

Extending Simple Drawings

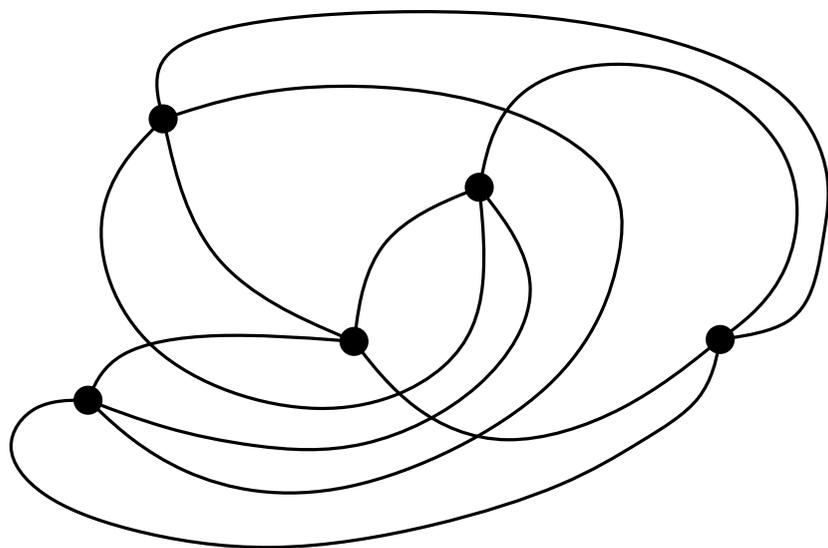
Alan Arroyo¹, Martin Derka², and Irene Parada³

¹ IST Austria

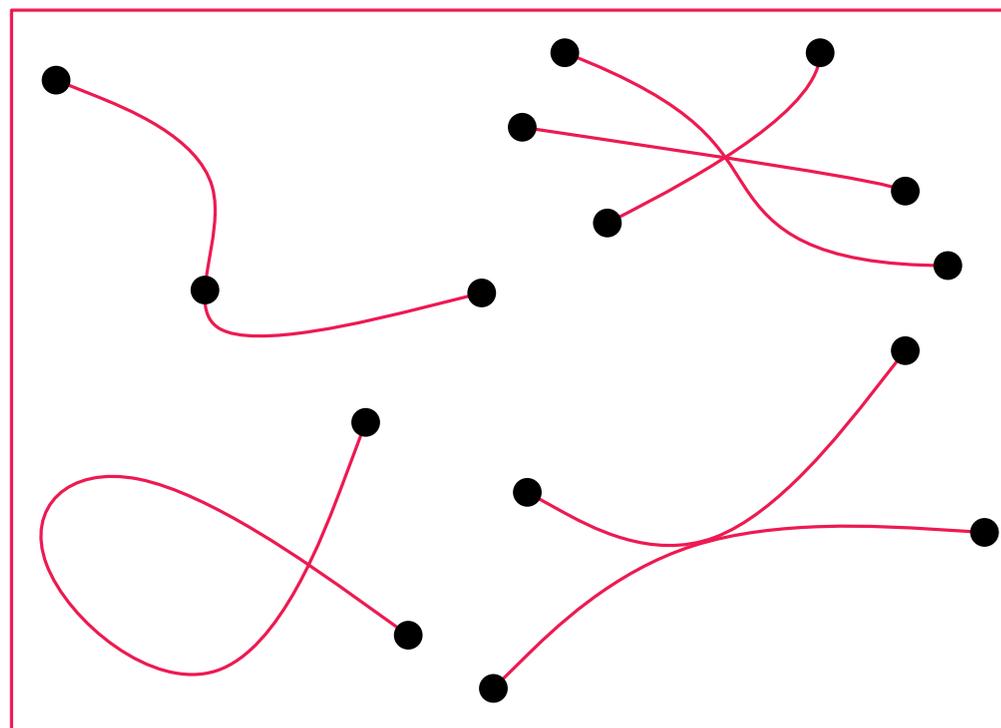
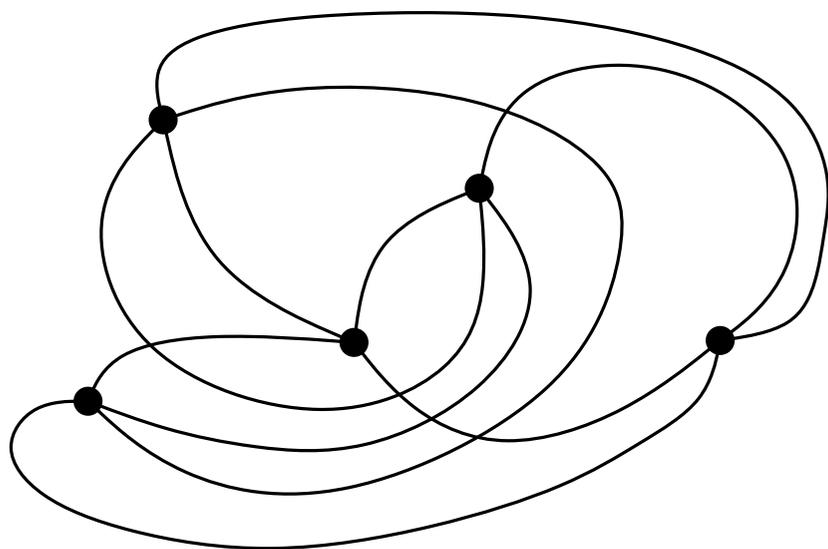
² Carleton University, Canada

