Homotopy height, grid-major height and graph-drawing height

Therese Biedl
Erin Chambers
David Eppstein
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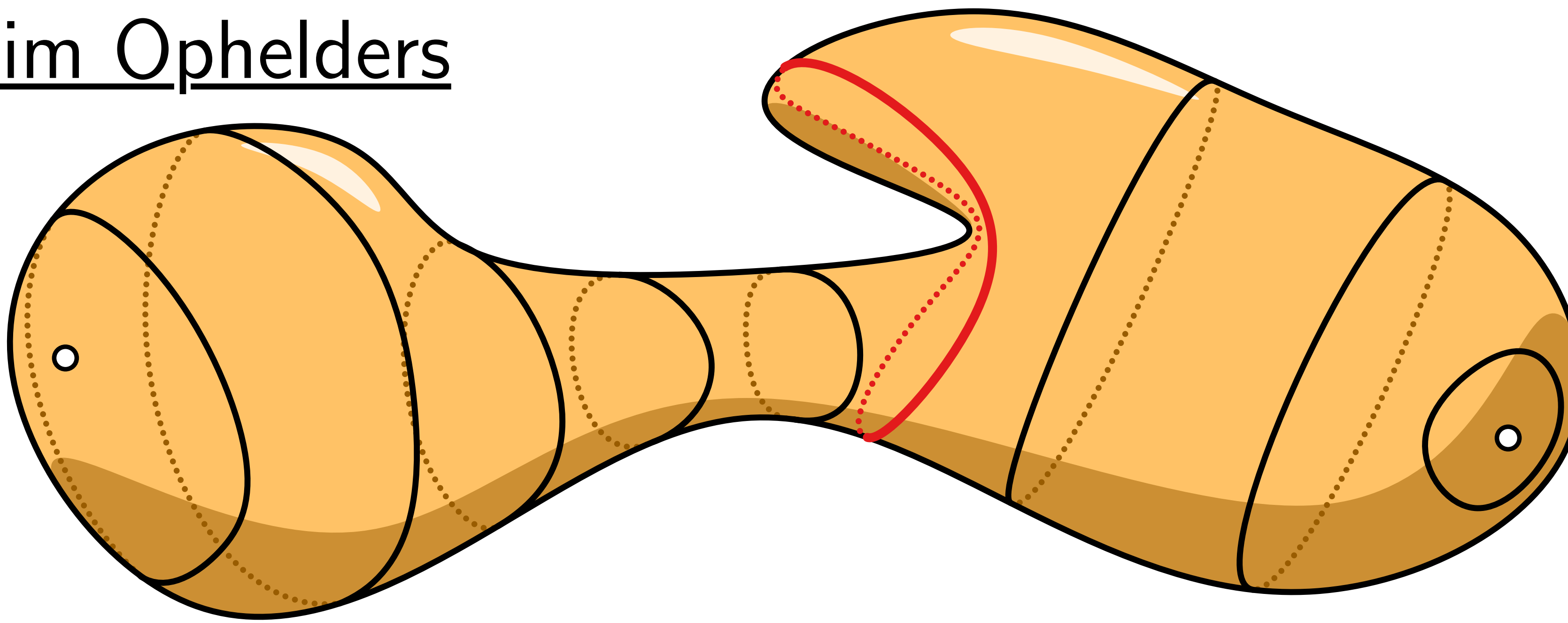
Tim Ophelders



Problem statement

Given a planar graph and a height h,
is there a planar straight line drawing of height h?

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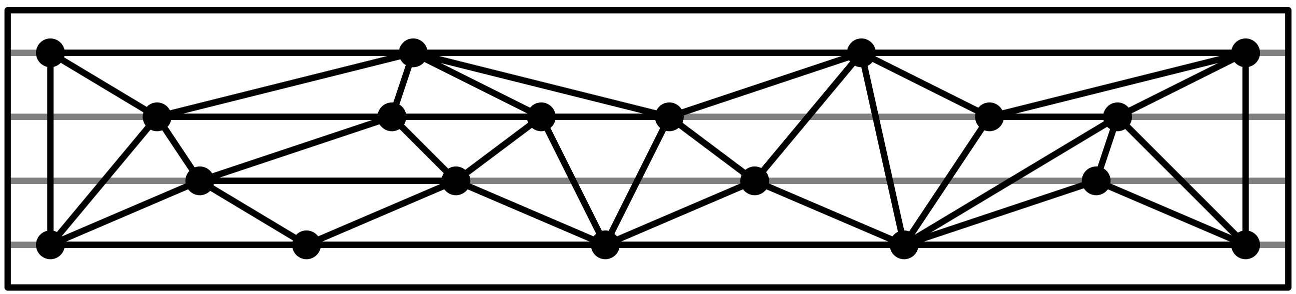
Drawing planar graphs on narrow strips of paper



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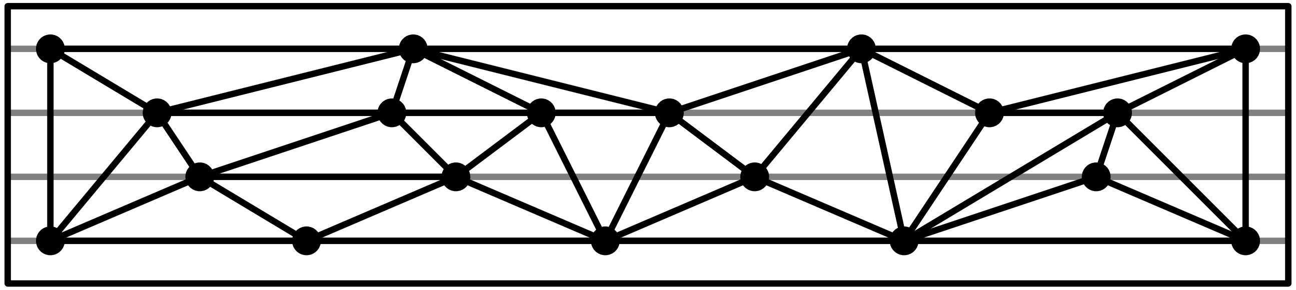
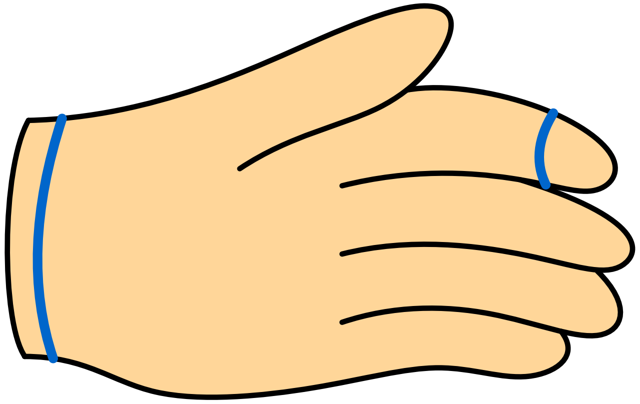
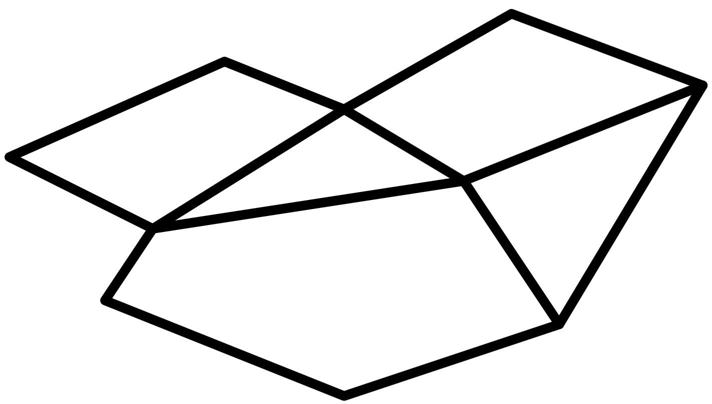


Measuring similarity between curves on surfaces



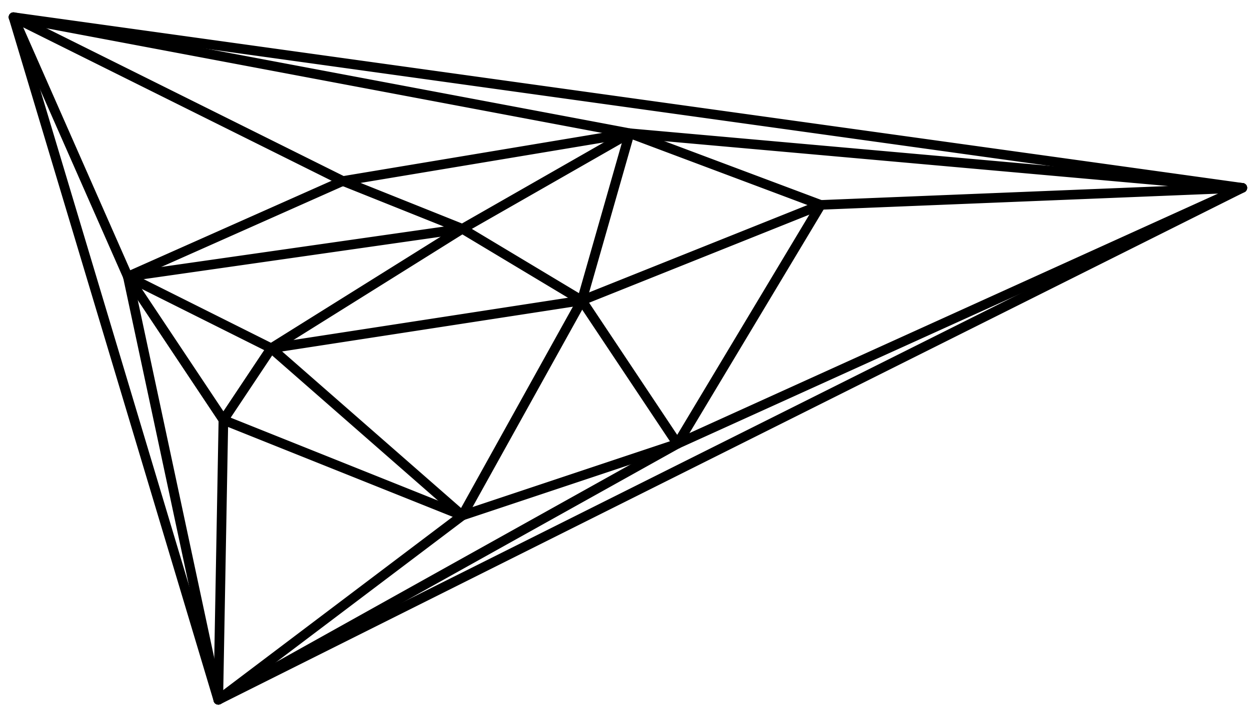
Assumptions on our graphs

All our graphs are planar



Assumptions on our graphs

All our graphs are planar
All faces (including the outer face) are triangular



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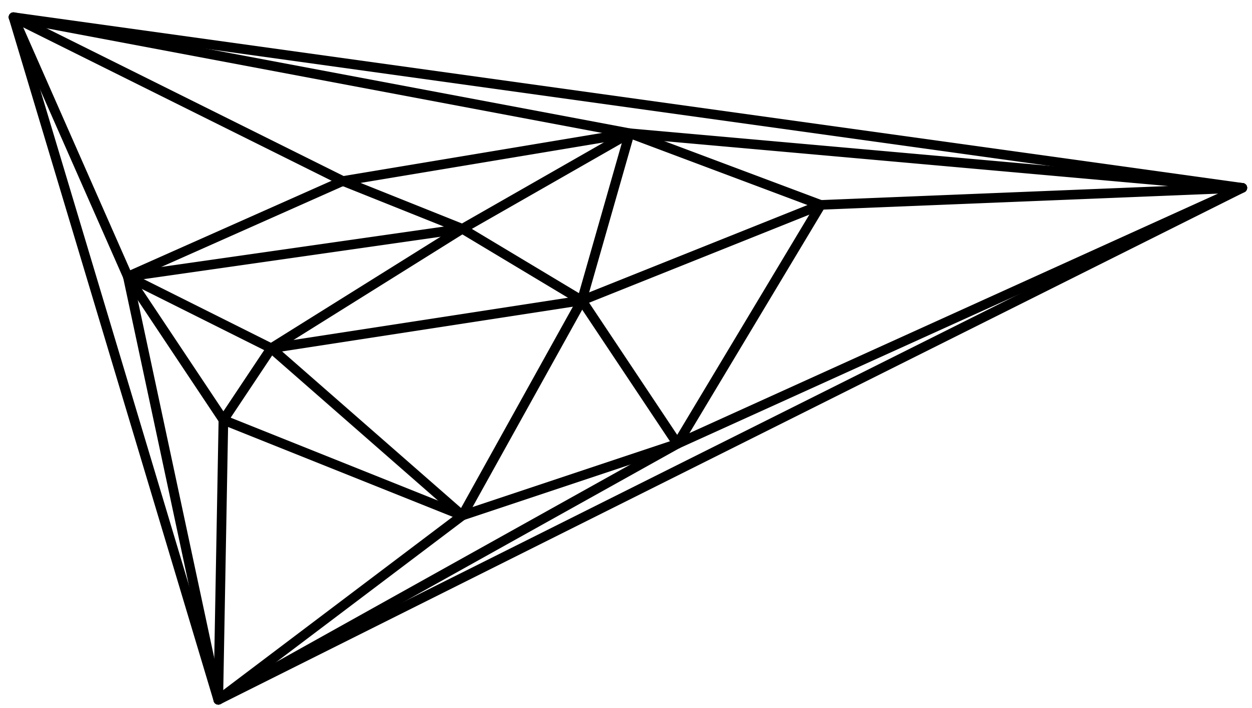

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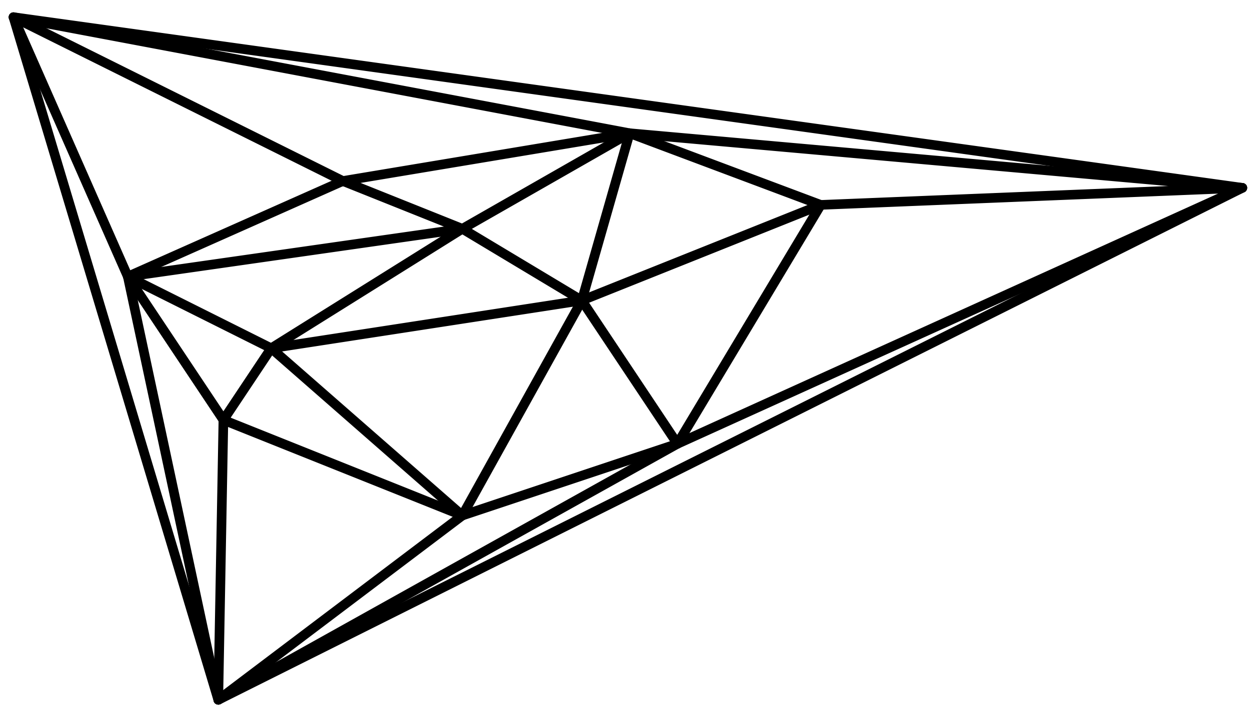
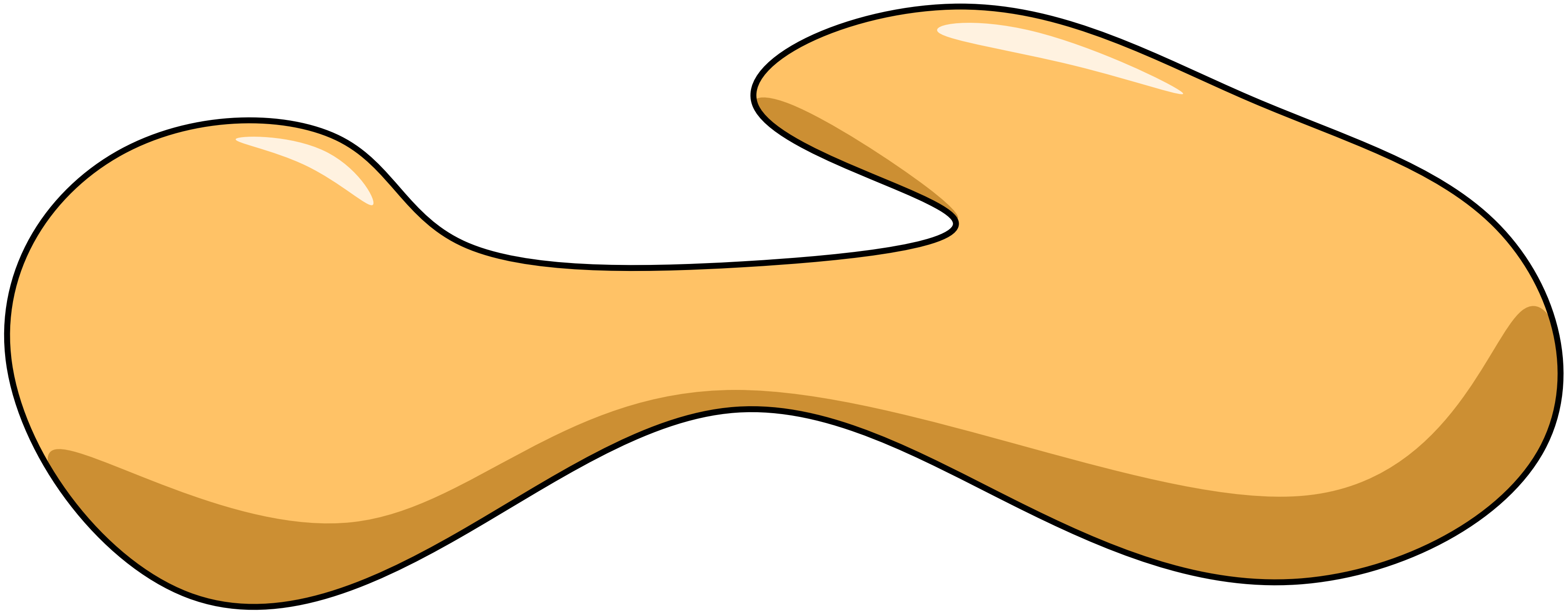
Models a triangulated sphere



Homotopy height

How short of a curve can sweep a topological sphere?



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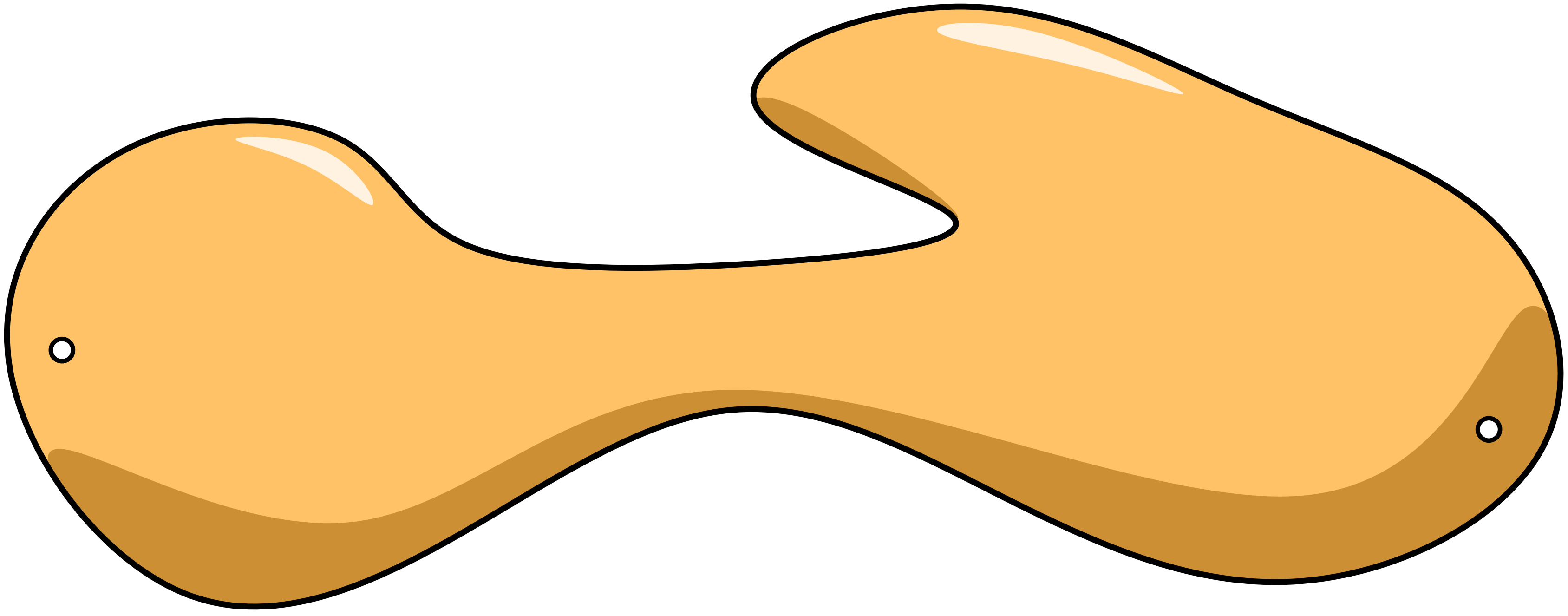

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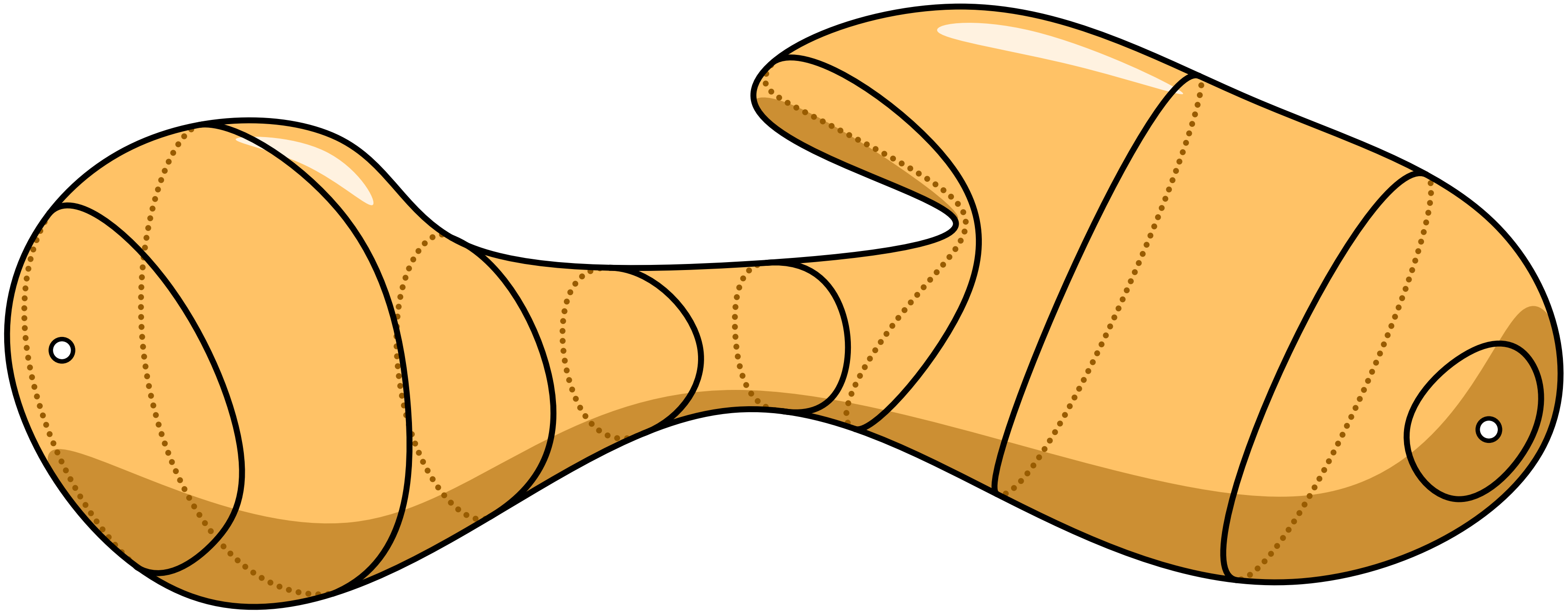

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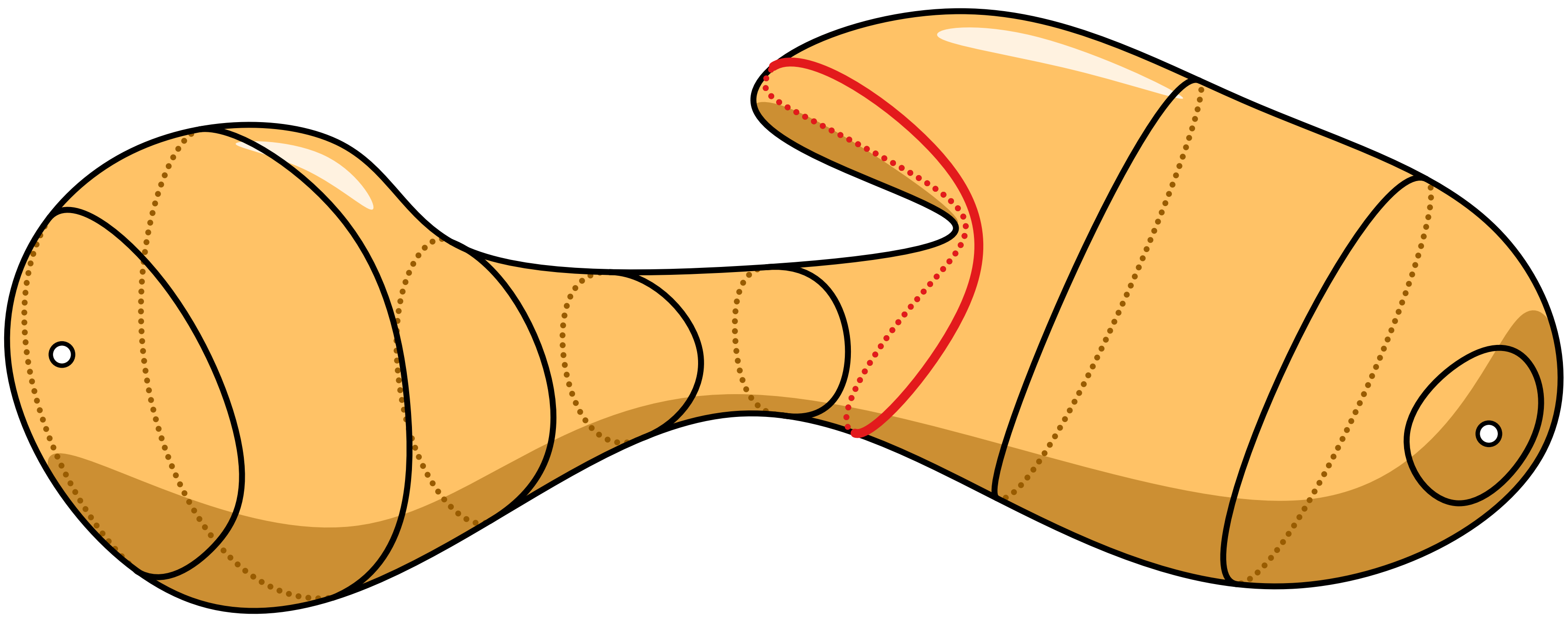

