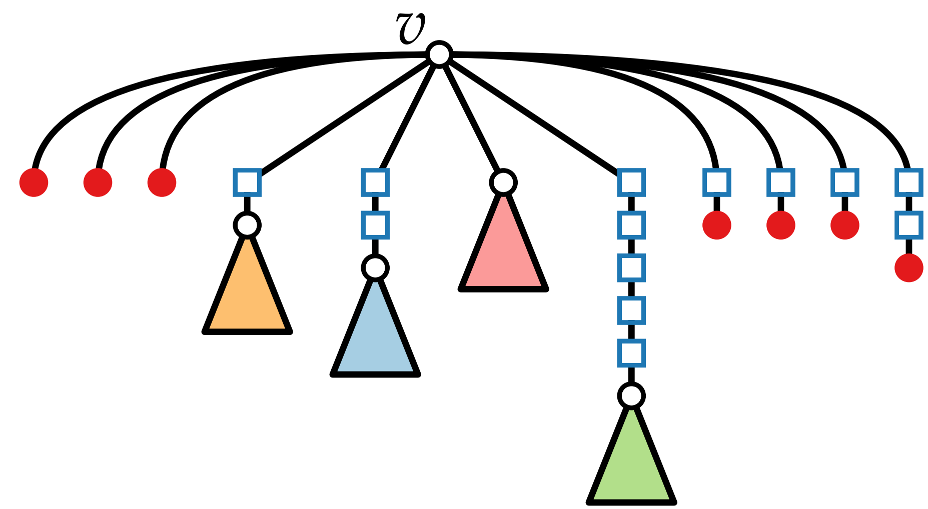
(3) Add

(4) Sort by #

(5) Place + on common segments in order



Tree Drawings



(1) Draw

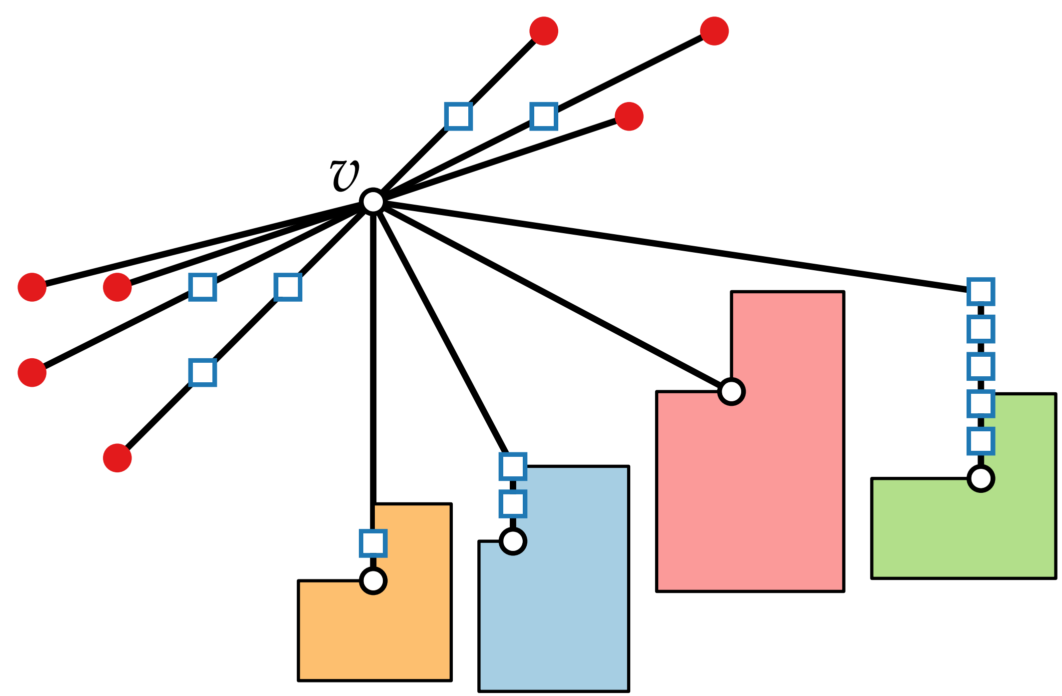
(2) Layout v +

(3) Add

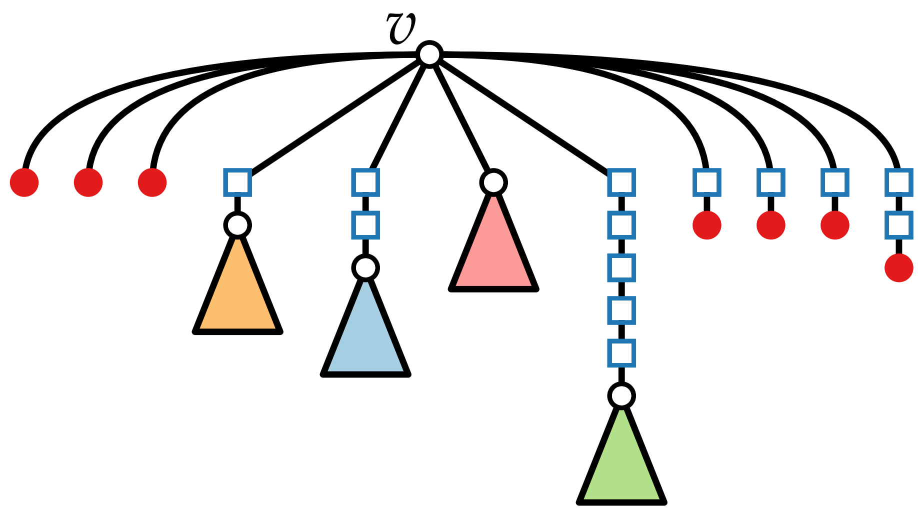
(4) Sort by #

(5) Place + on common segments in order

$3n/4$ segments



Tree Drawings



(1) Draw

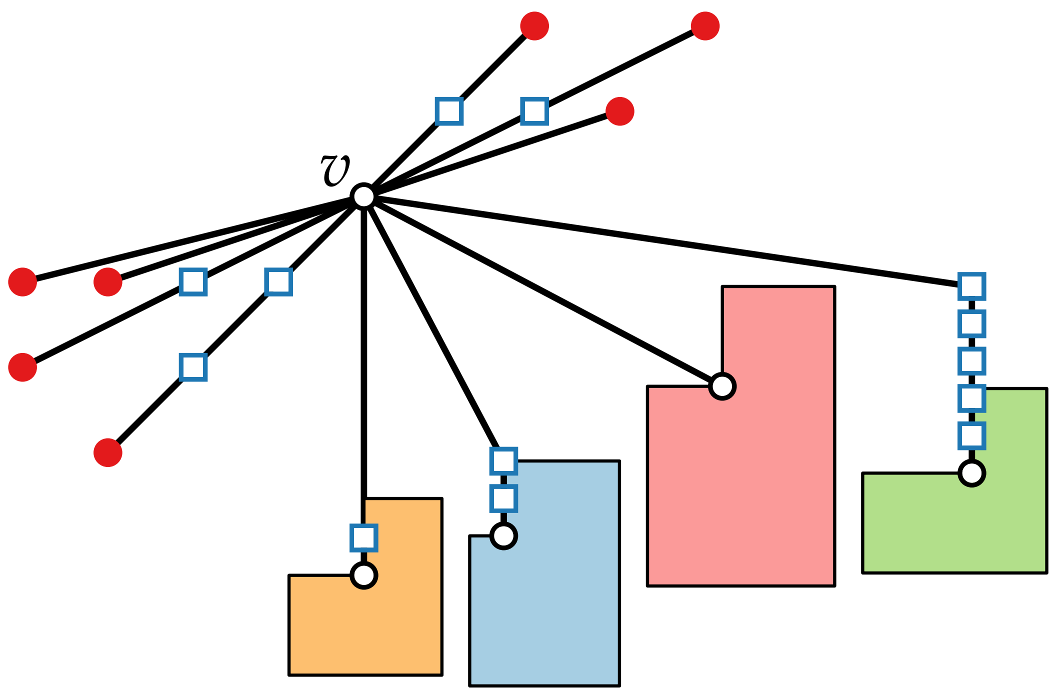
(2) Layout v +

(3) Add

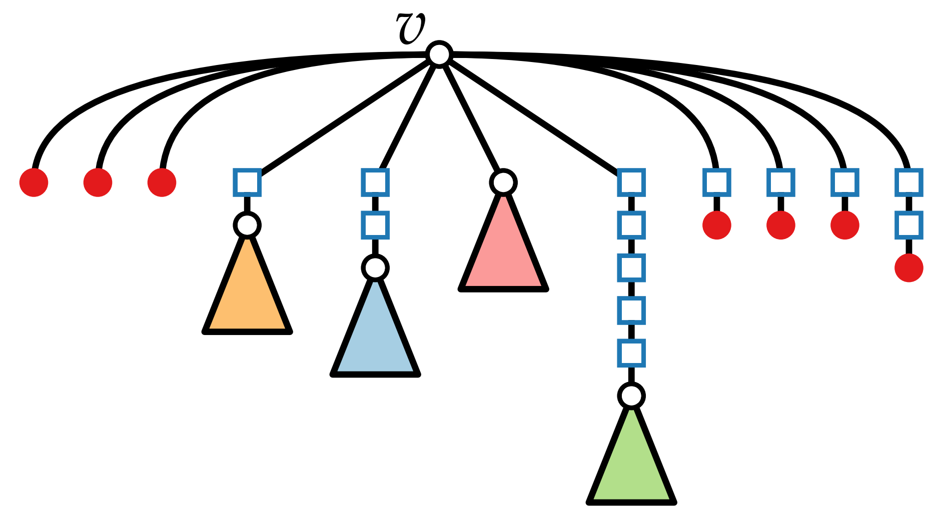
(4) Sort by #

(5) Place + on common segments in order

$3n/4$ segments
$n \times n$ grid



Tree Drawings



(1) Draw

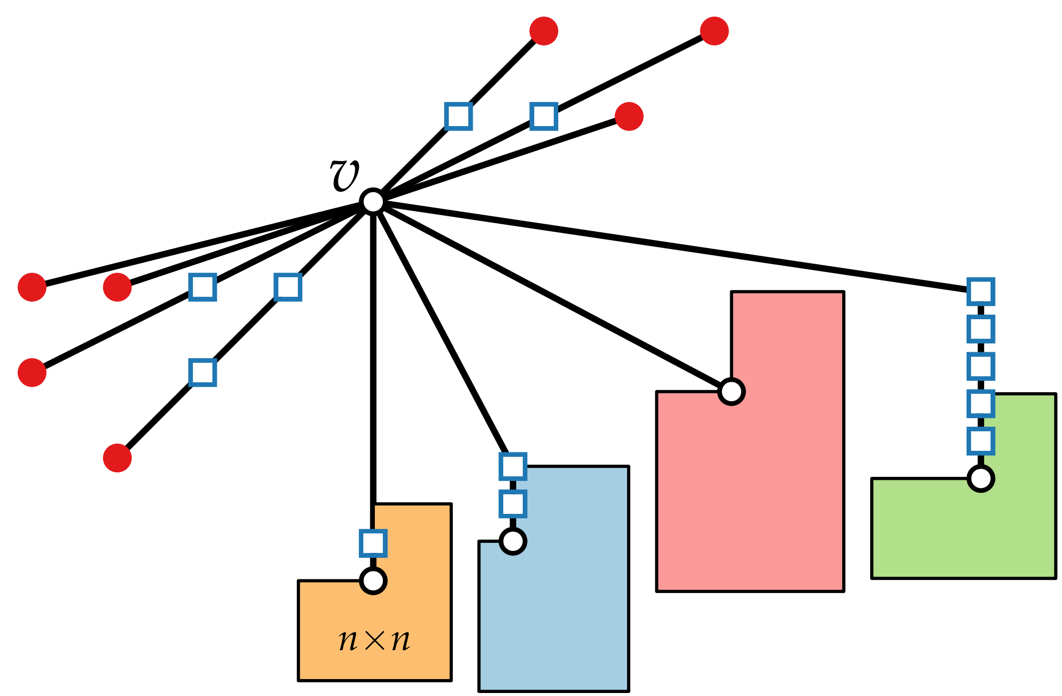
(2) Layout v +

(3) Add

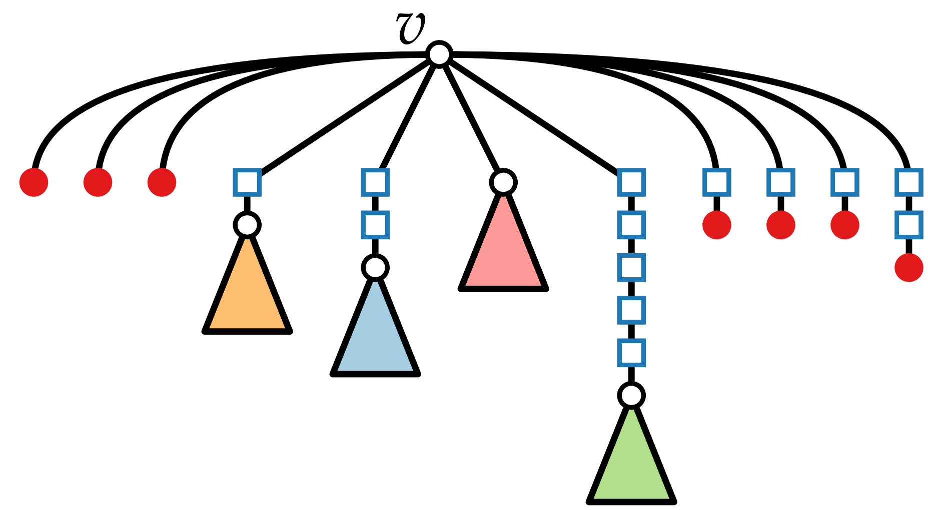
(4) Sort by #

(5) Place + on common segments in order

3n/4 segments
$n \times n$ grid



Tree Drawings



(1) Draw

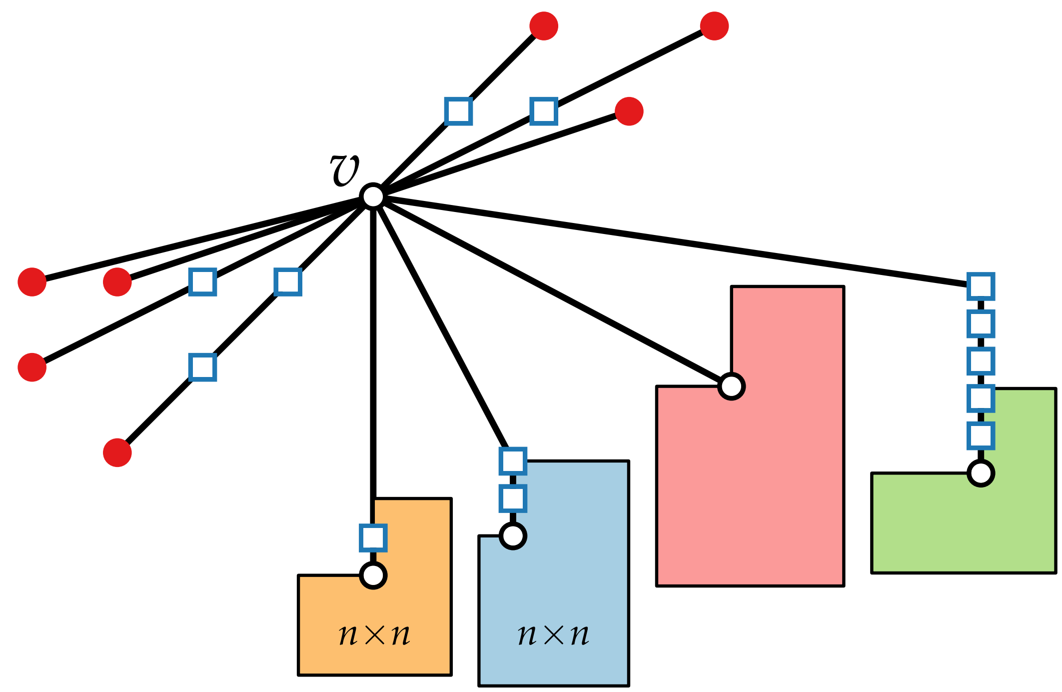
(2) Layout v +

(3) Add

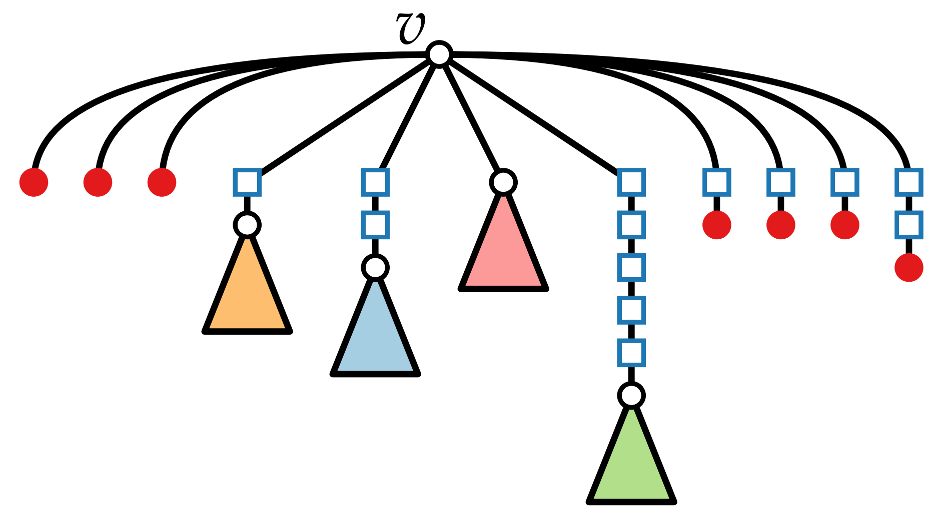
(4) Sort by #

(5) Place + on common segments in order

$3n/4$ segments
$n \times n$ grid



Tree Drawings



(1) Draw

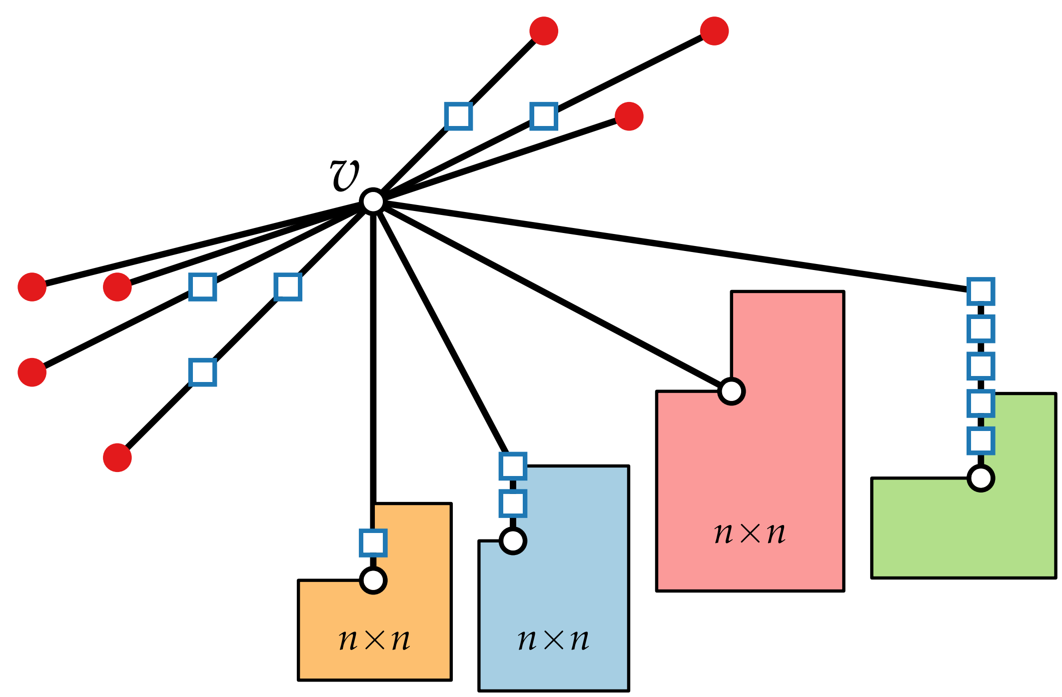
(2) Layout v +

(3) Add

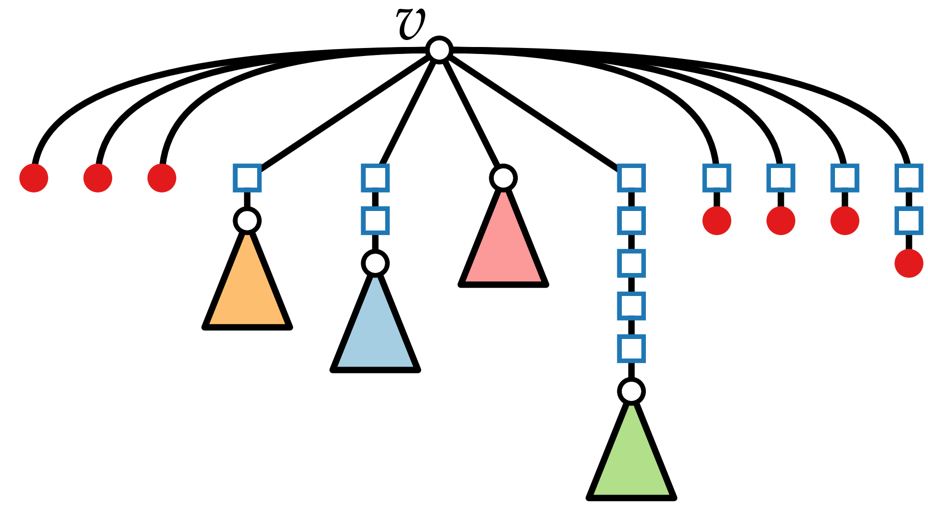
(4) Sort by #

(5) Place + on common segments in order

$3n/4$ segments
$n \times n$ grid



Tree Drawings



(1) Draw

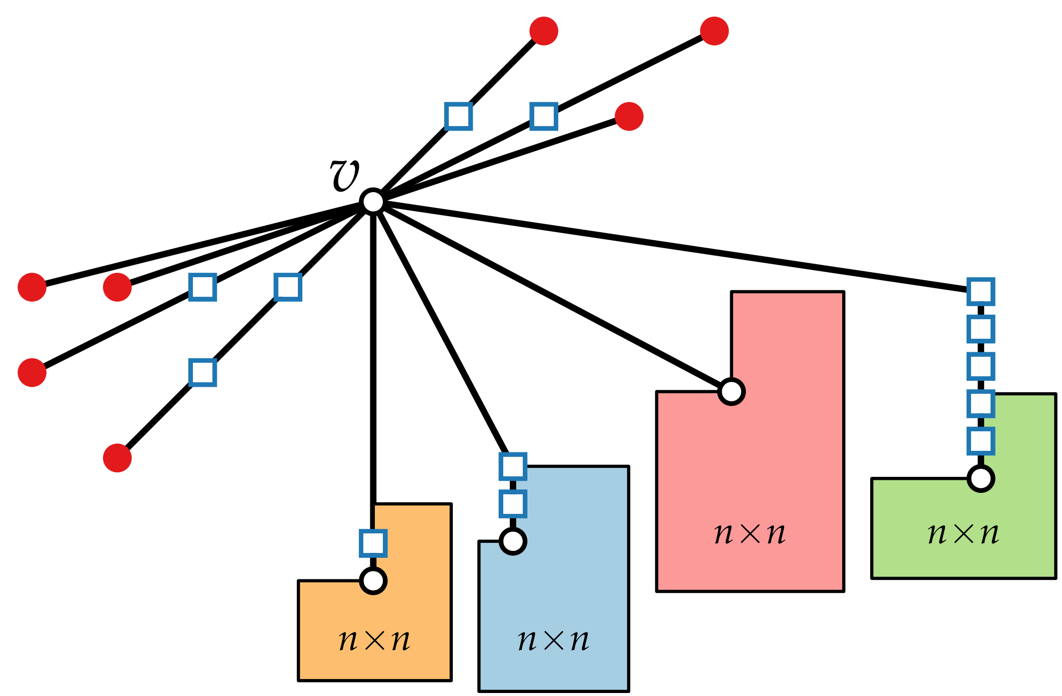
(2) Layout v +

(3) Add □

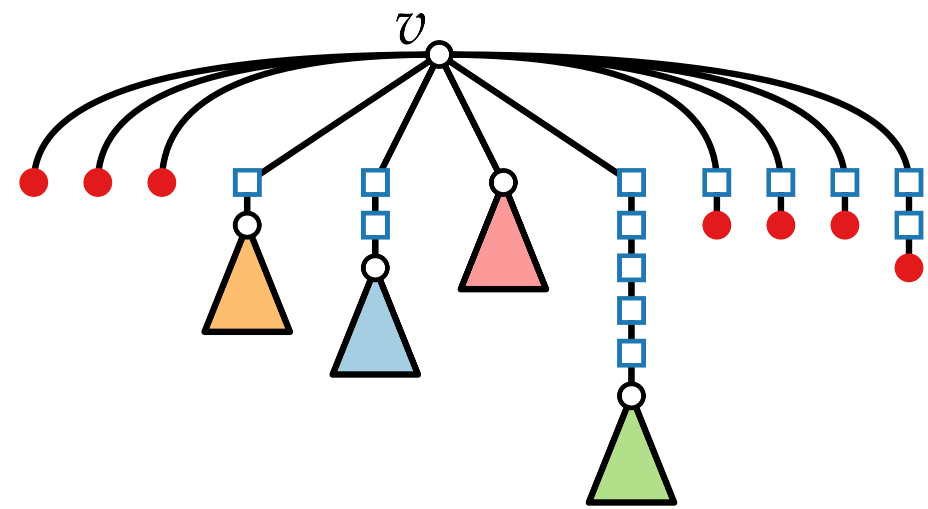
(4) Sort ● by #

(5) Place ● + □ on common segments in order

$3n/4$ segments
$n \times n$ grid



Tree Drawings