³ Graz University of Technology, Austria

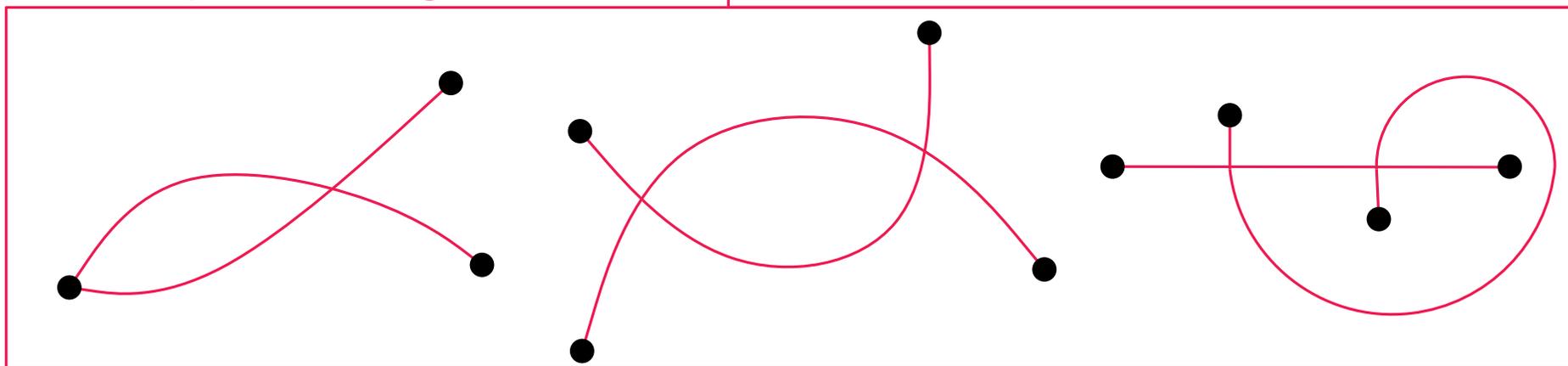
Simple drawings



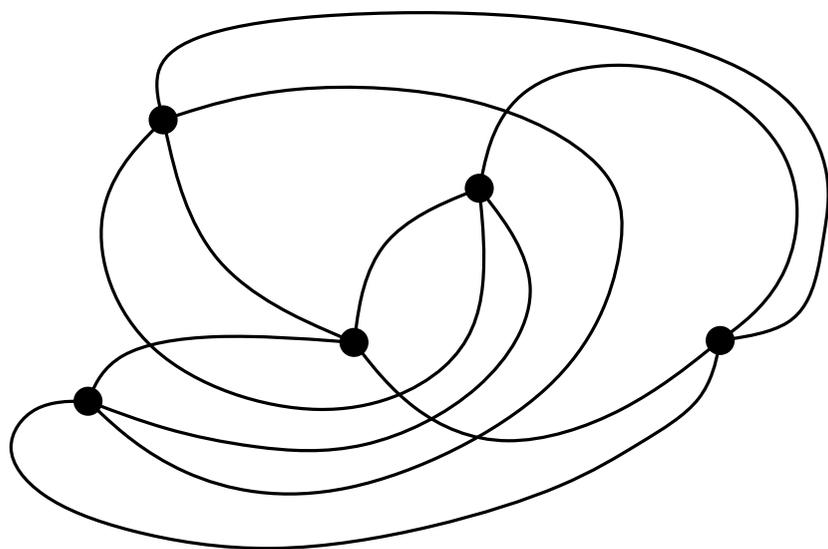
Simple drawings



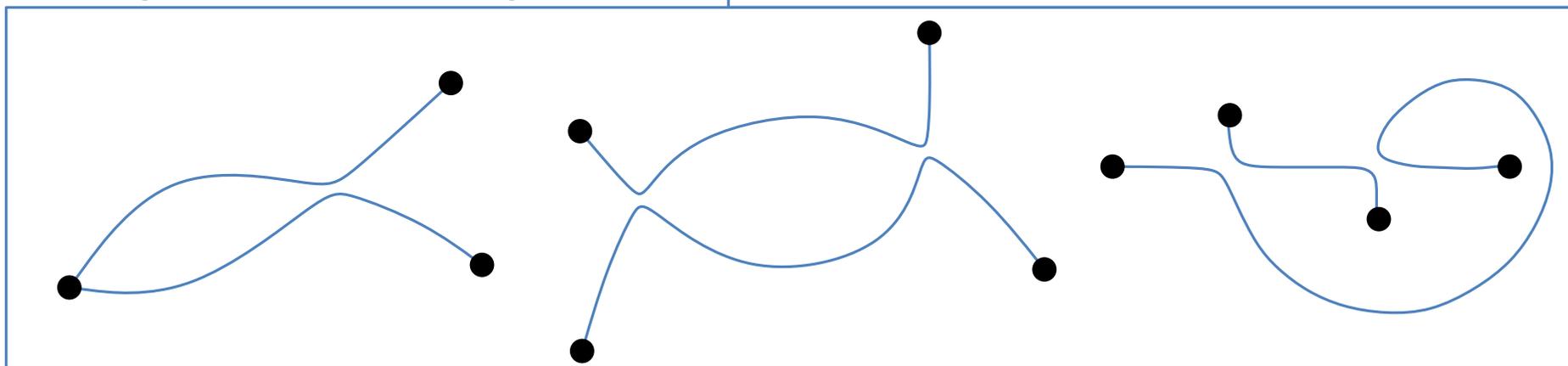
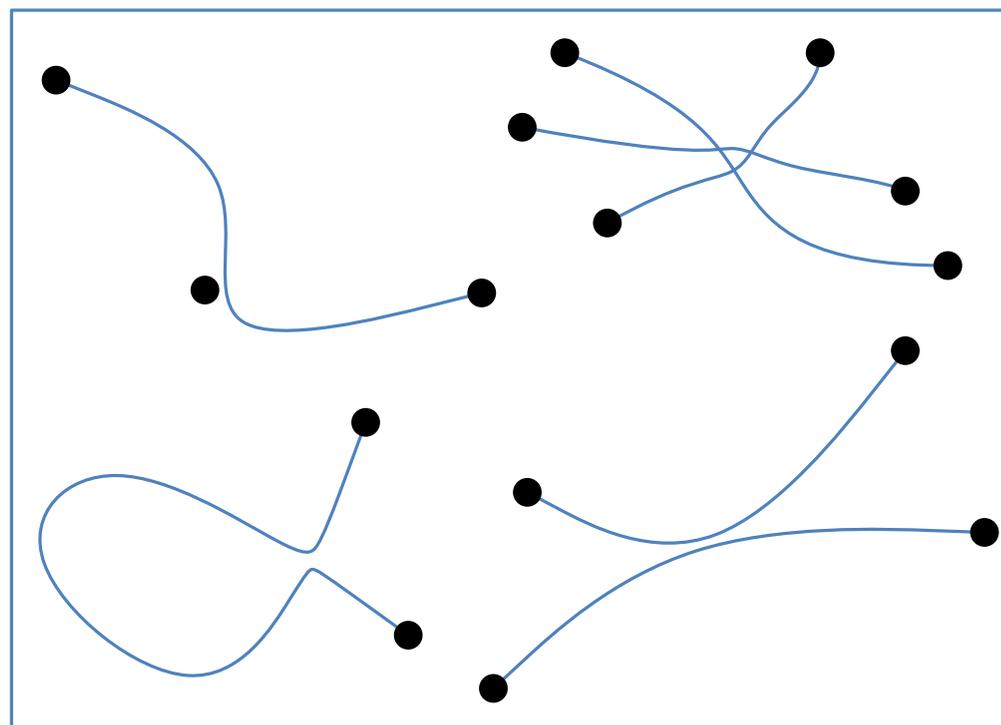
Not simple drawings:



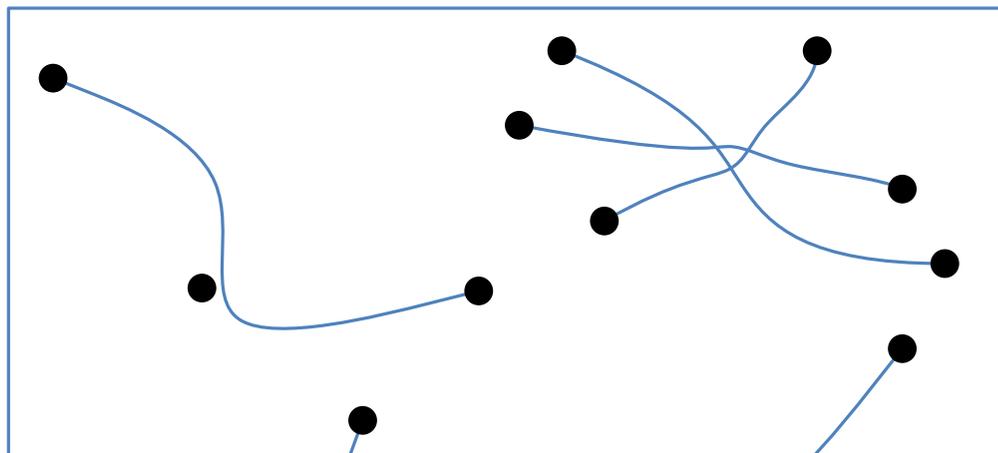
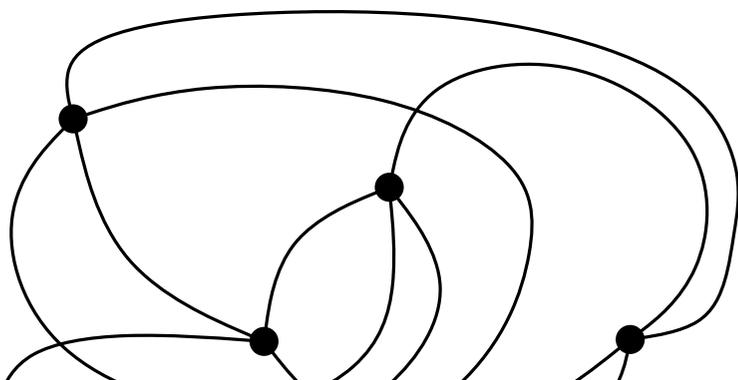
Simple drawings



Locally fixed: now they are!

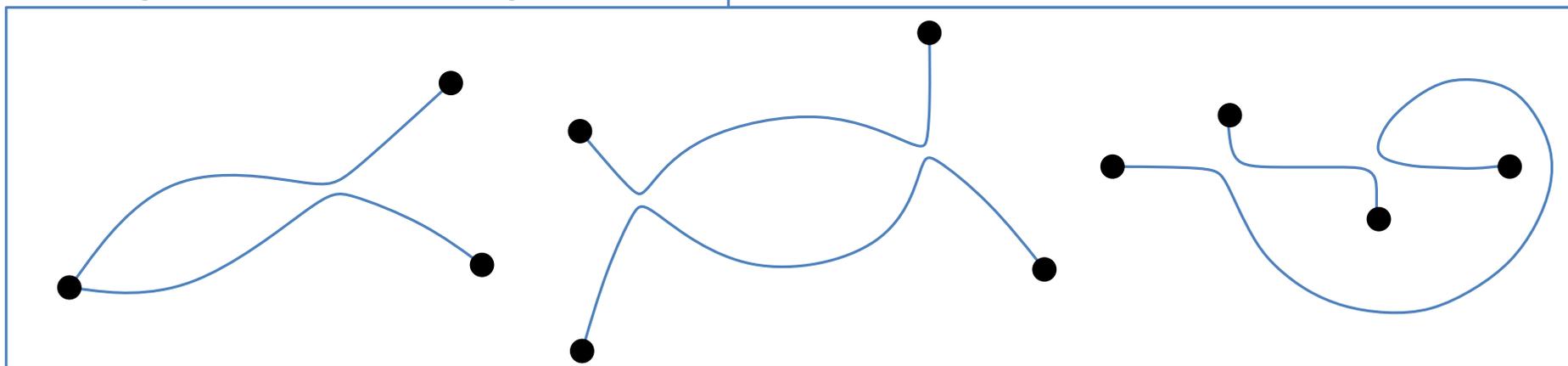


Simple drawings

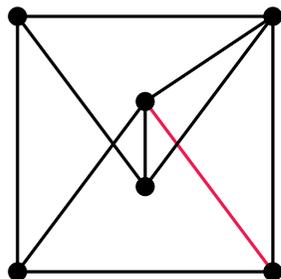


Drawings that minimize the total number of crossings are simple.