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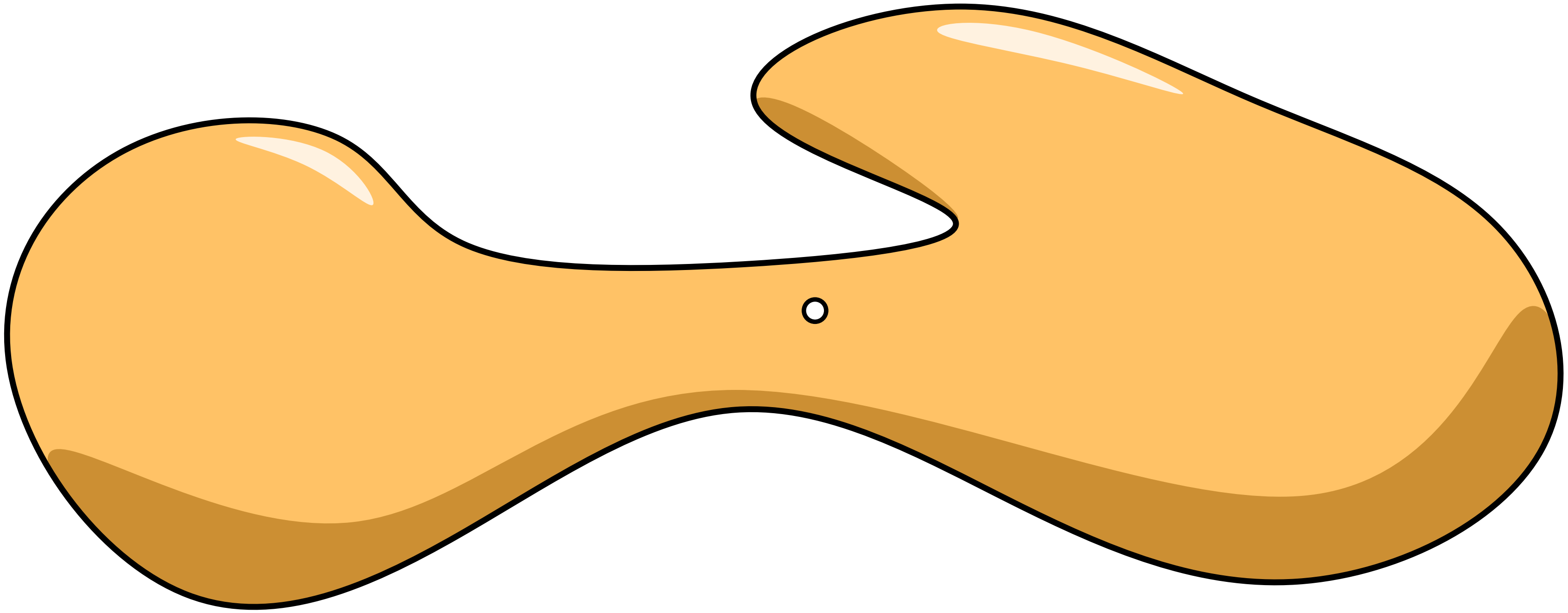
Variant in this talk: curve fixed to arbitrary basepoint



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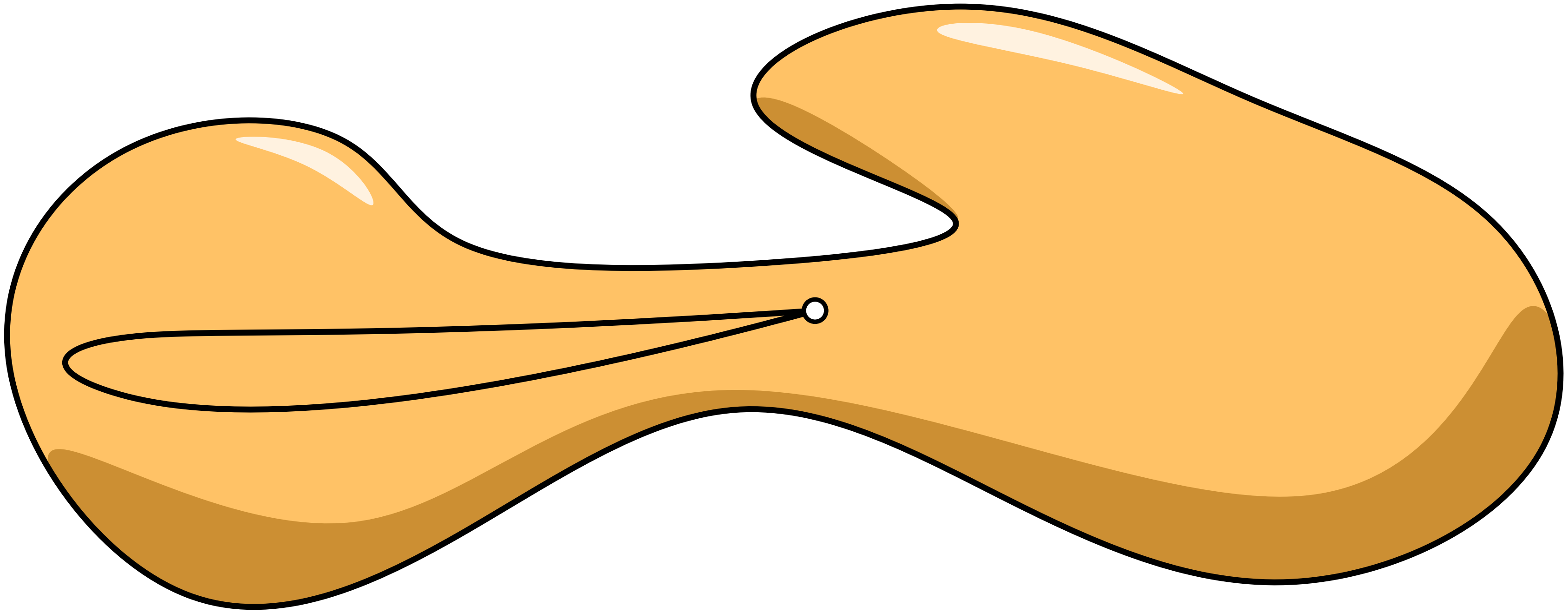

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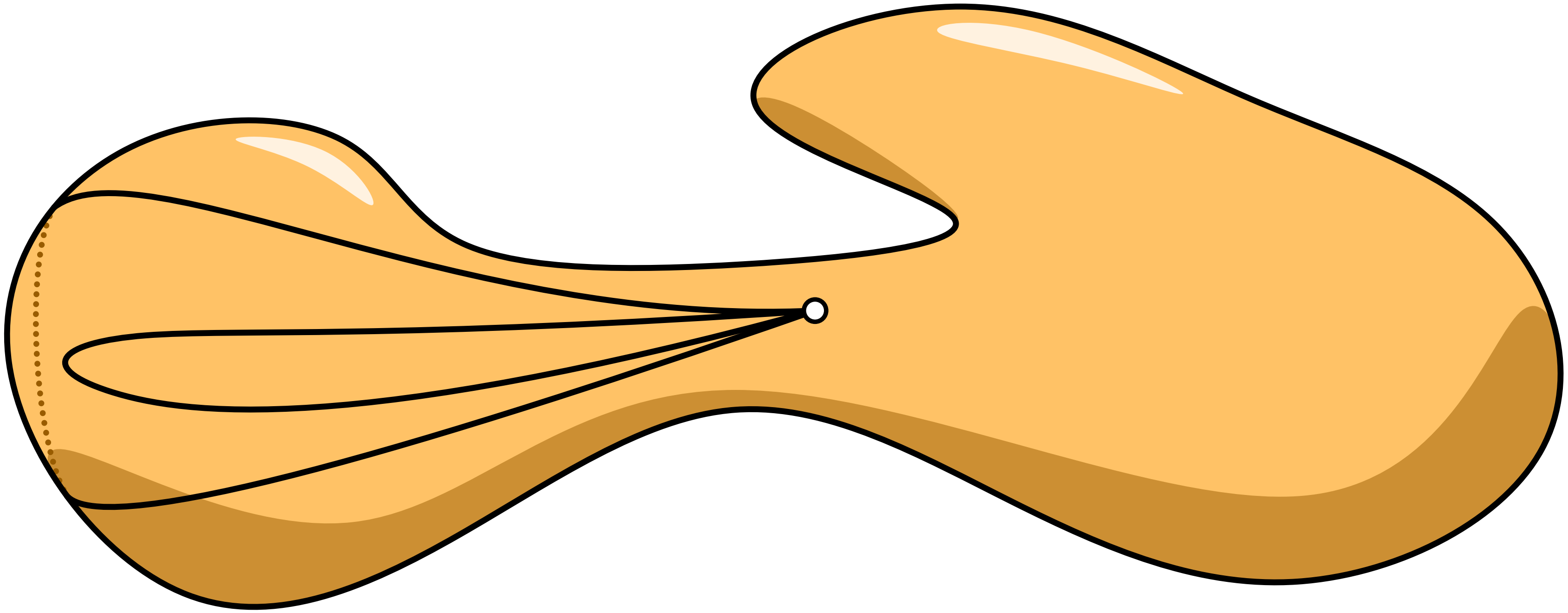

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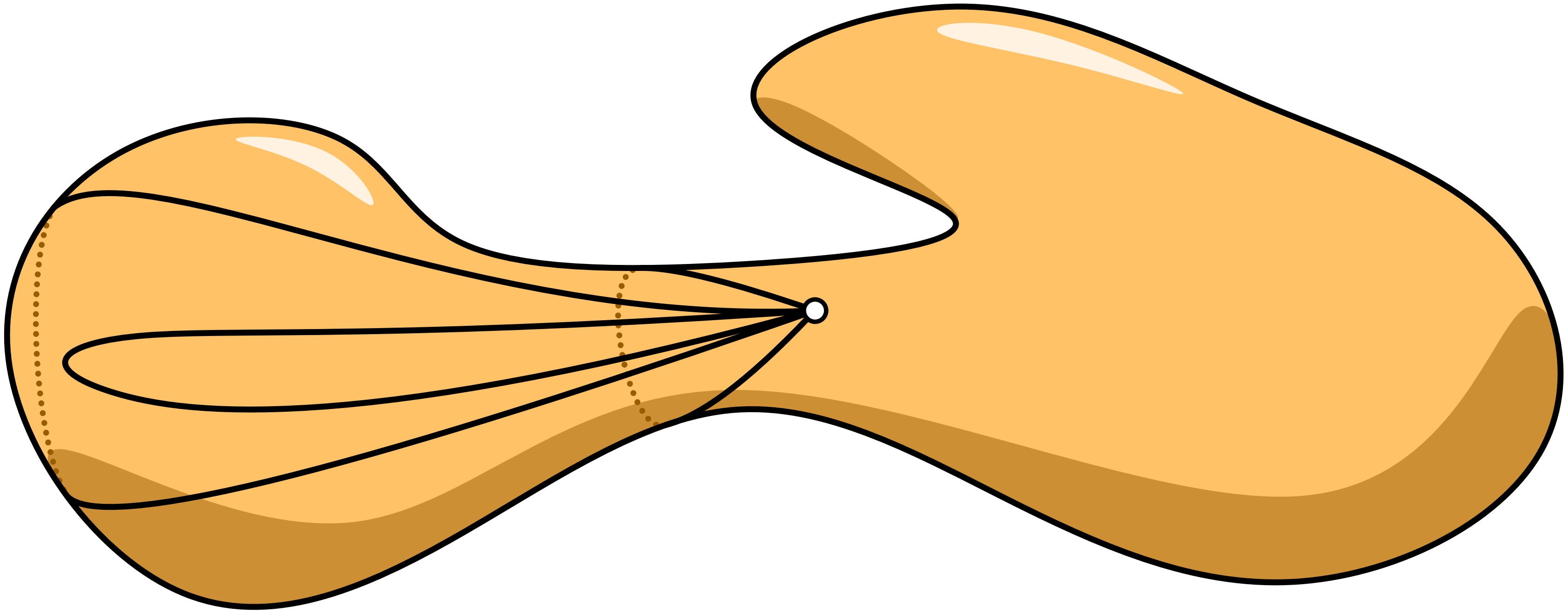

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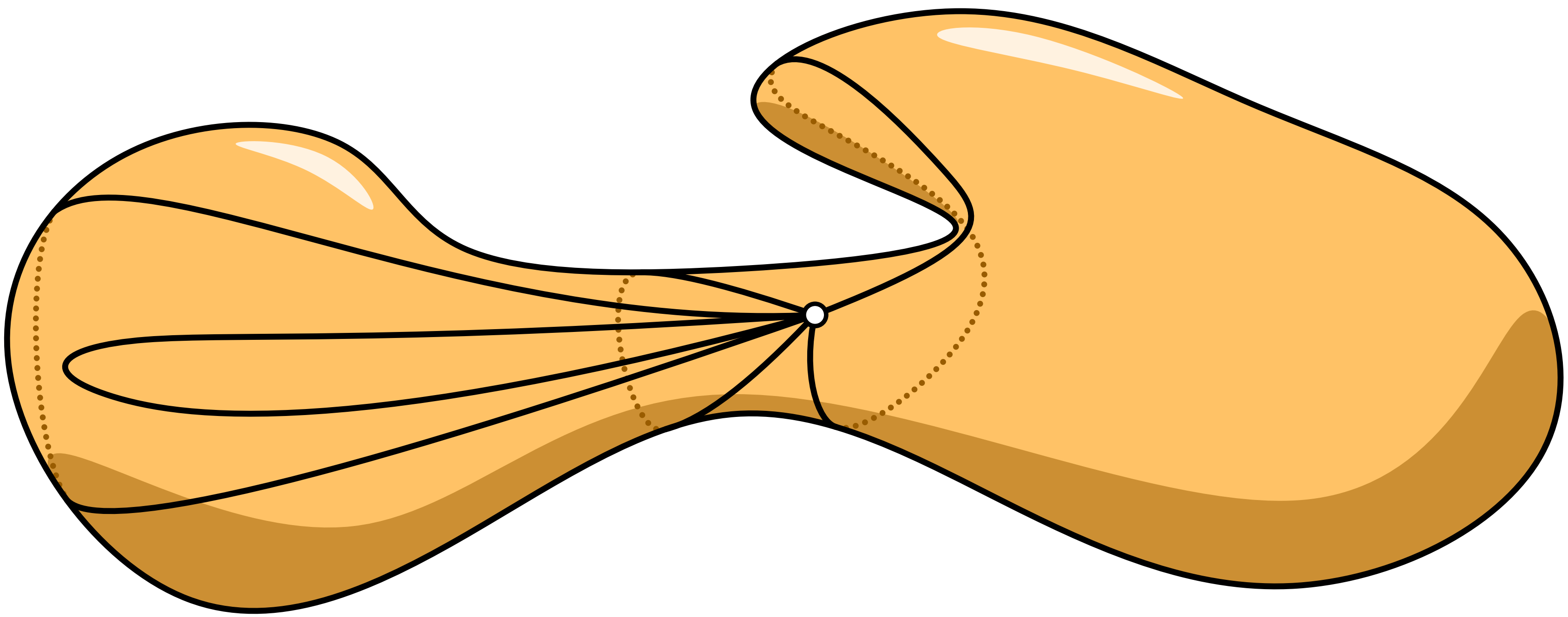

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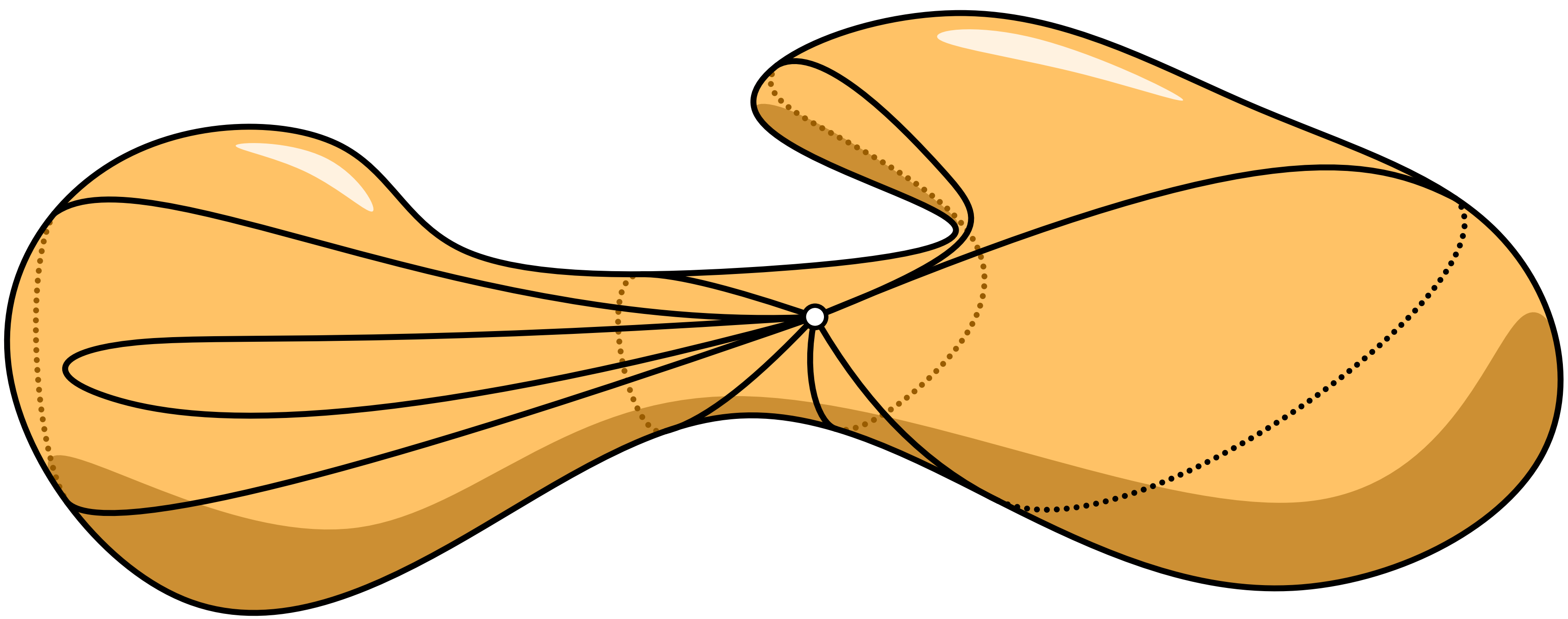

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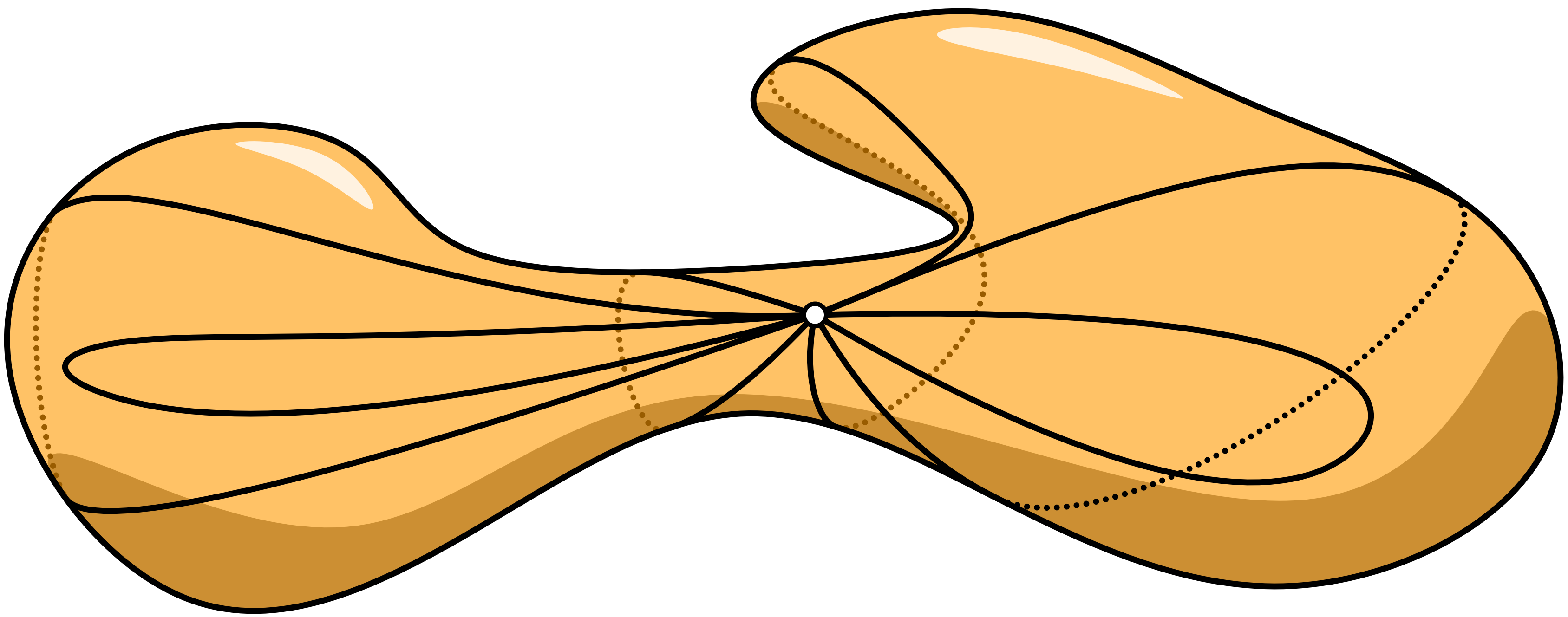
Homotopy height $= \inf_{\textit{basepoint}} \inf_{\textit{sweep}} \sup_t \|\textit{sweep}(t)\|$



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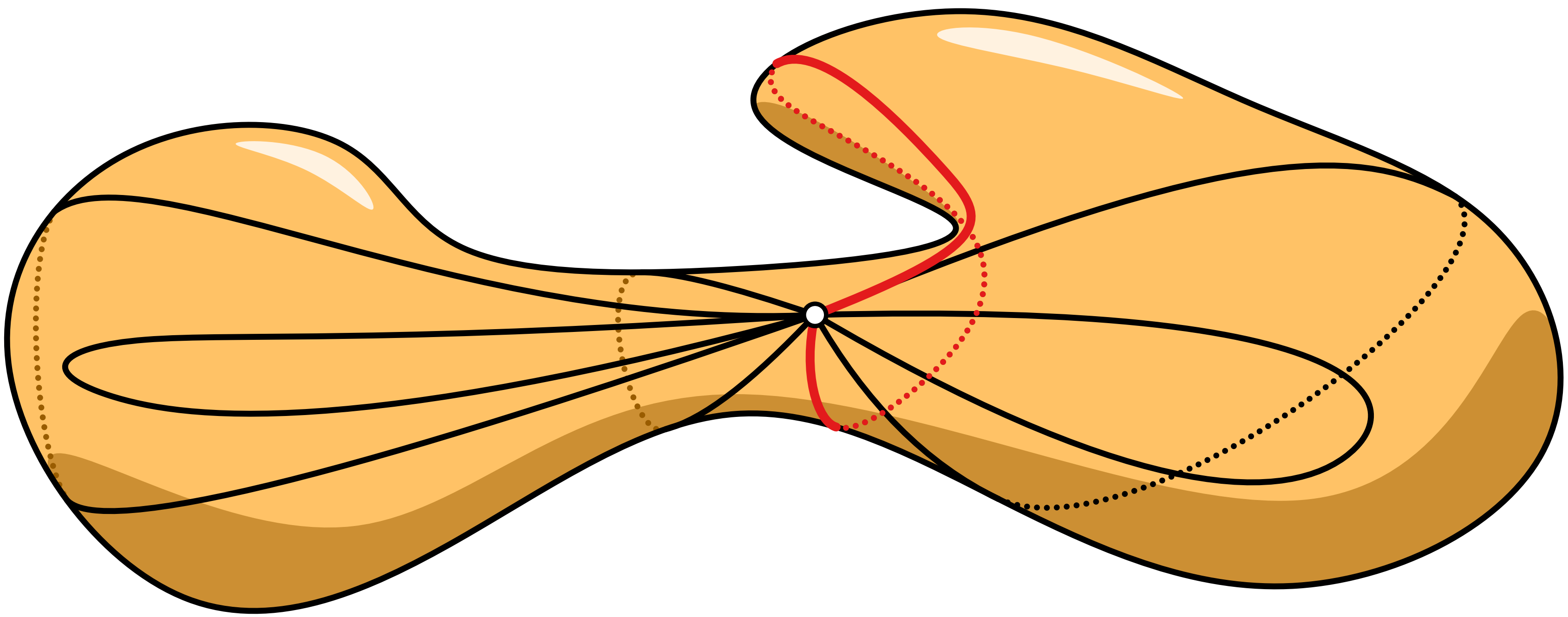
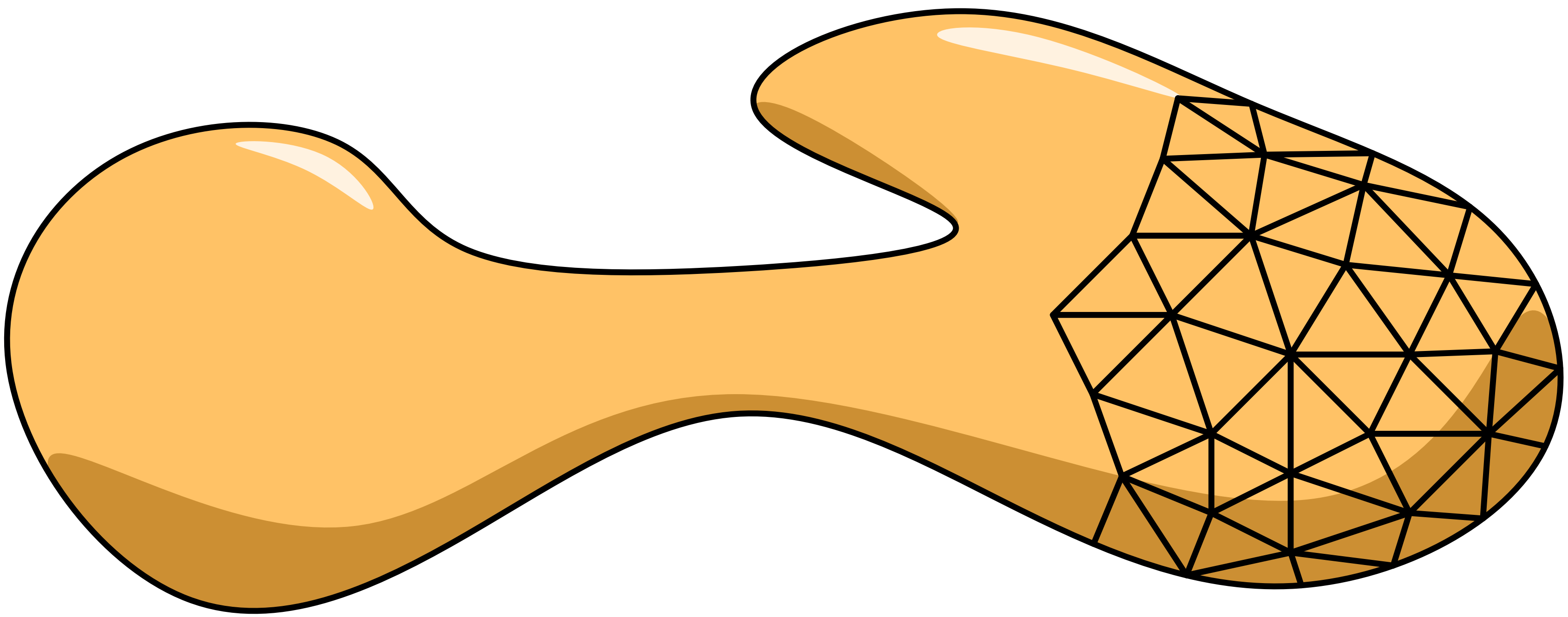