(1) Draw

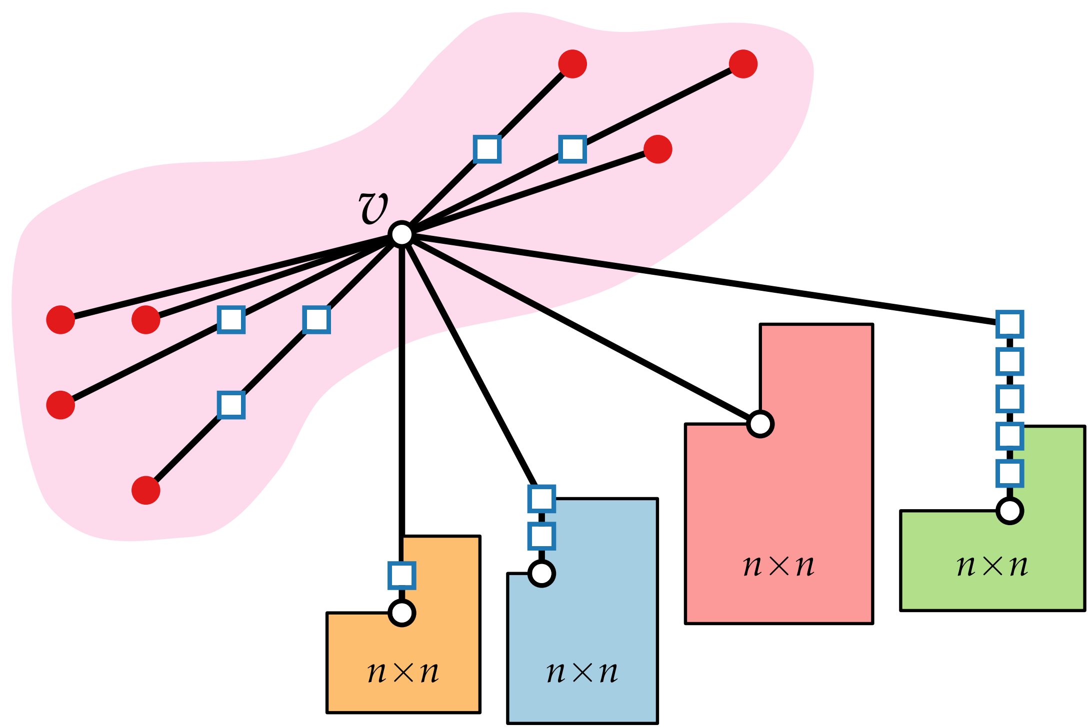
(2) Layout v +

(3) Add

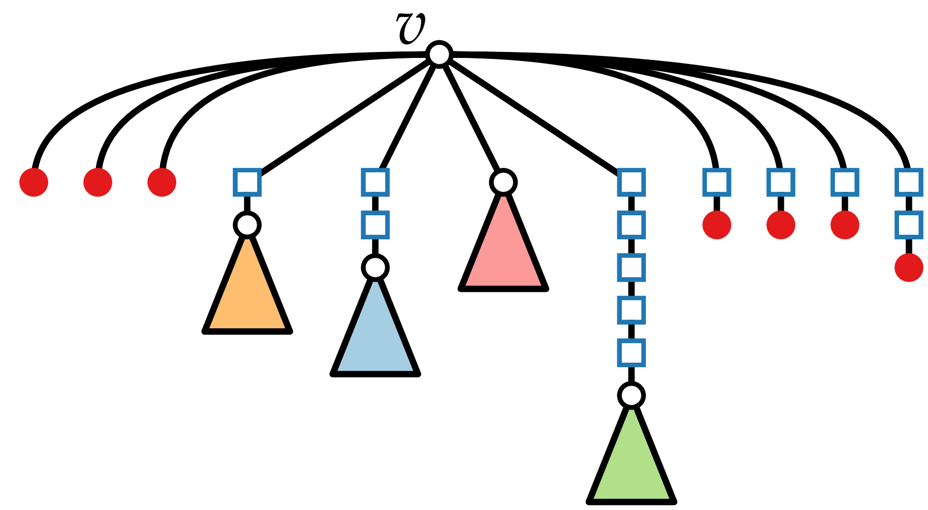
(4) Sort by #

(5) Place + on common segments in order

3n/4 segments
$n \times n$ grid



Tree Drawings



(1) Draw

(2) Layout v +

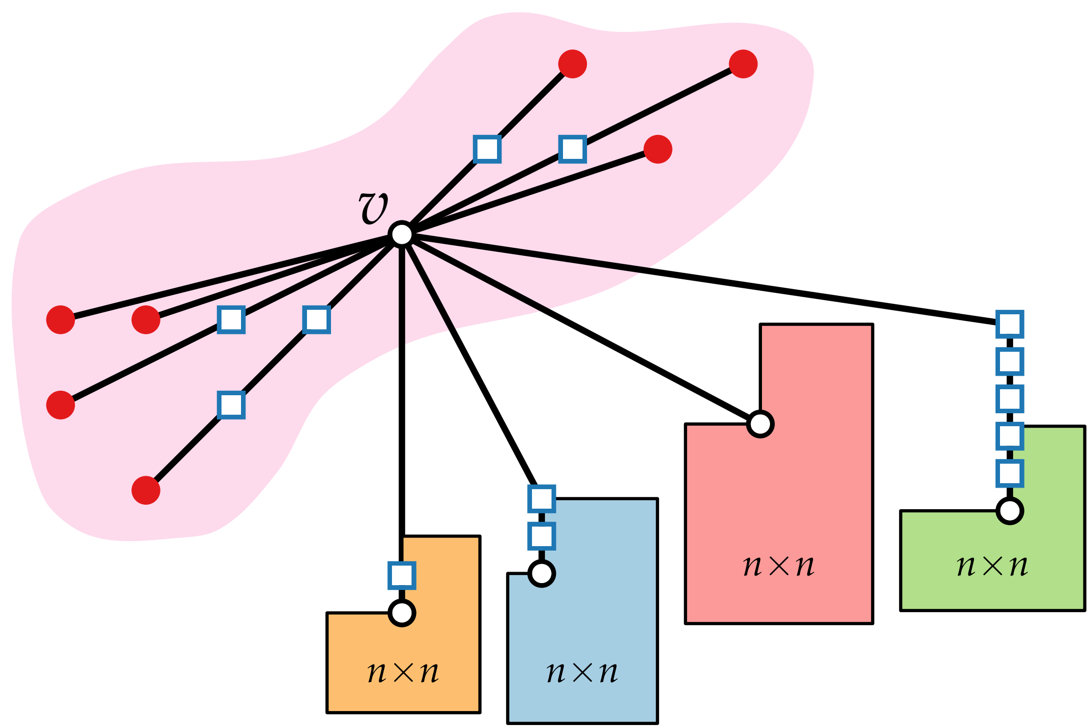
(3) Add

(4) Sort by #

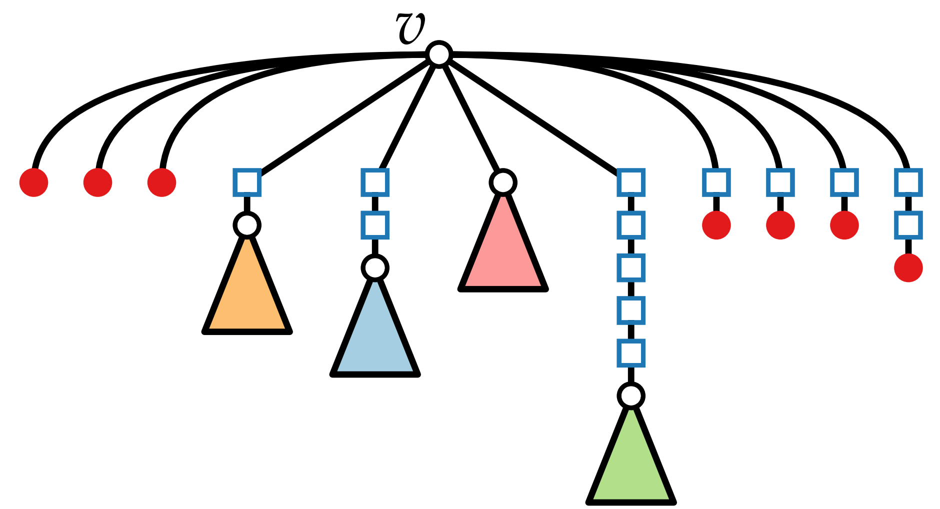
(5) Place + on common segments in order

$3n/4$ segments
$n \times n$ grid

height ✓



Tree Drawings



(1) Draw

(2) Layout v +

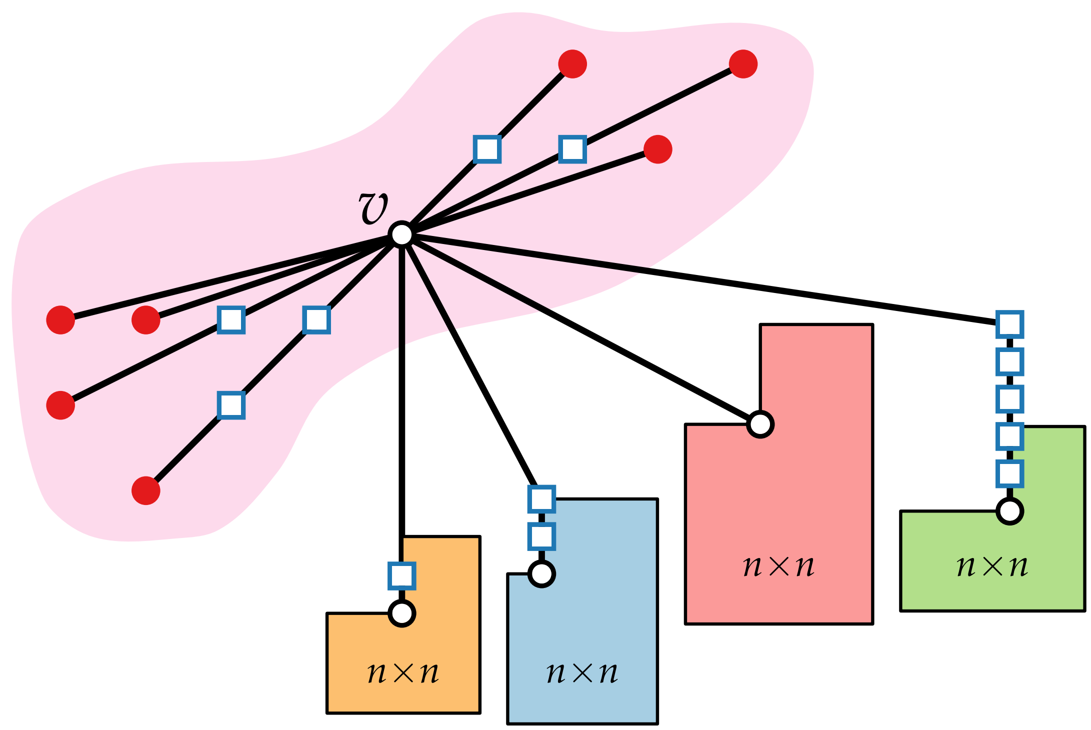
(3) Add

(4) Sort by #

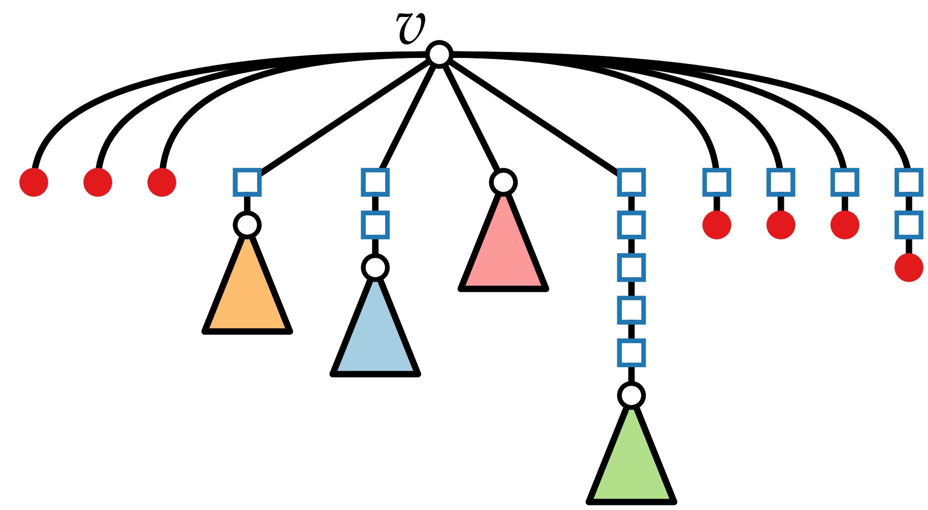
(5) Place + on common segments in order

$3n/4$ segments
$n \times n$ grid

height ✓
width



Tree Drawings



(1) Draw

(2) Layout v +

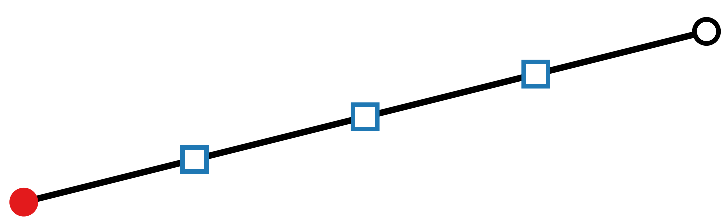
(3) Add

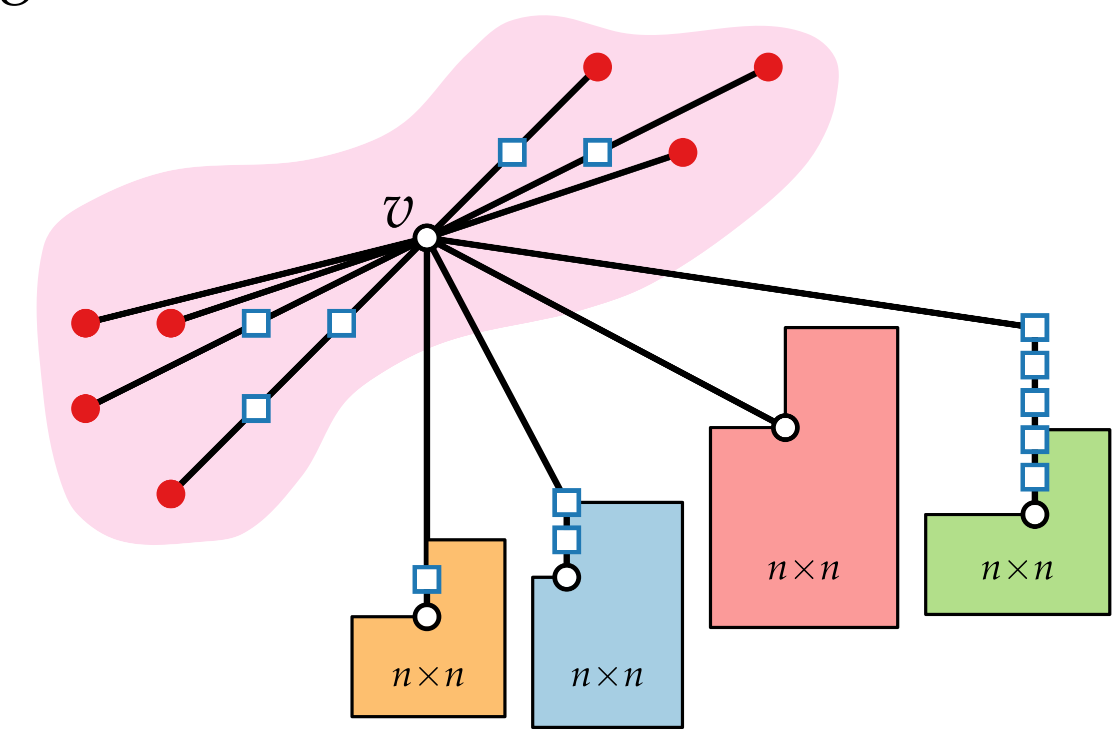
(4) Sort by #

(5) Place + on common segments in order

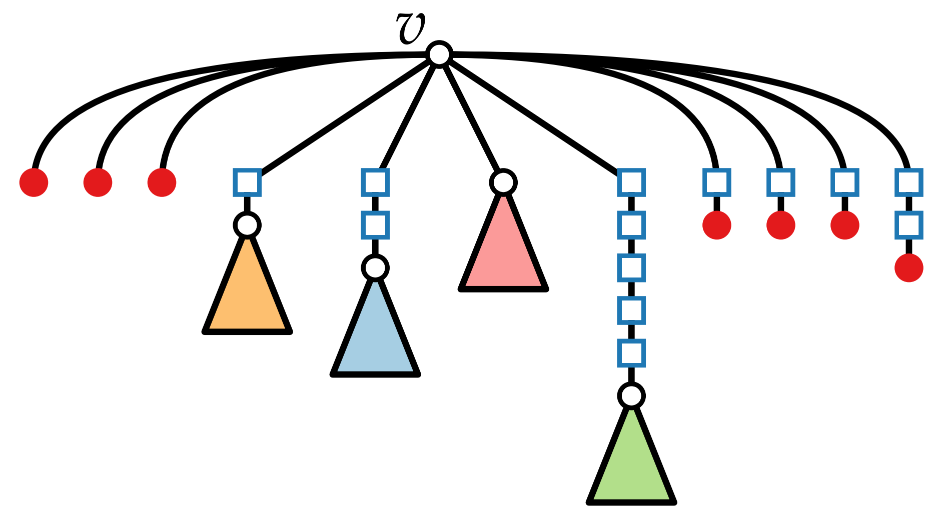
$3n/4$ segments
$n \times n$ grid

height ✓
width





Tree Drawings



(1) Draw

(2) Layout v +

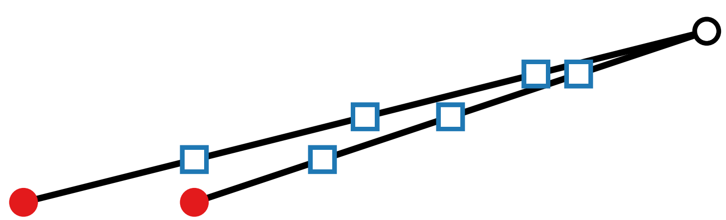
(3) Add

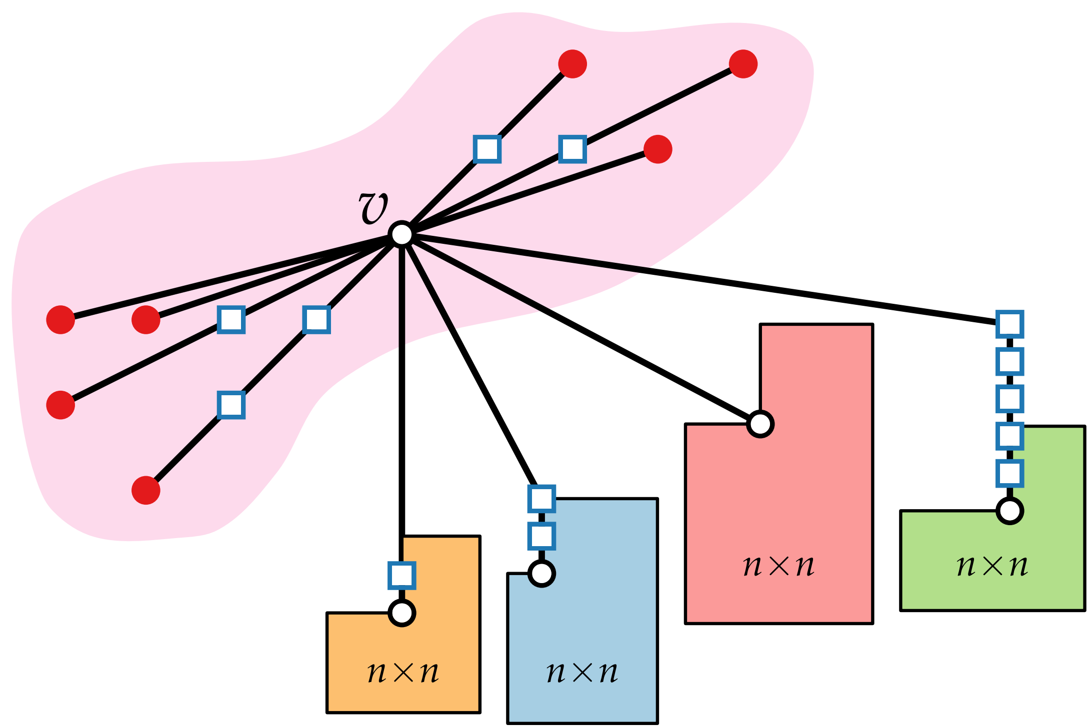
(4) Sort by #

(5) Place + on common segments in order

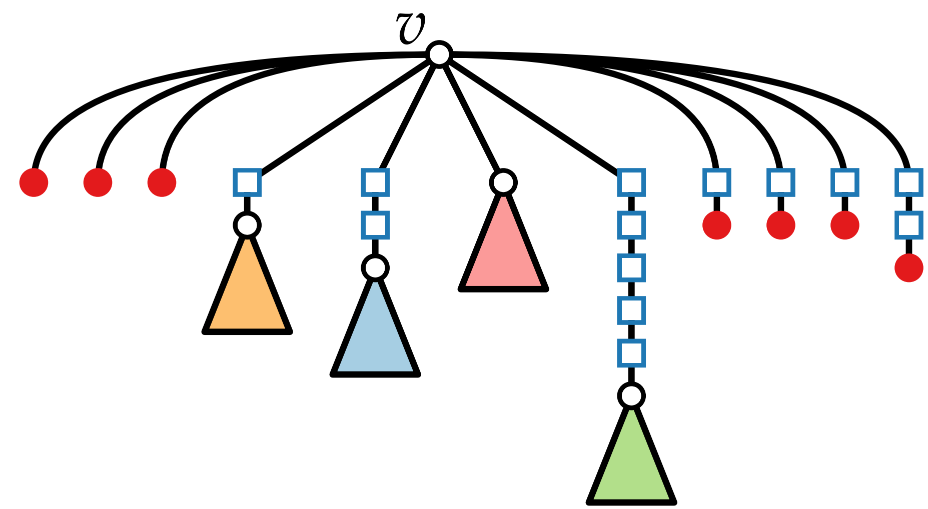
$3n/4$ segments
$n \times n$ grid

height ✓
width





Tree Drawings



(1) Draw

(2) Layout v +

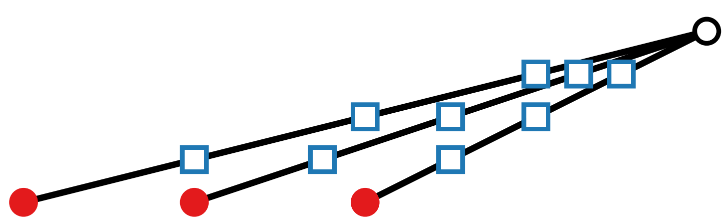
(3) Add

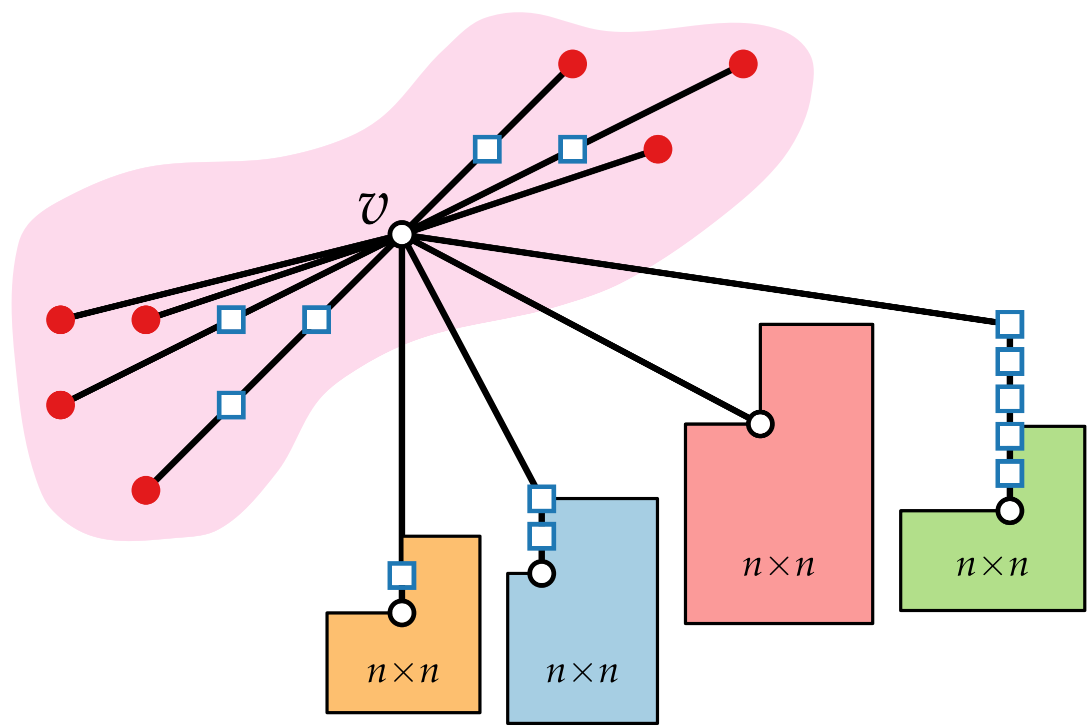
(4) Sort by #

(5) Place + on common segments in order