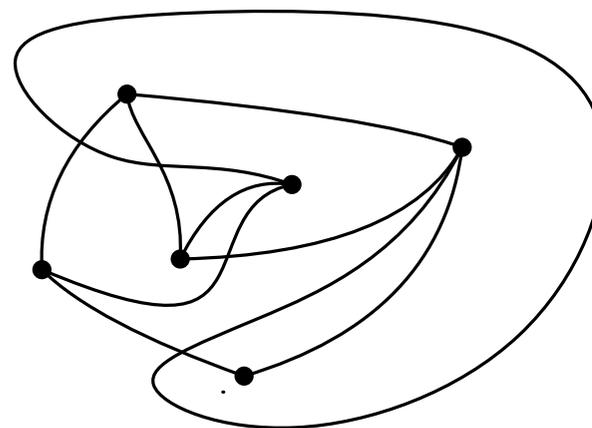
Locally fixed: now they are!



Extending a partial representation

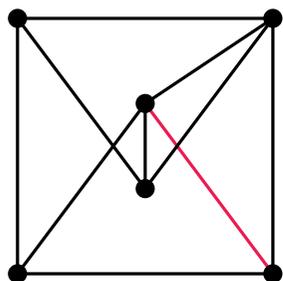


Abstract graph G

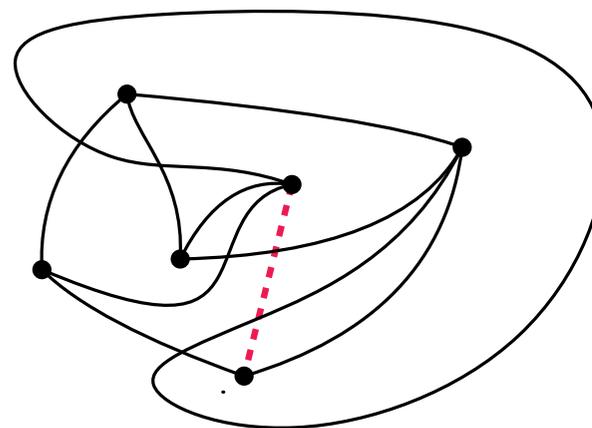


(Partial)
representation of
a subgraph of G

Extending a partial representation

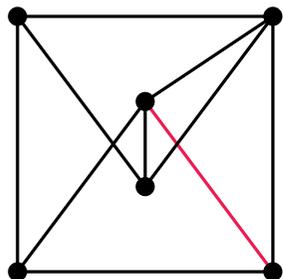


Abstract graph G

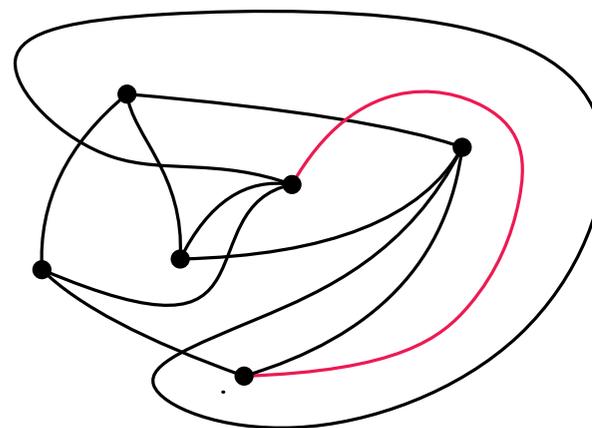


(Partial)
representation of
a subgraph of G

Extending a partial representation



Abstract graph G



(Partial)
representation of
a subgraph of G

Extending a partial representation

Extending partial drawings of planar graphs:

- [Bagheri, Razzazi '10]
- [Jelínek, Kratochvíl, Rutter '13]
- [Angelini et. al. '15]
- [Mchedlidze, Nöllenburg, Rutter '15]
- [Brückner, Rutter '17]
- [Da Lozzo, Di Battista, Frati '19]
- [Patrignani '06]

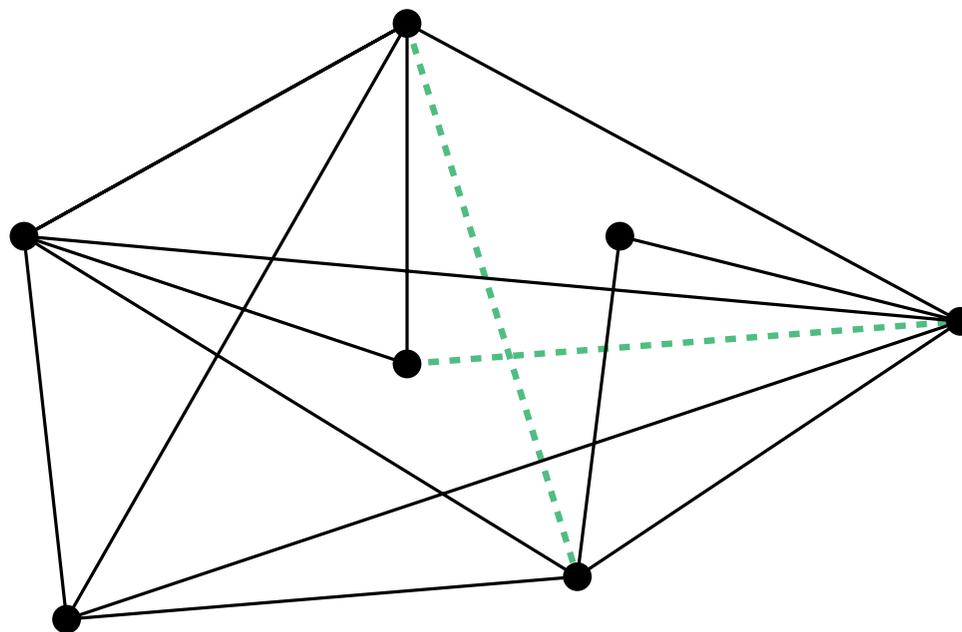
Extending partial rep. that are not drawings:

- [Klavík, Kratochvíl, Krawczyk, Walczak '12]
- [Chaplick et. al. '14]
- [Klavík, Kratochvíl, Otachi, Saitoh '15]
- [Klavík et. al. '17]
- [Klavík et. al. '17]
- [Chaplick et. al. '18]
- [Chaplick, Fulek, Klavík '19]

Can we always insert the remaining edges?