Discretizing Homotopy height

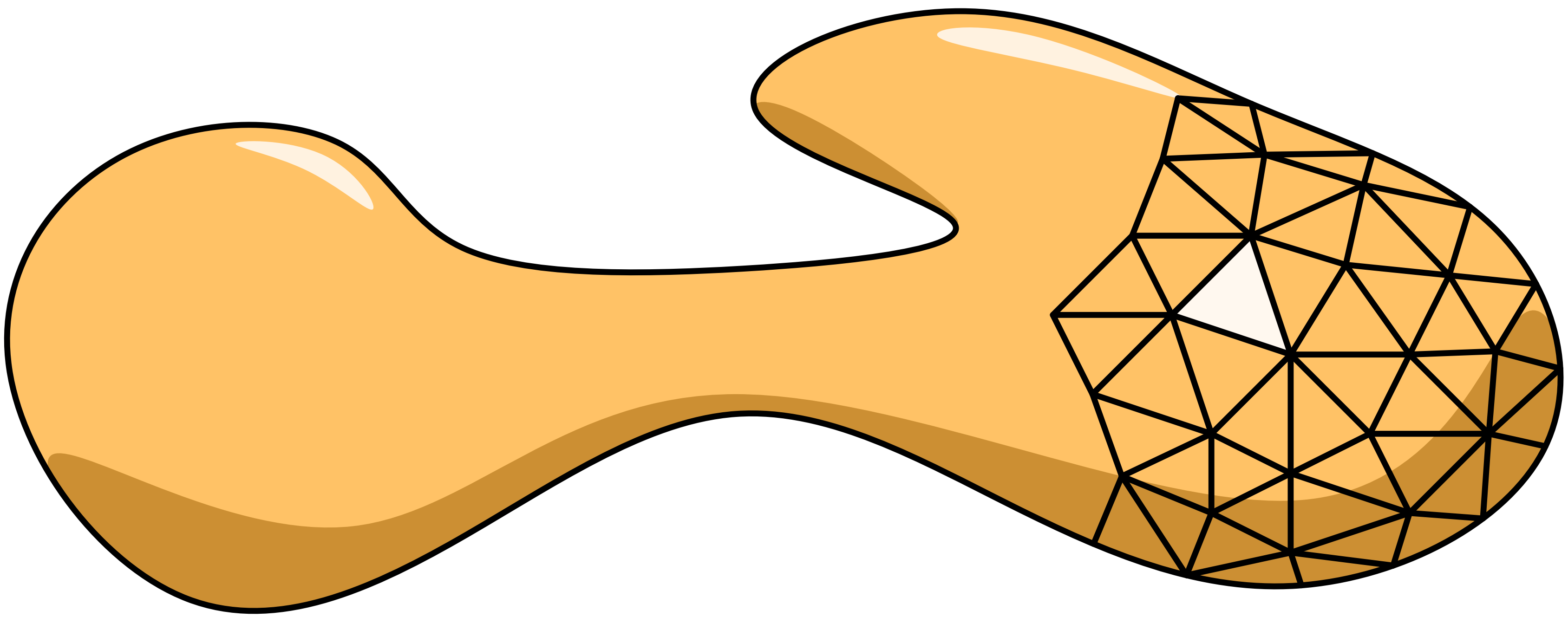
Triangulate surface to approximate metric



Discretizing Homotopy height

Triangulate surface to approximate metric

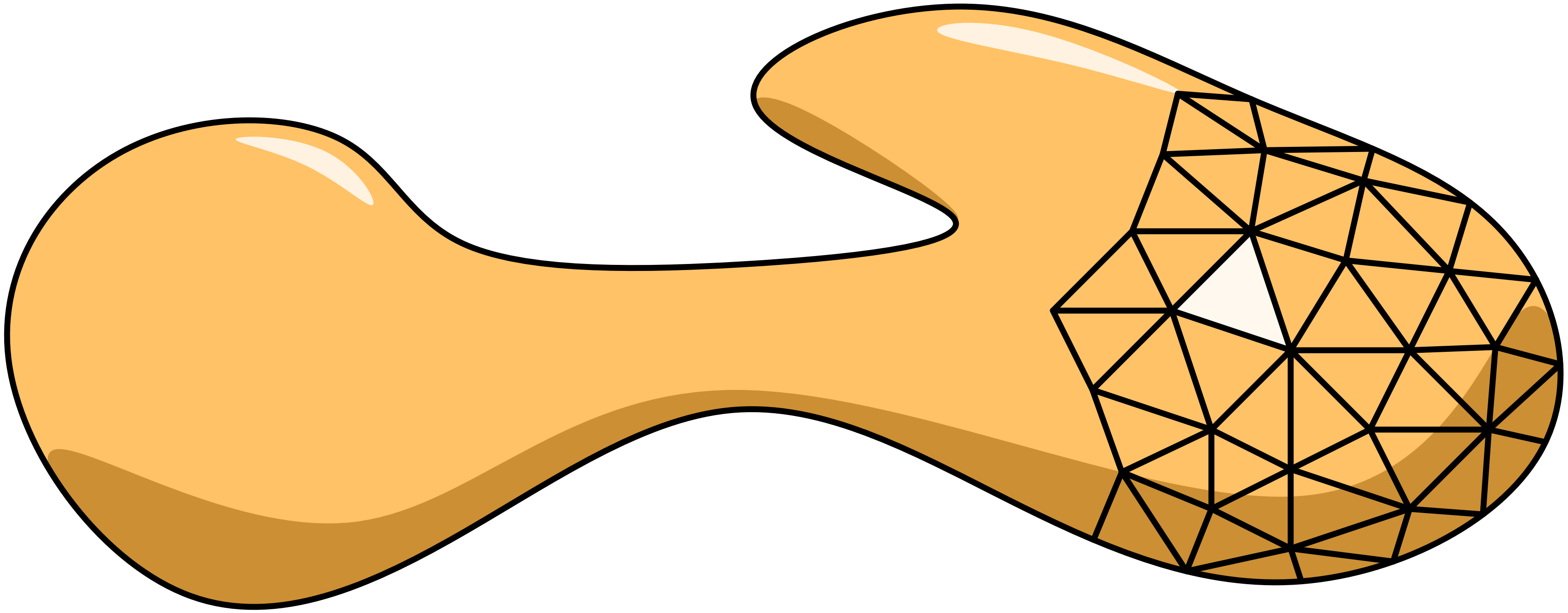
Basepoint $=$ face of triangulation



Discretizing Homotopy height

Triangulate surface to approximate metric

Basepoint = face of triangulation = outer face



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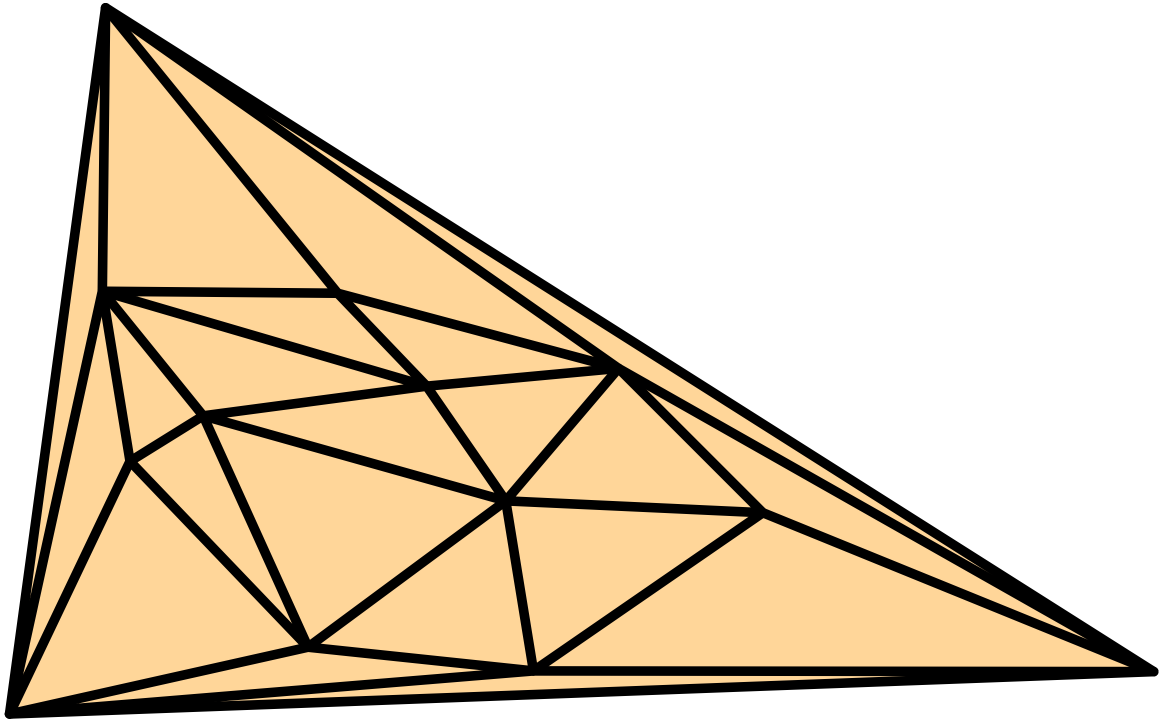
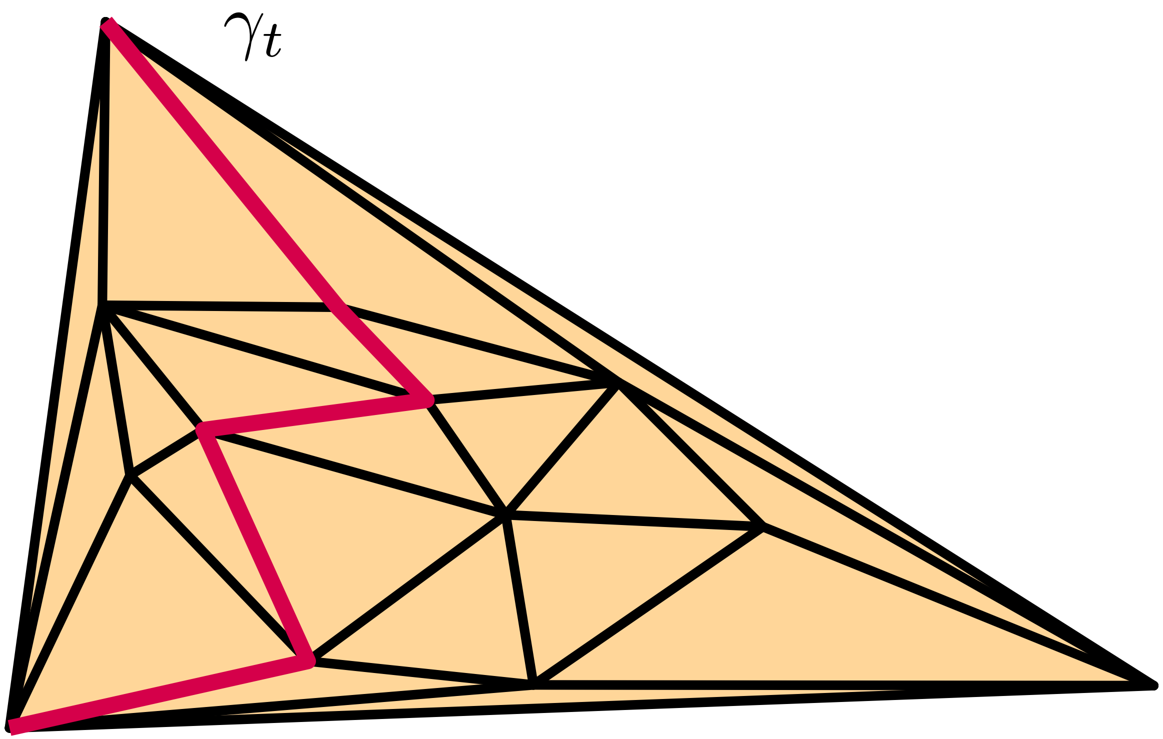


Discretizing Homotopy height

Triangulate surface to approximate metric

Basepoint = face of triangulation = outer face
All curves γ_t of sweep start and end on outer face



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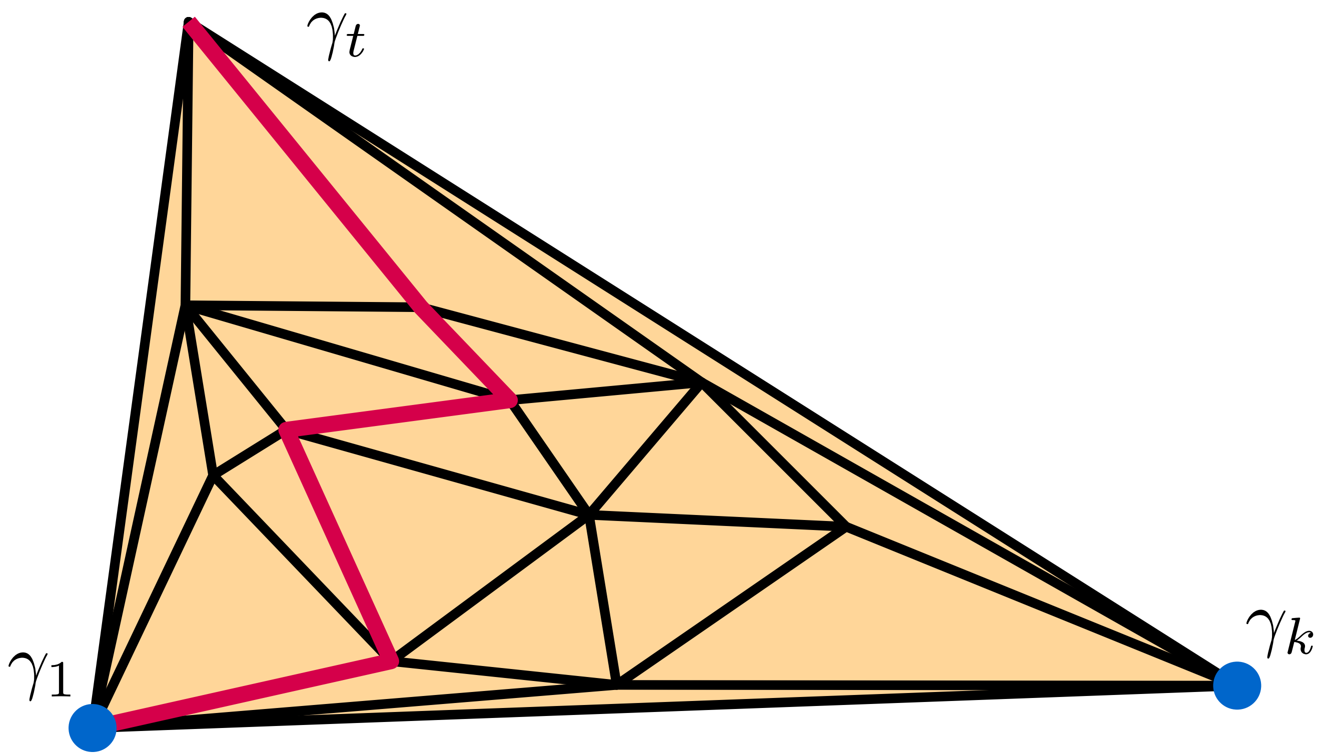
First and last curves of sweep consist of single (distinct) vertex



Discretizing Homotopy height

Triangulate surface to approximate metric

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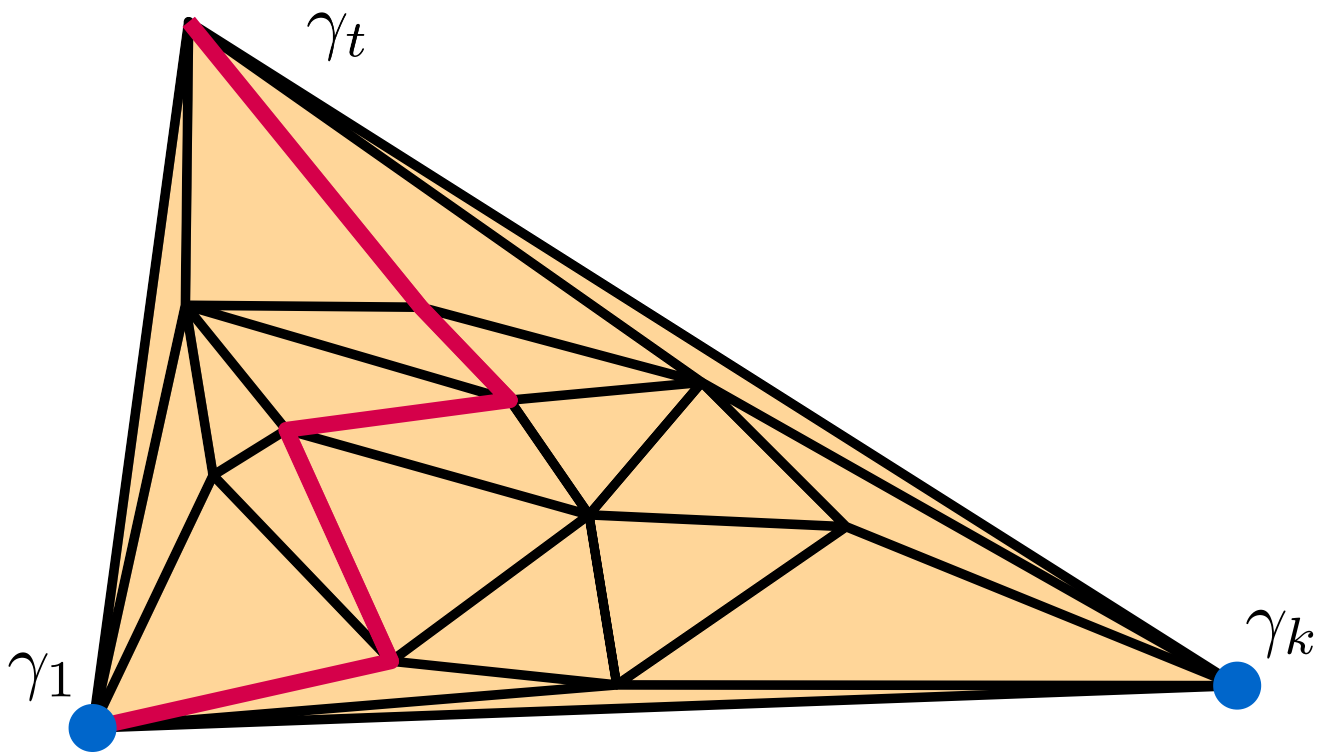
All curves γ_t of sweep start and end on outer face
First and last curves of sweep consist of single (distinct) vertex
Consecutive curves differ by a (simple) homotopy move



Simple homotopy moves

Any curve in simple sweep uses any vertex $\leq$ once

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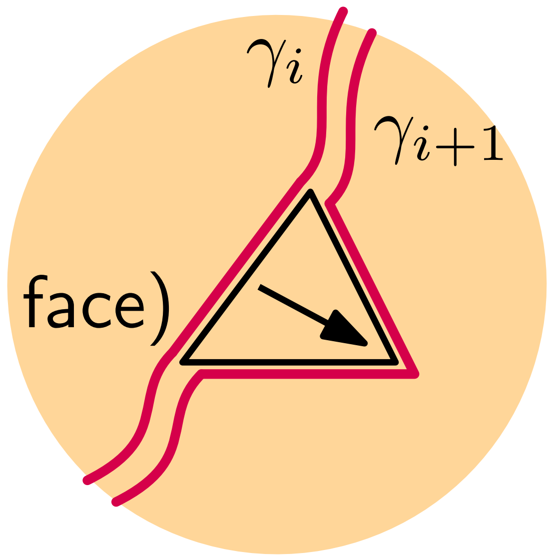
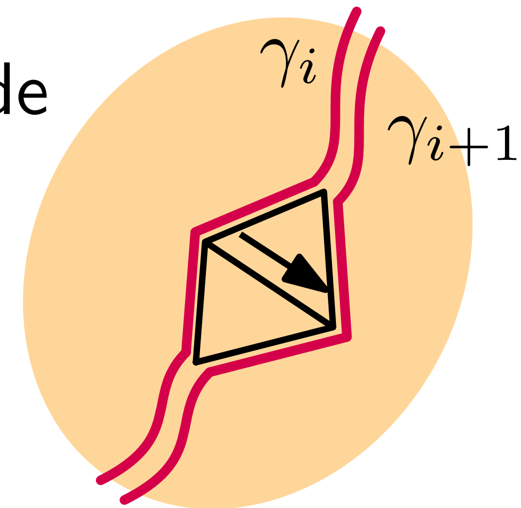
Face-flip

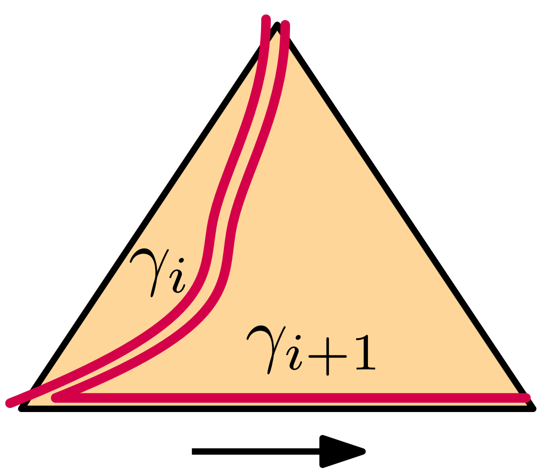
(not outer face)



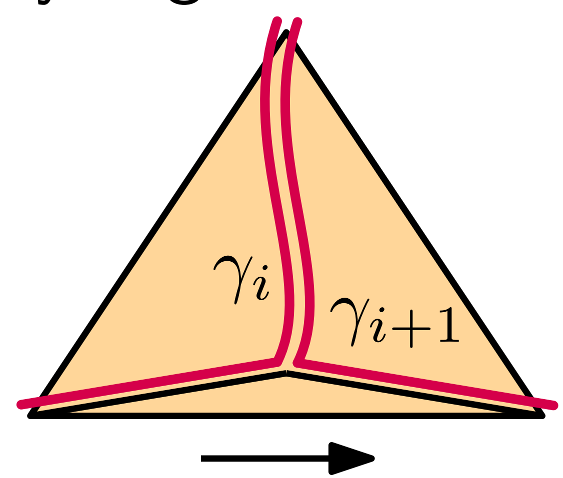
Edge-slide



Boundary-move



Boundary-edge-slide



Simple homotopy moves

Any curve in simple sweep uses any vertex $\leq$ once

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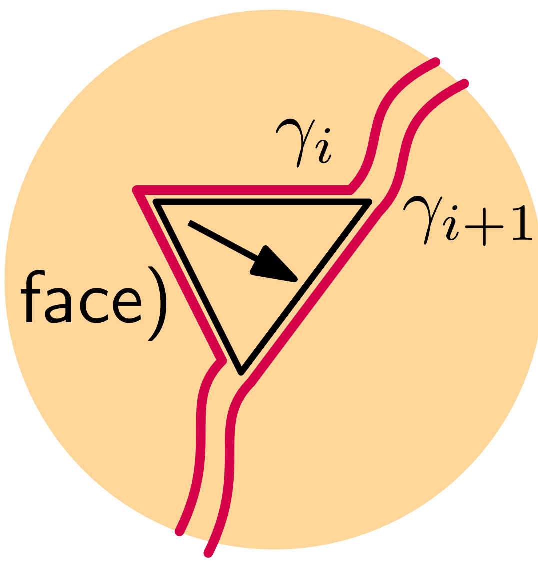

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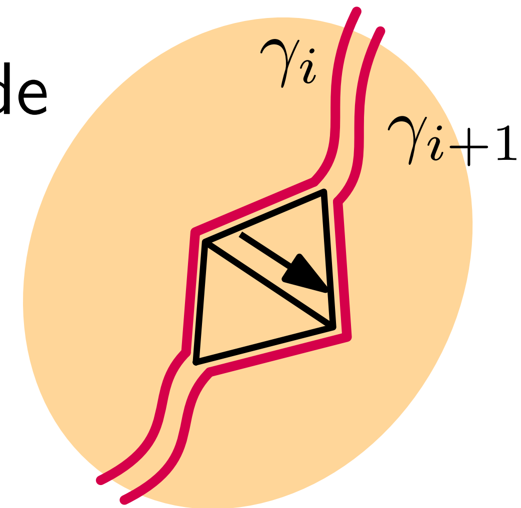

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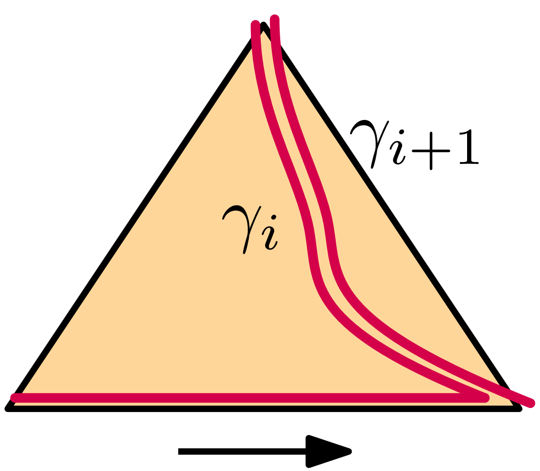
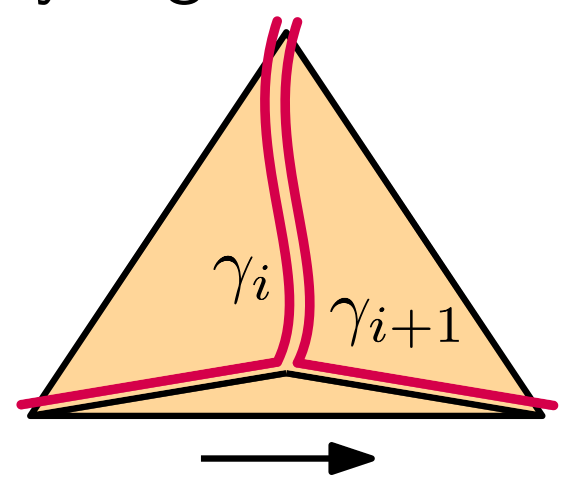


Boundary-edge-slide



Homotopy moves (nonsimple)

Vertices can be reused

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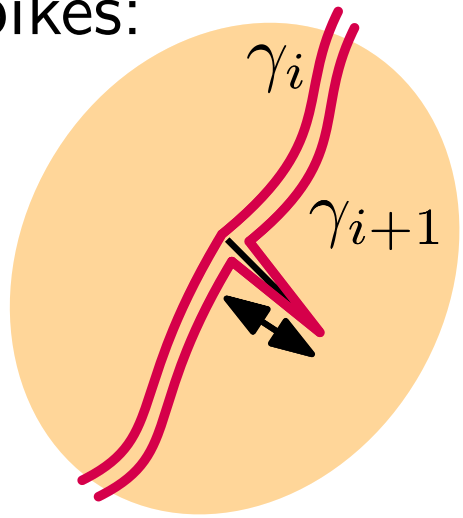
Simple homotopy moves + edge spikes:



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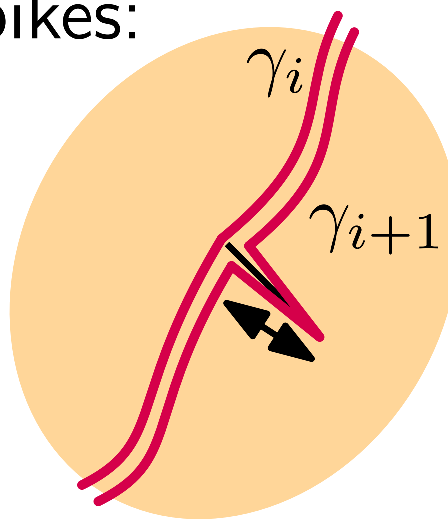


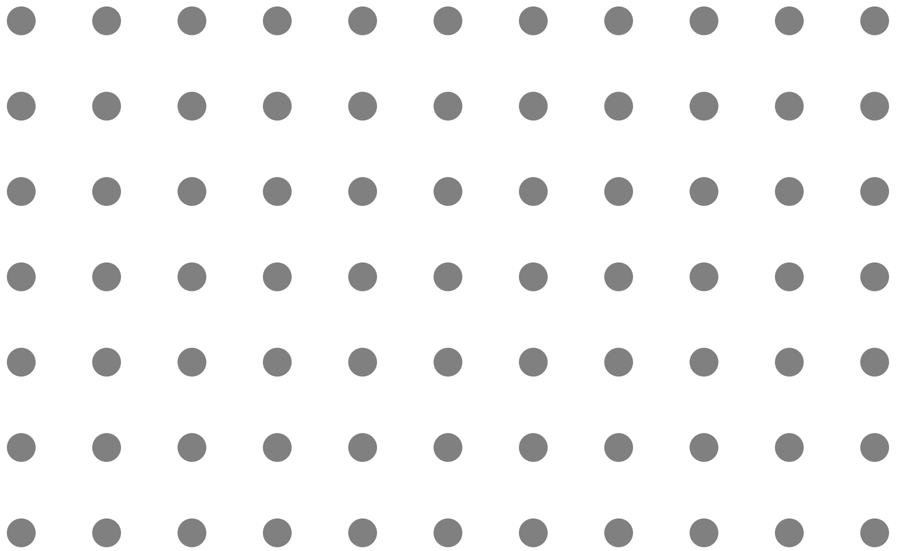
Sweep must flip (or slide) across each face ‘from-left-to-right’ once more than ‘from-right-to-left’

Grid-major height

NEW!