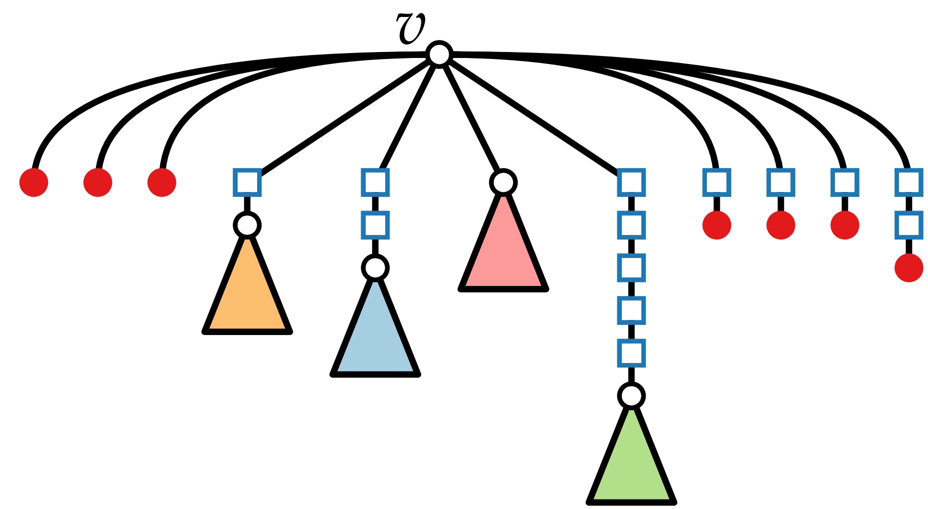
$3n/4$ segments
$n \times n$ grid

height ✓
width





Tree Drawings



(1) Draw

(2) Layout v +

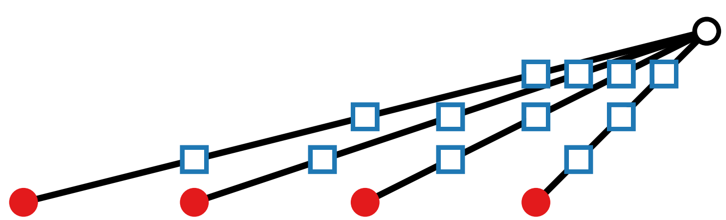
(3) Add

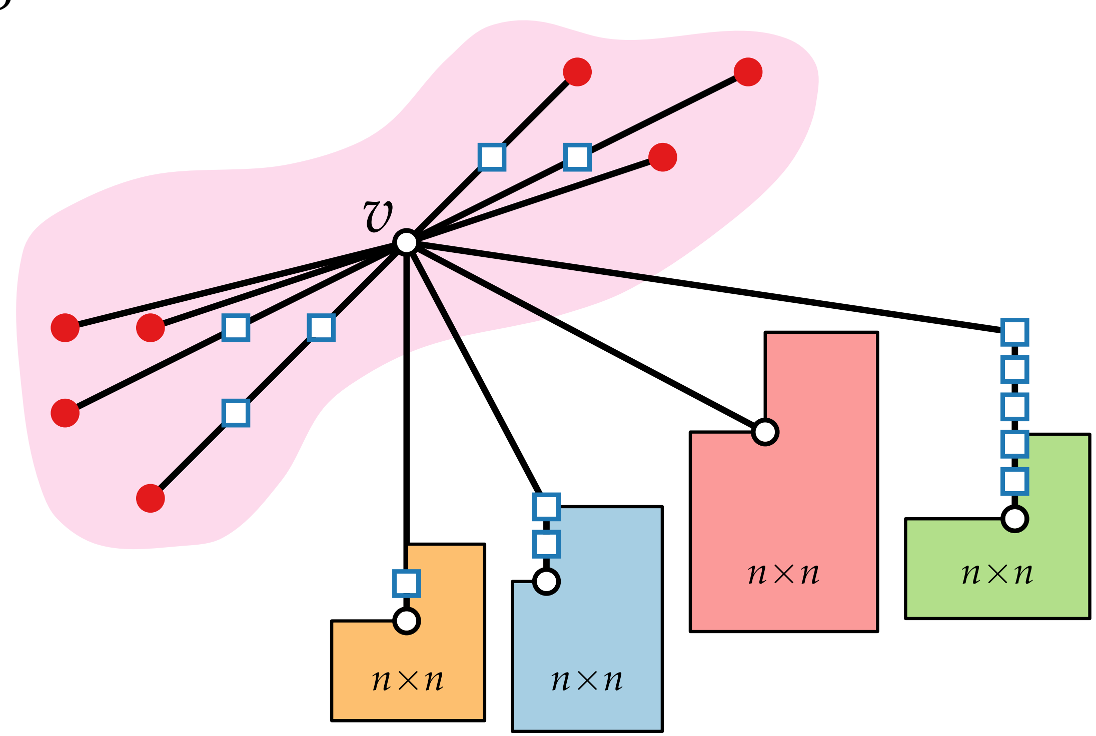
(4) Sort by #

(5) Place + on common segments in order

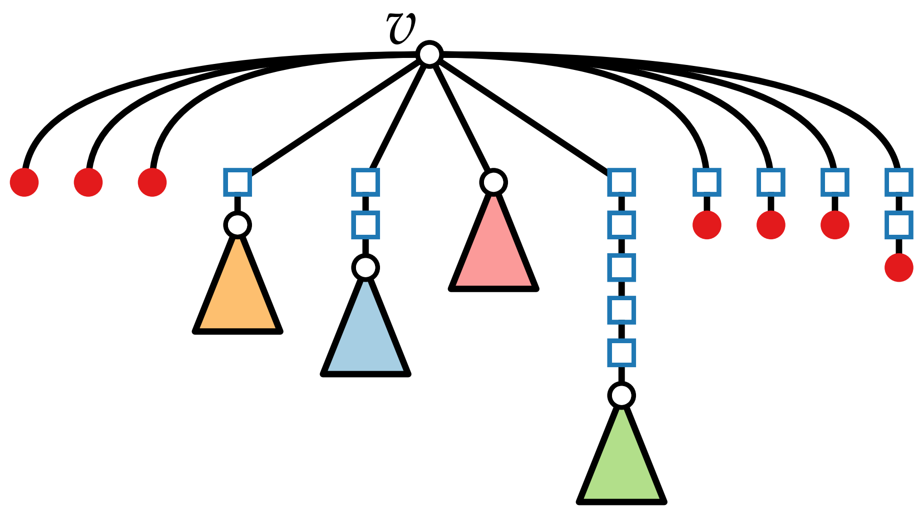
$3n/4$ segments
$n \times n$ grid

height ✓
width





Tree Drawings



(1) Draw

(2) Layout v +

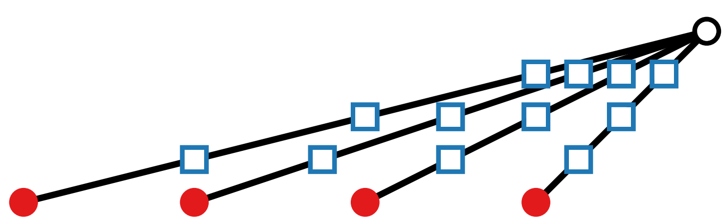
(3) Add

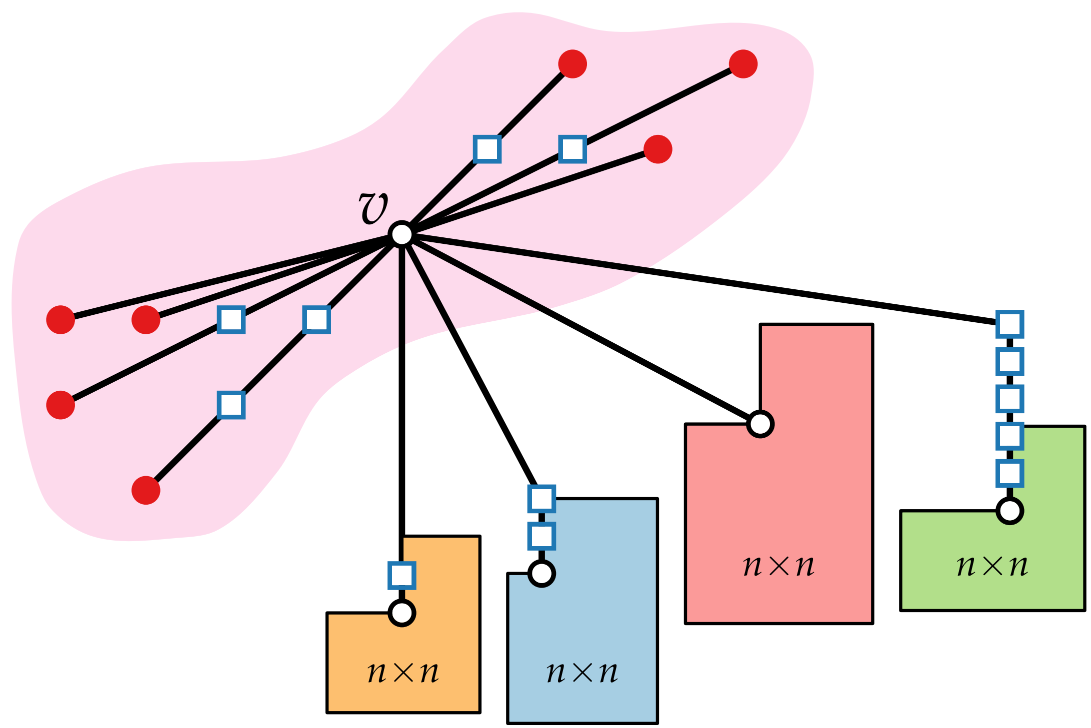
(4) Sort by #

(5) Place + on common segments in order

$3n/4$ segments
$n \times n$ grid

height ✓
width ✓





Improved Results

Class	Segments: Lower	Segments: Upper	Grid Segments: Segm.	Grid Segments: Area
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$3n/4$ [6]	$O(n^2) \times O(n^{1.58})$
			$\vartheta/2$ [6]	quasipolynomial
outerplanar	n [1]			
max. outerp.	n [1]	n [1]	$3n/2$ [6]	$O(n) \times O(n^2)$
3-trees	$2n$ [1]	$2n$ [1]	$8n/3$ [6]	$O(n) \times O(n^2)$
2-connected	$2n$ [1]			
3-connected	$2n$ [1]	$5n/2$ [1]		
cubic 3-conn.	$n/2$ [3]	$n/2$ [2]	$n/2$ [2]	$O(n) \times O(n)$
triangulation	$2n$ [4]	$7n/3$ [4]	$8n/3$ [5]	$2^{O}(n \log n)$
4-conn. triang.	$2n$ [4]	$9n/3$ [4]		
planar	$2n$ [4]	$8n/3$ [4]		

[1] Dujmović et al. 2007 [2] Igamberdiev et al. 2015 [3] Mondal et al. 2013
[4] Durocher & Mondal 2014 [5] Mondal 2016 [6] Hültenschmidt et al. 2017

Improved Results

Class	Segments: Lower		Segments: Upper		Grid Segments: Segm.		Grid Segments: Area