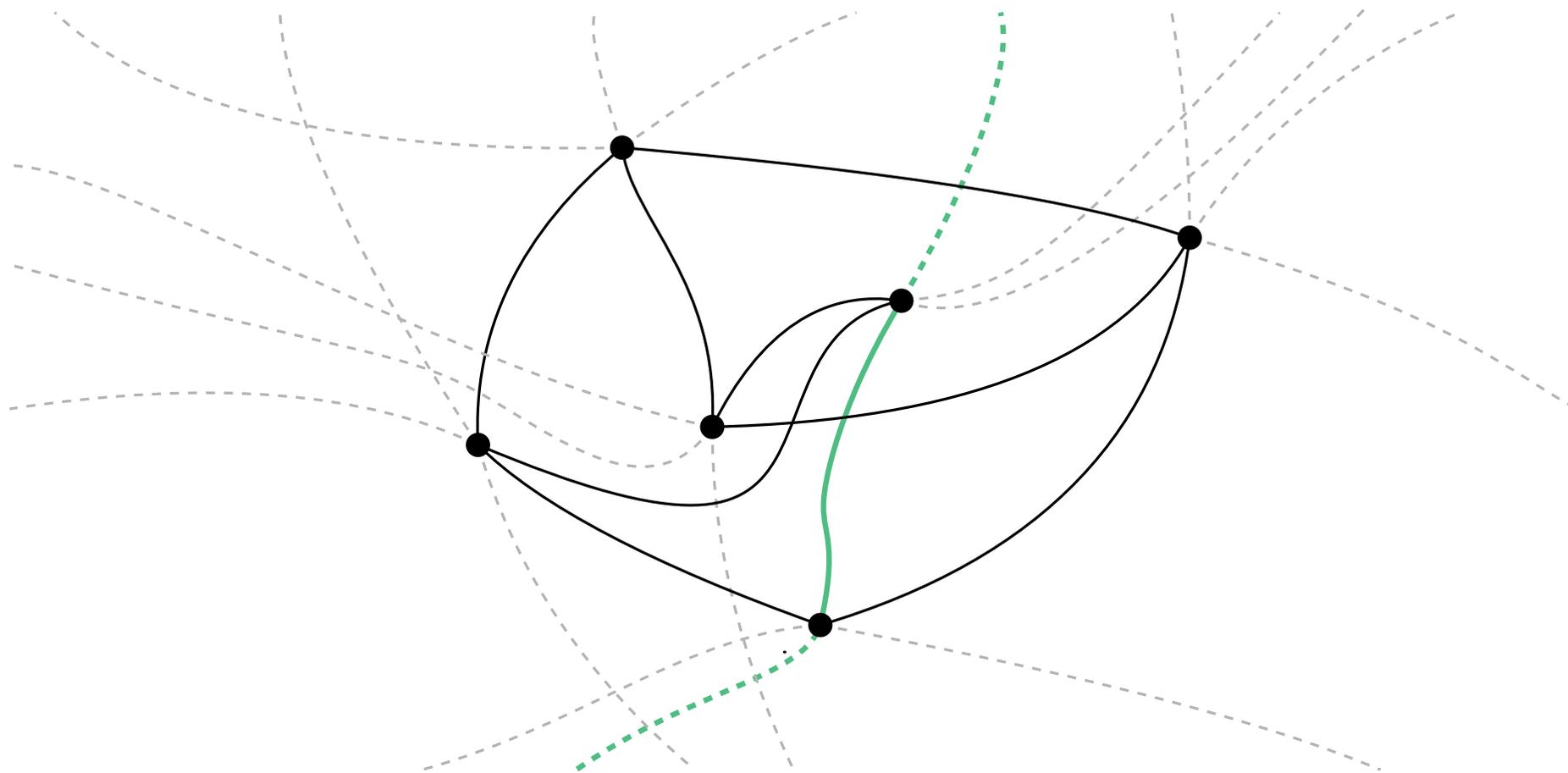
Given a simple drawing $D(G)$ of a graph $G = (V, E)$ we want to insert a set of edges (of the complement of G) s.t. the result is a simple drawing with $D(G)$ as a subdrawing.

Can we always insert the remaining edges?



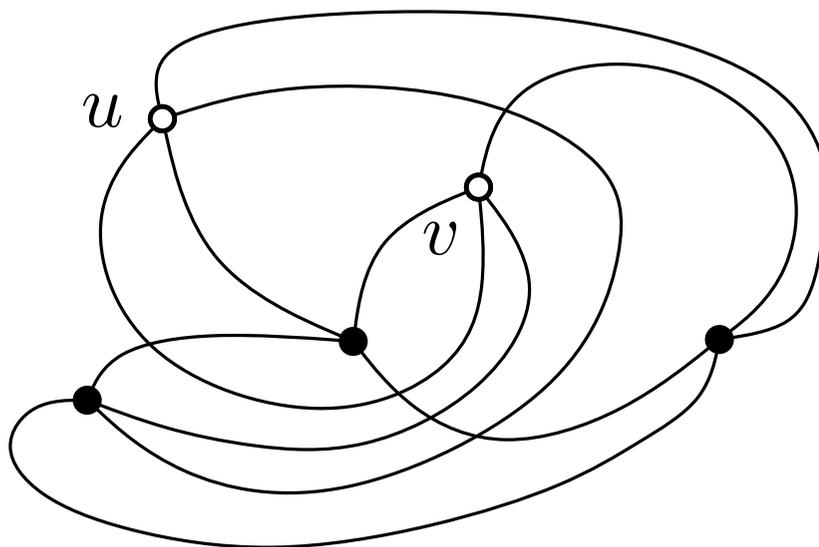
In straight-line drawings trivially YES

Can we always insert the remaining edges?

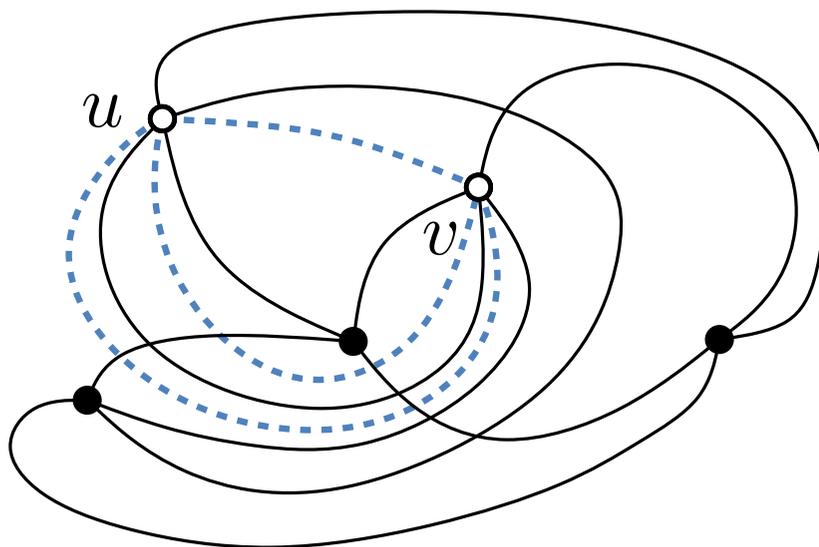


In pseudolinear drawings YES by Levis enlargement lemma

Can we always insert the remaining edges?

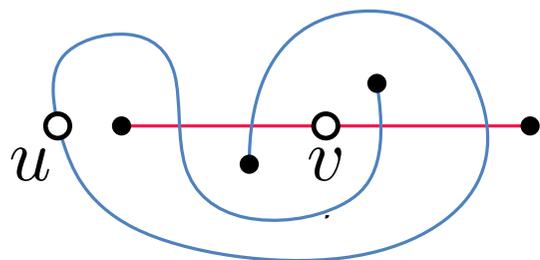


Can we always insert the remaining edges?



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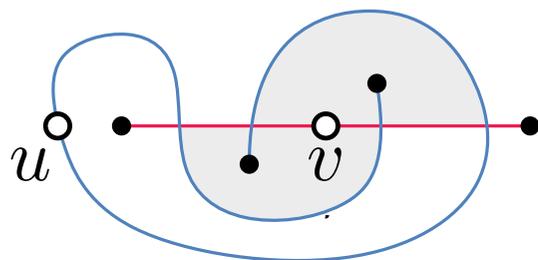
uv cannot be added



[Kynčl '13]

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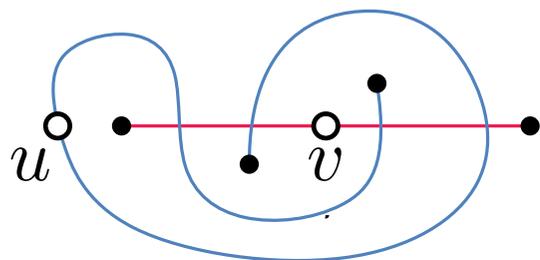
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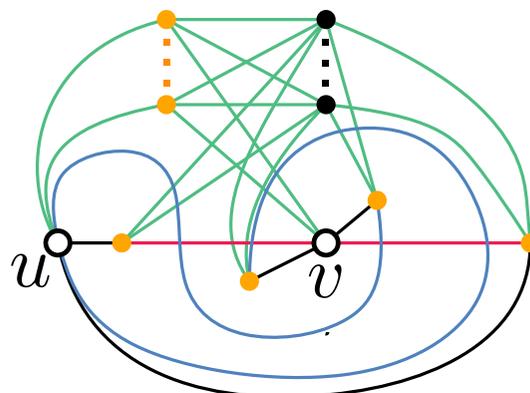
[Kynčl '13]

Can we always insert the remaining edges?