WxH gridpoints

$$\{1,\ldots,W\} \times \{1,\ldots,H\}$$



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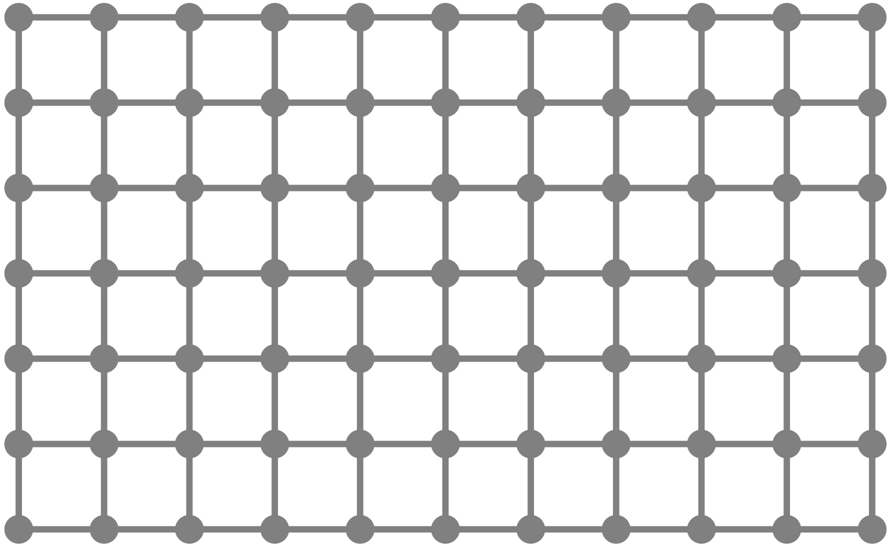

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WxH grid

graph on gridpoints, edges between points at distance 1

Grid-major height

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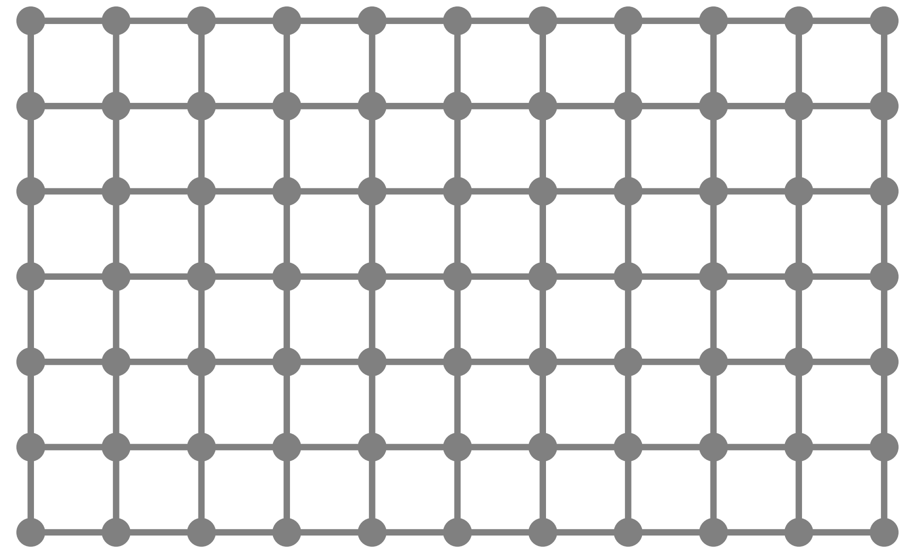

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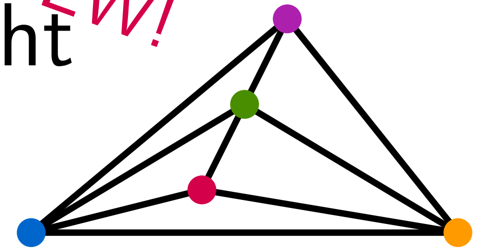
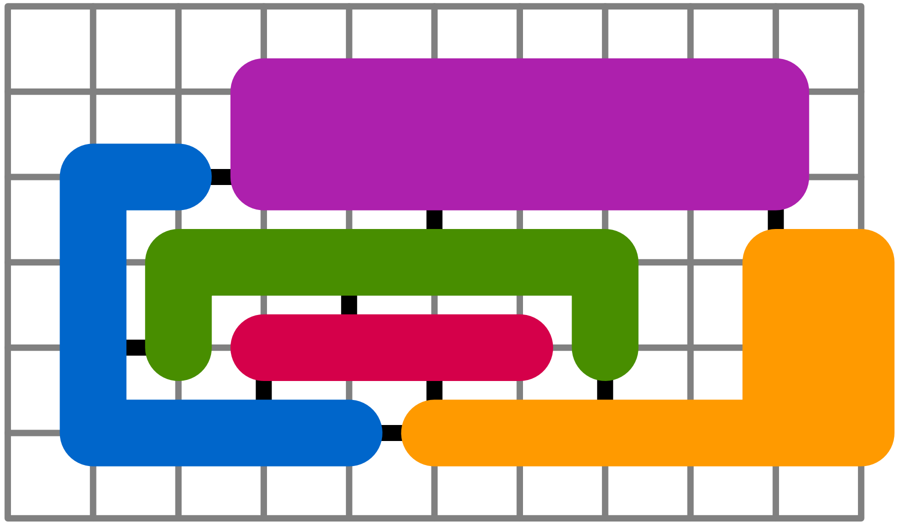
Grid-major height (of a planar graph G)

minimum h s.t. G is a minor of Wxh grid

Grid-major height





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Minor (of graph H)

graph obtained from H by

contracting edges

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Grid-major height

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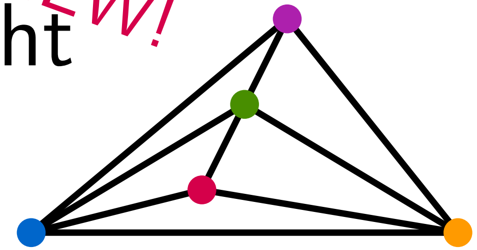

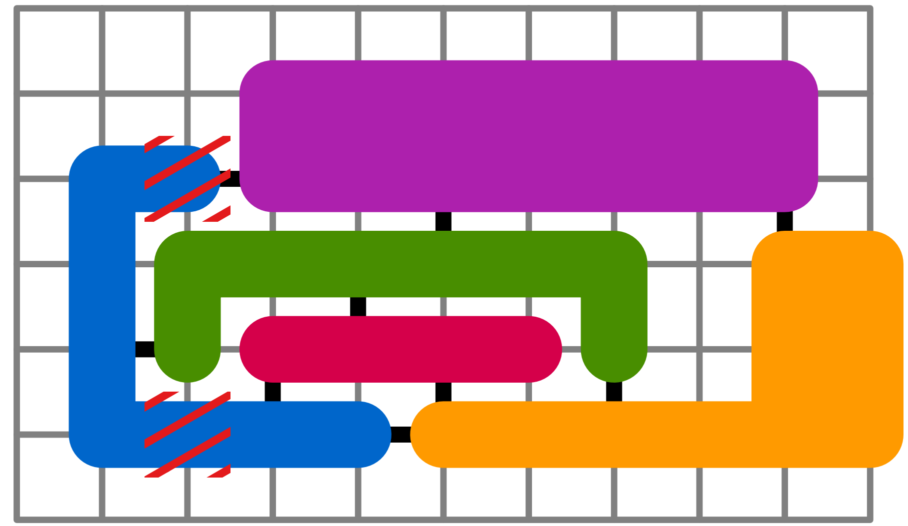

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Simple grid-major height

each label in a column appears consecutively

Some graph parameters...

(Simple) homotopy height

(Simple) grid-major height

(Simple) contact representation height

Visibility representation height

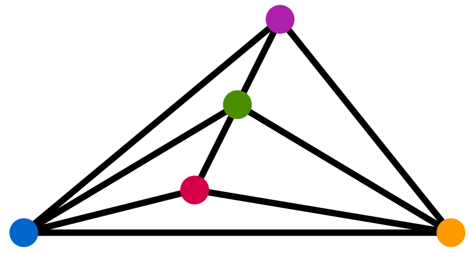
Straight-line drawing height

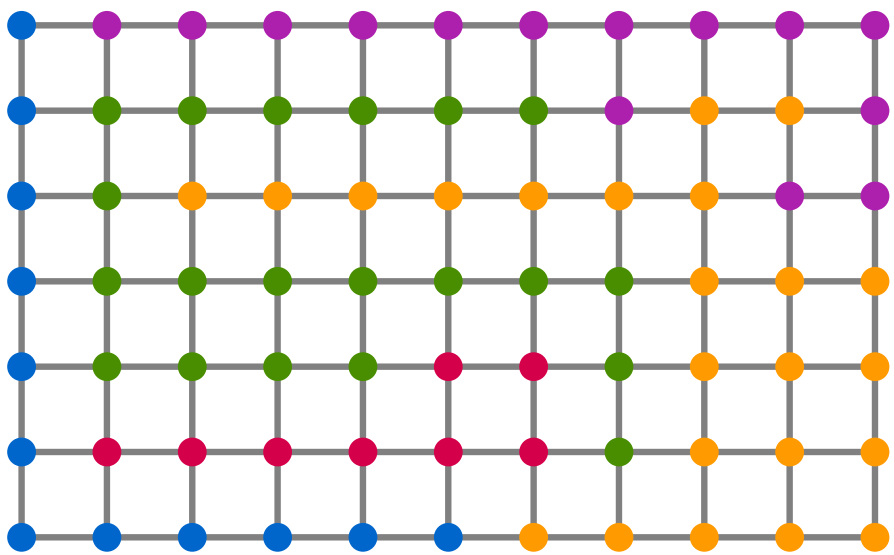
Pathwidth

Outerplanarity

Contact representation

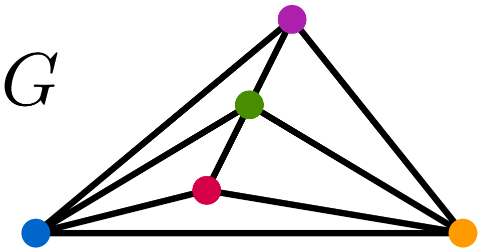
each gridpoint labeled by a vertex of G

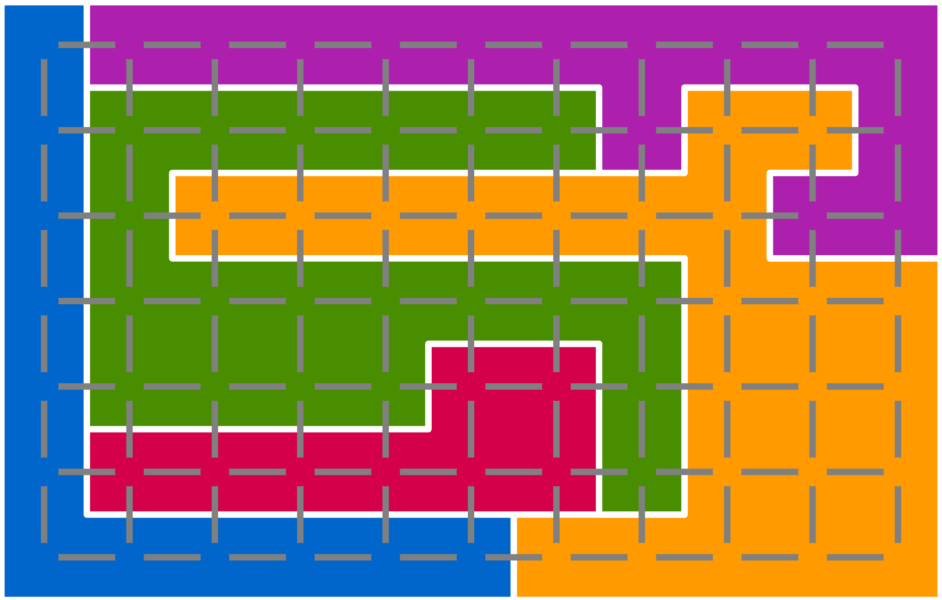




Contact representation

each gridpoint labeled by a vertex of G
each label forms connected subgraph
two labels adjacent if and only if edge in G





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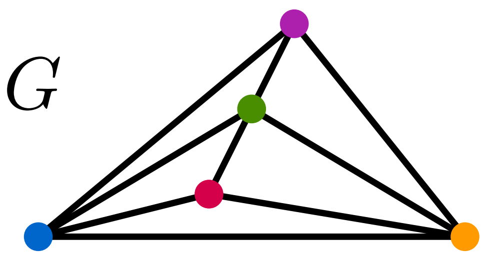

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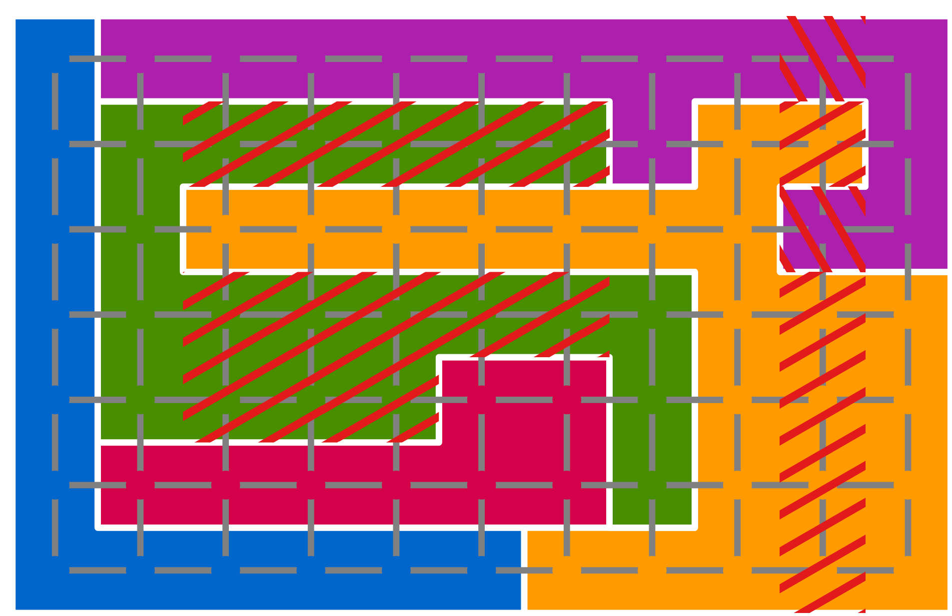

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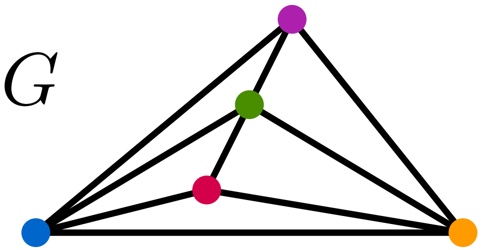

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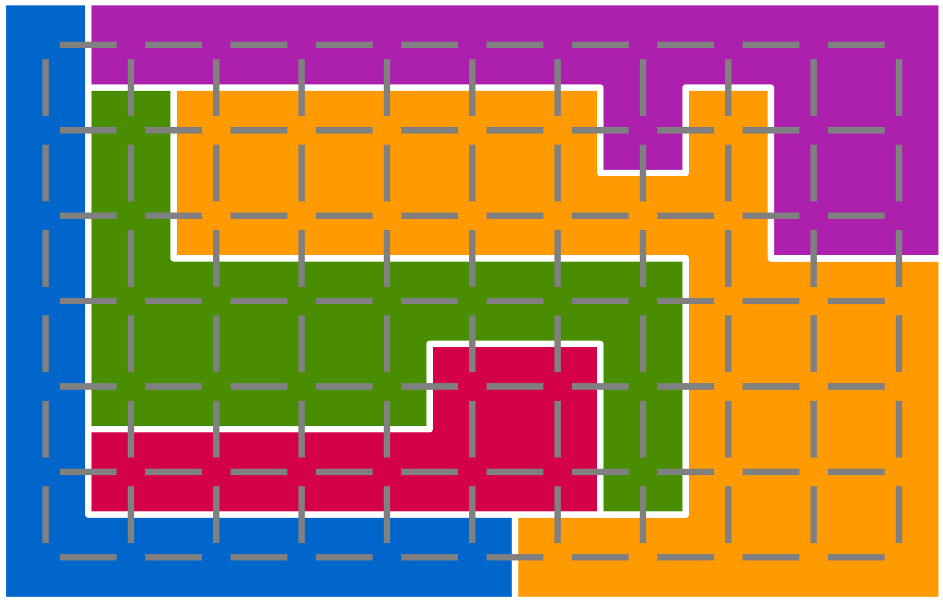

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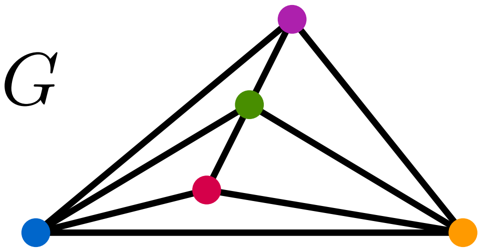
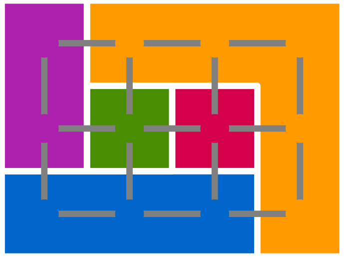


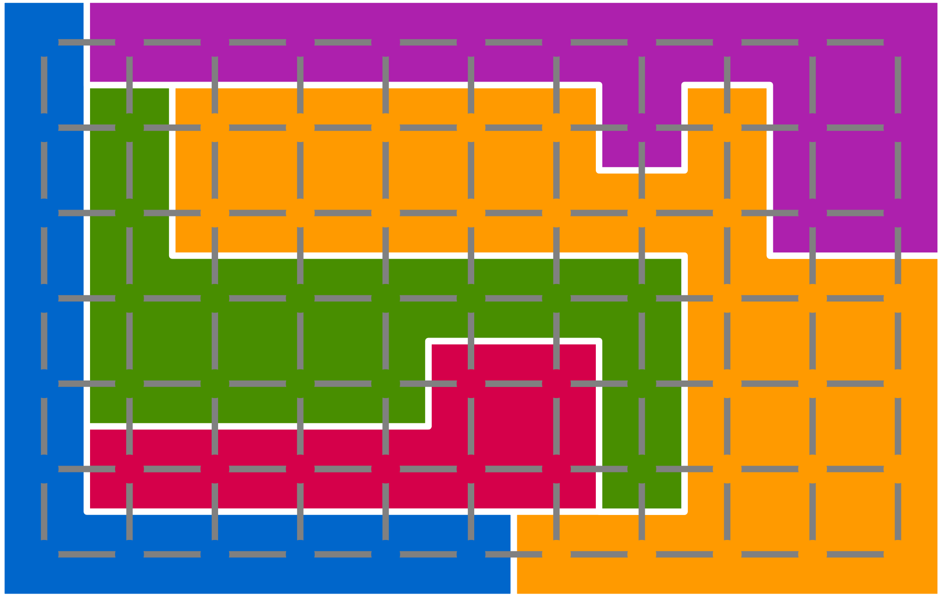
Simple contact representation

each label appears consecutively in each column

(Simple) contact representation height

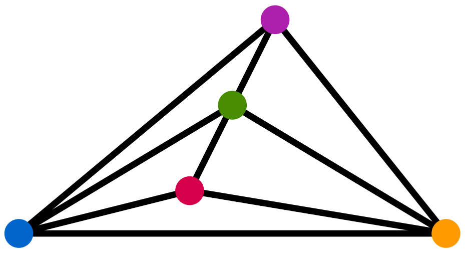
min h s.t. Wxh grid has (simple) contact representation

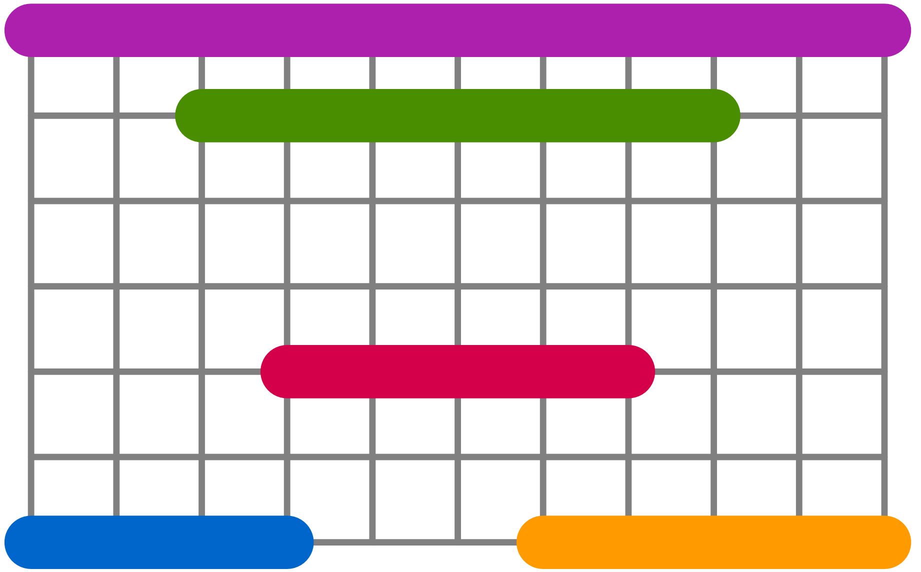




Flat visibility representation

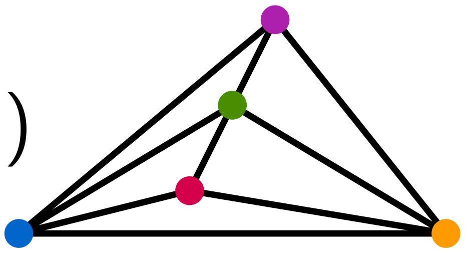
each vertex corresponds to a horizontal bar

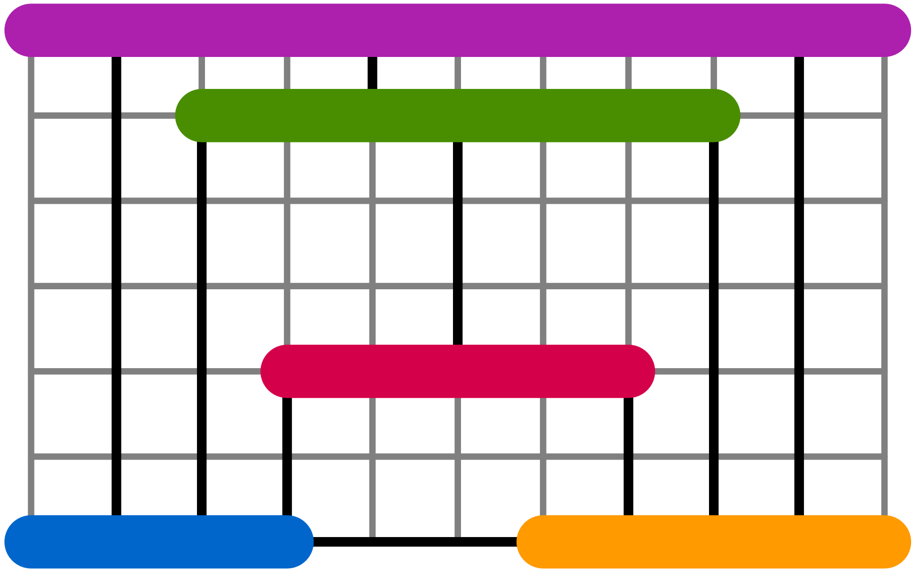




Flat visibility representation

each vertex corresponds to a horizontal bar
for each edge there is a line of visibility
(horizontal or vertical)
bars and lines of visibility do not cross