tree	$\vartheta/2$	[1]	$\vartheta/2$	[1]	$3n/4$		$n \times n$
					$\vartheta/2$	[6]	quasipolynomial
outerplanar	n	[1]					
max. outerp.	n	[1]	n	[1]	$3n/2$	[6]	$O(n) \times O(n^2)$
3-trees	$2n$	[1]	$2n$	[1]	$8n/3$	[6]	$O(n) \times O(n^2)$
2-connected	$2n$	[1]					
3-connected	$2n$	[1]	$5n/2$	[1]			
cubic 3-conn.	$n/2$	[3]	$n/2$	[2]	$n/2$	[2]	$O(n) \times O(n)$
triangulation	$2n$	[4]	$7n/3$	[4]	$8n/3$	[5]	$2^{O}(n \log n)$
4-conn. triang.	$2n$	[4]	$9n/3$	[4]			
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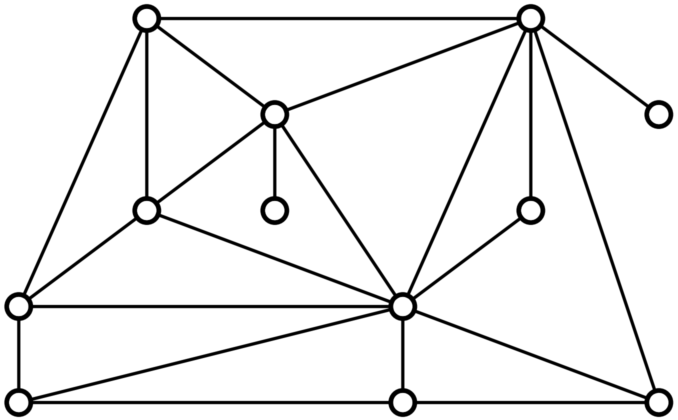
Orderly Spanning Trees

[Chiang, Lin, Lu '05]



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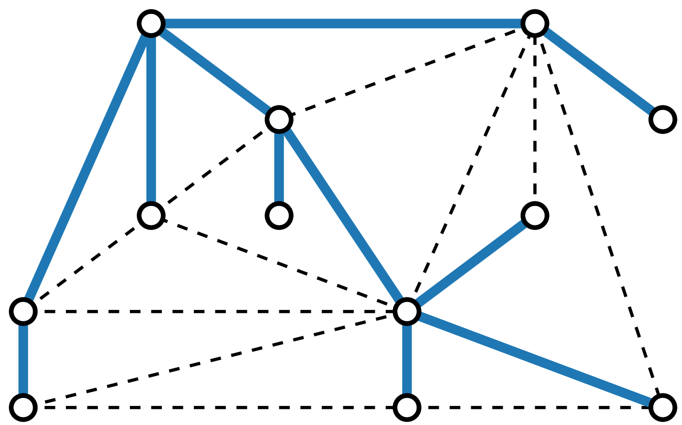

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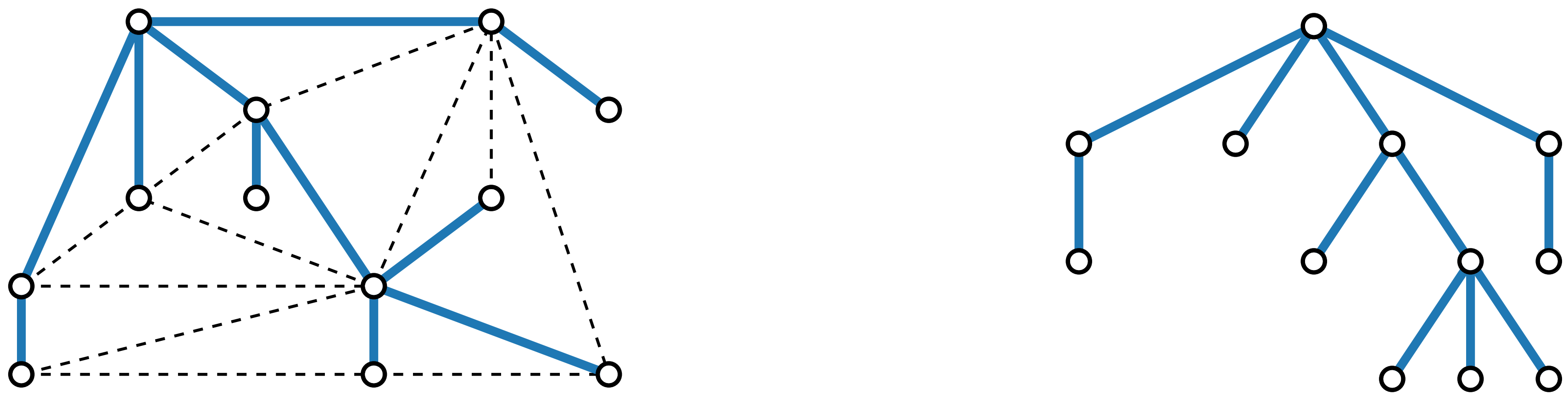

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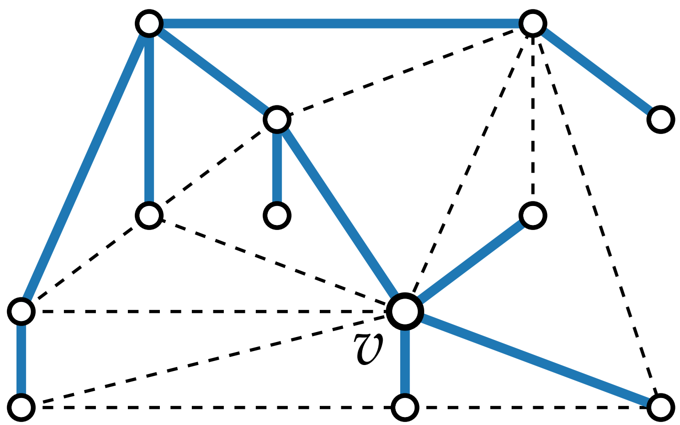

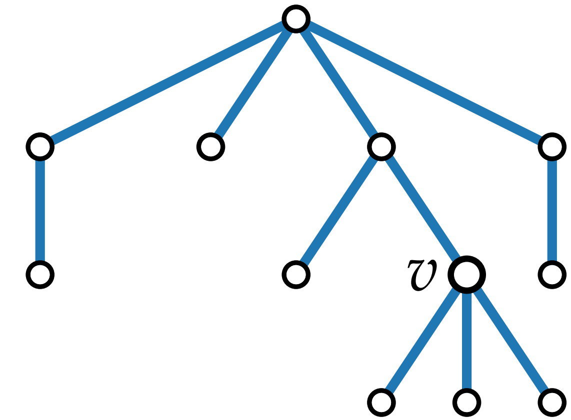

Orderly Spanning Trees

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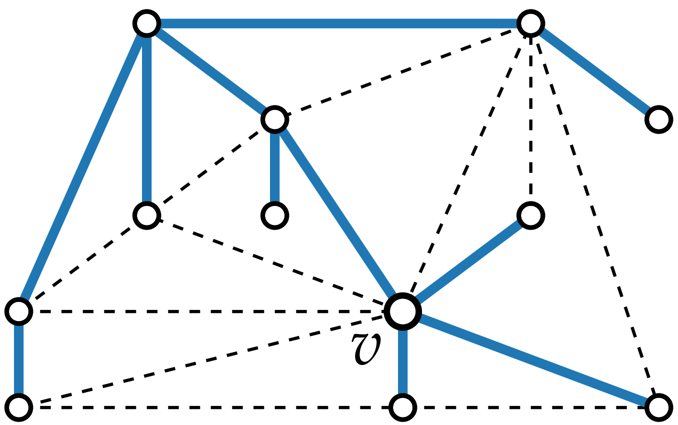
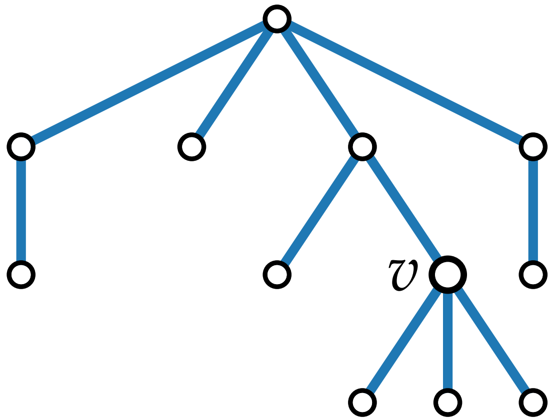
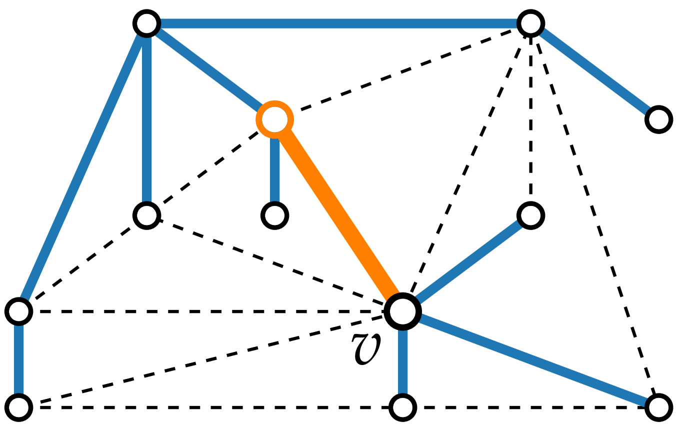


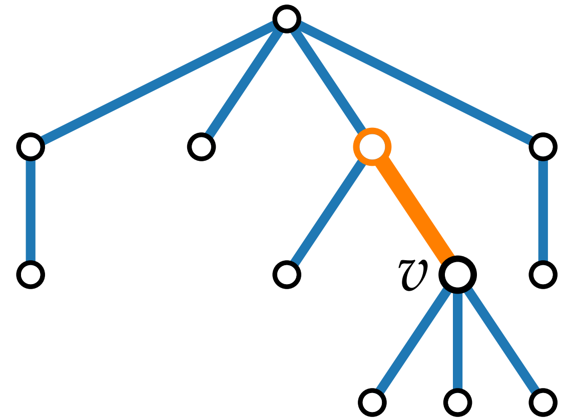


neighbors of v in circ. order:

Orderly Spanning Trees

[Chiang, Lin, Lu '05]



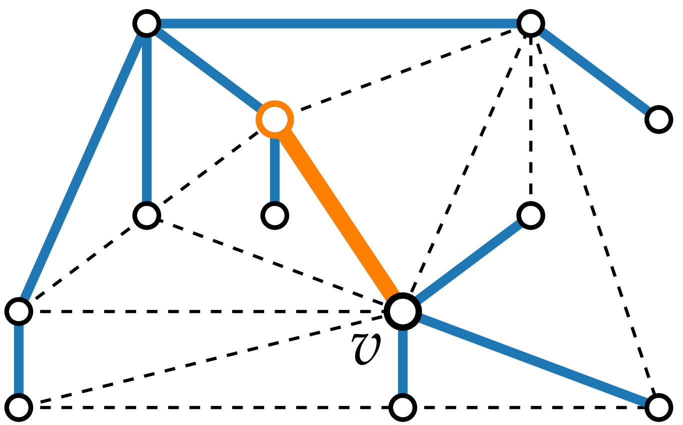


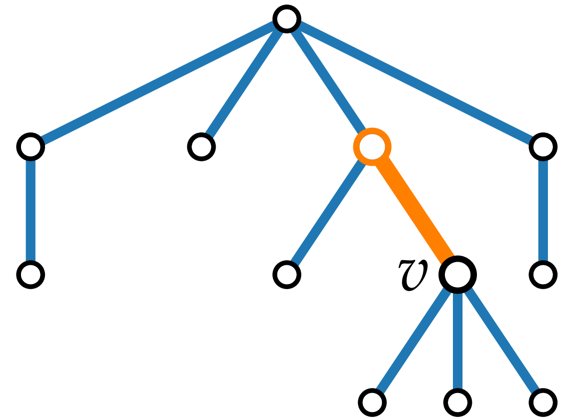
neighbors of v in circ. order:

(1) parent

Orderly Spanning Trees

[Chiang, Lin, Lu ’05]





neighbors of v in circ. order:

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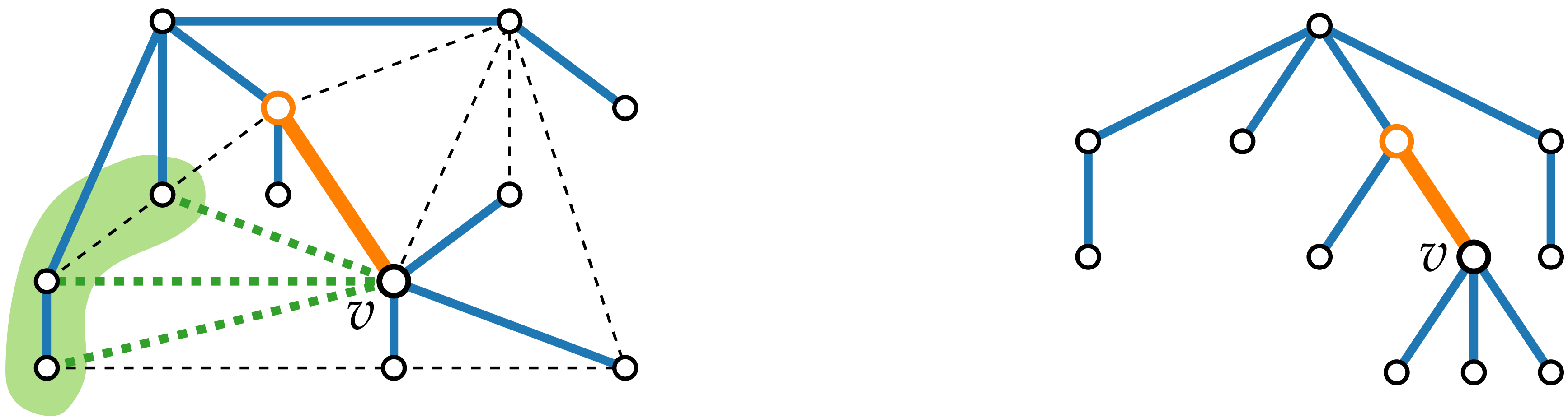
(2) $N^+(v)$: diff. subtree (left)

Orderly Spanning Trees

[Chiang, Lin, Lu ’05]



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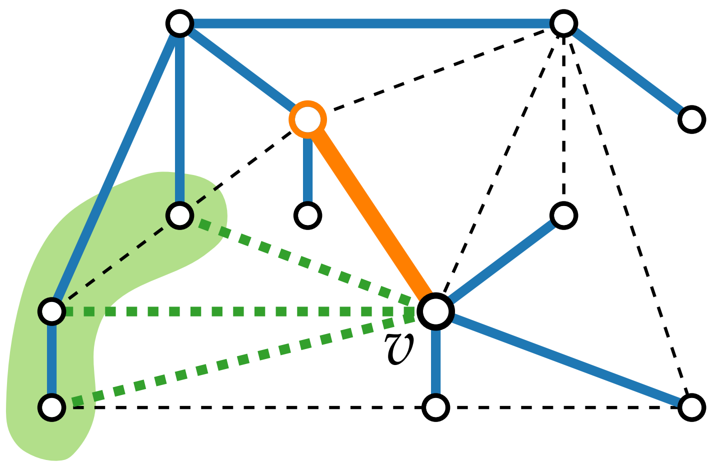

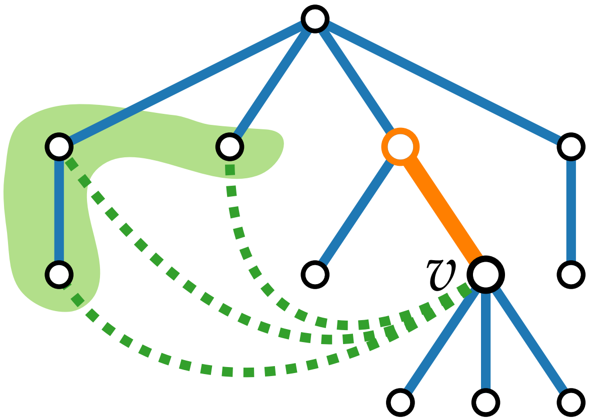

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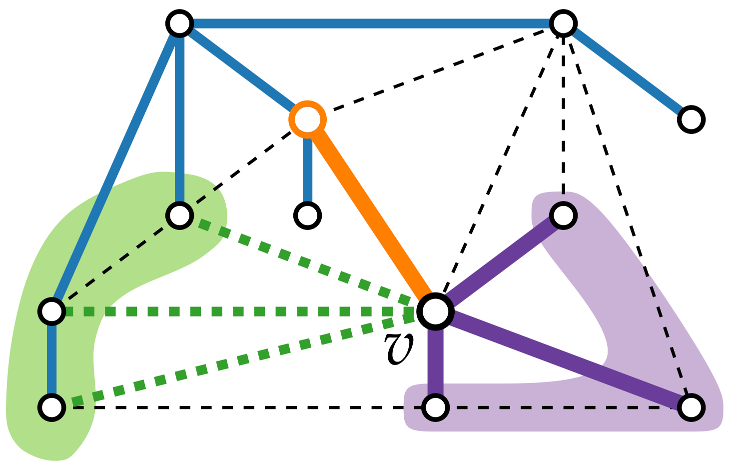

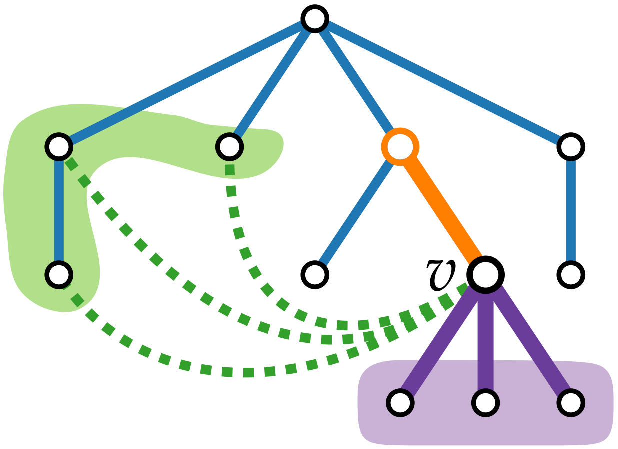

neighbors of v in circ. order:

(1) parent

(2) $N^+(v)$: diff. subtree (left)

(3) children

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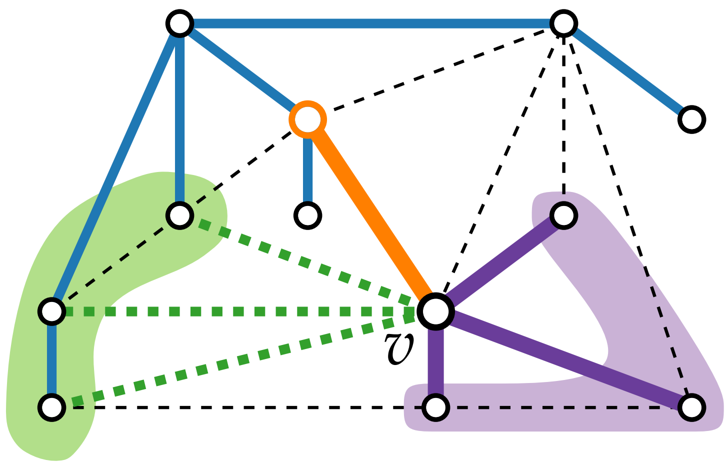
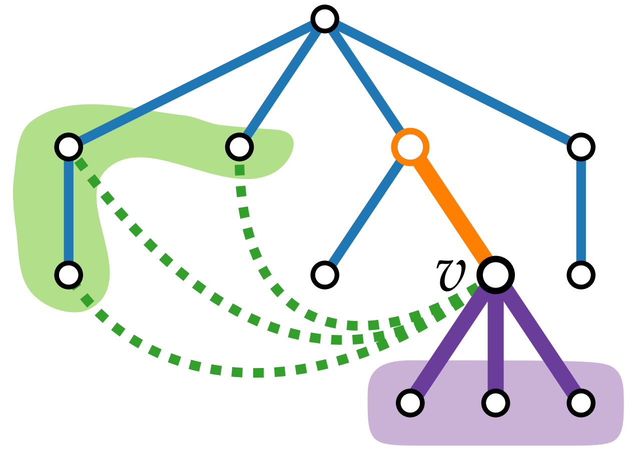
[Chiang, Lin, Lu '05]





neighbors of v in circ. order:

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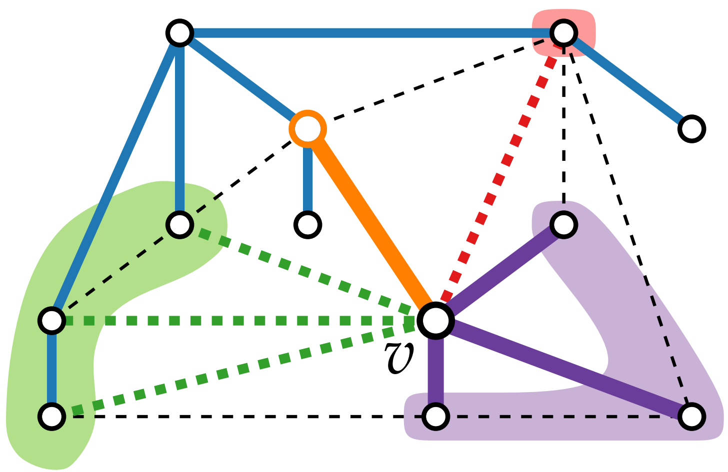

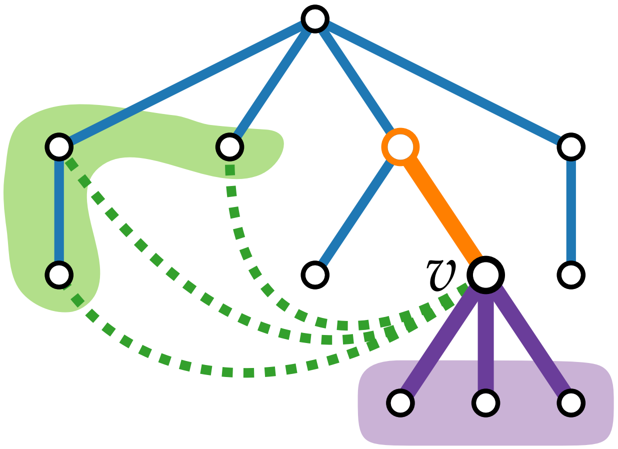

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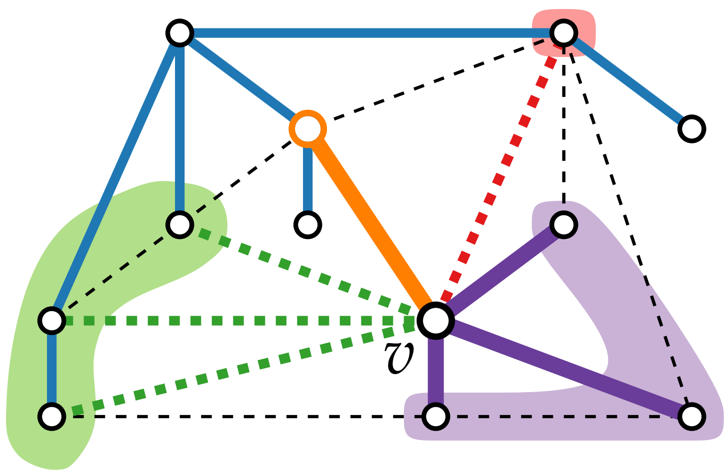

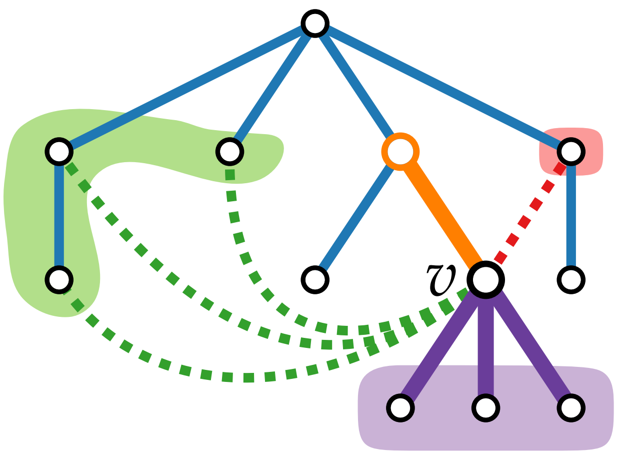

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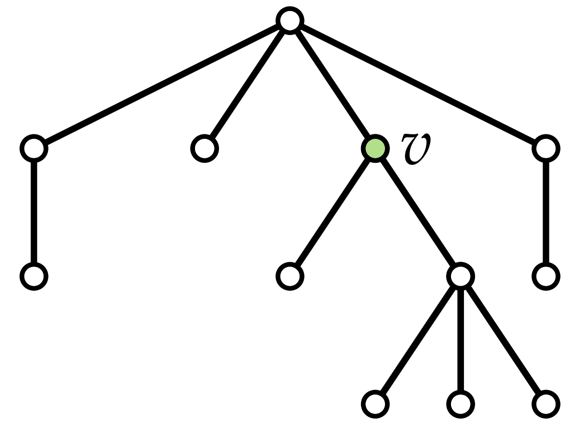
Slope-Disjoint Drawing of a Tree

[Angelini et al. '12]

Assign angle interval to each vtx



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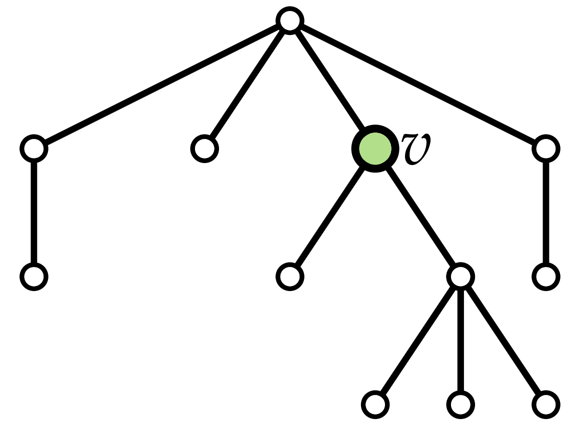

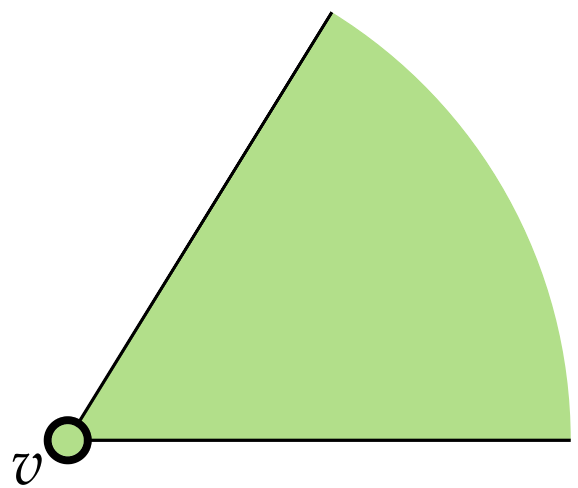

Slope-Disjoint Drawing of a Tree

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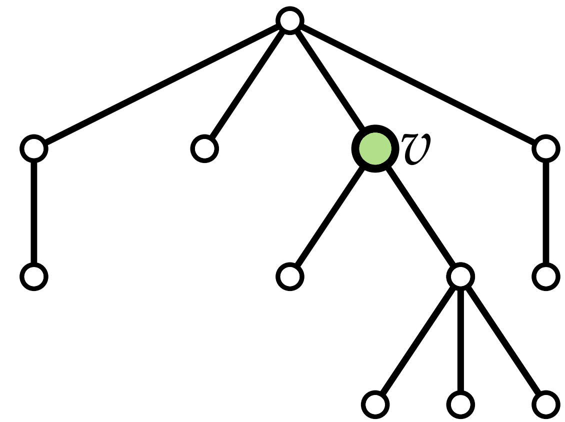
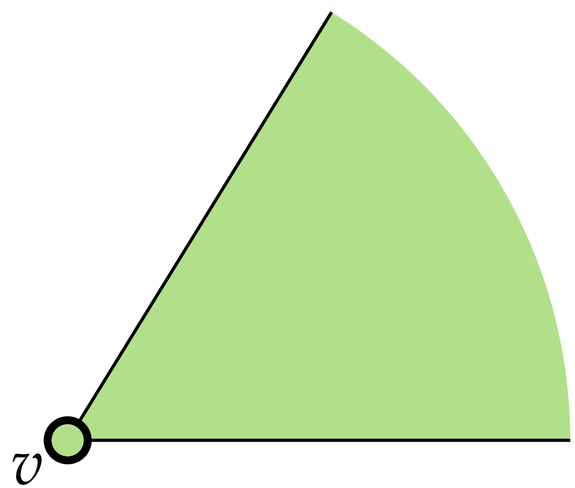
Assign angle interval to each vtx

All segments in $T[v]$ in interval





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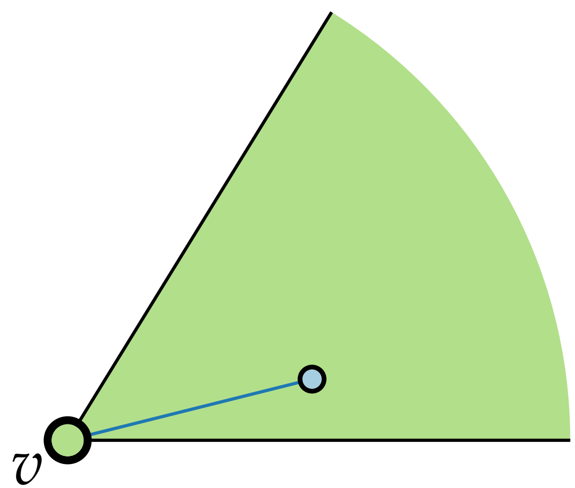

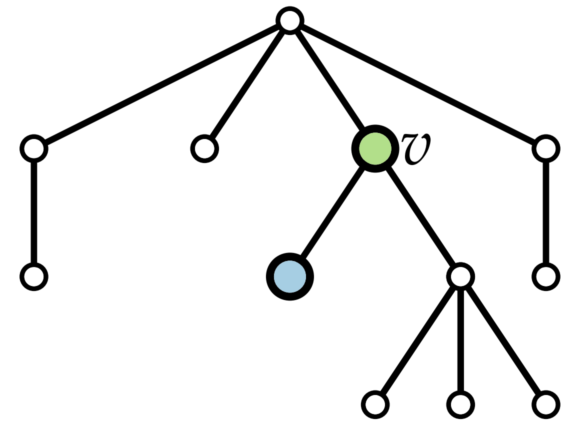

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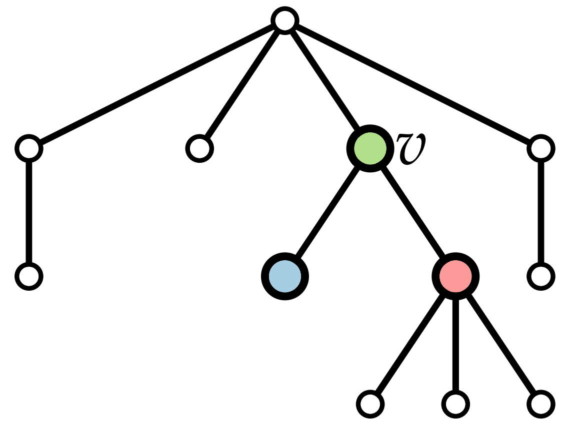