uv cannot be added



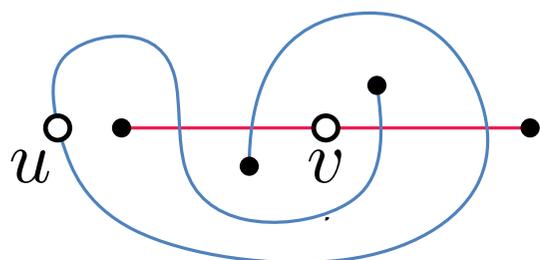
[Kynčl '13]



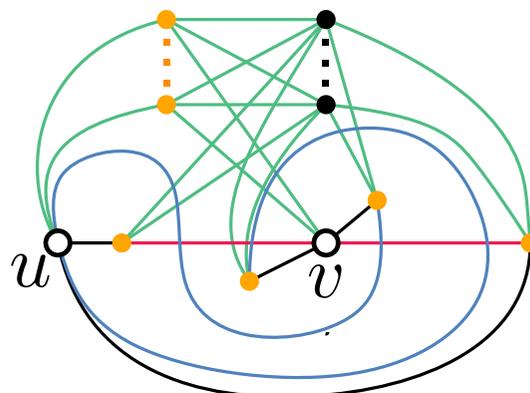
$K_{m,n}$

Can we always insert the remaining edges?

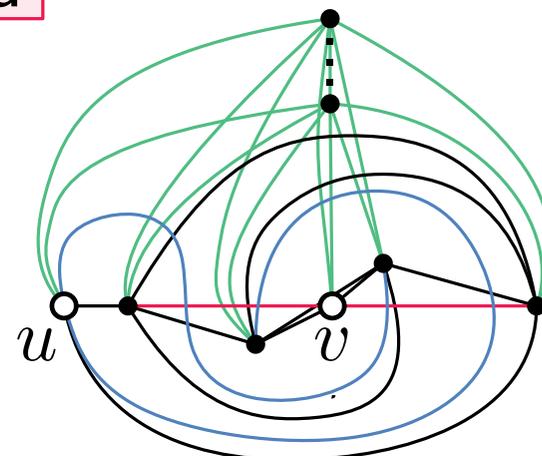
uv cannot be added



[Kynčl '13]



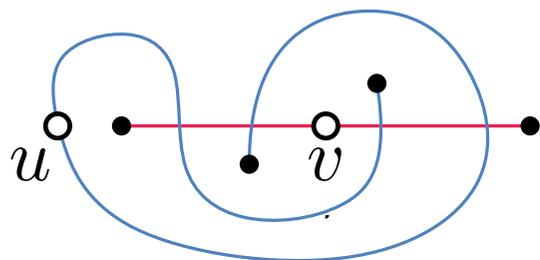
$K_{m,n}$



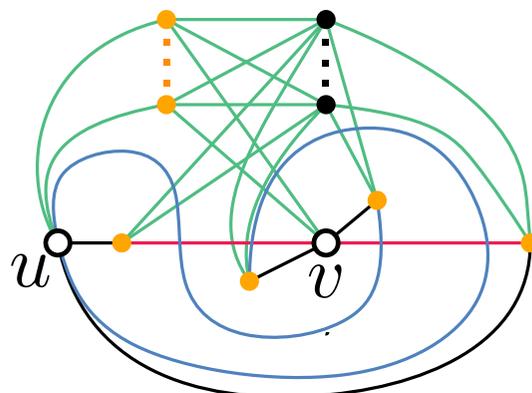
$K_n \setminus uv$

Can we always insert the remaining edges?

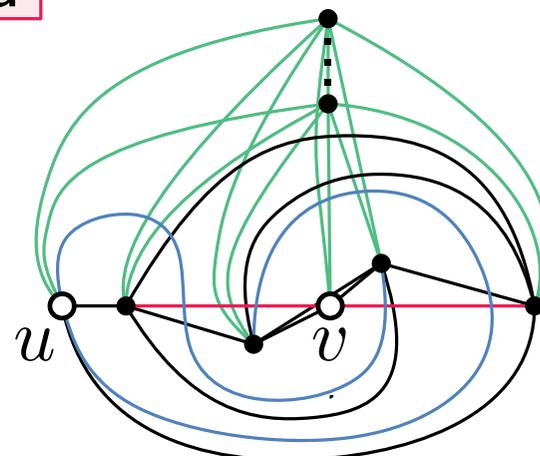
uv cannot be added



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$K_{m,n}$

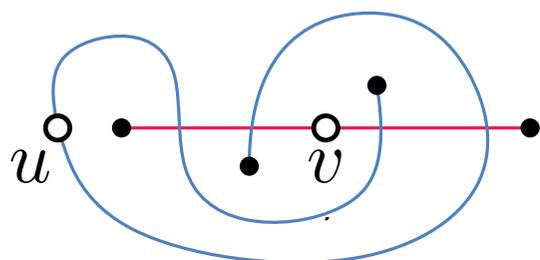


$K_n \setminus uv$

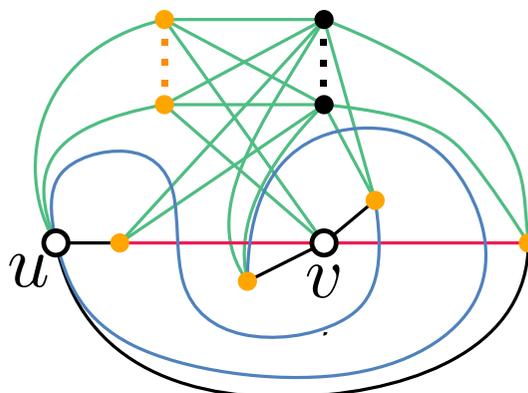
What about matchings?

Can we always insert the remaining edges?

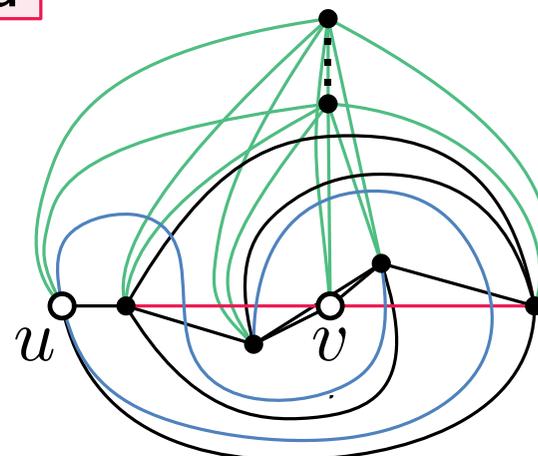
uv cannot be added



[Kynčl '13]



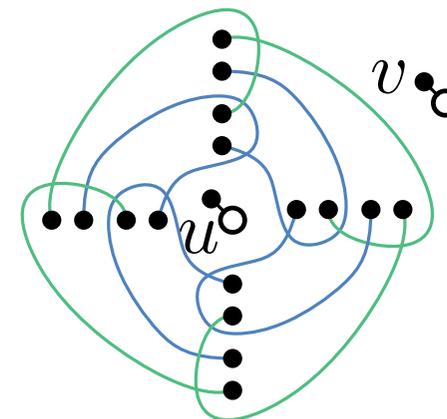
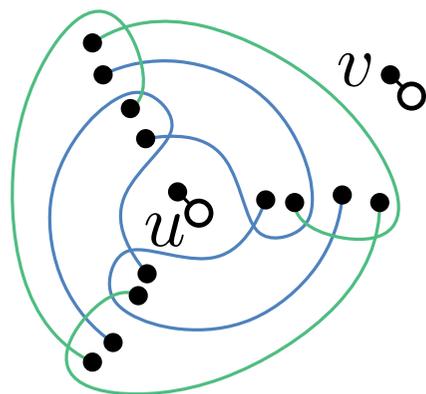
$K_{m,n}$



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What about matchings?

[Kynčl,
Pach,
Radoičić,
Tóth '14]

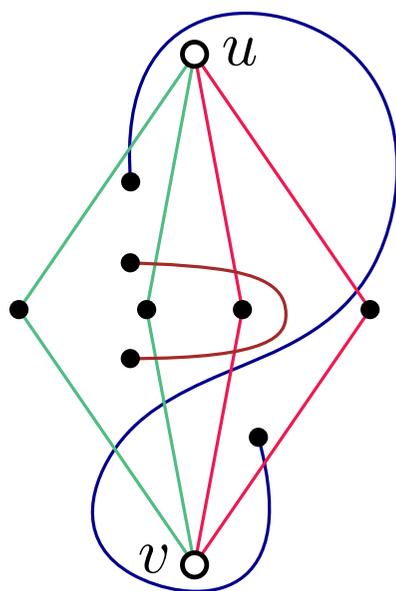


Inserting a set of edges is NP-complete

Reduction from monotone 3SAT.