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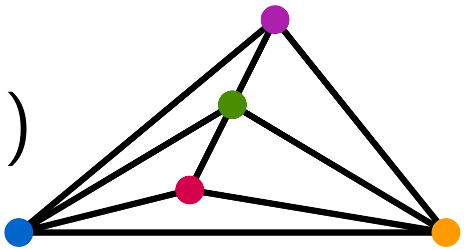
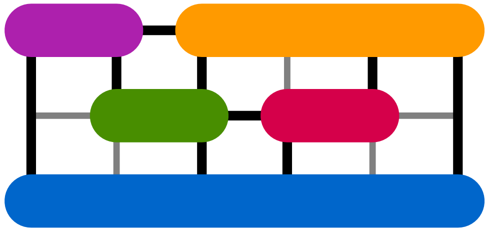
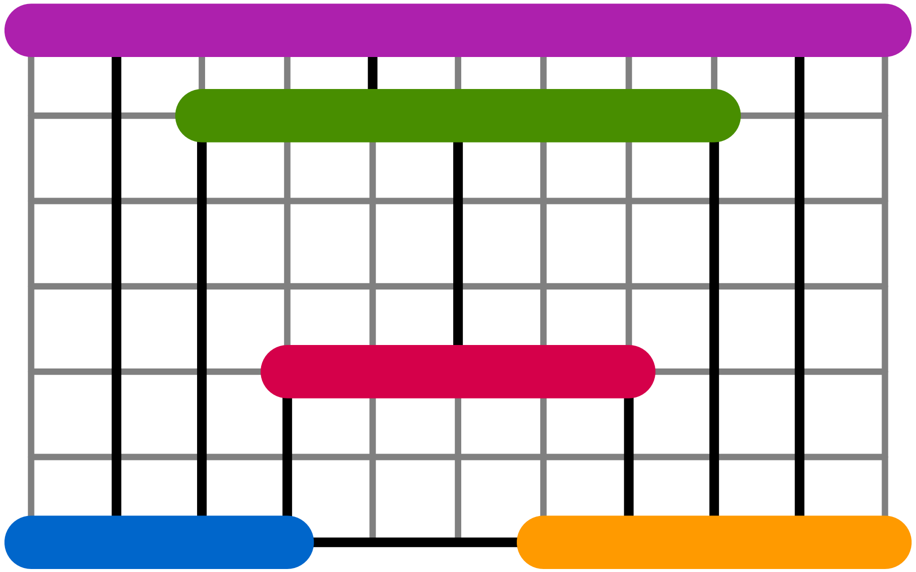
bars and lines of visibility do not cross



Visibility representation height

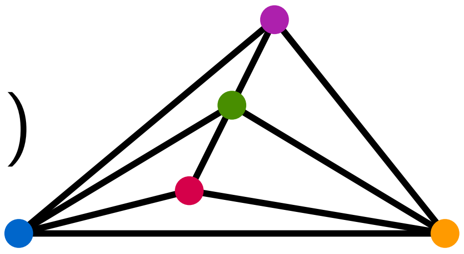
min h s.t. Wxh grid has flat visibility representation





Flat visibility representation

each vertex corresponds to a horizontal bar
for each edge there is a line of visibility
(horizontal or vertical)
bars and lines of visibility do not cross
we allow additional visibilities (without edge in G)



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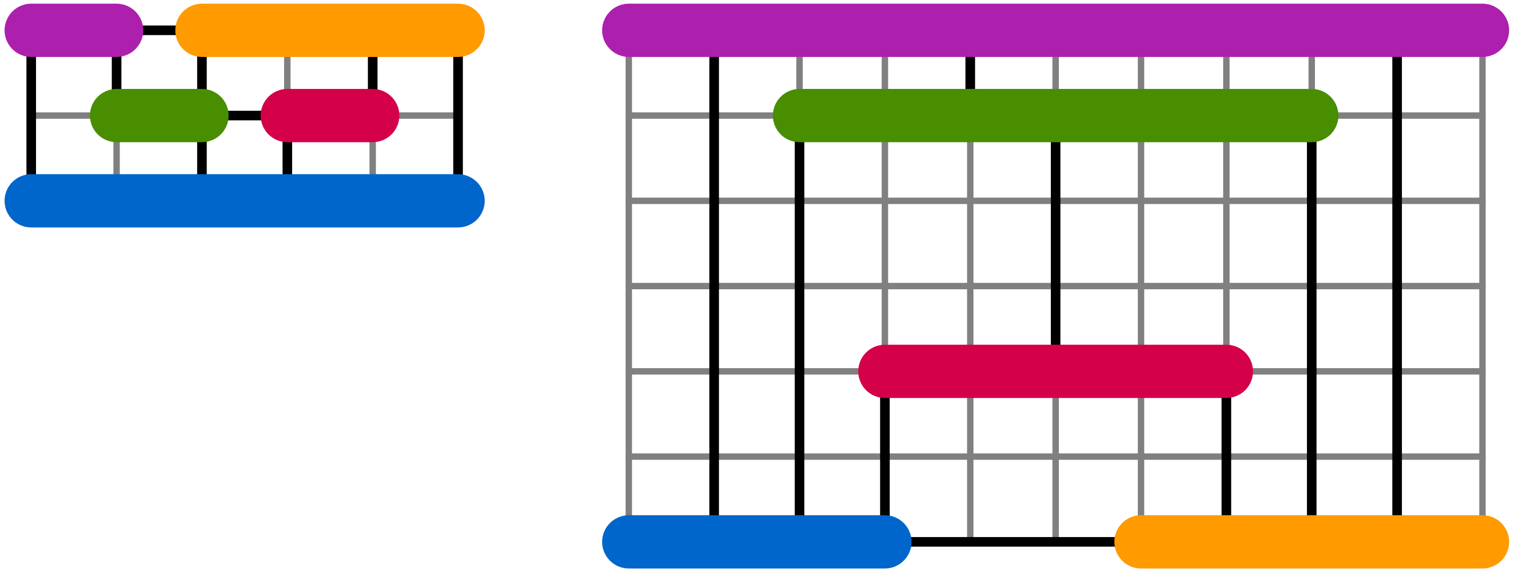
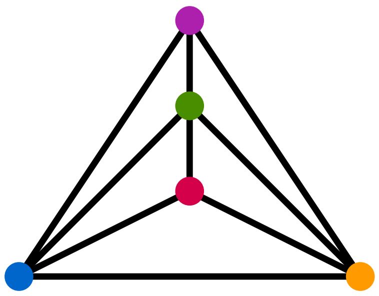


Straight-line height

min h with planar straight line drawing
that has all vertices on Wxh gridpoints



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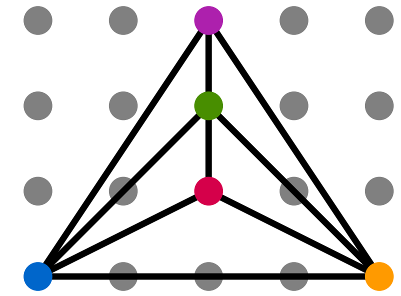
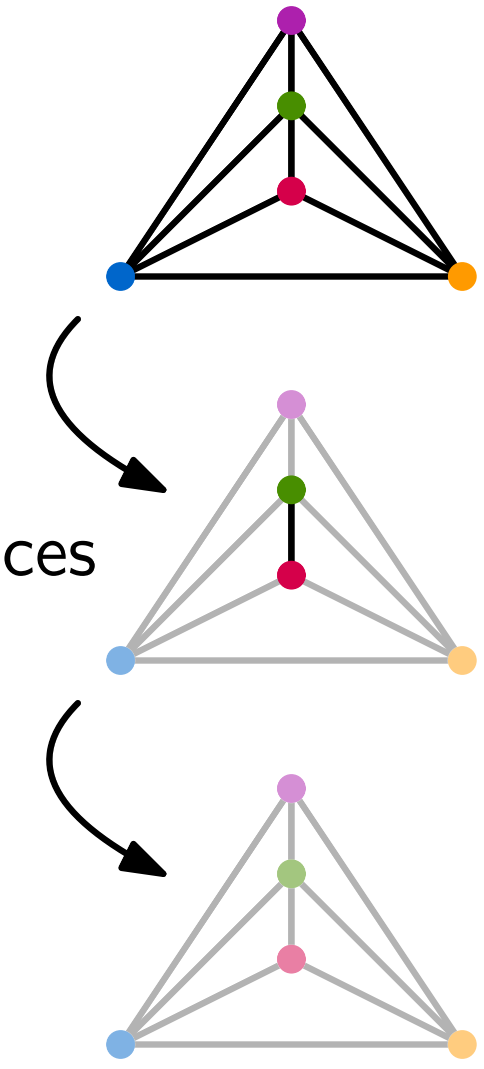


Outerplanarity

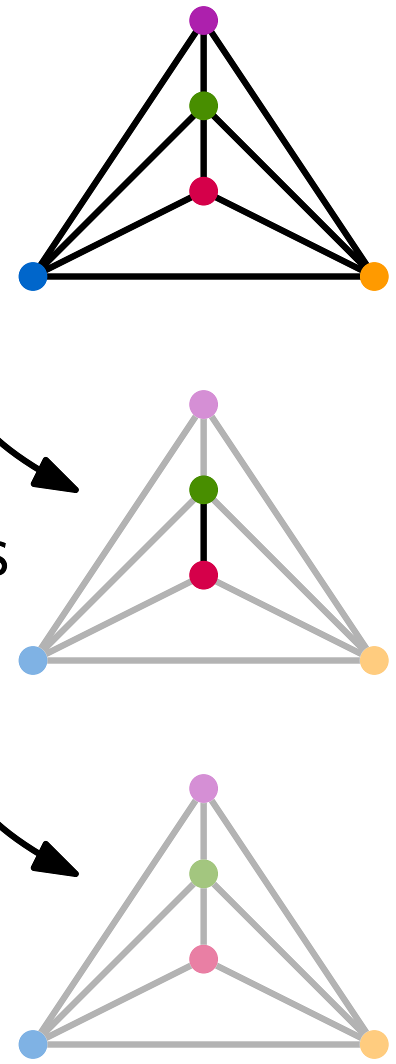
Outerplanarity (of a planar embedding)

number of steps needed to remove all vertices

each step: remove vertices of outer face



Outerplanarity



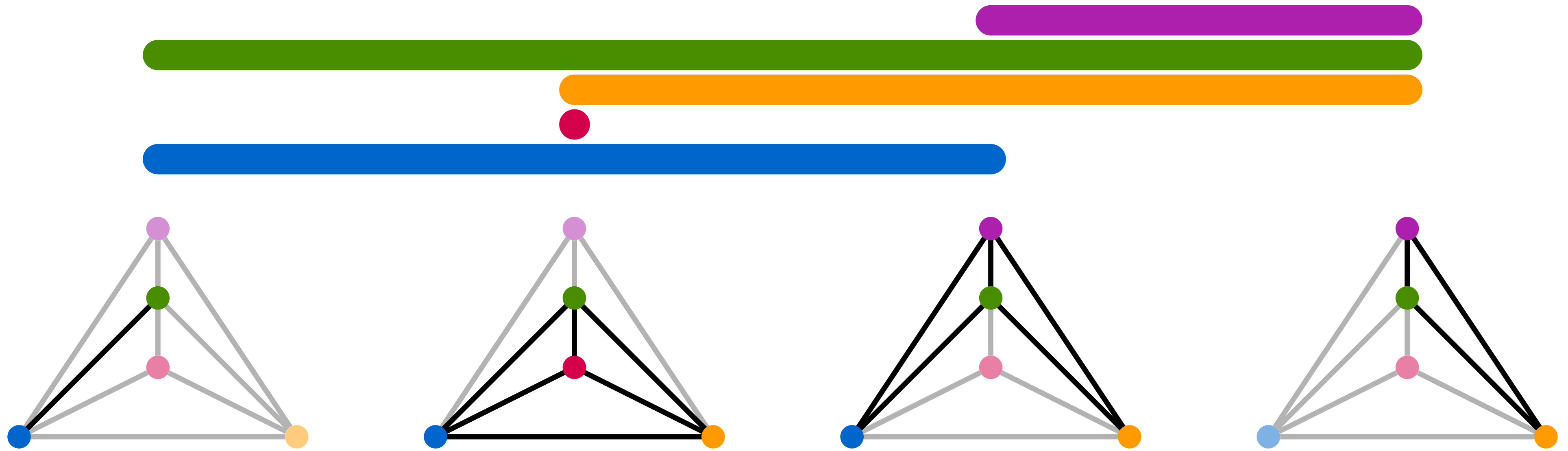
Outerplanarity (of a planar embedding)
number of steps needed to remove all vertices
each step: remove vertices of outer face

Outerplanarity (of a planar graph)
minimum outerplanarity over all embeddings

Pathwidth

Path decomposition

Form groups of vertices and put groups on a path
Each vertex belongs to a subpath of groups
For any edge, endpoints lie in a common group



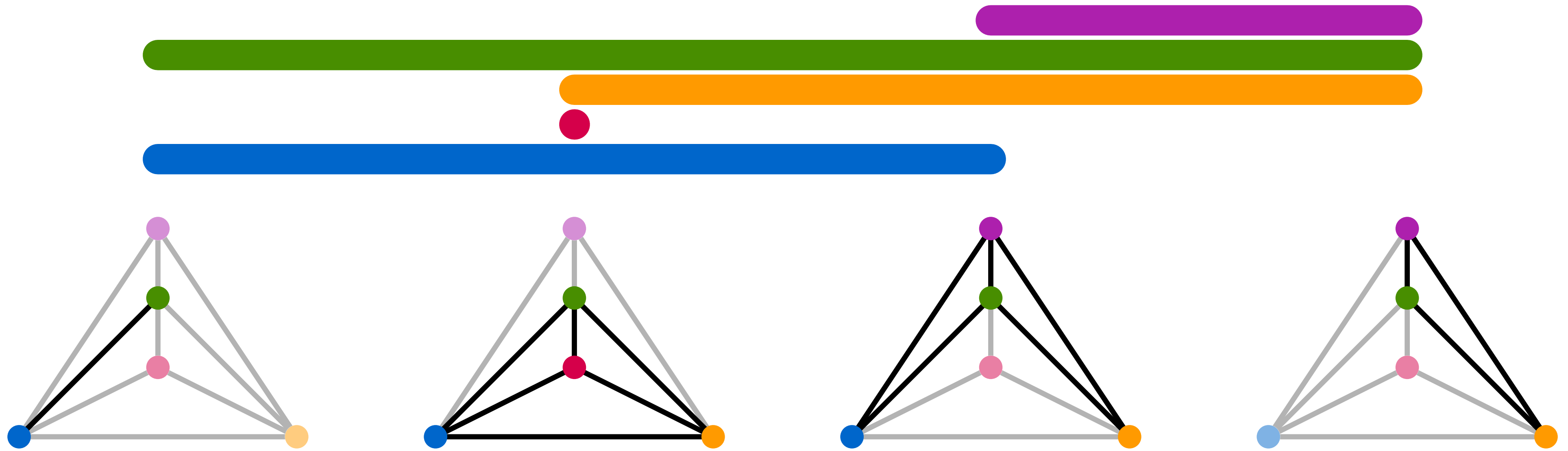
Pathwidth

Path decomposition

Form groups of vertices and put groups on a path
Each vertex belongs to a subpath of groups
For any edge, endpoints lie in a common group

Pathwidth

Minimum largest group size -1 over all decompositions



Relations between graph parameters...

(Simple) homotopy height

(Simple) grid-major height

(Simple) contact representation height

Visibility representation height

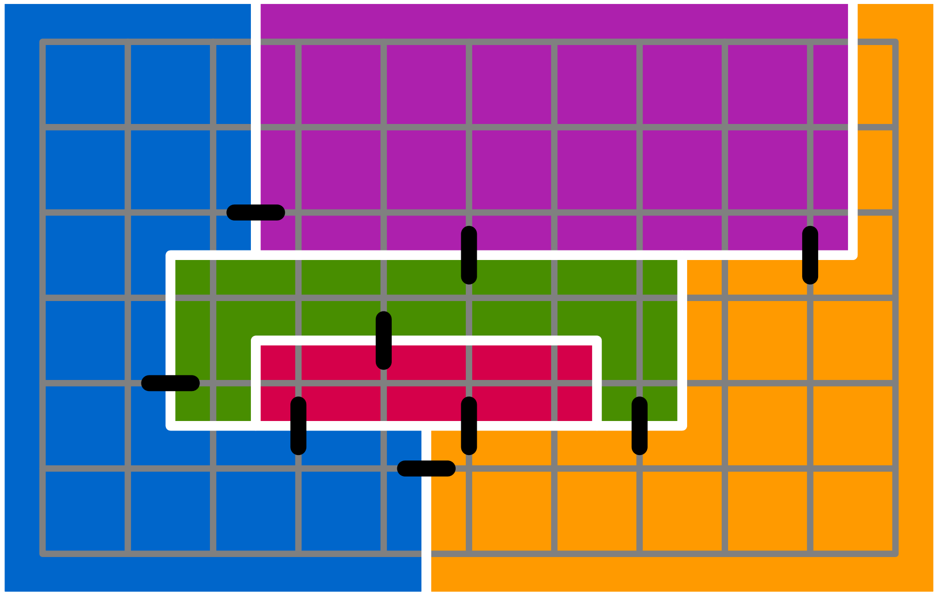
Straight-line drawing height

Pathwidth

Outerplanarity

Bounds

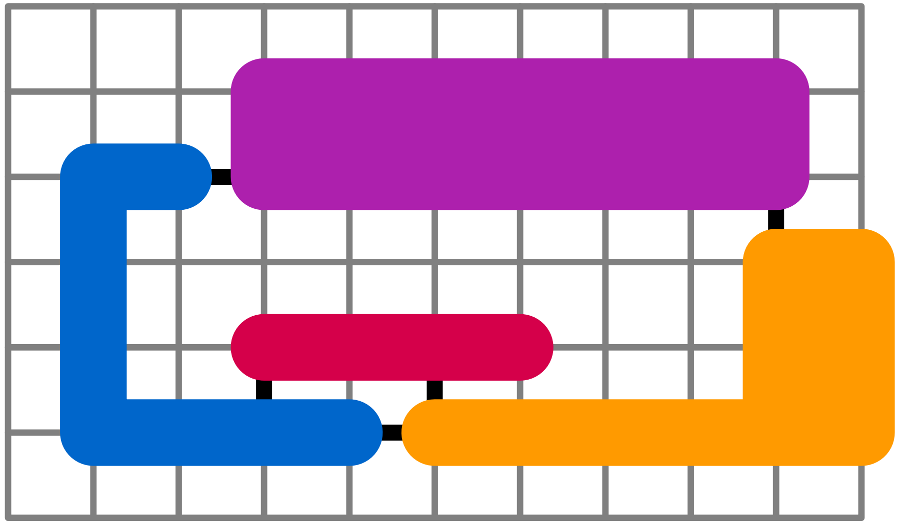
Every contact representation is a grid-major representation



Bounds

Every contact representation is a grid-major representation

Reverse is not necessarily true:
Grid-major repr. can have unwanted contacts and empty spots



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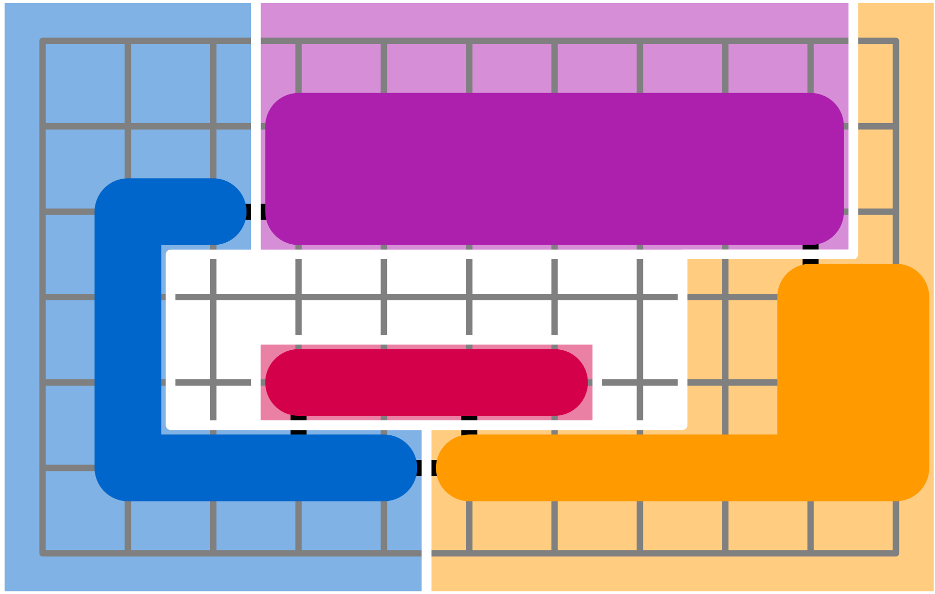

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Every contact representation is a grid-major representation

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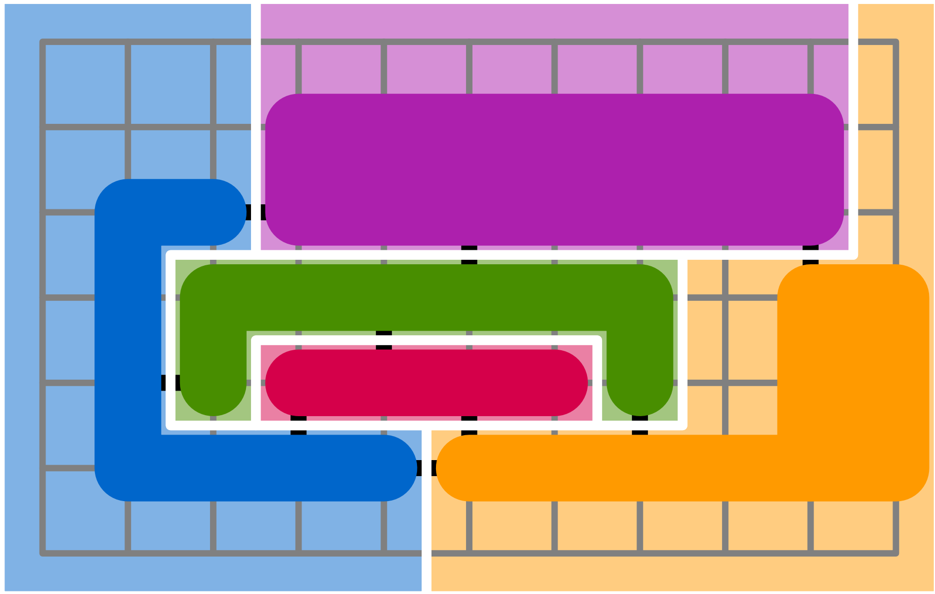
Our assumptions on the graph
$\Rightarrow$ empty space can be filled without unwanted contacts



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Every contact representation is a grid-major representation

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Our assumptions on the graph
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contact representation height $=$ grid-major height

Bounds

Every contact representation is a grid-major representation

Reverse is not necessarily true:
Grid-major repr. can have unwanted contacts and empty spots

Our assumptions on the graph
$\Rightarrow$ empty space can be filled without unwanted contacts

contact representation height = grid-major height

simple contact representation height = simple grid-major height

Bounds

Every contact representation is a grid-major representation

Reverse is not necessarily true:
Grid-major repr. can have unwanted contacts and empty spots

Our assumptions on the graph
$\Rightarrow$ empty space can be filled without unwanted contacts

contact representation height $=$ grid-major height

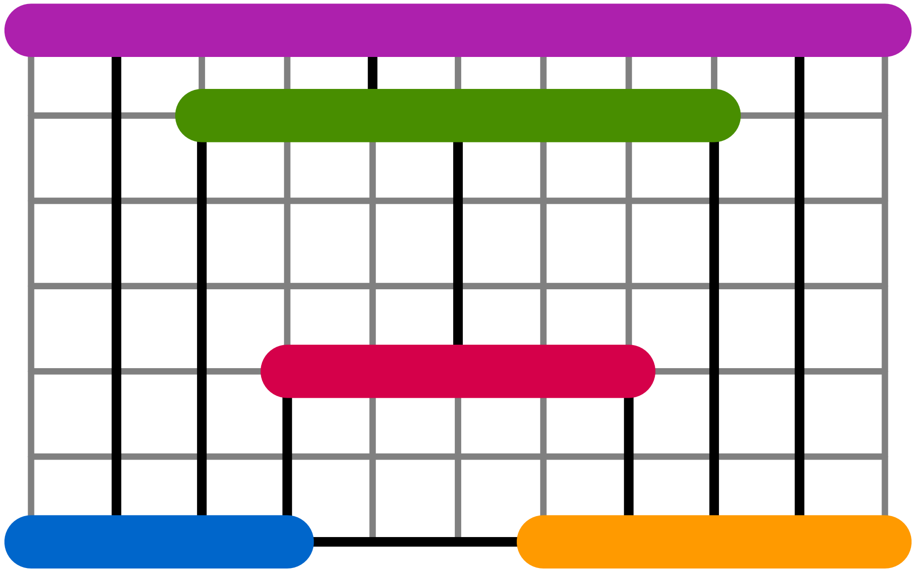
simple contact representation height $=$ simple grid-major height

Requiring that regions are x-monotone can only increase height

grid-major height $\leq$ simple grid-major height

Bounds

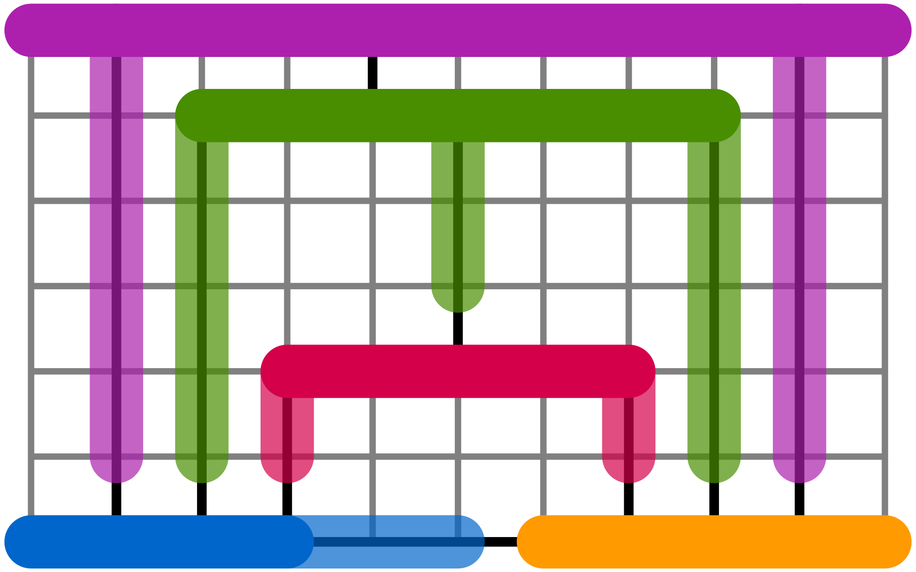
Every flat visibility representation can be turned into a simple grid-major representation



Bounds

Every flat visibility representation can be turned into a simple grid-major representation

simple grid-major height $\leq$ visibility representation height



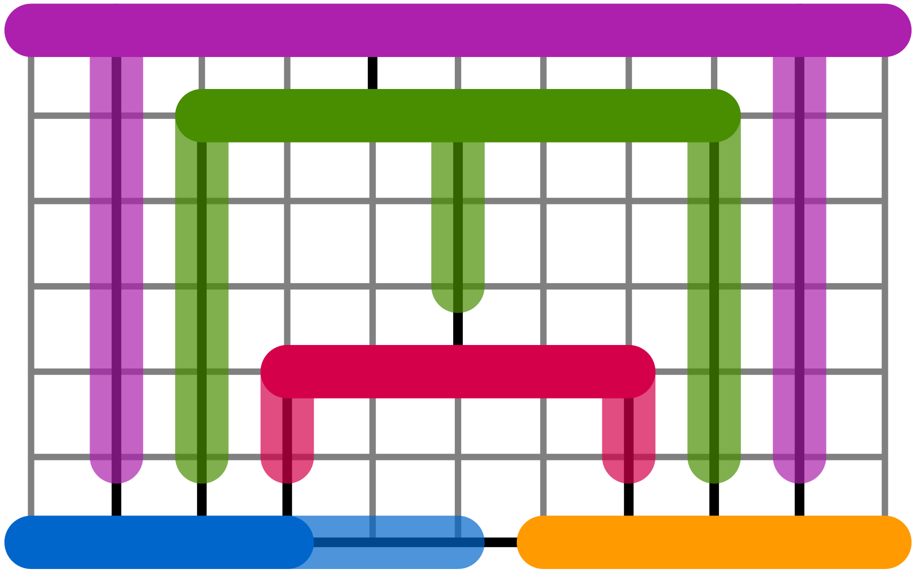
Bounds

Every flat visibility representation can be turned into a simple grid-major representation

simple grid-major height $\leq$ visibility representation height

Previously shown [Biedl14]:

visibility representation height $=$ straight-line drawing height



Bounds

Pathwidth of Wxh grid minor $\leq$ pathwidth of Wxh grid $\leq h$

Bounds

Pathwidth of Wxh grid minor $\leq$ pathwidth of Wxh grid $\leq h$

pathwidth $\leq$ grid-major height

Bounds

Pathwidth of Wxh grid minor $\leq$ pathwidth of Wxh grid $\leq h$

pathwidth $\leq$ grid-major height

Outerplanarity of Wxh grid minor $\leq$ that of Wxh grid $\leq \lceil h/2 \rceil$