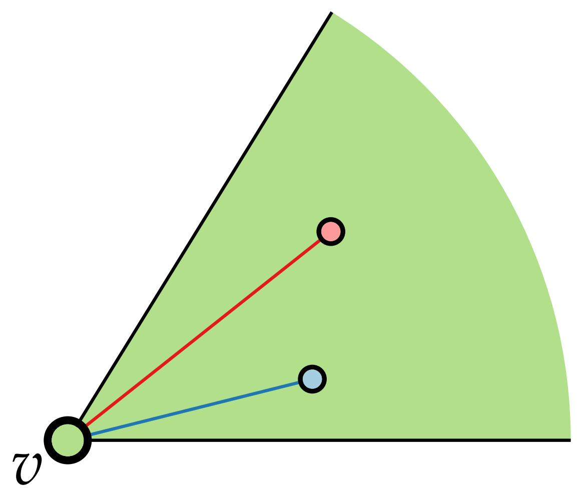

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[Angelini et al. ’12]

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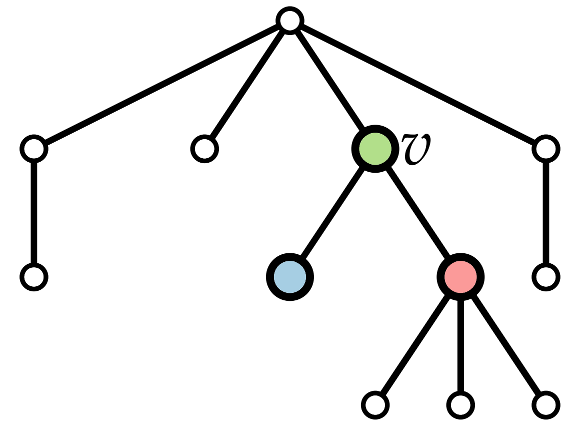
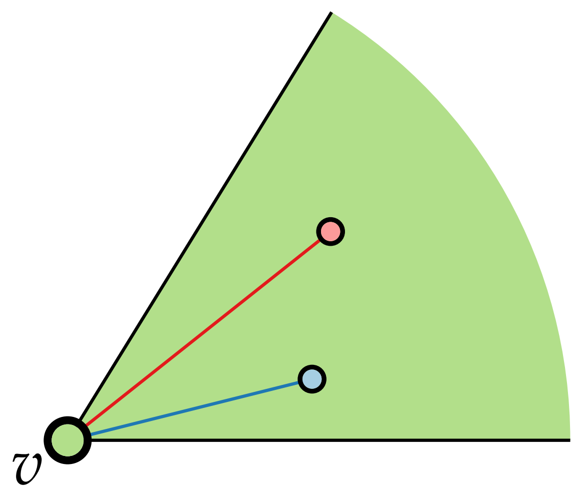
Intervals of children: disjoint subintervals that contain parent edge





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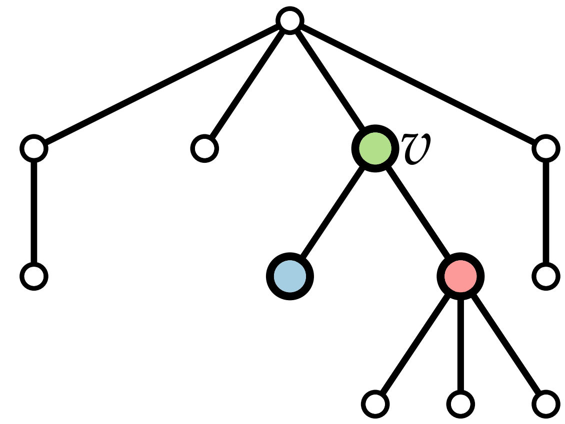

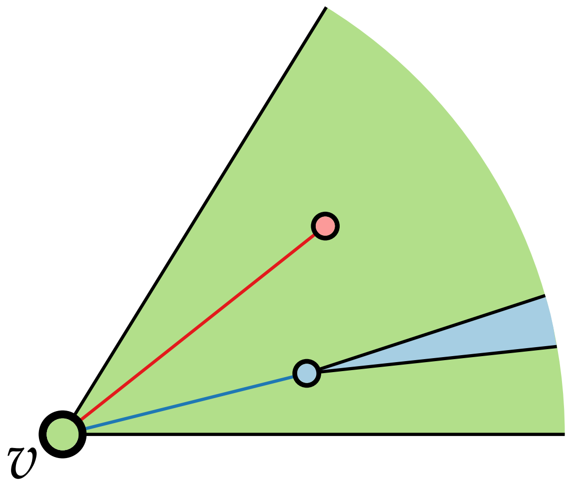

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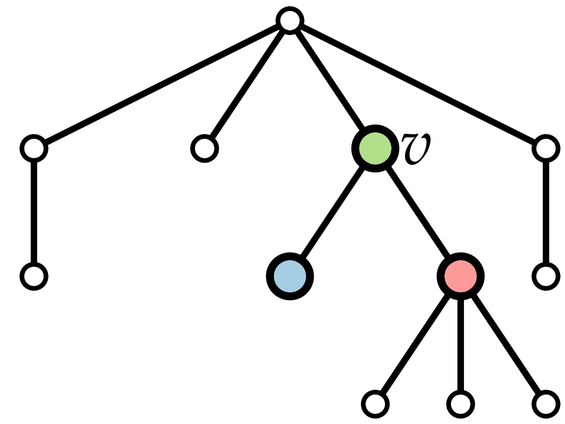

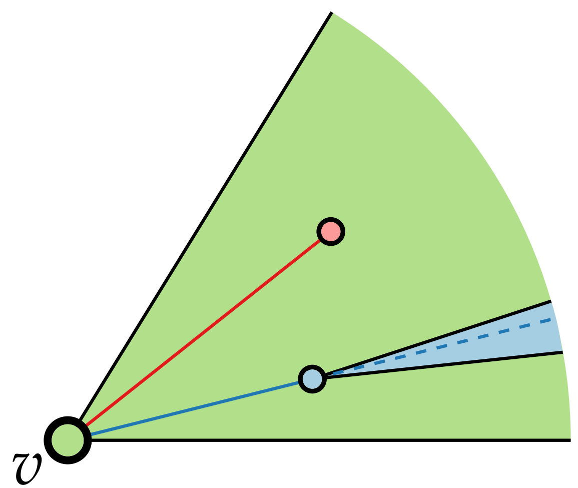

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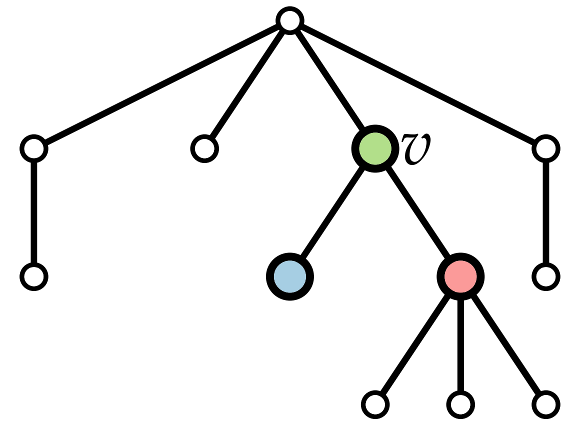

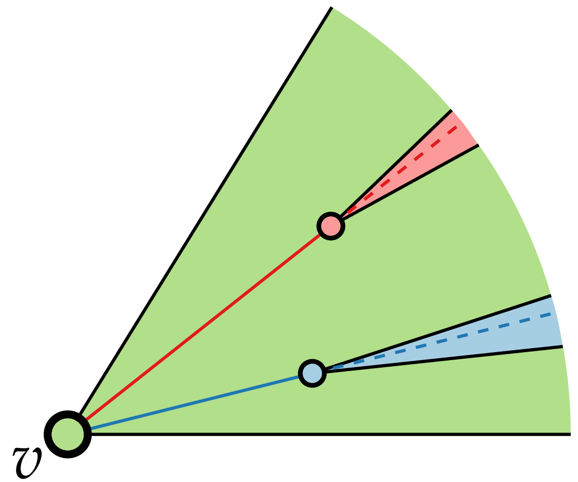

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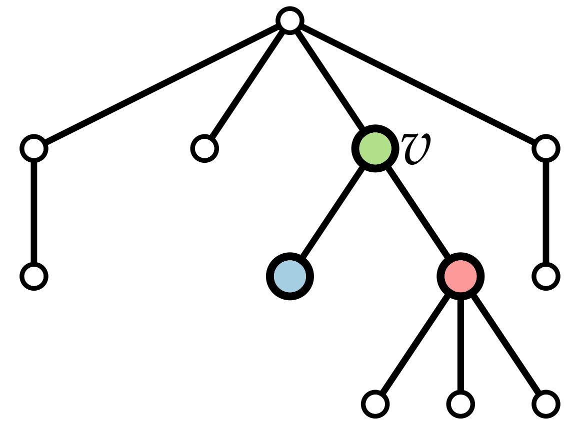

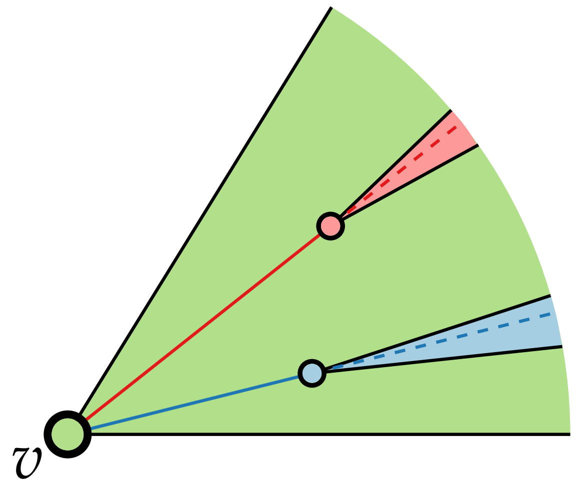

[Hossain & Rahman ’15]

Slope-disjoint drawing of orderly spanning tree on $O(n) \times O(n^2)$ grid
$\Rightarrow$ planar (monotone) drawing on $O(n) \times O(n^2)$ grid

Slope-Disjoint Drawing of a Tree

[Angelini et al. ’12]

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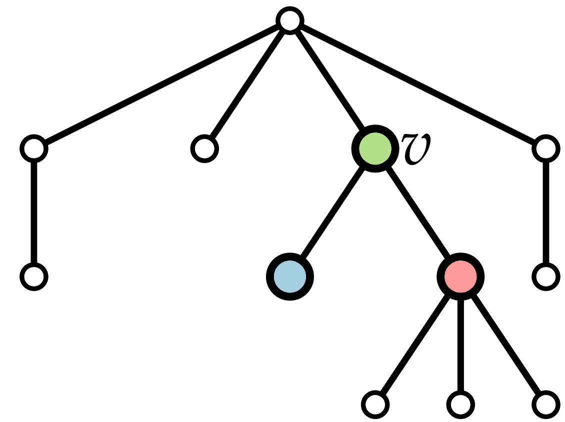
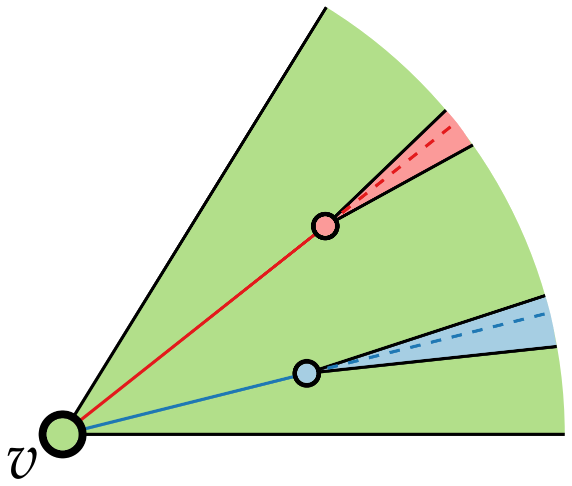


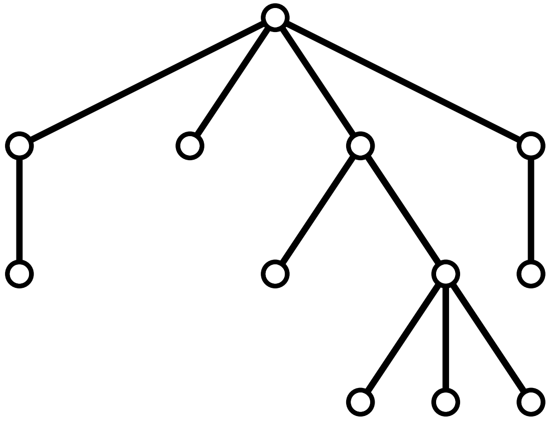


[Hossain & Rahman ’15]

Slope-disjoint drawing of orderly spanning tree on $O(n) \times O(n^2)$ grid
$\Rightarrow$ planar (monotone) drawing on $O(n) \times O(n^2)$ grid

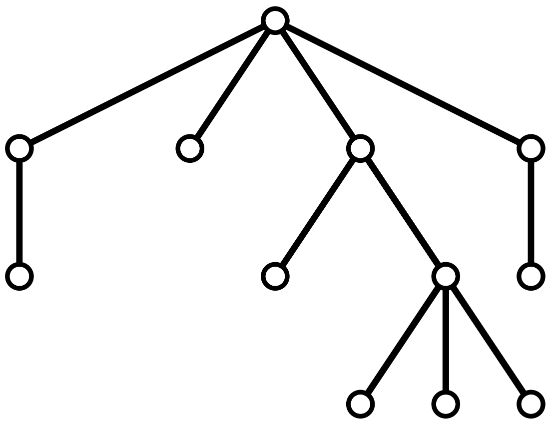
doesn’t change the slopes!

Obtaining a slope-disjoint drawing



Obtaining a slope-disjoint drawing

ccw pre-order traversal



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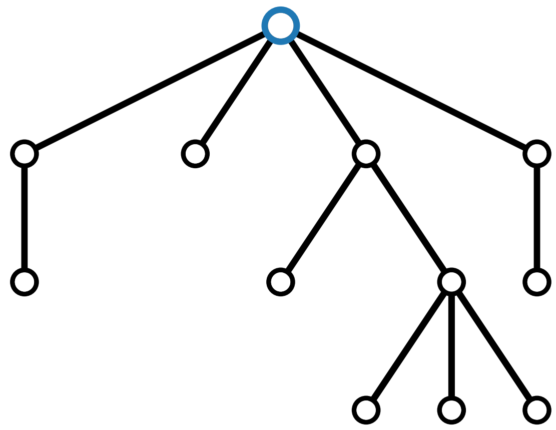

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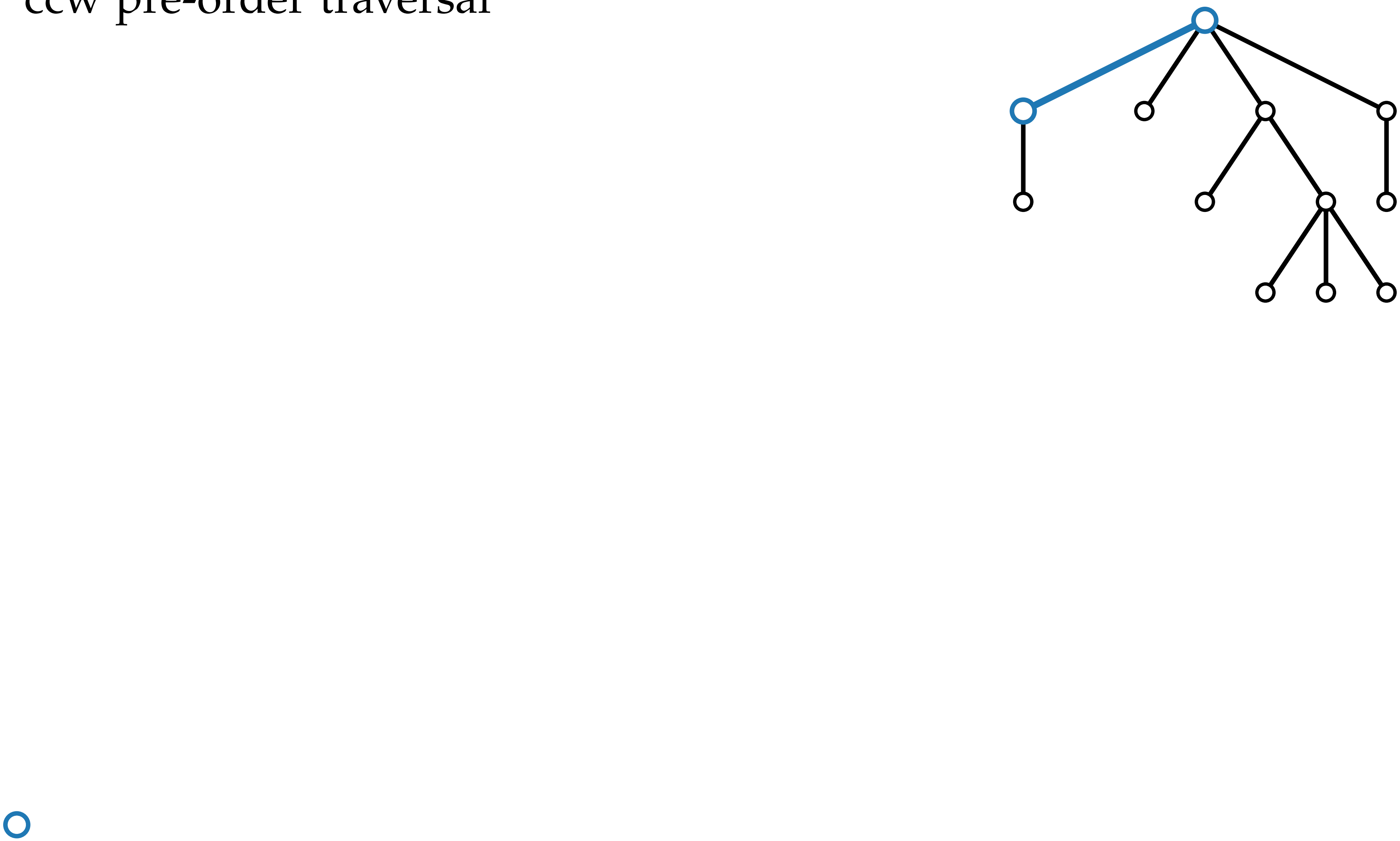