Inserting a set of edges is NP-complete

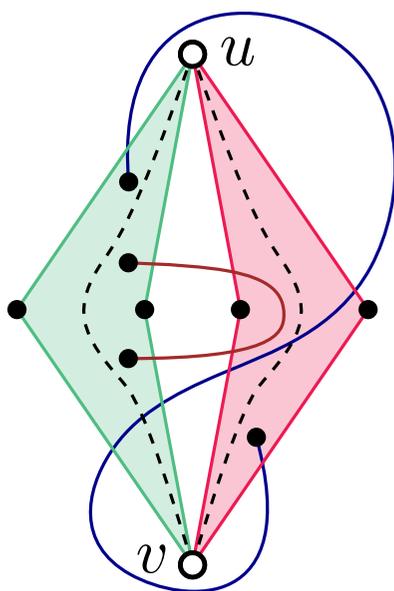
Reduction from monotone 3SAT.



Variable gadget

Inserting a set of edges is NP-complete

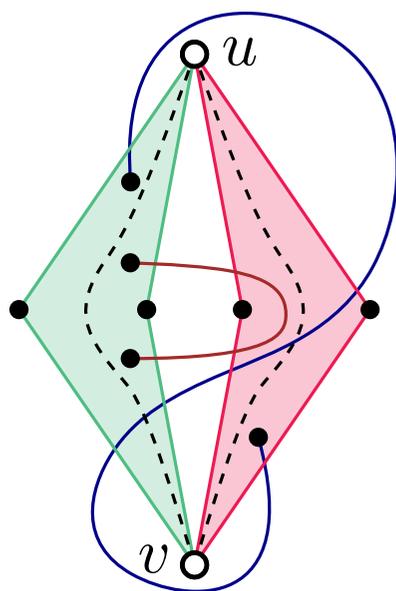
Reduction from monotone 3SAT.



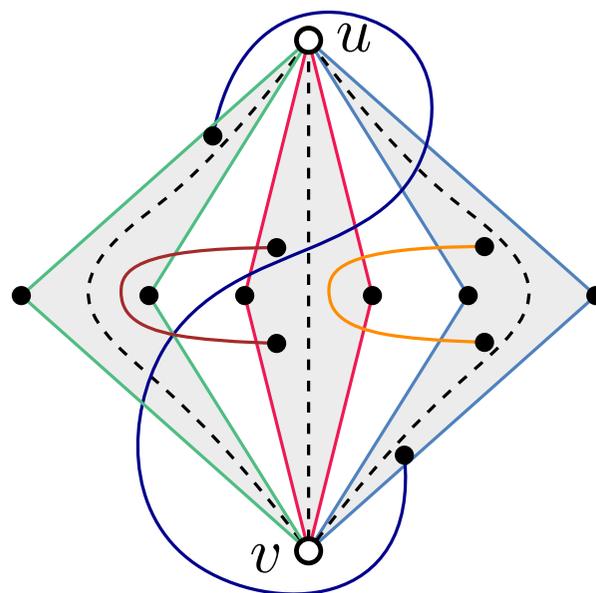
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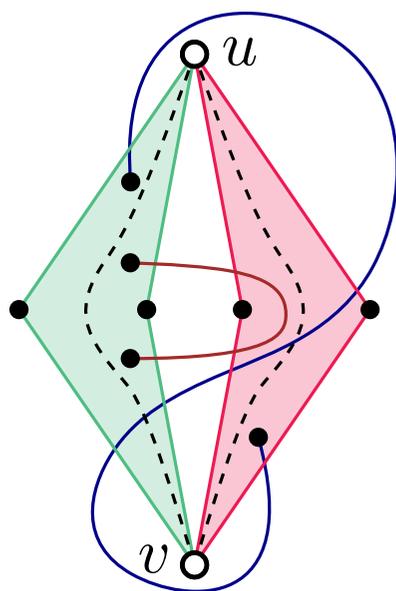
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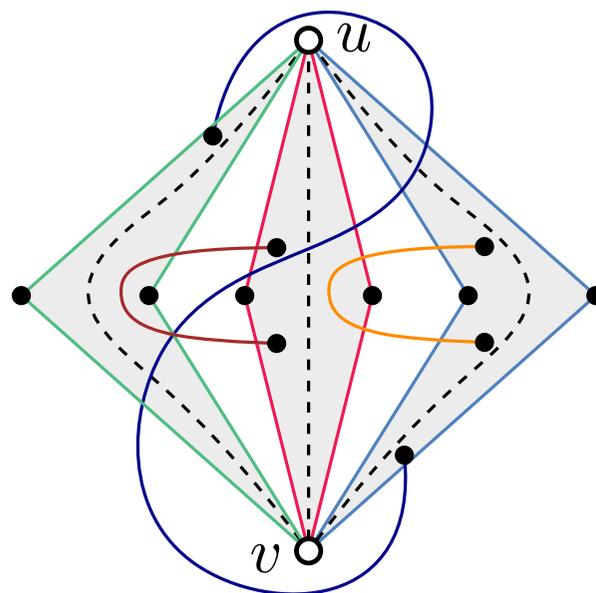
Clause gadget

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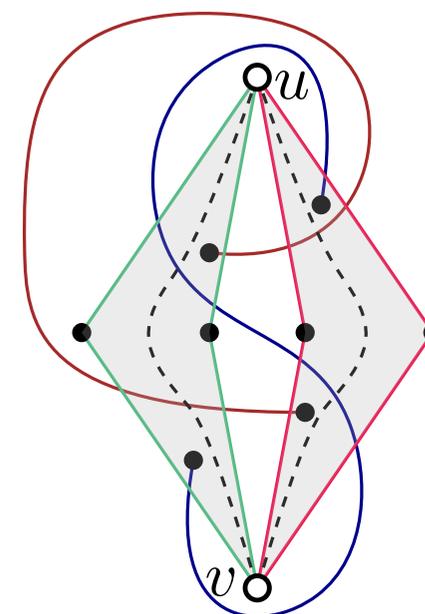
Reduction from monotone 3SAT.