2 outerplanarity $-1 \leq$ grid-major height

Overview of bounds

2 outerplanarity -1 and pathwidth

$\leq$

grid-major height

$=$

contact representation height

$\leq$

simple grid-major height

$=$

simple contact representation height

$\leq$

visibility representation height

$=$

straight-line drawing height

Overview of bounds

2 outerplanarity -1 and pathwidth

$\leq$

grid-major height

$=$

contact representation height $=$ homotopy height

$\leq$

simple grid-major height

$=$

simple contact representation height $=$ simple homotopy height

$\leq$

visibility representation height

$=$

straight-line drawing height

Simple grid-major height = simple homotopy height

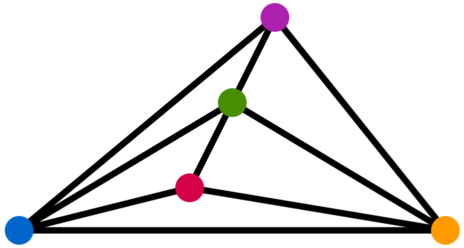
Sweep can be assumed monotone based on [CMO et al. 17]

curve does not sweep backwards

Simple grid-major height $=$ simple homotopy height

Sweep can be assumed monotone based on [CMO et al. 17]
curve does not sweep backwards

Simple homotopy height $\geq$ simple grid-major height:



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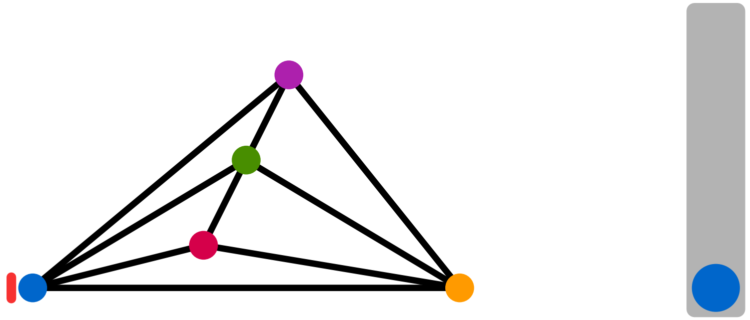

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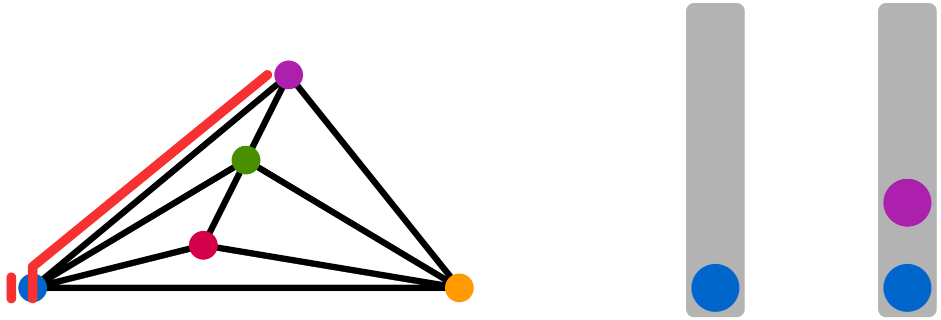

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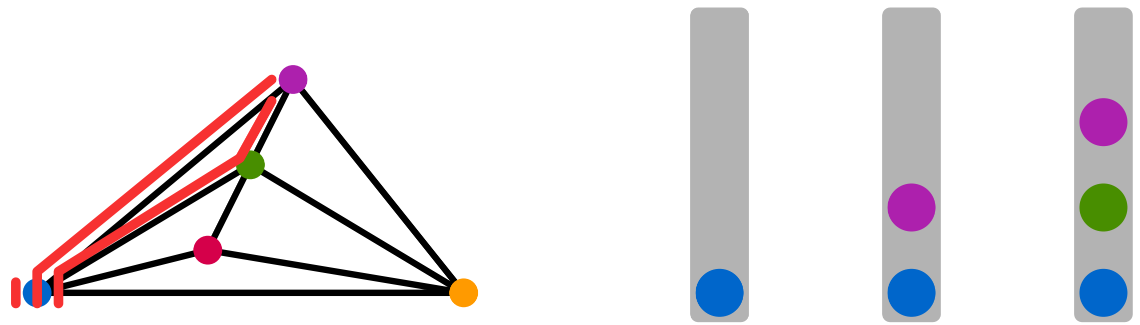

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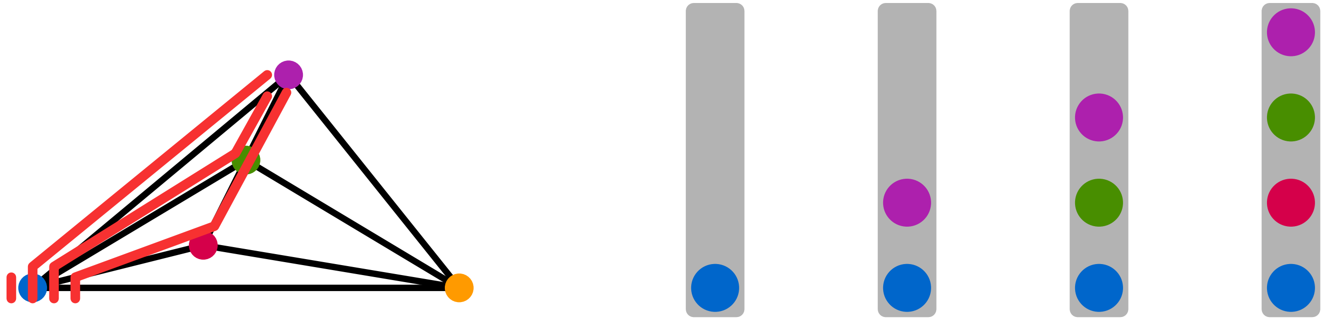

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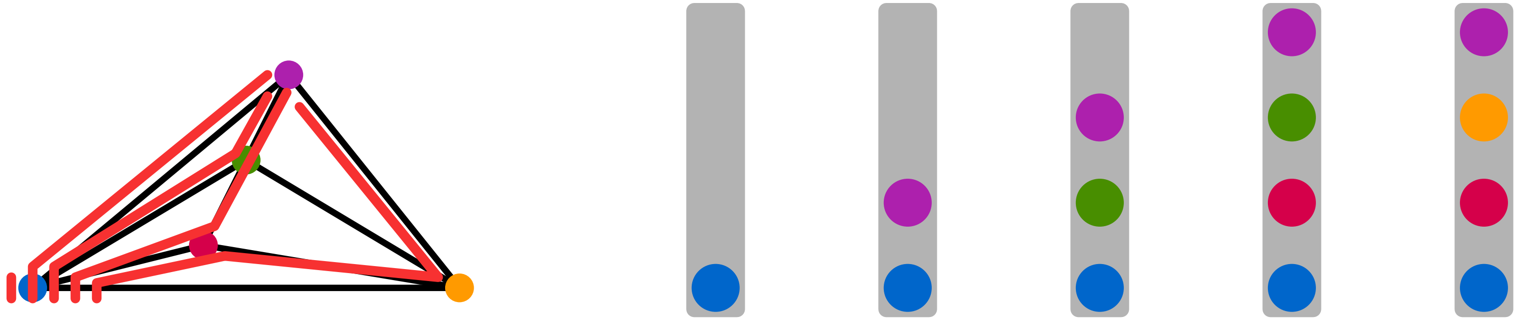

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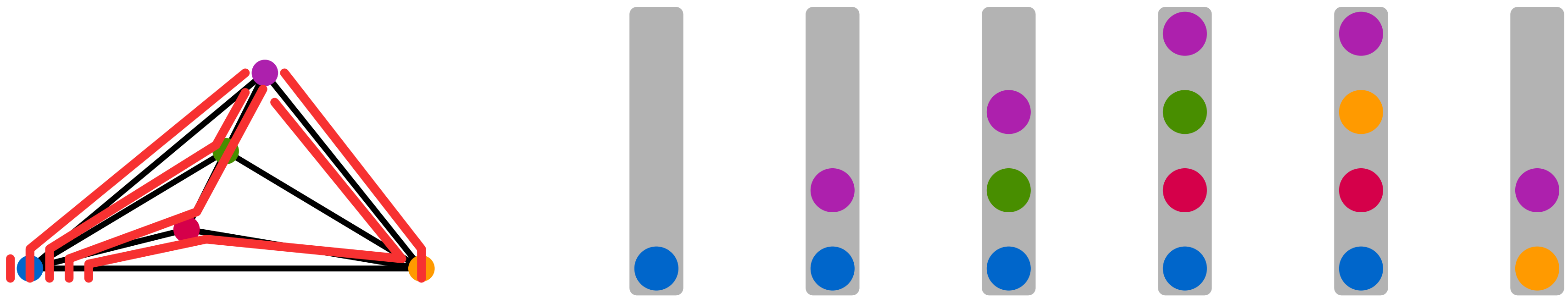

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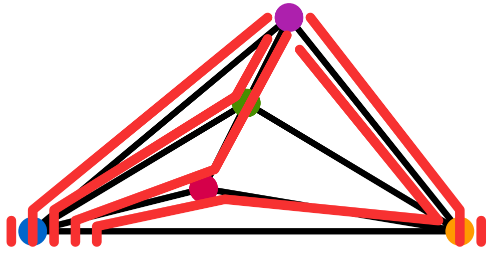

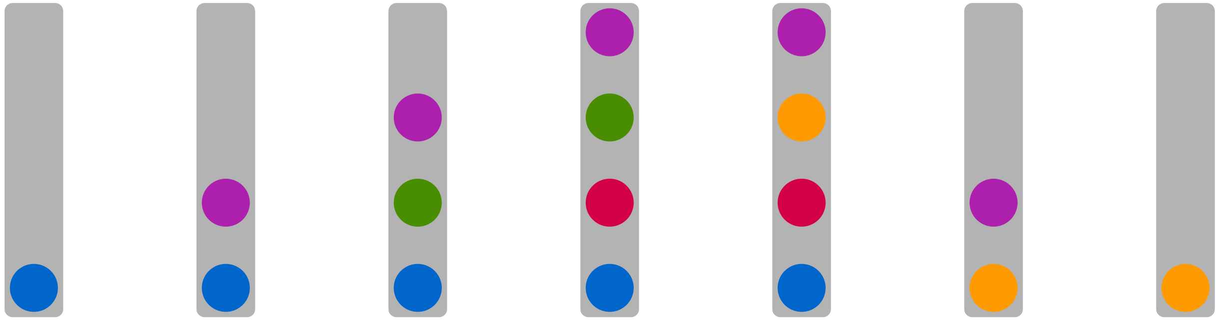

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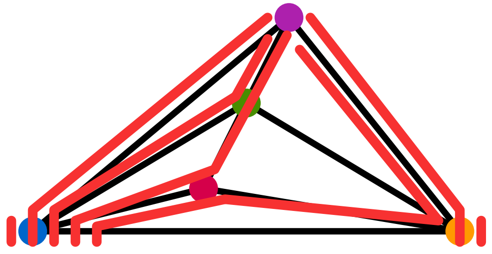

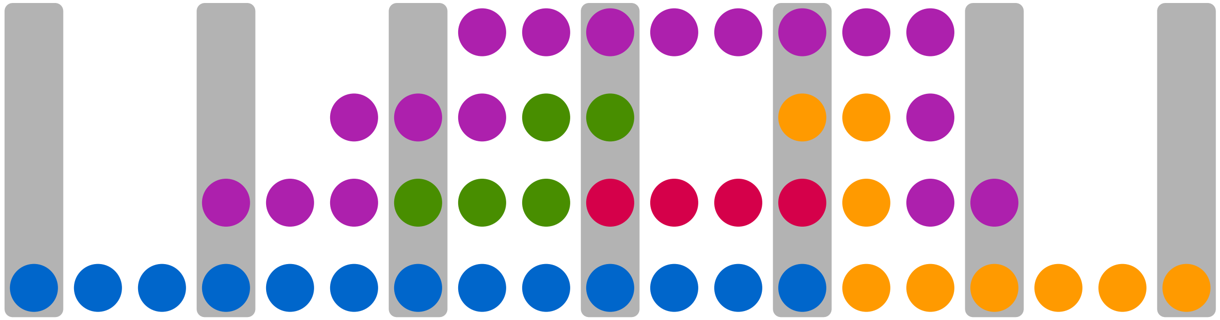

Simple grid-major height $=$ simple homotopy height

Sweep can be assumed monotone based on [CMO et al. 17]

curve does not sweep backwards

Simple homotopy height $\leq$ simple grid-major height:

Simple grid-major height = simple homotopy height

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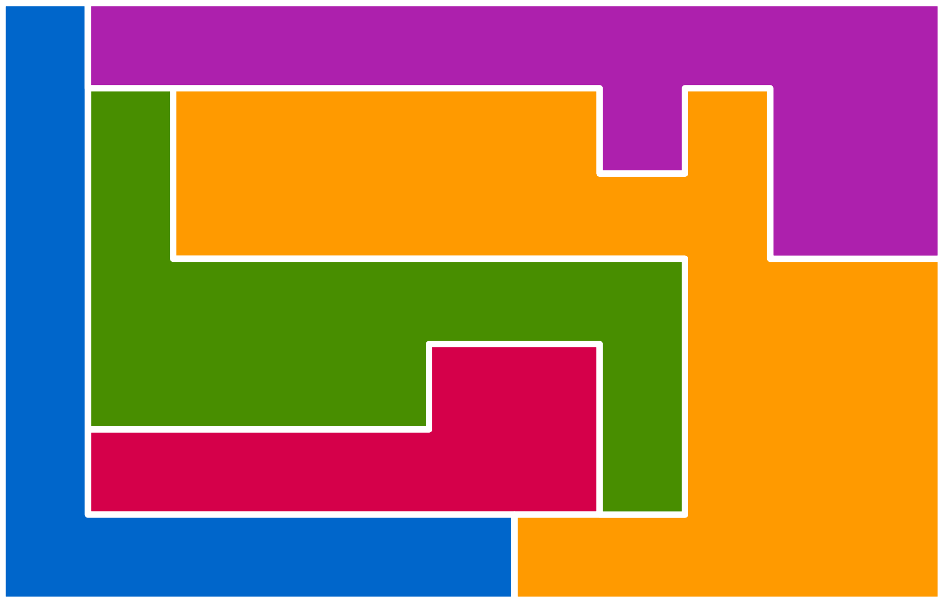
curve does not sweep backwards

Simple homotopy height $\leq$ simple grid-major height:

Take contact representation

wlog 3 colors on boundary



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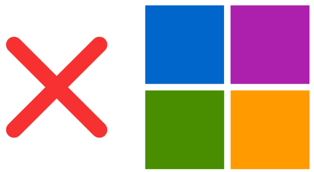
Simple homotopy height $\leq$ simple grid-major height:

Take contact representation

wlog 3 colors on boundary

No four polygons meet at a point



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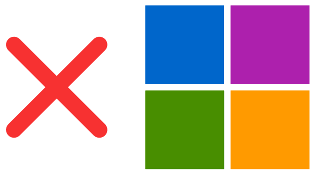

Remove interior vertical junctions

Simple grid-major height $=$ simple homotopy height

Sweep can be assumed monotone based on [CMO et al. 17]

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Simple homotopy height $\leq$ simple grid-major height:

Take contact representation

wlog 3 colors on boundary

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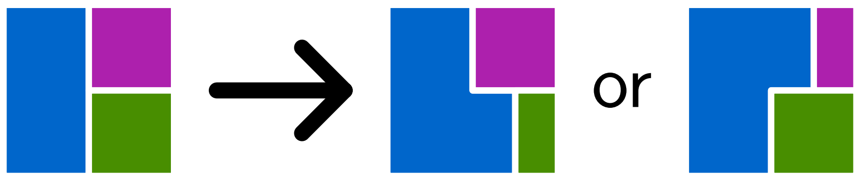

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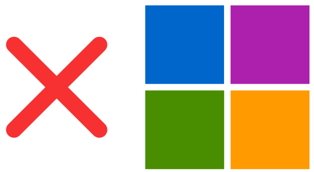


Remove interior vertical junctions

Make x-coordinates distinct

Simple grid-major height = simple homotopy height

Sweep can be assumed monotone based on [CMO et al. 17]

  curve does not sweep backwards

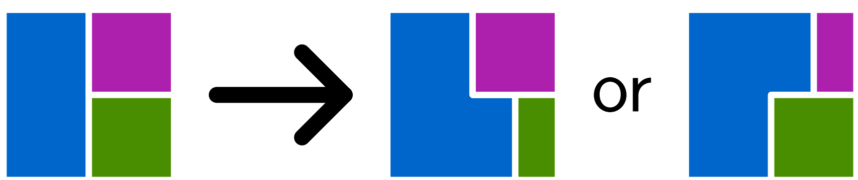
Simple homotopy height $\leq$ simple grid-major height:

Take contact representation

  wlog 3 colors on boundary

No four polygons meet at a point

Remove interior vertical junctions



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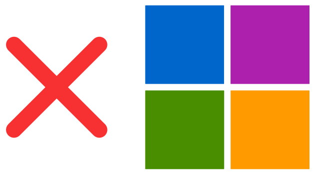

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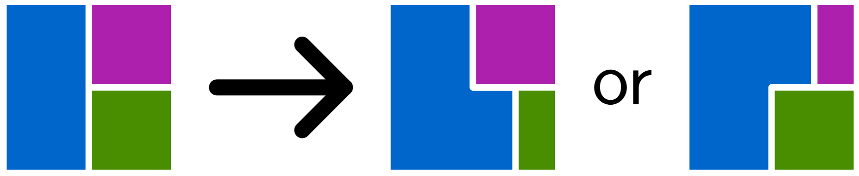


Make x-coordinates distinct

Make left and right boundary single (but distinct) color

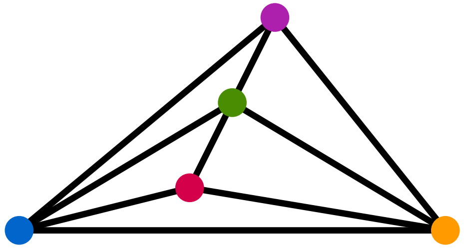
Simple grid-major height $=$ simple homotopy height

Sweep can be assumed monotone based on [CMO et al. 17]

curve does not sweep backwards

Simple homotopy height $\leq$ simple grid-major height:

Extract sweep



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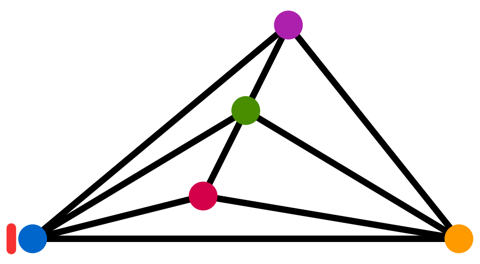

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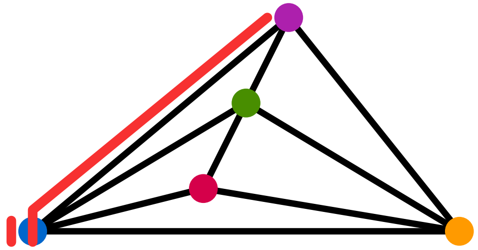

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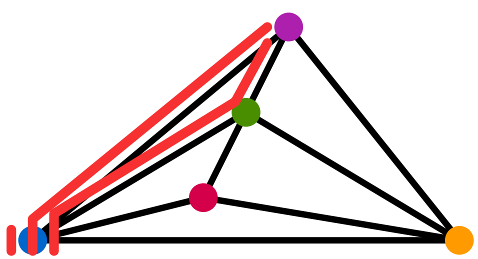

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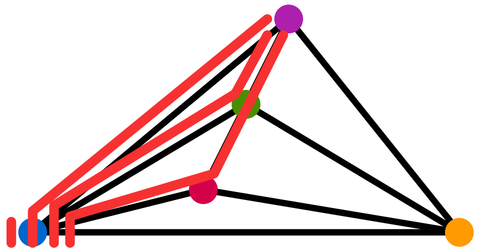

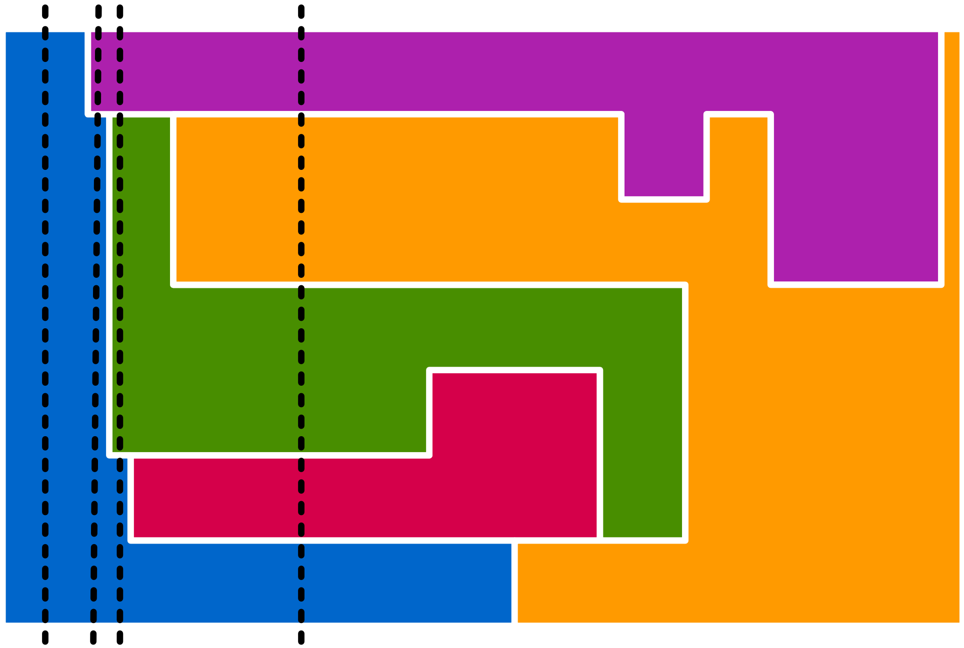

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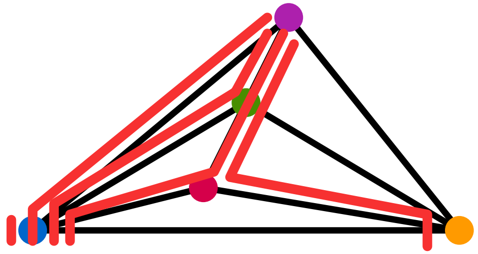

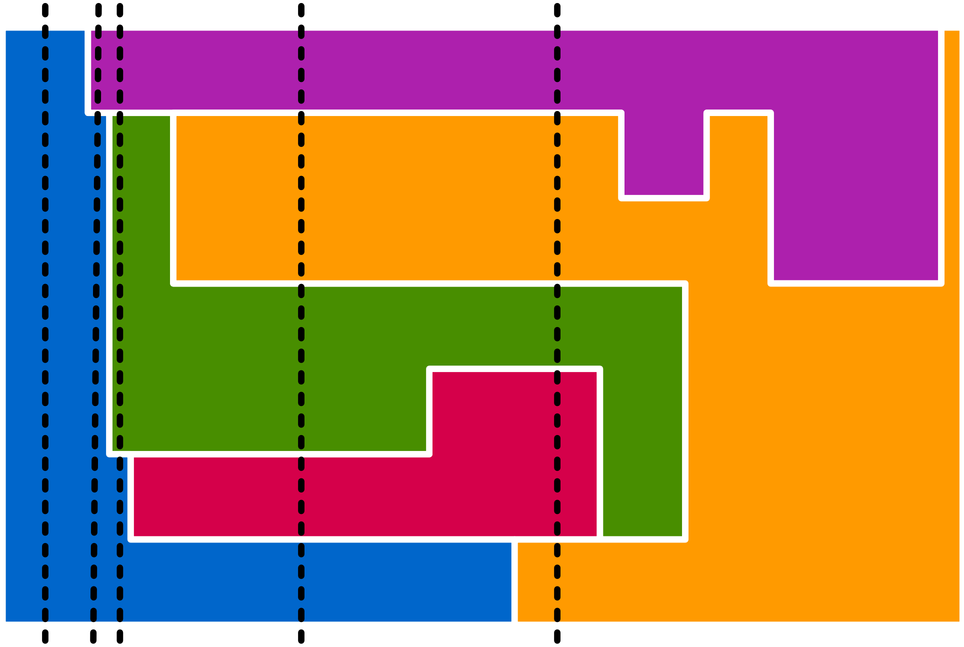

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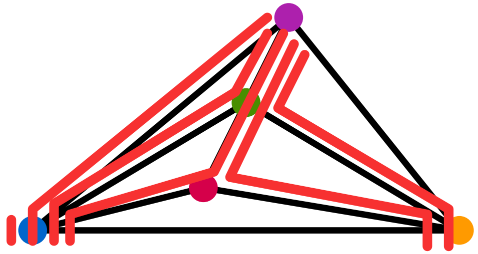