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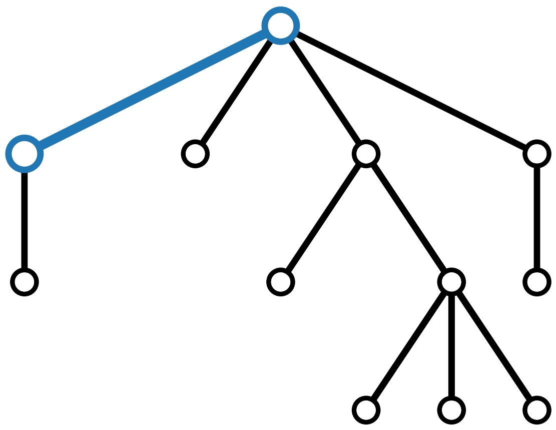

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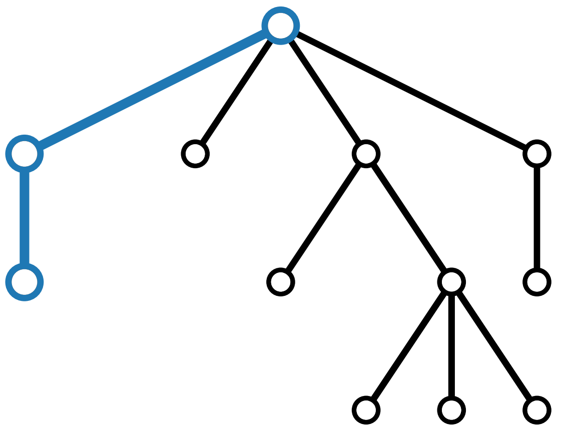
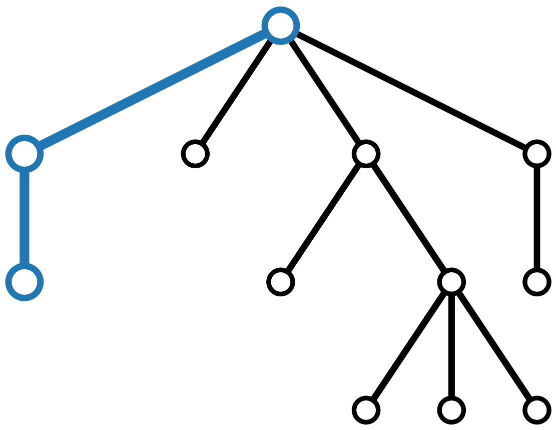
ccw pre-order traversal



Obtaining a slope-disjoint drawing

ccw pre-order traversal

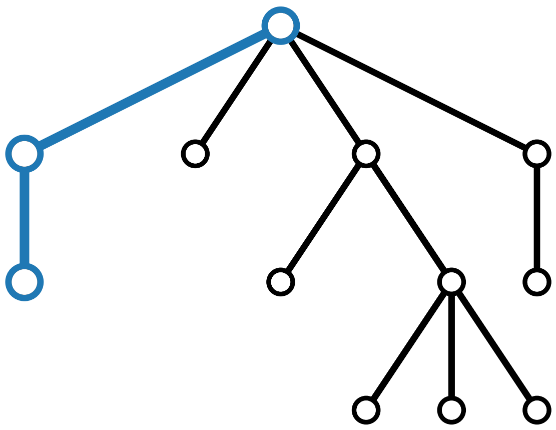
reuse slope whenever possible

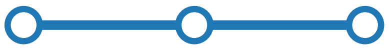


Obtaining a slope-disjoint drawing

ccw pre-order traversal

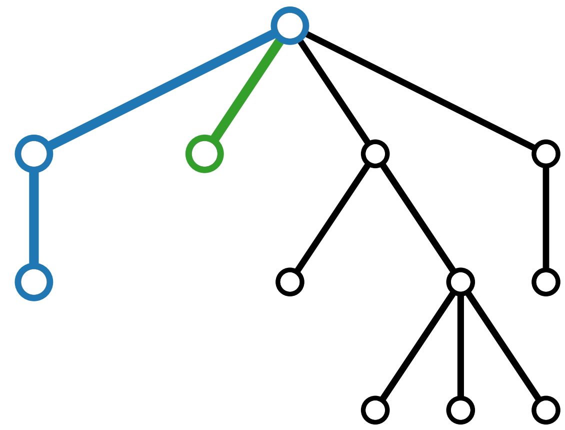
reuse slope whenever possible





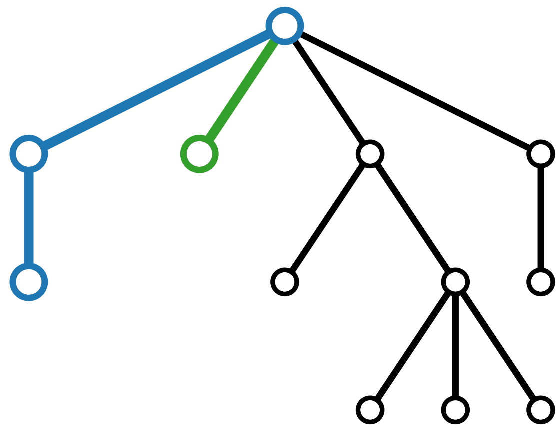
Obtaining a slope-disjoint drawing

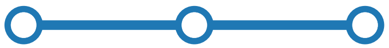
ccw pre-order traversal
reuse slope whenever possible



Obtaining a slope-disjoint drawing

ccw pre-order traversal
reuse slope whenever possible
otherwise use highest slope +1





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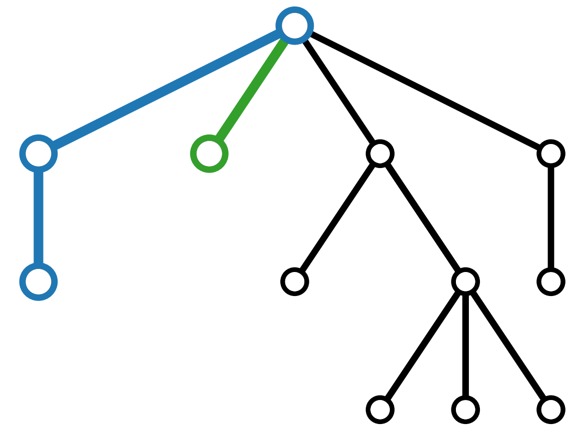

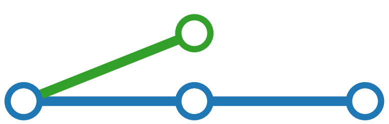

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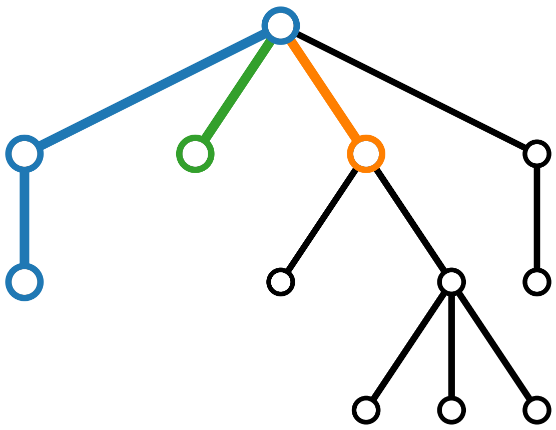

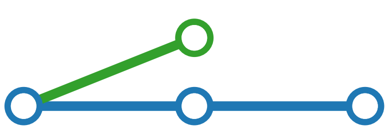

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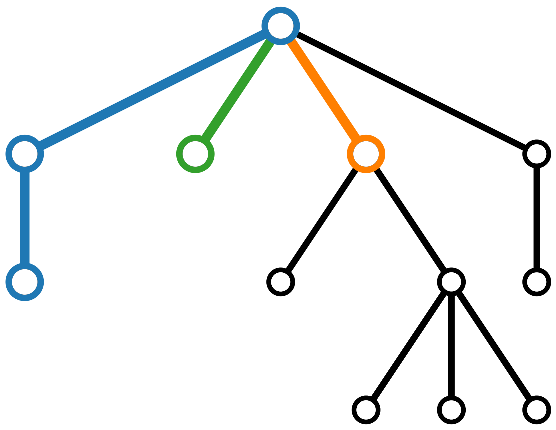

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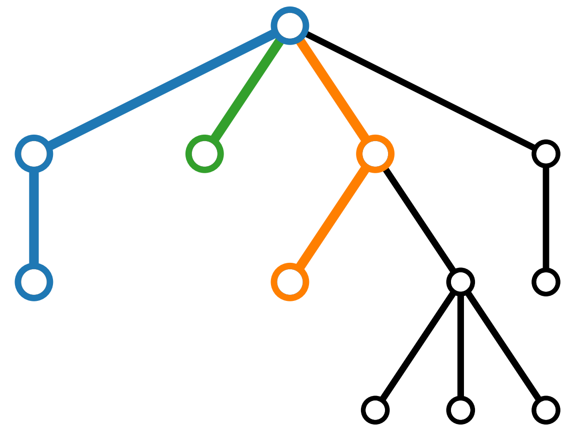

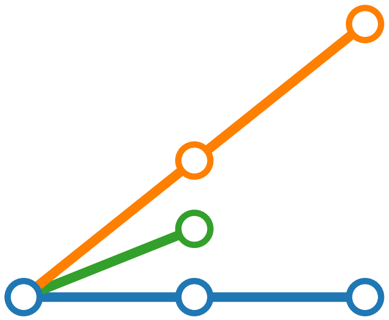

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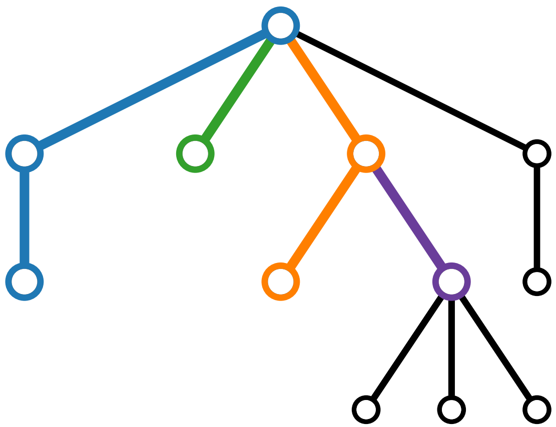

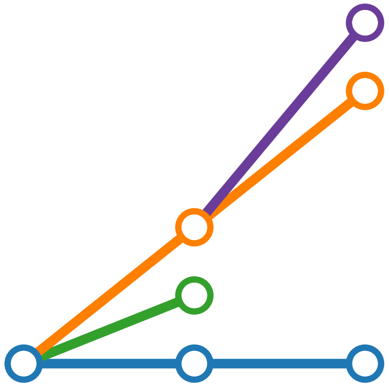

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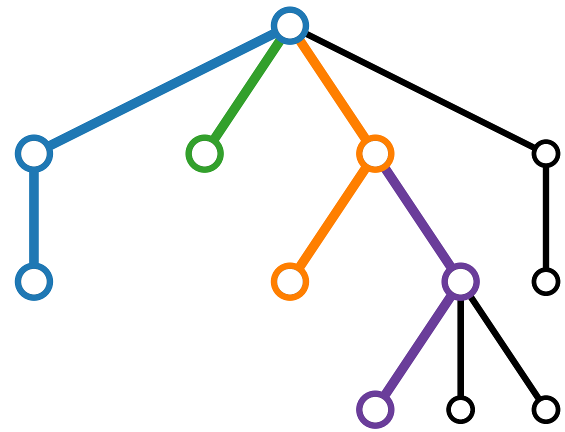

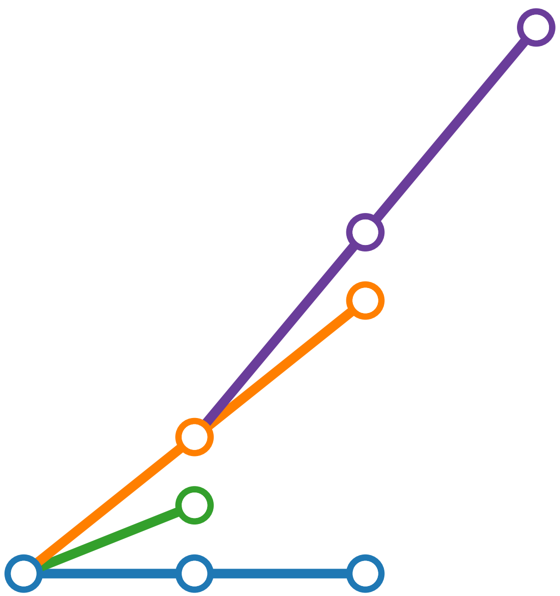

Obtaining a slope-disjoint drawing

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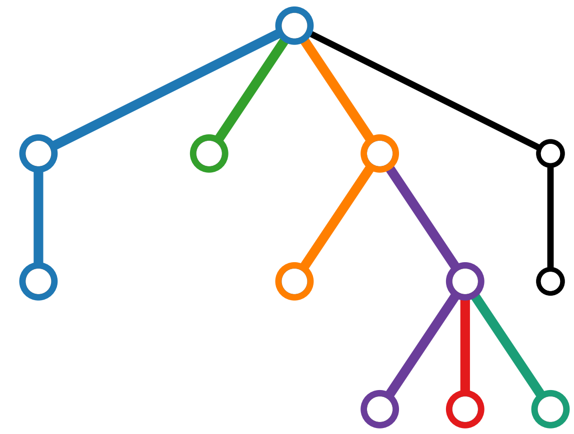

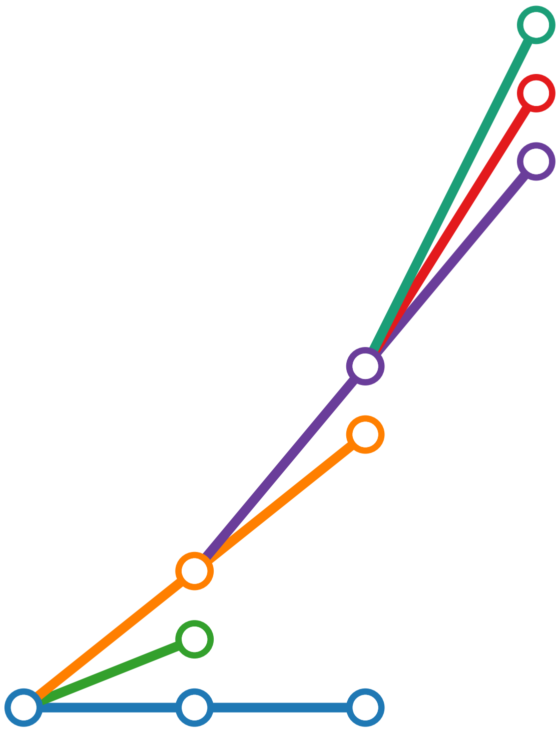

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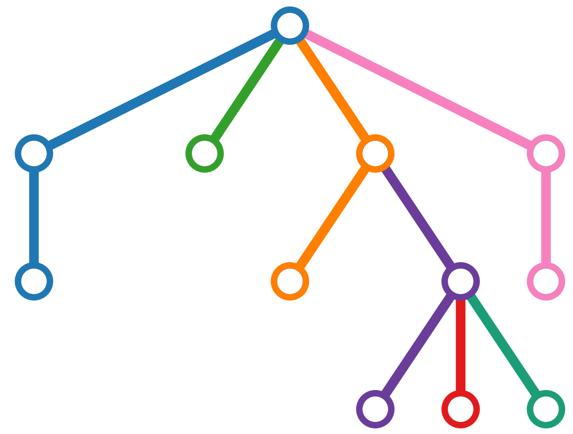

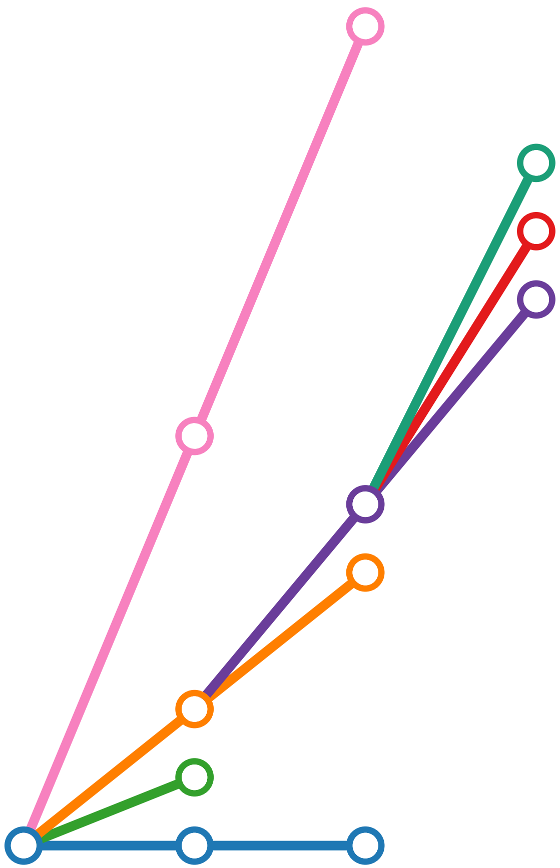

Obtaining a slope-disjoint drawing

ccw pre-order traversal
reuse slope whenever possible
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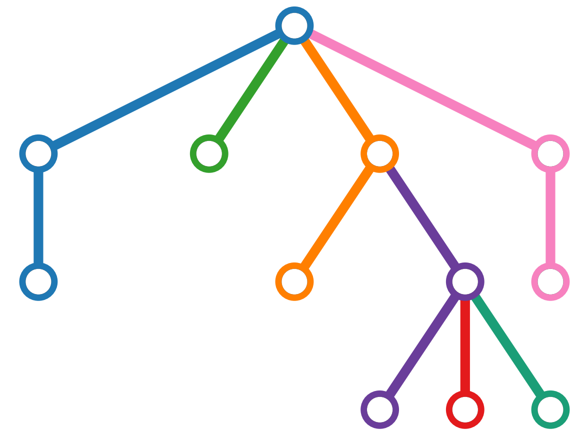
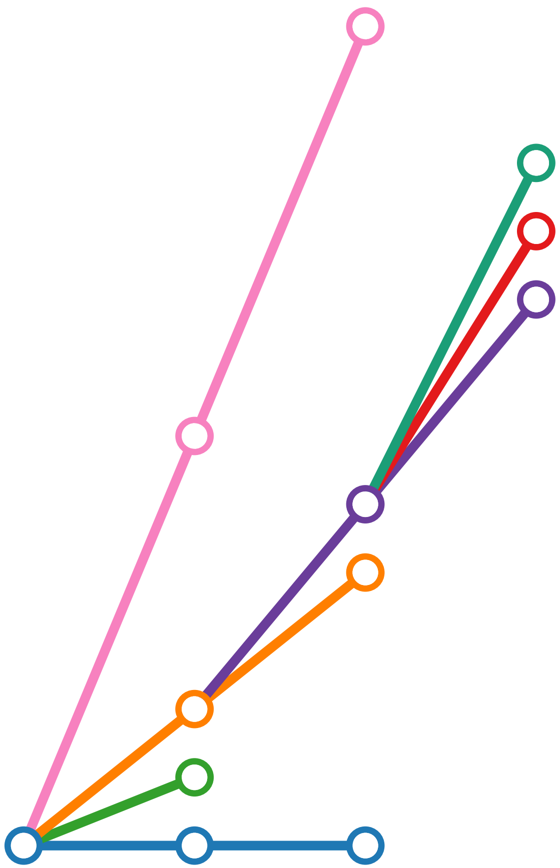