Variable gadget



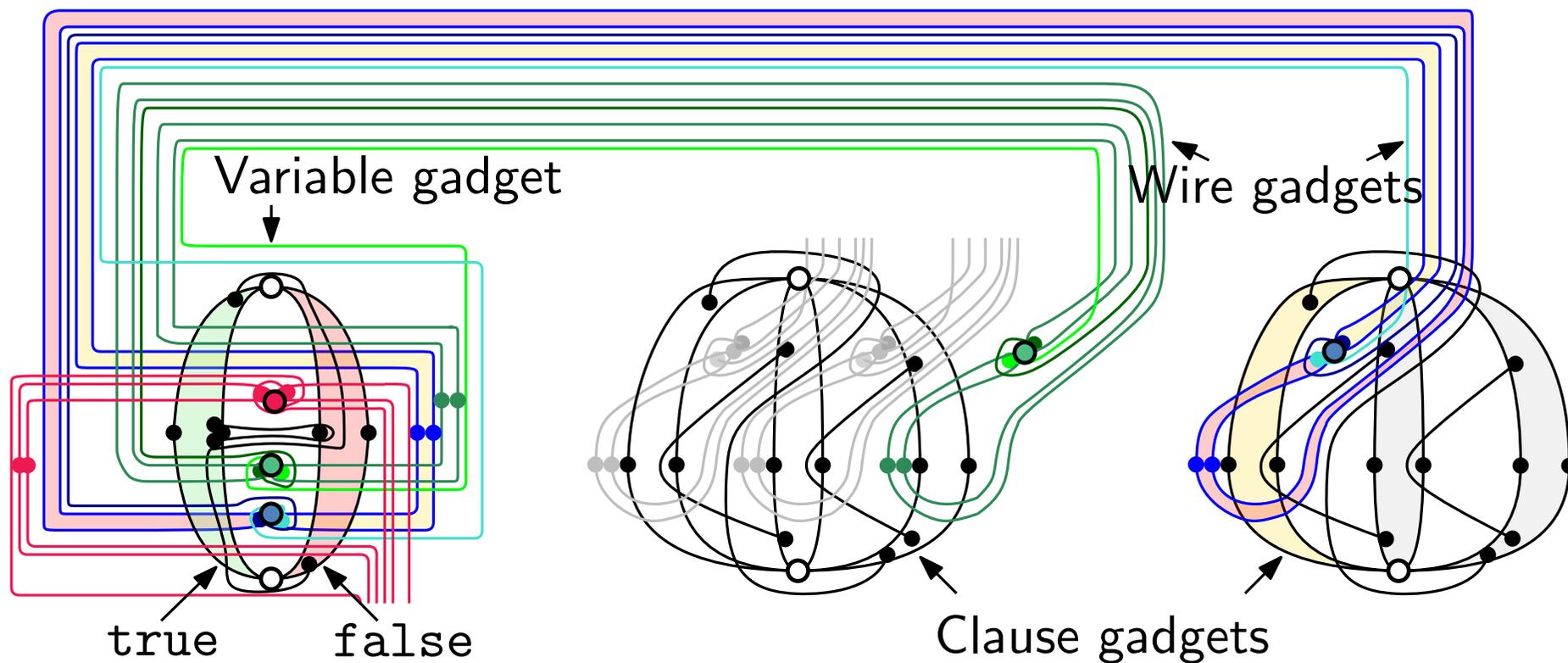
Clause gadget



Wire gadget

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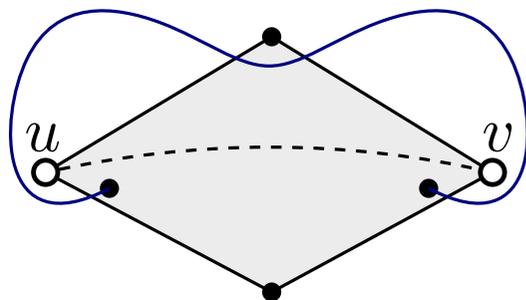


Finding the largest extension is APX-hard

Reduction from maximum indep. set in max. deg. ≤ 3 .

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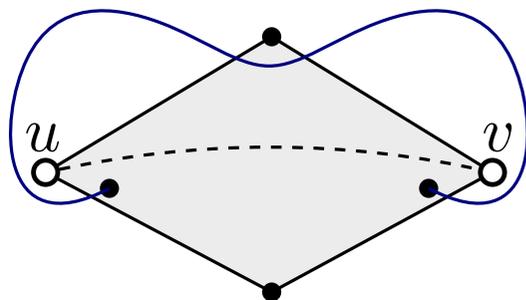
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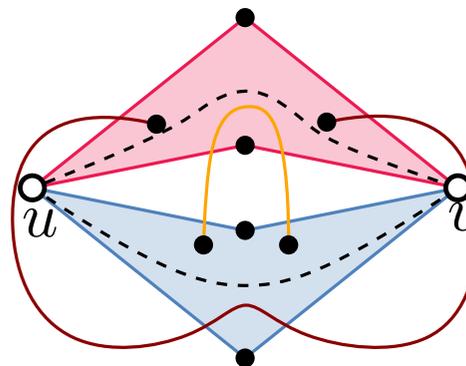
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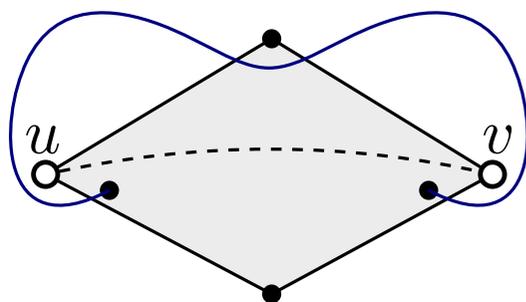
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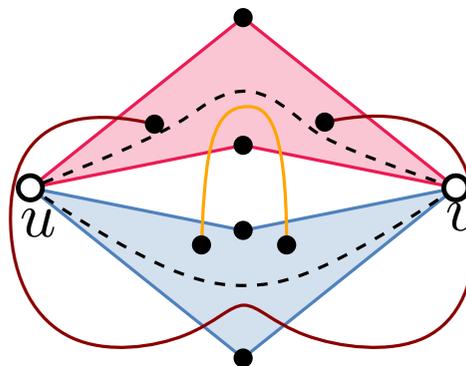
Edge gadget

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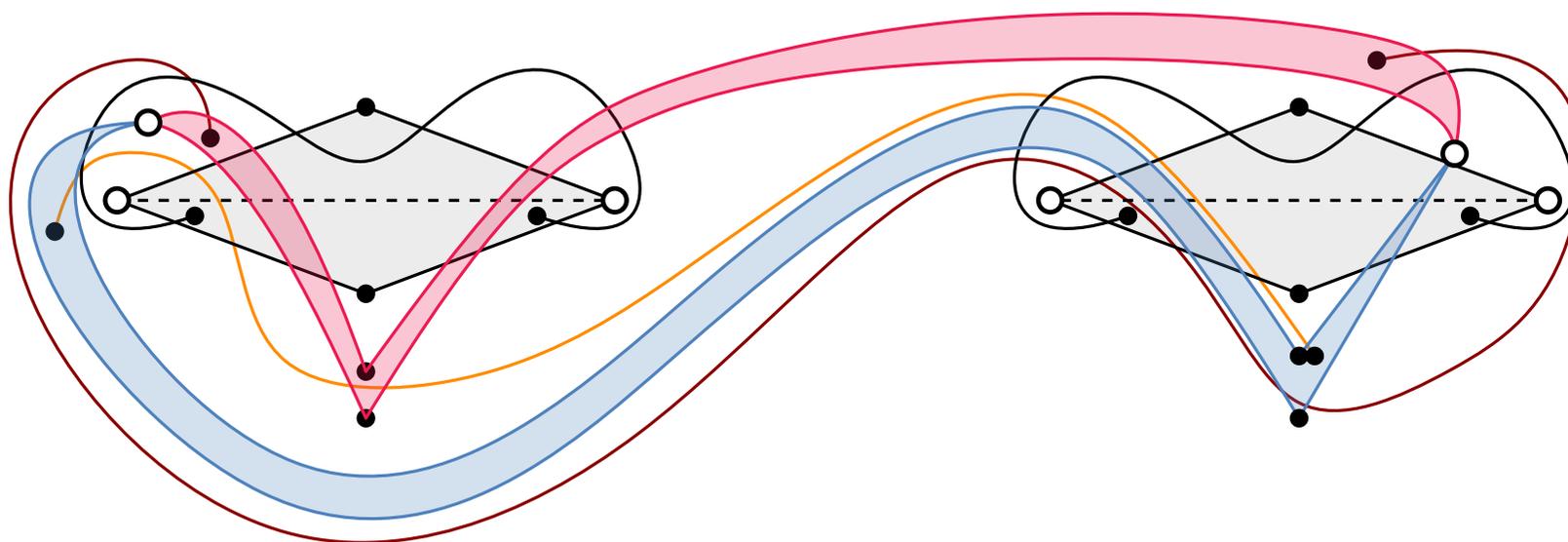
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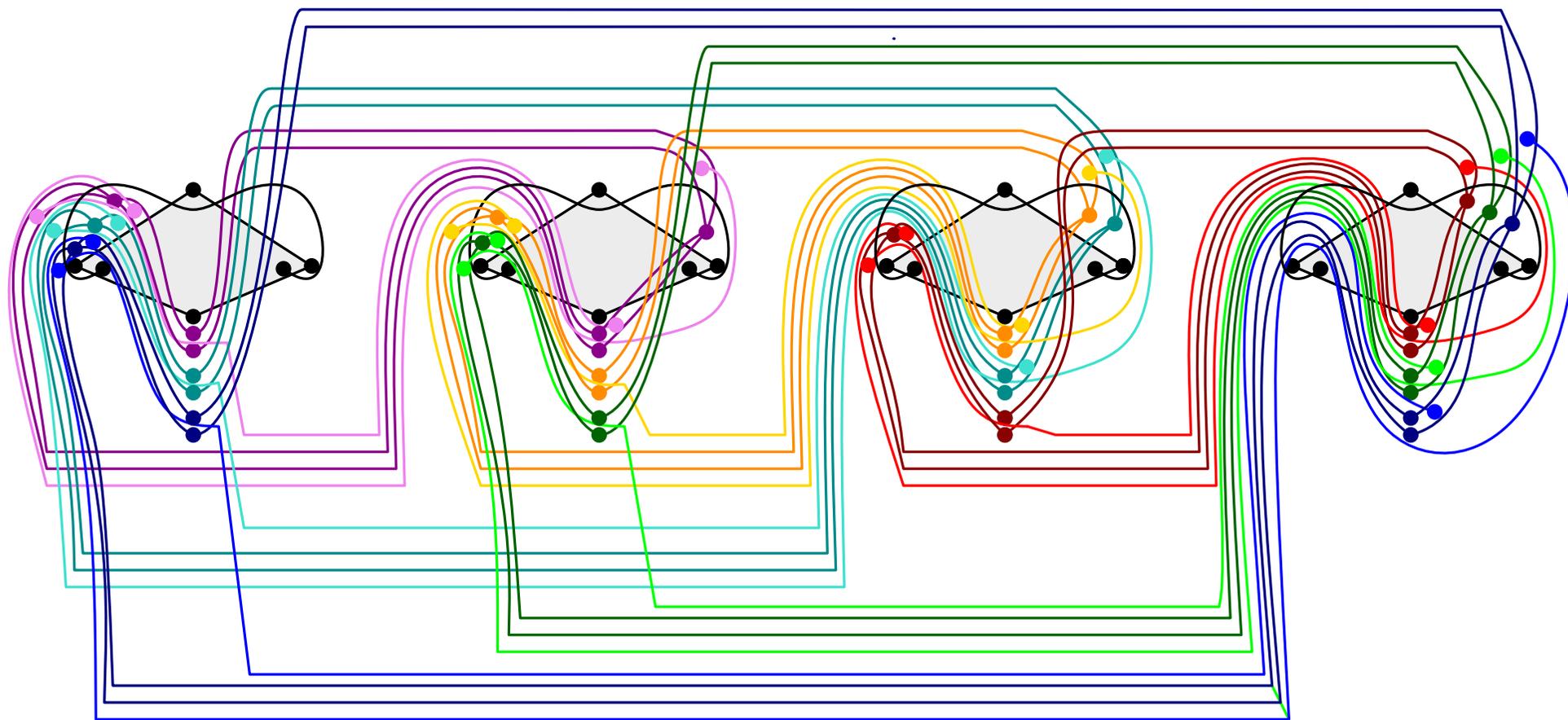