highest slope: n



Obtaining a slope-disjoint drawing

ccw pre-order traversal
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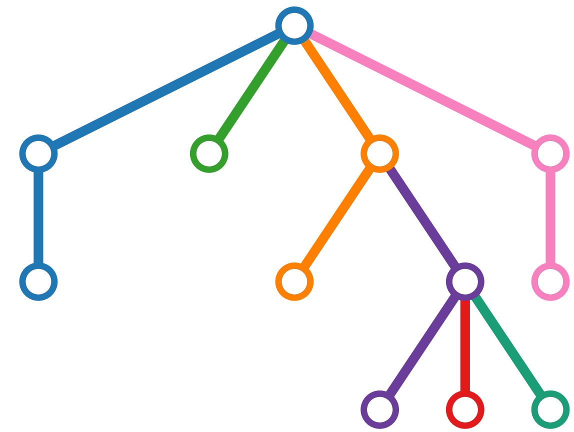
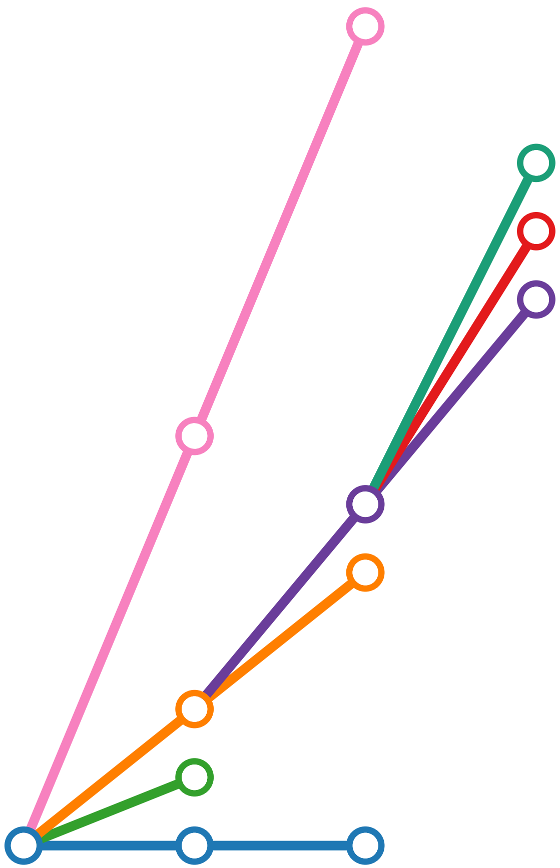
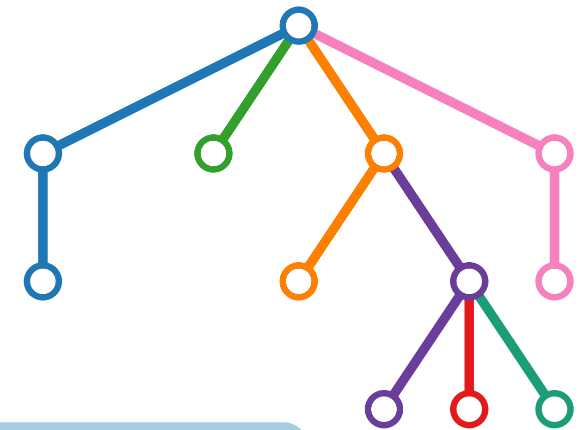


highest slope: n
max. width: n



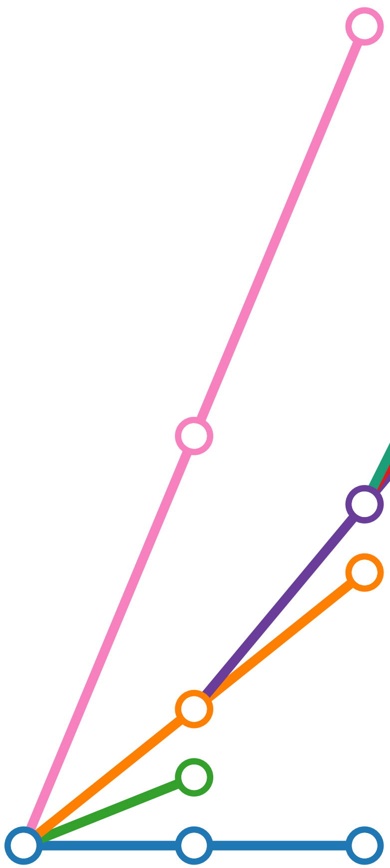
Obtaining a slope-disjoint drawing

ccw pre-order traversal
reuse slope whenever possible
otherwise use highest slope $+1$



highest slope: n
max. width: n

$\Rightarrow n \times n^2$ grid



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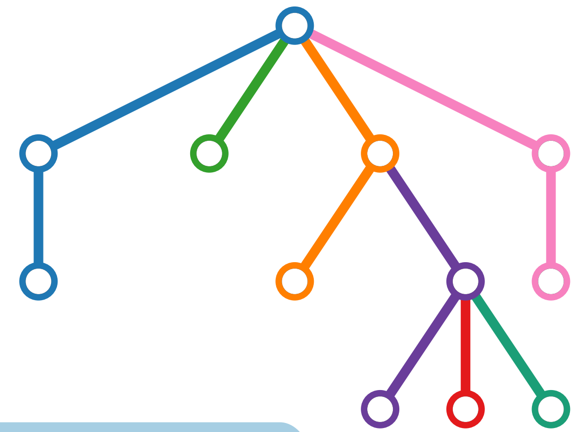
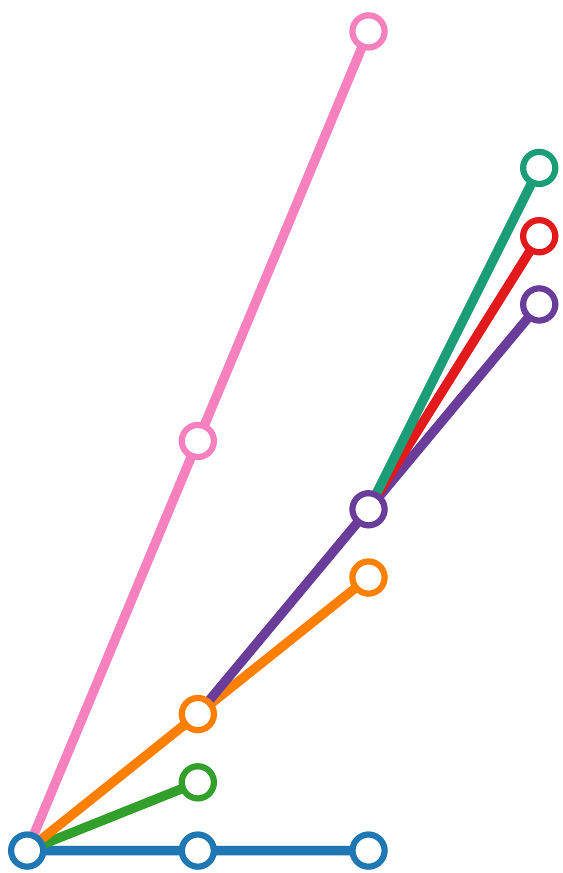


highest slope: n
max. width: n

$\Rightarrow n \times n^2$ grid , 1 segment per leaf



Obtaining a slope-disjoint drawing

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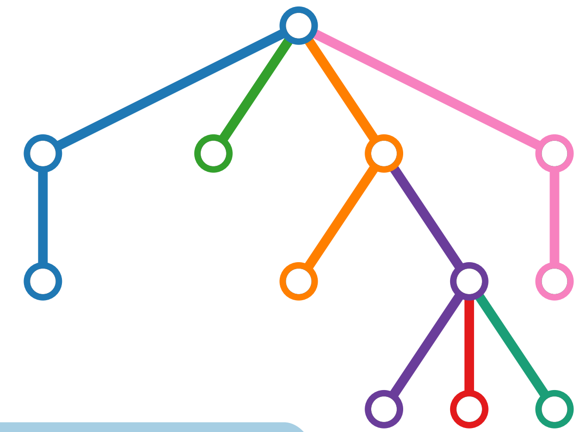

highest slope: n
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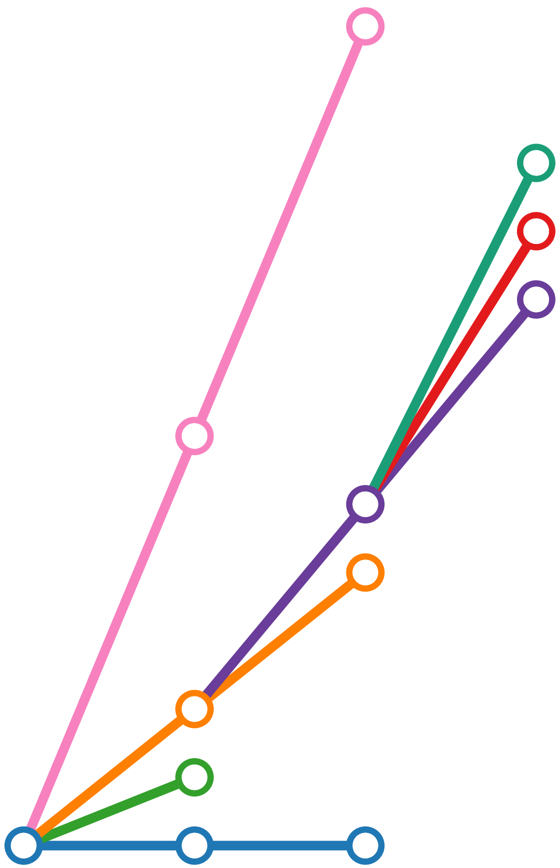
$\Rightarrow n \times n^2$ grid , 1 segment per leaf

[Miura, Azuma, Nishizeki '05]
Every Schnyder tree is an orderly spanning tree



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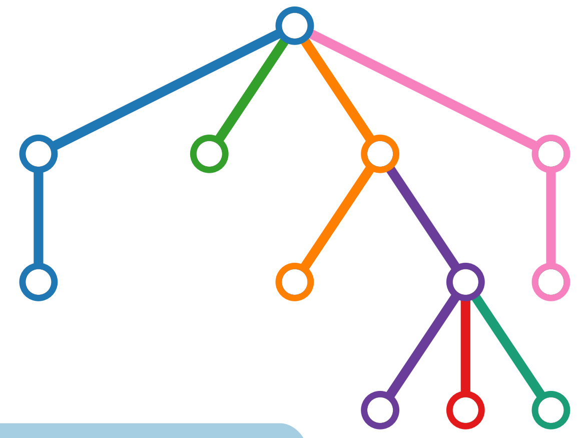

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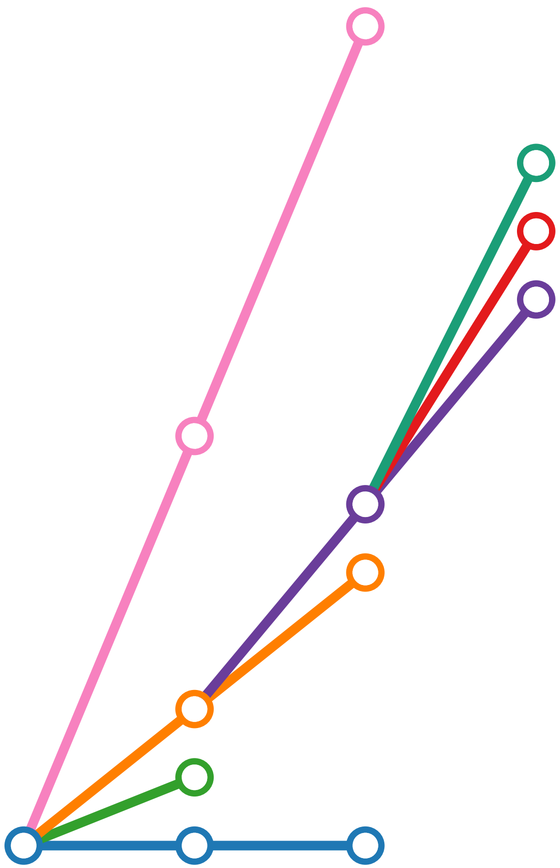
T_1, T_2, T_3 Schnyder realizer of 3-conn. planar graph
$\Rightarrow \leq 2n+1$ leaves in total in T_1, T_2, T_3



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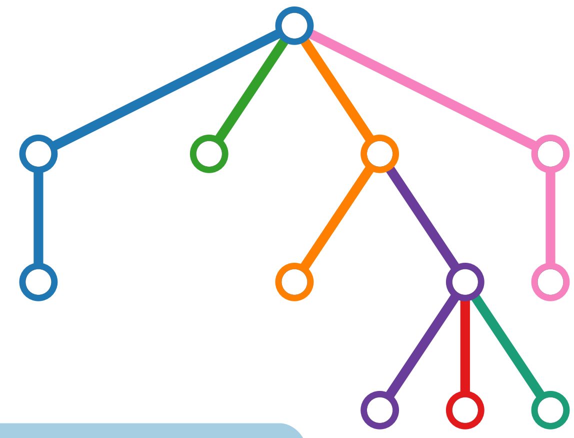
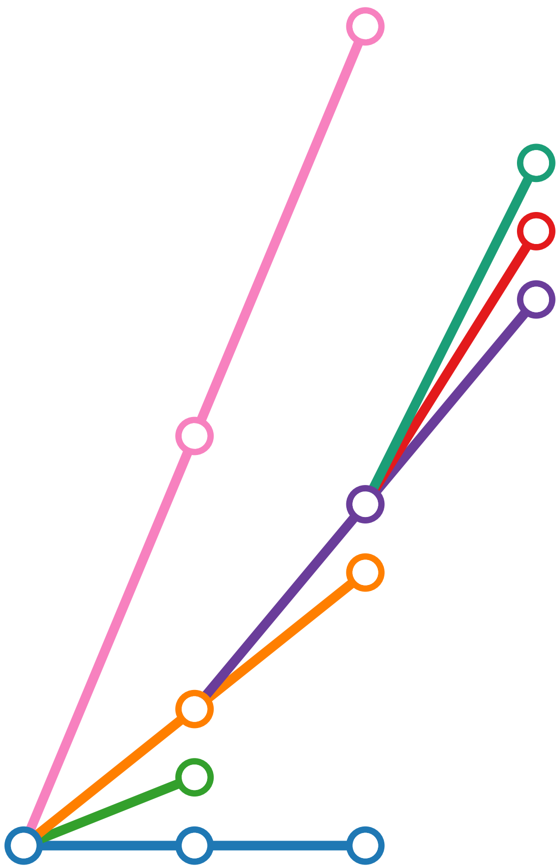


highest slope: n
max. width: n

$\Rightarrow n \times n^2$ grid , 1 segment per leaf



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$\Rightarrow \leq 2n+1$ leaves in total in T_1, T_2, T_3

3-conn. planar graph
$\Rightarrow (8n-14)/3$ segments, $O(n) \times O(n^2)$ grid

New Results

Class	Segments		Grid Segments	
	Lower	Upper	Segm.	Area
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$3n/4$	$n \times n$
			$\vartheta/2$ [6]	quasipolynomial
outerplanar	n [1]			
max. outerp.	n [1]	n [1]	$3n/2$ [6]	$O(n) \times O(n^2)$
3-trees	$2n$ [1]	$2n$ [1]	$8n/3$ [6]	$O(n) \times O(n^2)$
2-connected	$2n$ [1]			
3-connected	$2n$ [1]	$5n/2$ [1]		
cubic 3-conn.	$n/2$ [3]	$n/2$ [2]	$n/2$ [2]	$O(n) \times O(n)$
triangulation	$2n$ [4]	$7n/3$ [4]	$8n/3$ [5]	$2^{O}(n \log n)$
4-conn. triang.	$2n$ [4]	$9n/3$ [4]		
planar	$2n$ [4]	$8n/3$ [4]		

[1] Dujmović et al. 2007 [2] Igamberdiev et al. 2015 [3] Mondal et al. 2013
[4] Durocher & Mondal 2014 [5] Mondal 2016 [6] Hültenschmidt et al. 2017

New Results

Class	Segments: Lower	Segments: Upper	Grid Segments: Segm.	Grid Segments: Area
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$3n/4$	$n \times n$
			$\vartheta/2$ [6]	quasipolynomial
outerplanar	n [1]			
max. outerp.	n [1]	n [1]	$3n/2$ [6]	$O(n) \times O(n^2)$
3-trees	$2n$ [1]	$2n$ [1]	$8n/3$ [6]	$O(n) \times O(n^2)$
2-connected	$2n$ [1]			
3-connected	$2n$ [1]	$5n/2$ [1]	$8n/3$	$O(n) \times O(n^2)$
cubic 3-conn.	$n/2$ [3]	$n/2$ [2]	$n/2$ [2]	$O(n) \times O(n)$
triangulation	$2n$ [4]	$7n/3$ [4]	$8n/3$ [5]	$2^{O}(n \log n)$
4-conn. triang.	$2n$ [4]	$9n/3$ [4]		
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3-trees	$2n$ [1]	$2n$ [1]	$8n/3$ [6]	$O(n) \times O(n^2)$
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New Results

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outerplanar	n [1]		$7n/4$	$O(n) \times O(n^2)$
max. outerp.	n [1]	n [1]	$3n/2$ [6]	$O(n) \times O(n^2)$
3-trees	$2n$ [1]	$2n$ [1]	$8n/3$ [6]	$O(n) \times O(n^2)$
2-connected	$2n$ [1]			
3-connected	$2n$ [1]	$5n/2$ [1]	$8n/3$	$O(n) \times O(n^2)$
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3-trees	$2n$ [1]	$2n$ [1]	$8n/3$ [6]	$O(n) \times O(n^2)$
2-connected	$2n$ [1]			
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cubic 3-conn.	$n/2$ [3]	$n/2$ [2]	$n/2$ [2]	$O(n) \times O(n)$
triangulation	$2n$ [4]	$7n/3$ [4]	$8n/3$ [5]	$2^{O}(n \log n)$
4-conn. triang.	$2n$ [4]	$9n/3$ [4]	$5n/2$	$O(n) \times O(n^2)$
planar	$2n$ [4]	$8n/3$ [4]	$17n/6$	$O(n) \times O(n^2)$

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