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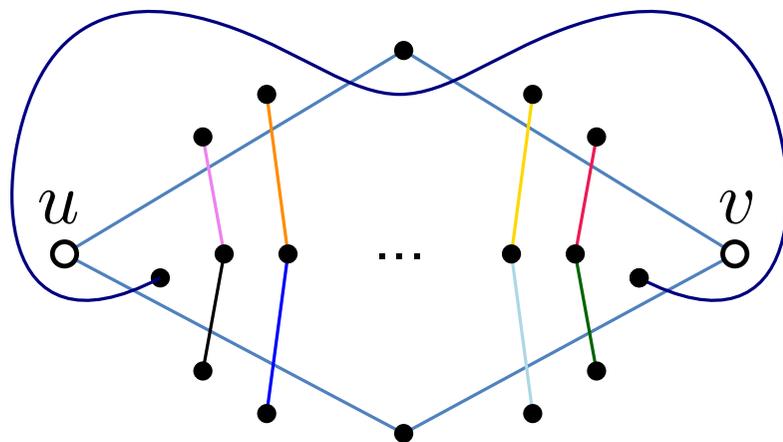


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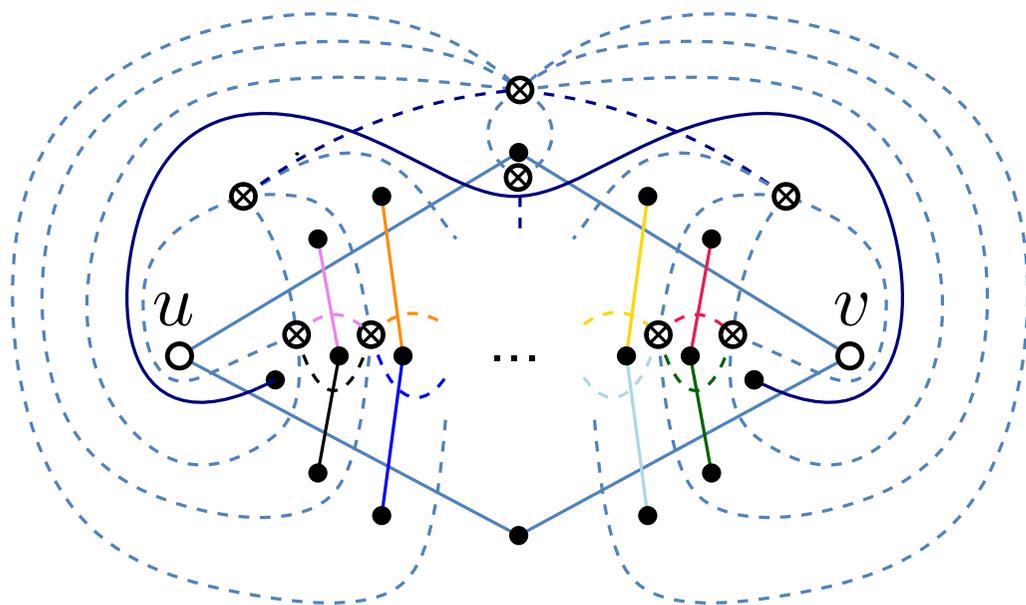


Inserting one single edge



An edge may be added in exponentially many ways.

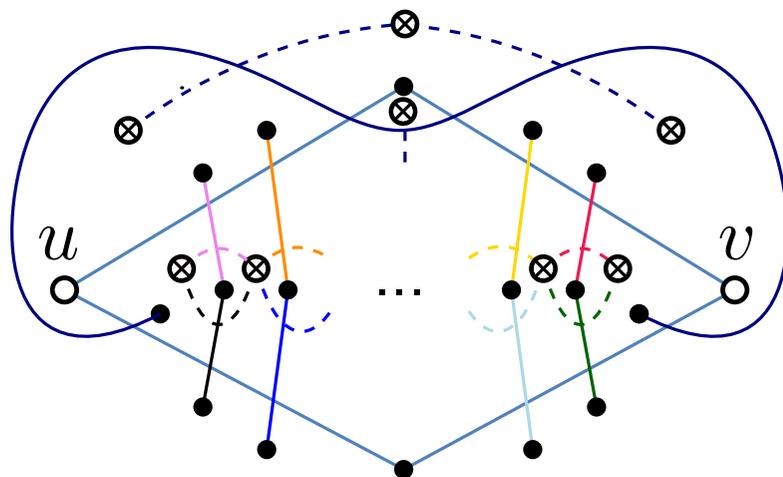
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View in the dual:
Heterochromatic path.

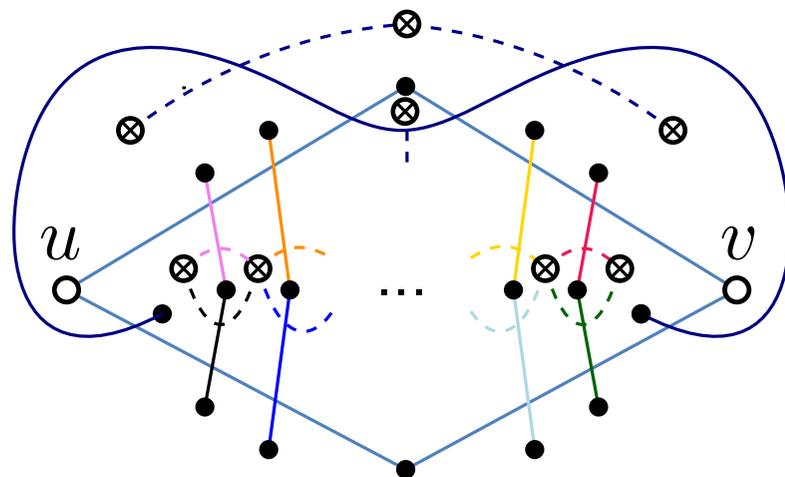
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View in the dual:
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Theorem: If $\{u, v\}$ is a dominating set for G then the problem of extending $D(G)$ with the edge uv can be decided in polynomial time.

Conclusions

Results:

- Deciding if we can insert a set of k edges is NP-complete.
- Maximizing the number of edges from a given set that we can insert is APX-hard.
- Under certain conditions we can decide in polynomial time if we can insert a particular edge.

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Question:

- Computational complexity of deciding whether a given edge can be inserted?

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Solved

Computational complexity of deciding whether a given edge can be inserted?

A. Arroyo, F. Klute, I. Parada, R. Seidel, B. Vogtenhuber, T. Wiedera.
 Extending simple drawings with one edge is hard. arXiv:1909.07347.

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