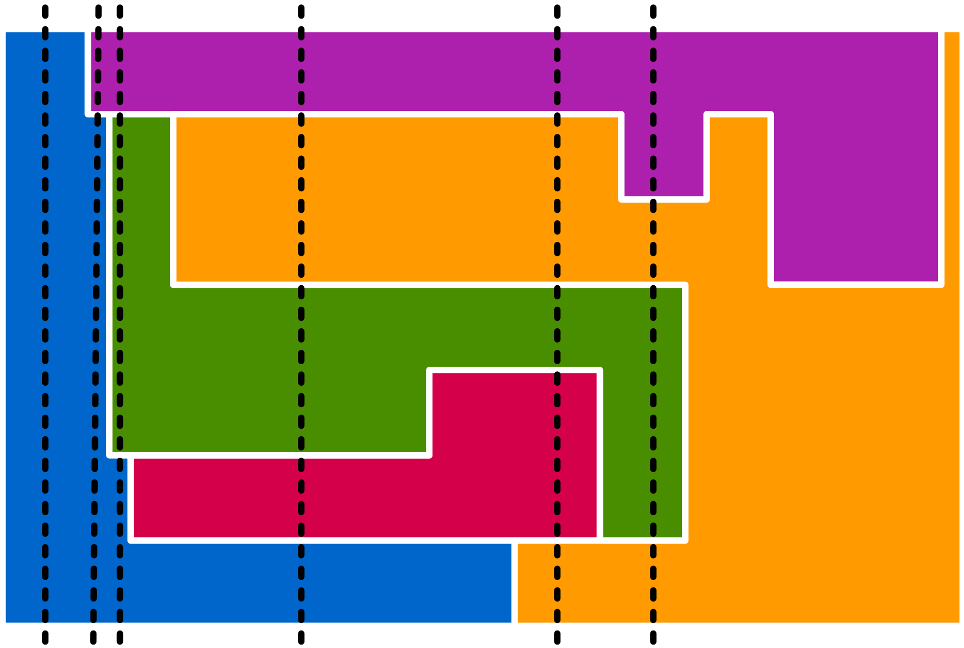

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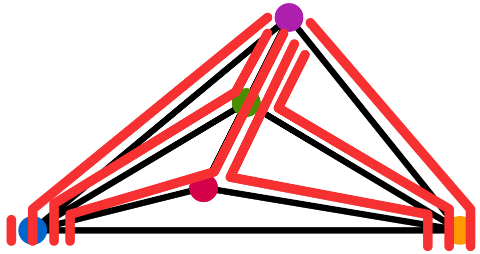
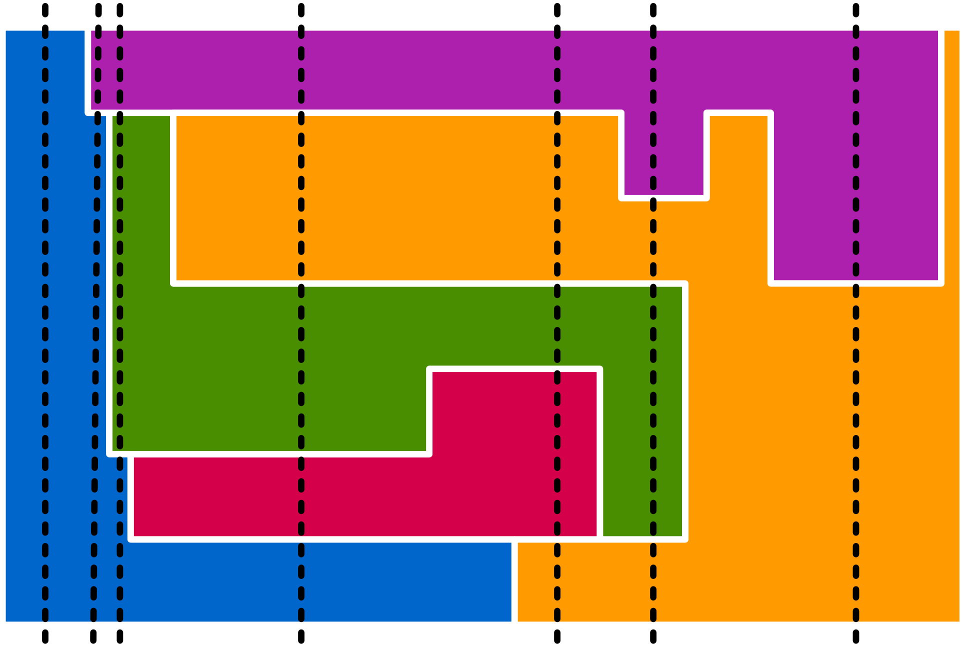
Sweep can be assumed monotone based on [CMO et al. 17]

curve does not sweep backwards

Simple homotopy height $\leq$ simple grid-major height:

Extract sweep





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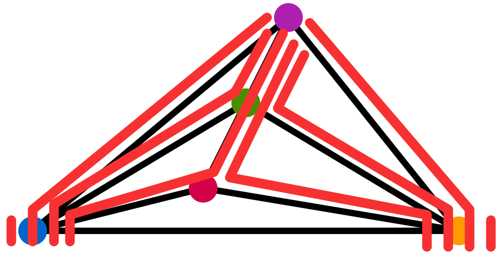
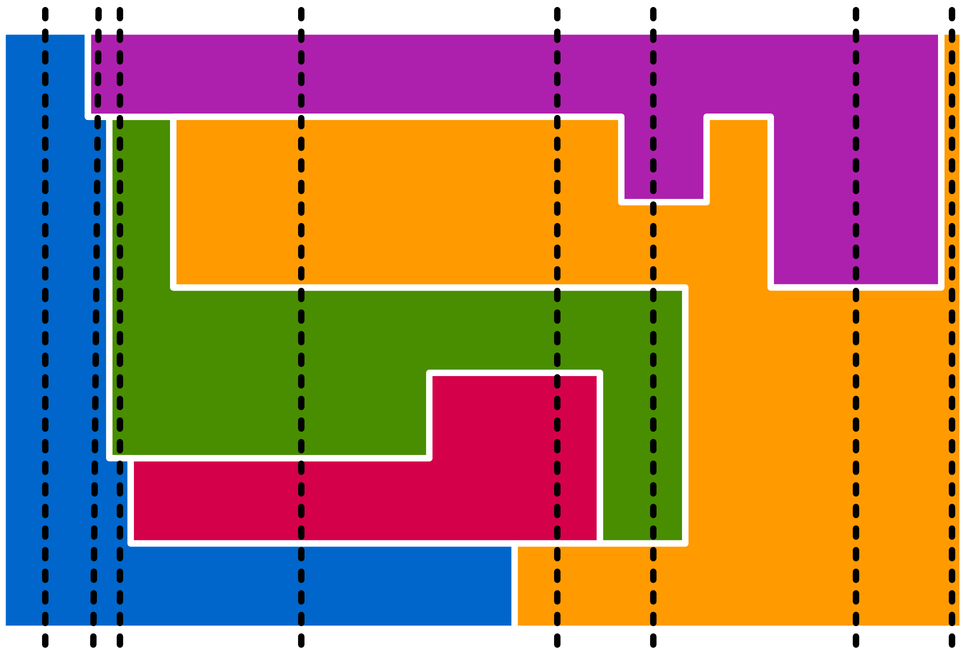
curve does not sweep backwards

Simple homotopy height ≤ simple grid-major height:

Extract sweep





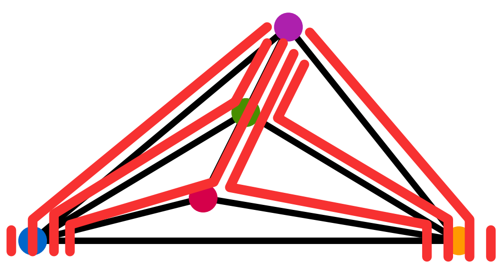
Simple grid-major height $=$ simple homotopy height

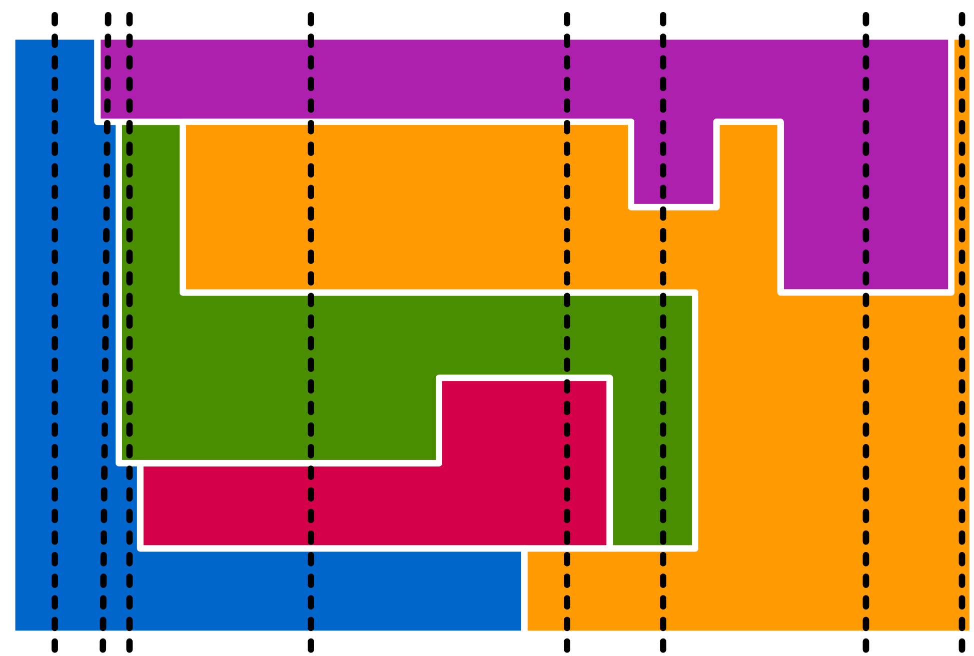
Sweep can be assumed monotone based on [CMO et al. 17]

curve does not sweep backwards

Simple homotopy height $\leq$ simple grid-major height:

Extract sweep





Similarly, grid-major height $=$ homotopy height

Overview of bounds

2 outerplanarity -1 and pathwidth

$\leq$

grid-major height

$=$

contact representation height $=$ homotopy height

$\leq$

simple grid-major height

$=$

simple contact representation height $=$ simple homotopy height

$\leq$

visibility representation height

$=$

straight-line drawing height

Overview of bounds

2 outerplanarity -1 and pathwidth

$\leq$

grid-major height

$=$

contact representation height $=$ homotopy height

$\leq$

simple grid-major height

$=$

simple contact representation height $=$ simple homotopy height

$\leq$

visibility representation height

$=$

straight-line drawing height

inequalities are strict

Overview of bounds

2 outerplanarity -1 and pathwidth

$\leq$

grid-major height

$=$

contact representation height $=$ homotopy height

$\leq$

simple grid-major height

$=$

simple contact representation height $=$ simple homotopy height

$\leq$

visibility representation height

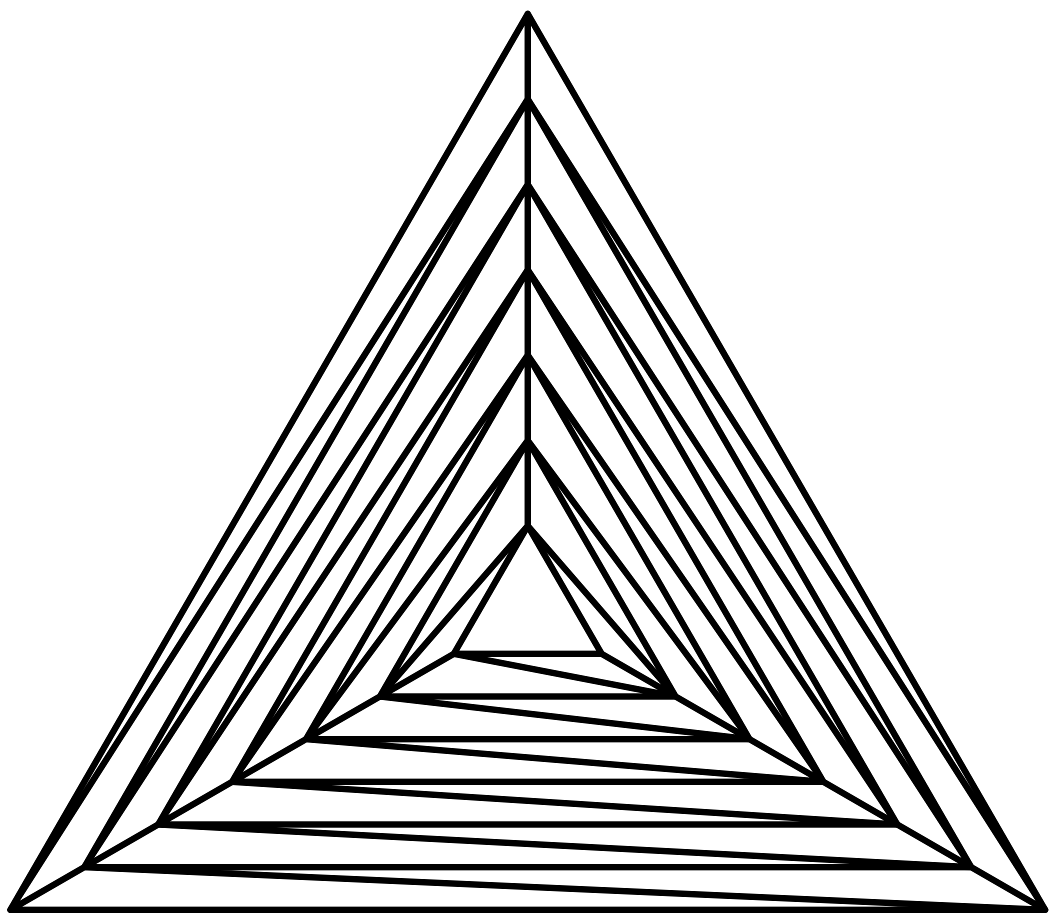
$=$

straight-line drawing height

inequalities are strict
gaps are nonconstant

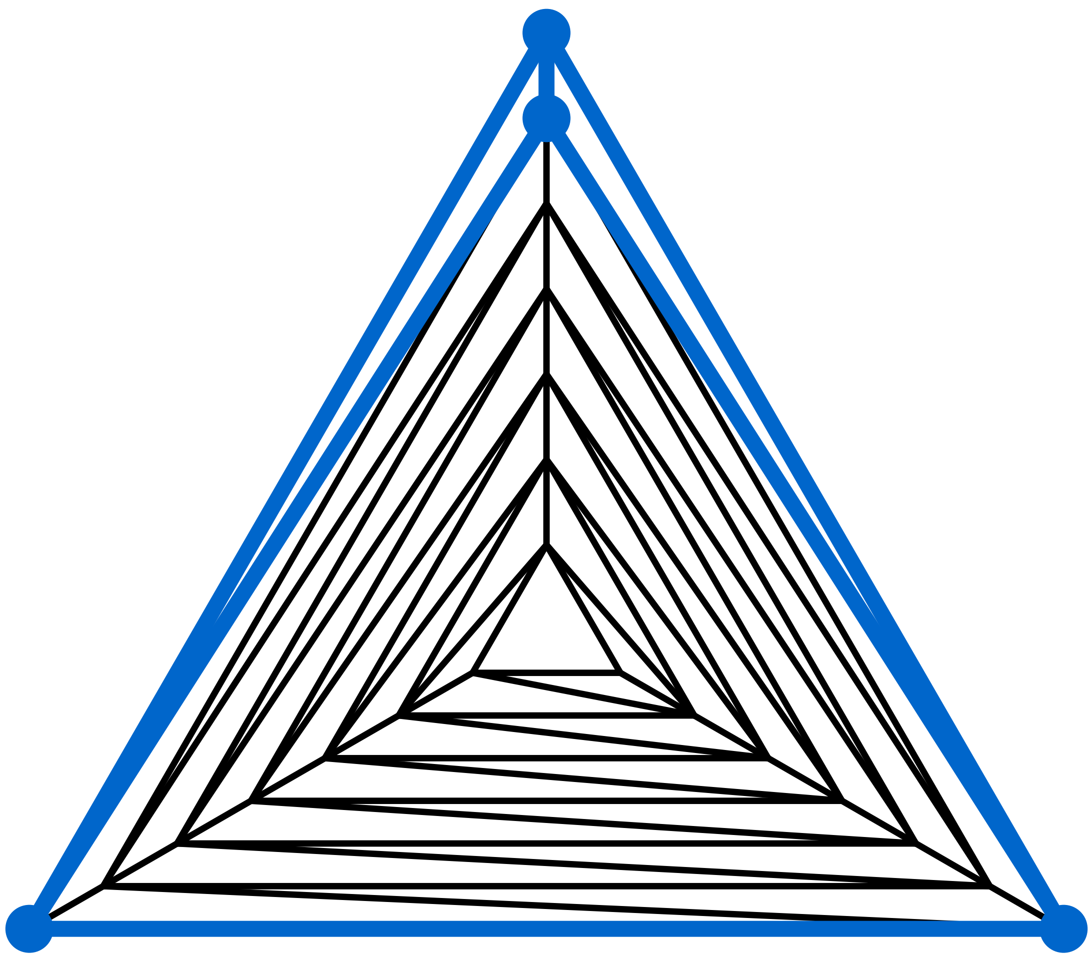
Pathwidth $\leq$ grid-major height

Pathwidth $= 3$



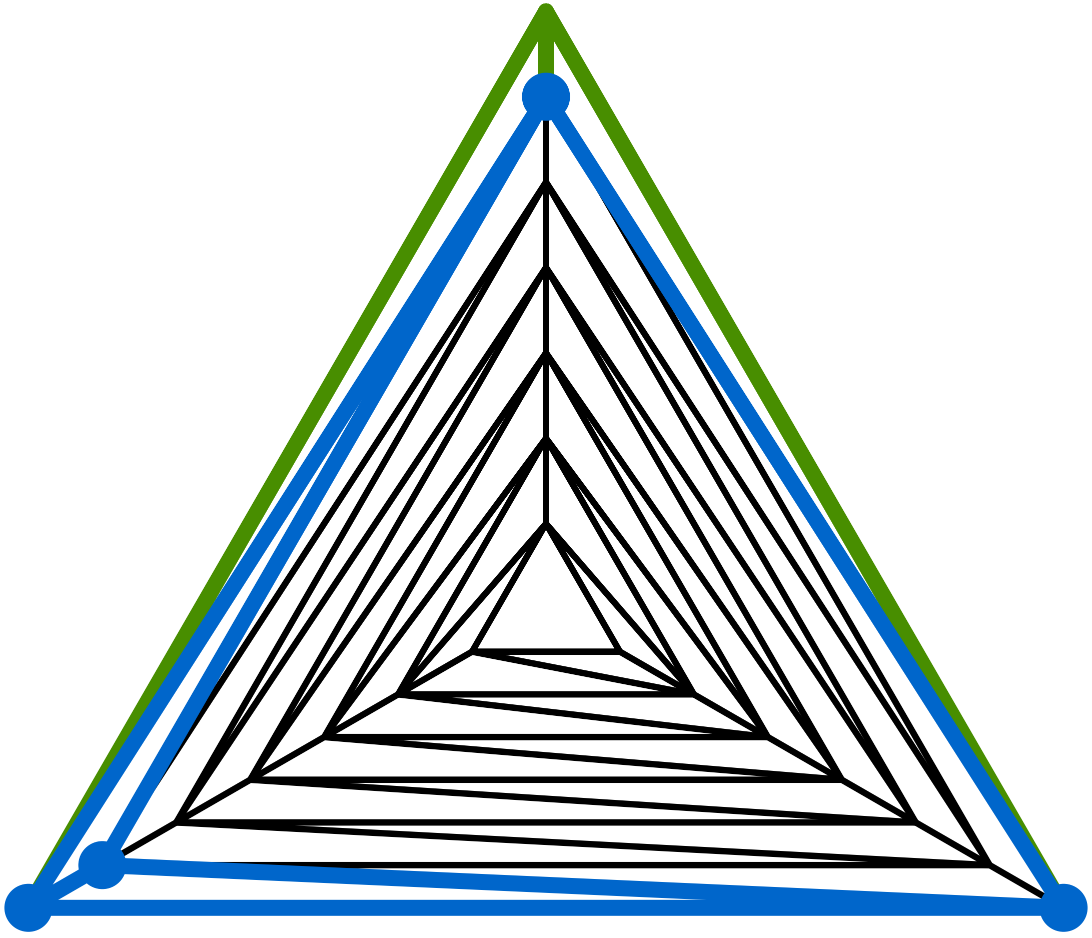
Pathwidth $\leq$ grid-major height

Pathwidth $= 3$



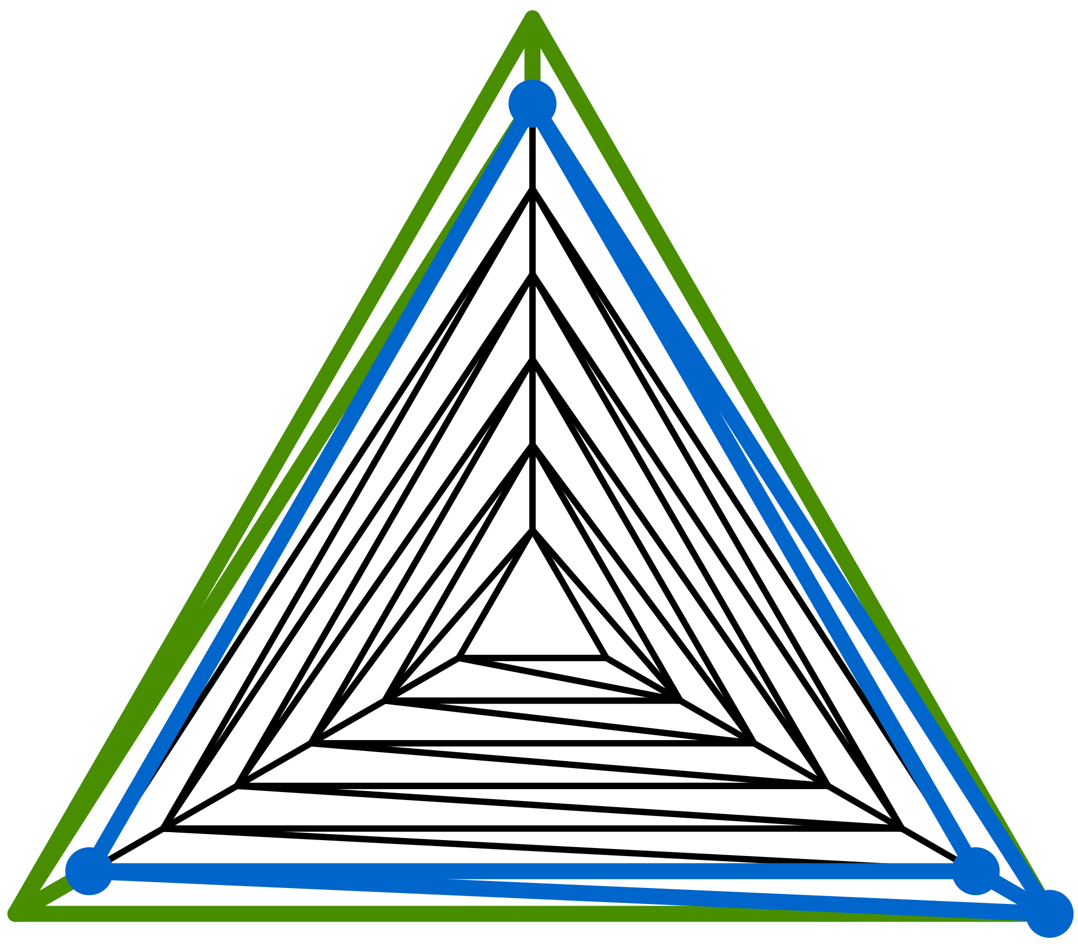
Pathwidth $\leq$ grid-major height

Pathwidth $= 3$



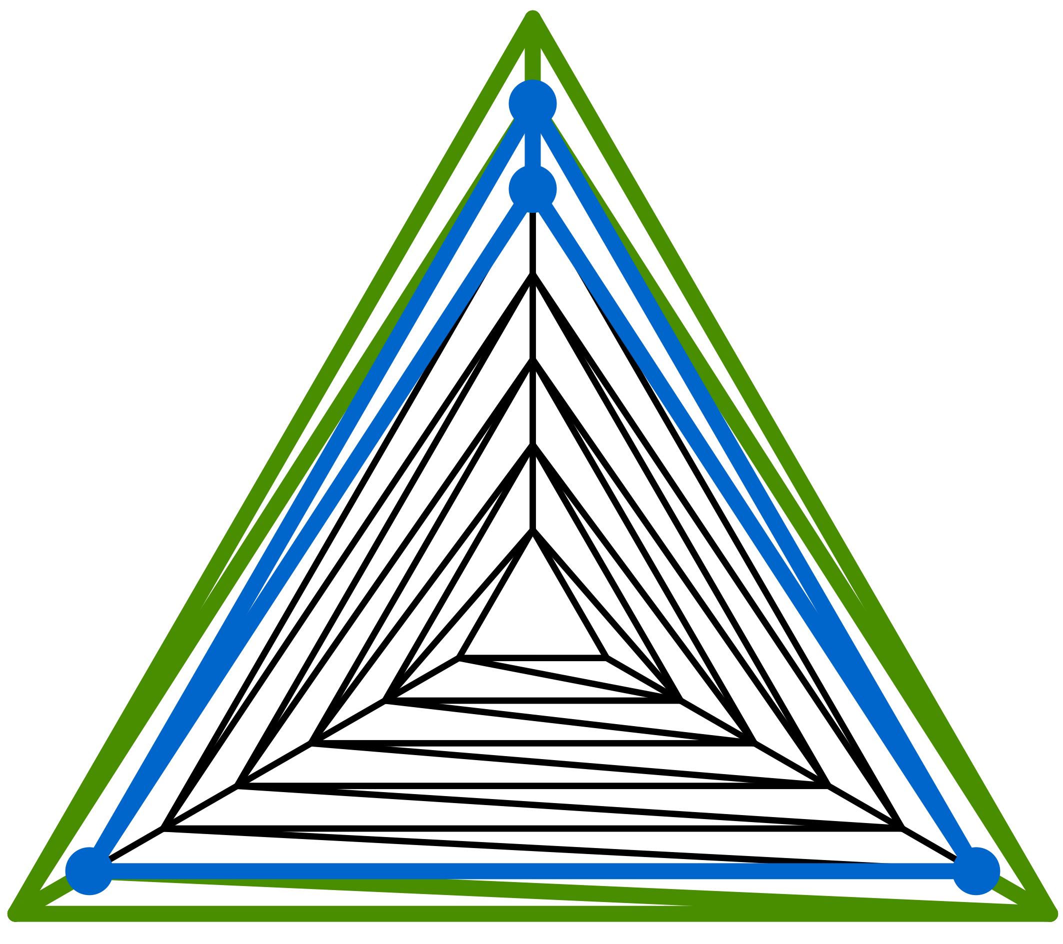
Pathwidth $\leq$ grid-major height

Pathwidth $= 3$



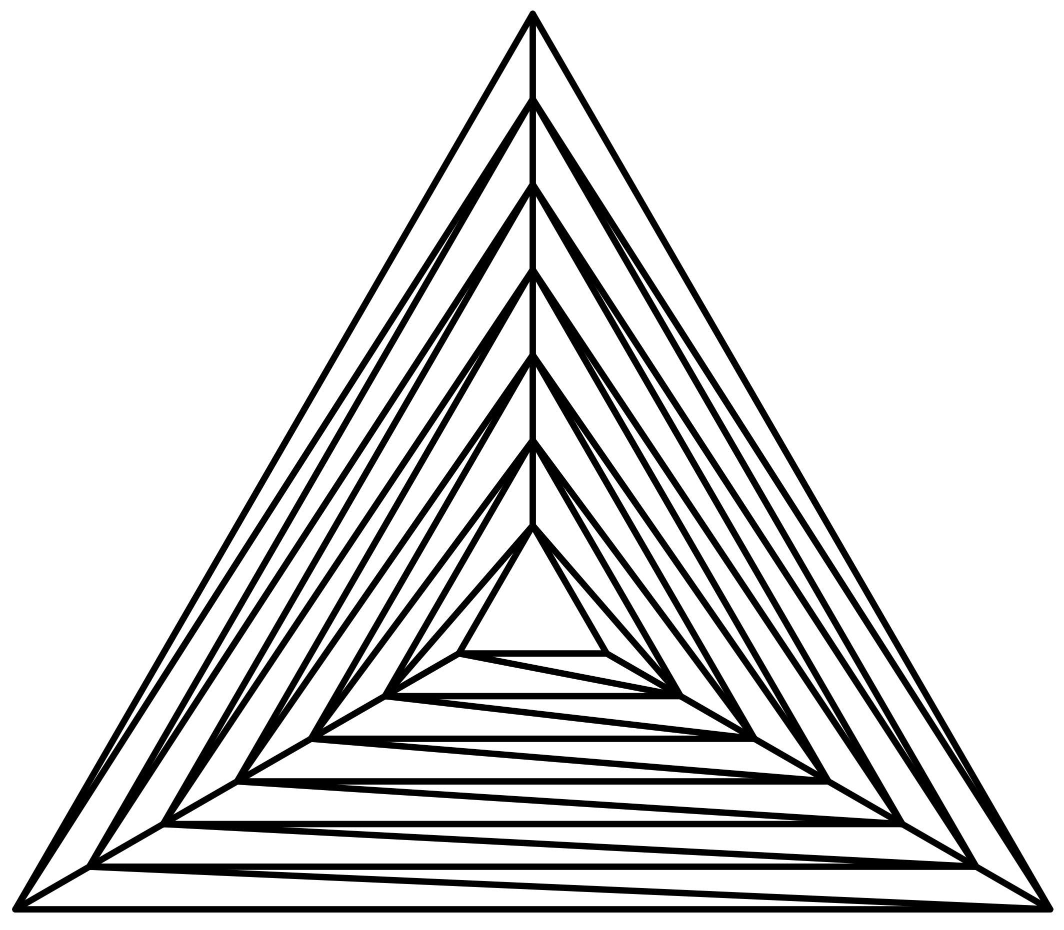
Pathwidth $\leq$ grid-major height

Pathwidth $= 3$



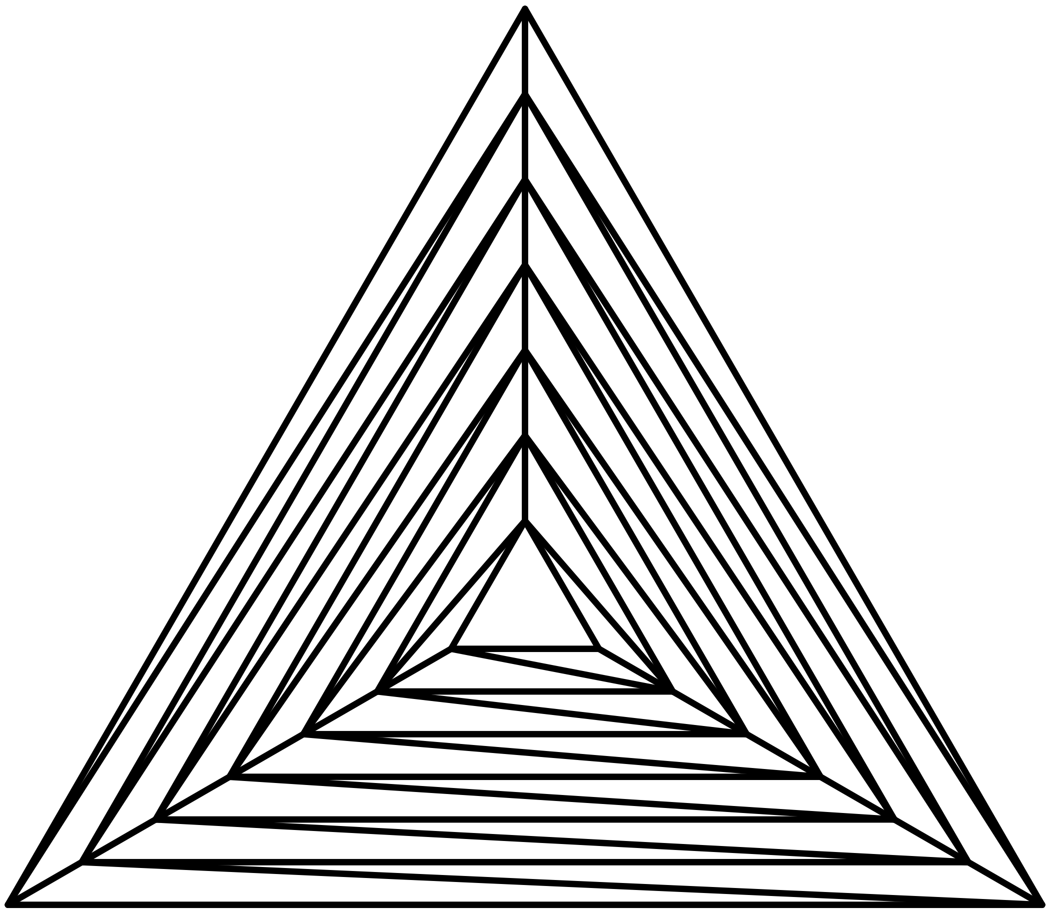
Pathwidth $\leq$ grid-major height

Pathwidth $= 3$



Pathwidth $\leq$ grid-major height

Pathwidth $= 3$
Grid-major height ≥ 2 outerplanarity $-1 \geq n/3 - 1$

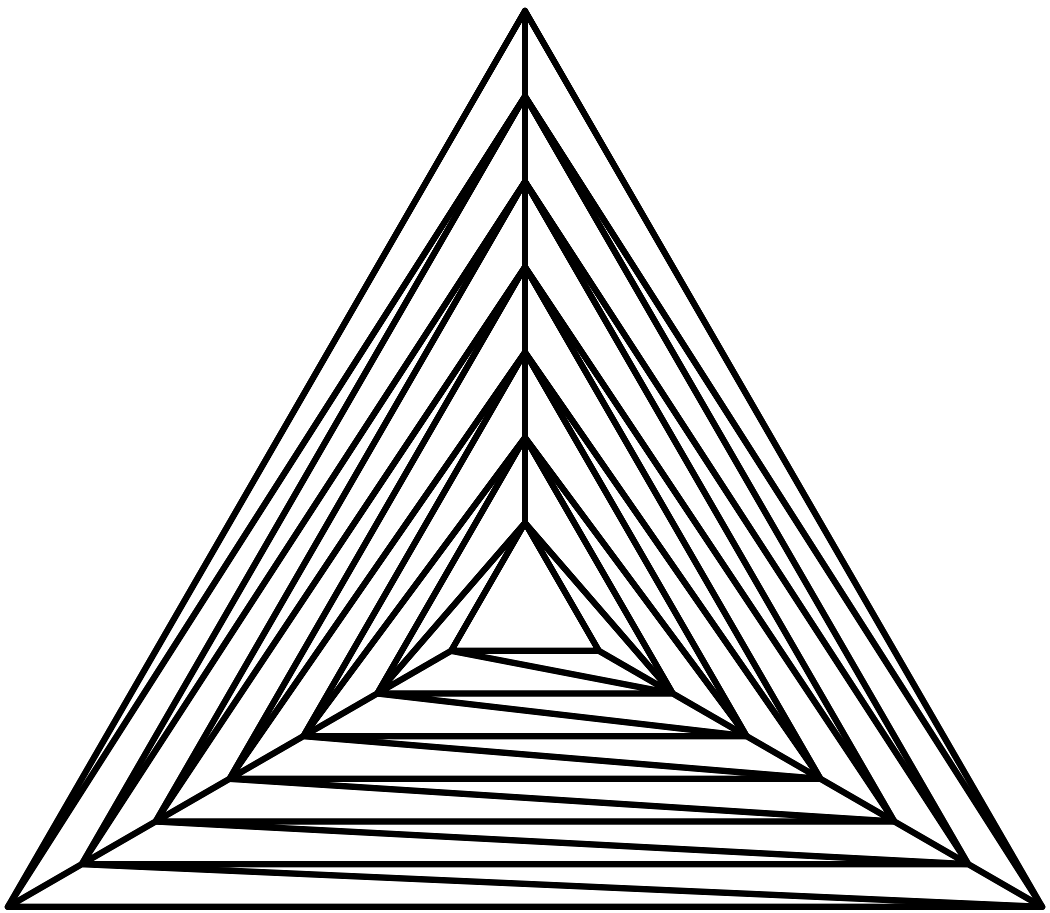


Pathwidth $\leq$ grid-major height

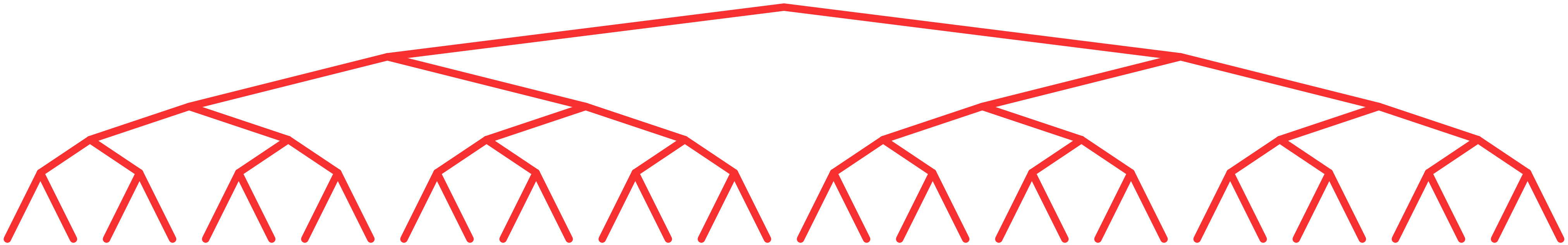
Pathwidth $= 3$

Grid-major height ≥ 2 outerplanarity $-1 \geq n/3 - 1$

$n/6$ triangles will be nested, no matter the outer face

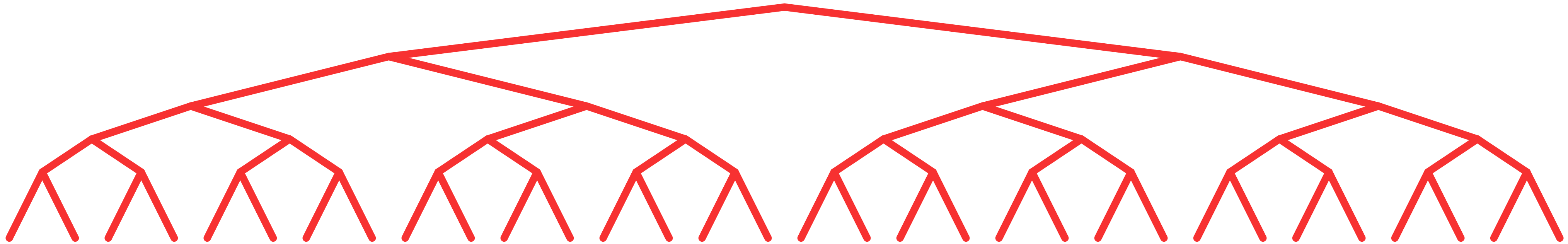


Outerplanarity $\leq$ grid-major height



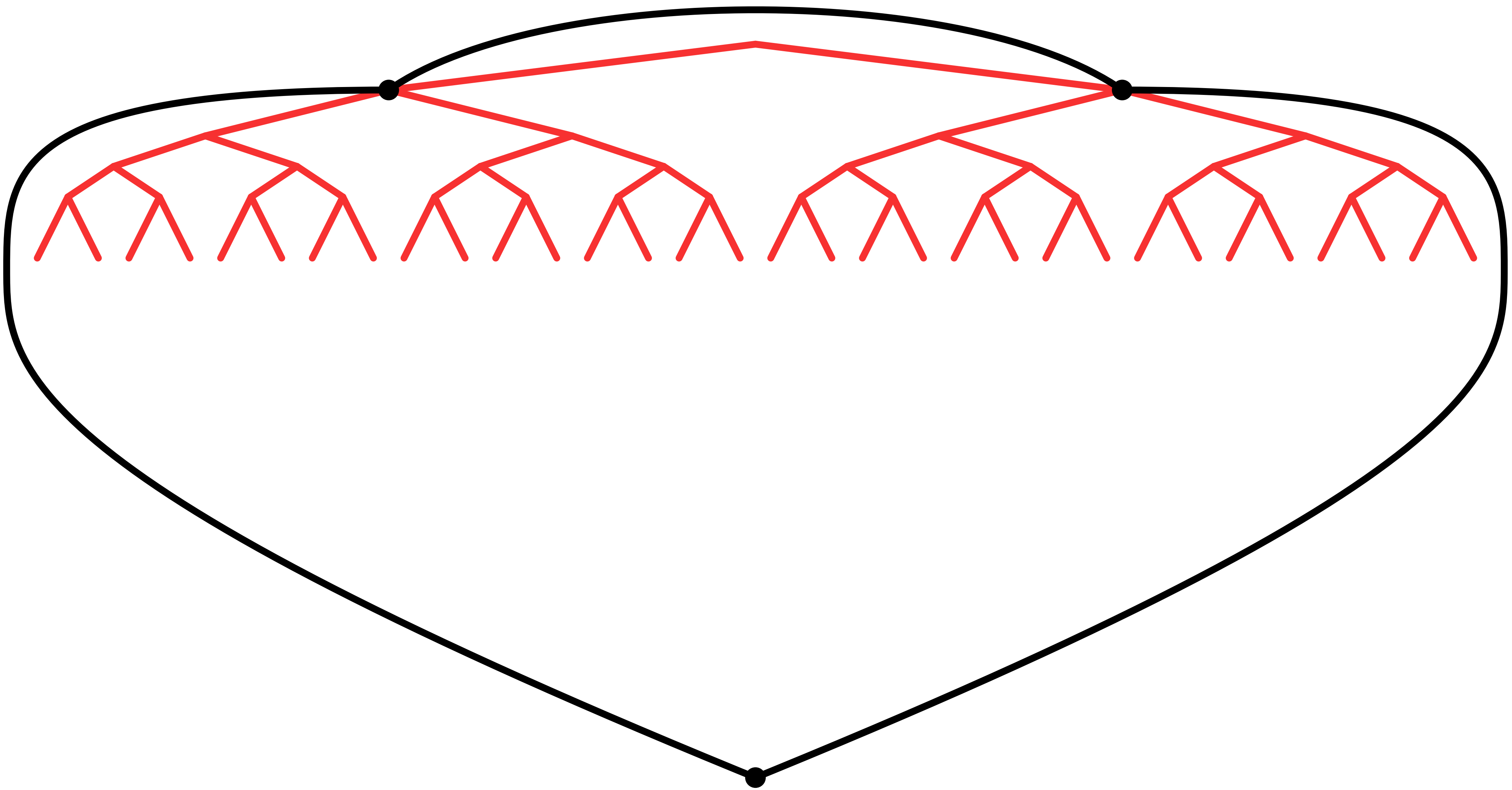
Outerplanarity $\leq$ grid-major height

Grid-major height $\geq$ pathwidth $= \Omega(\log n)$



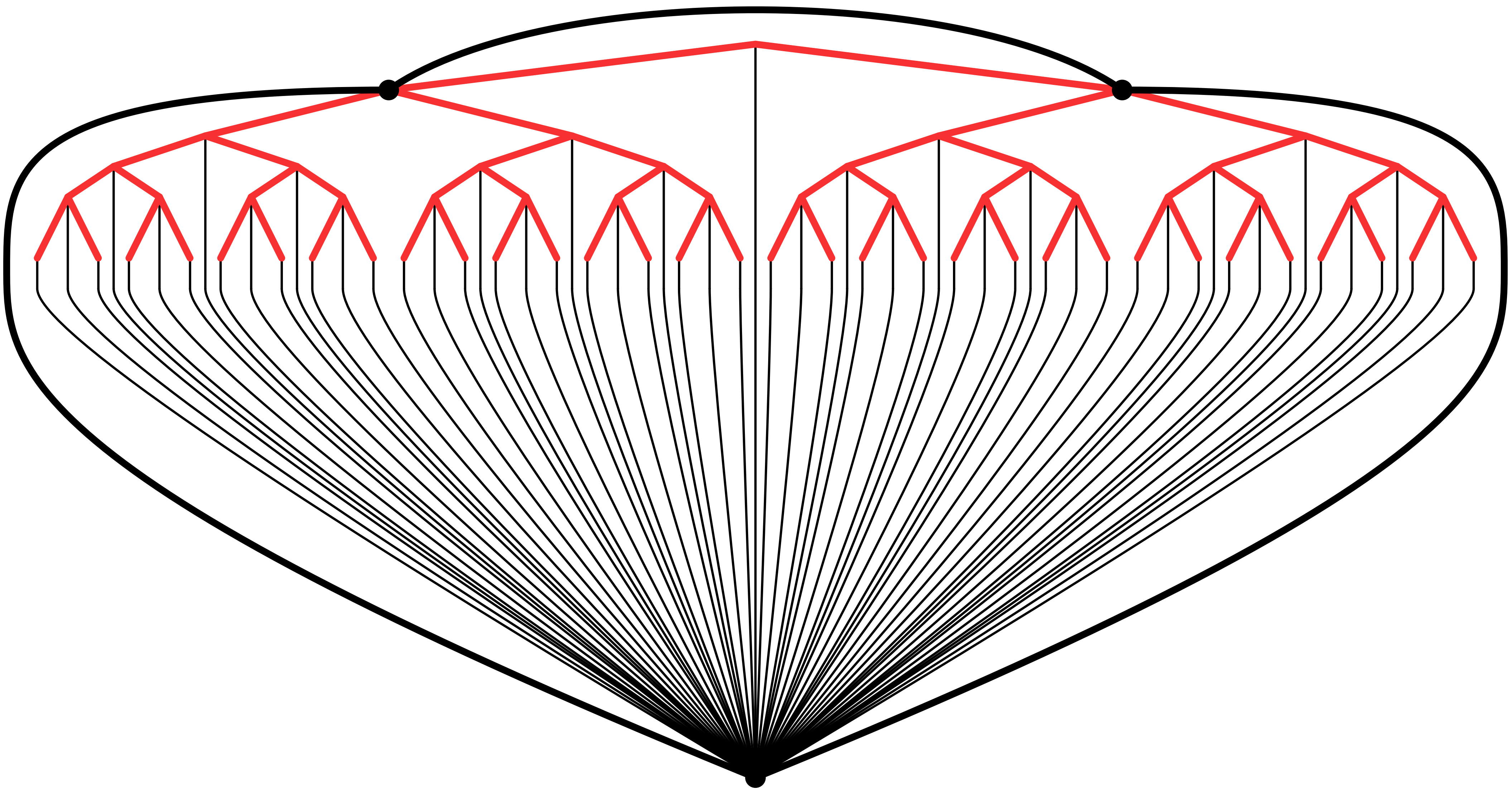
Outerplanarity $\leq$ grid-major height

Grid-major height $\geq$ pathwidth $= \Omega(\log n)$



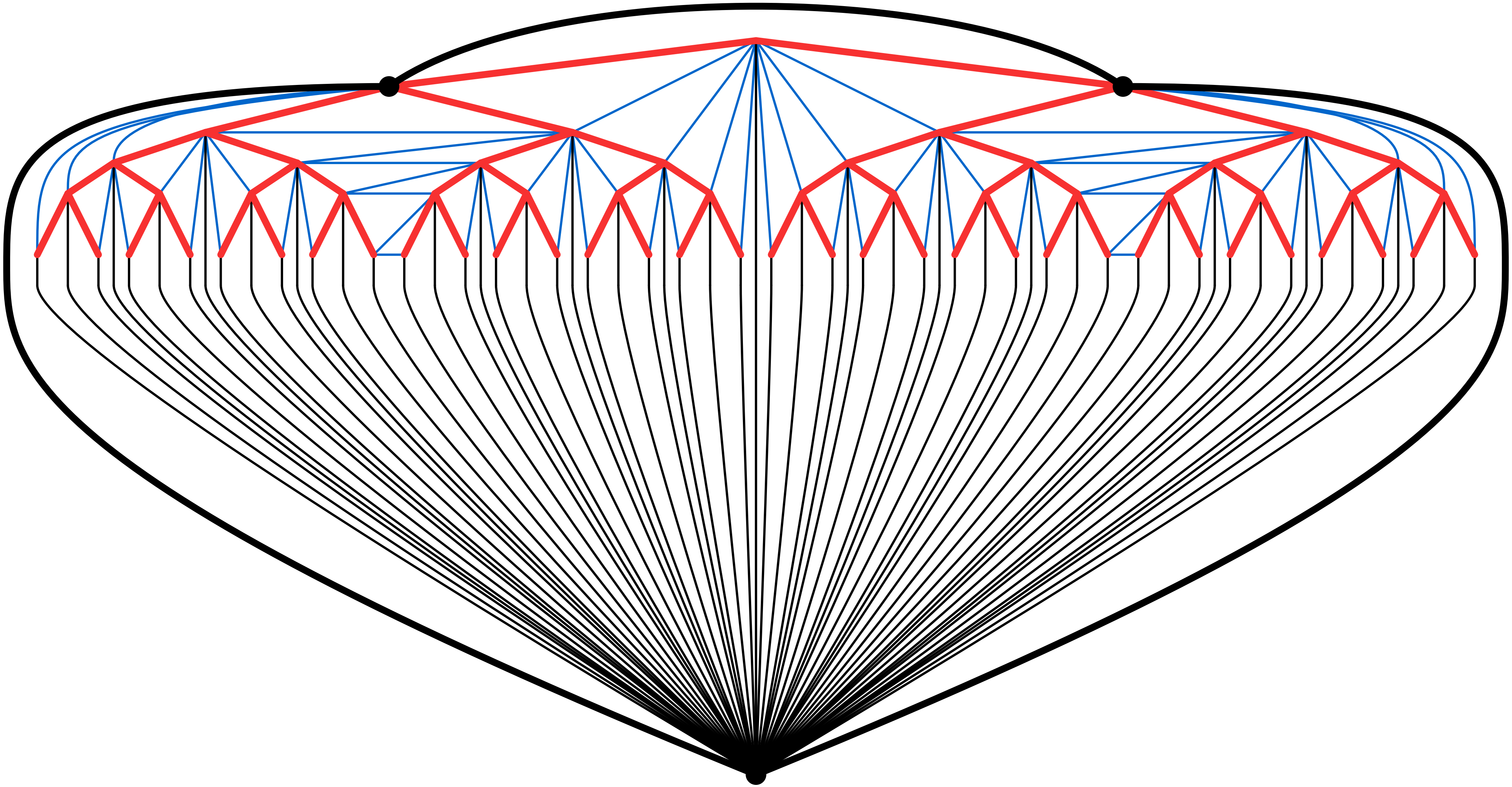
Outerplanarity $\leq$ grid-major height

Grid-major height $\geq$ pathwidth $= \Omega(\log n)$

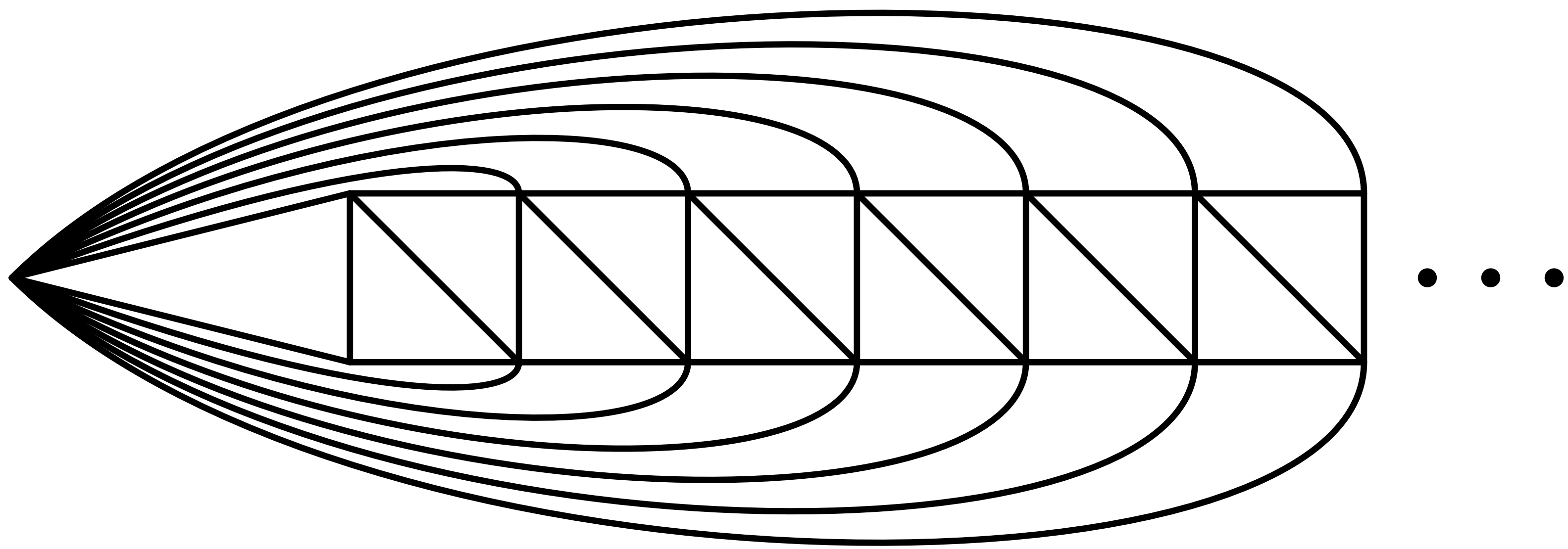


Outerplanarity $\leq$ grid-major height

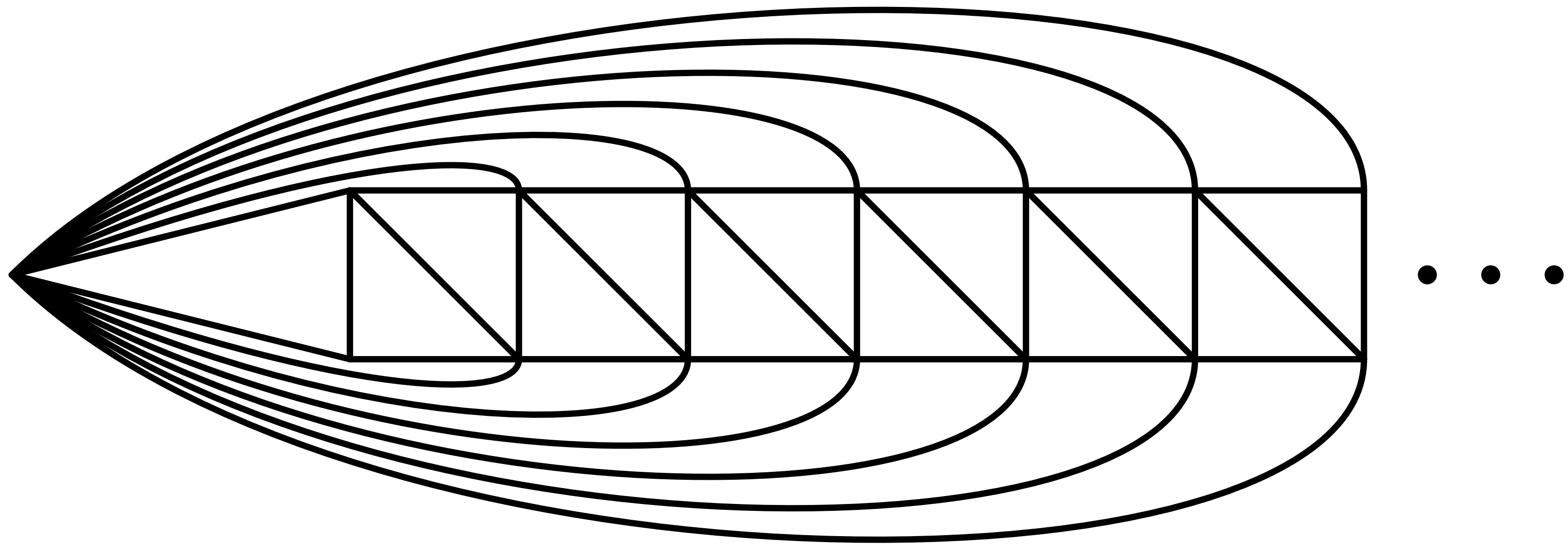
Grid-major height $\geq$ pathwidth $= \Omega(\log n)$
Outerplanarity $= 2$



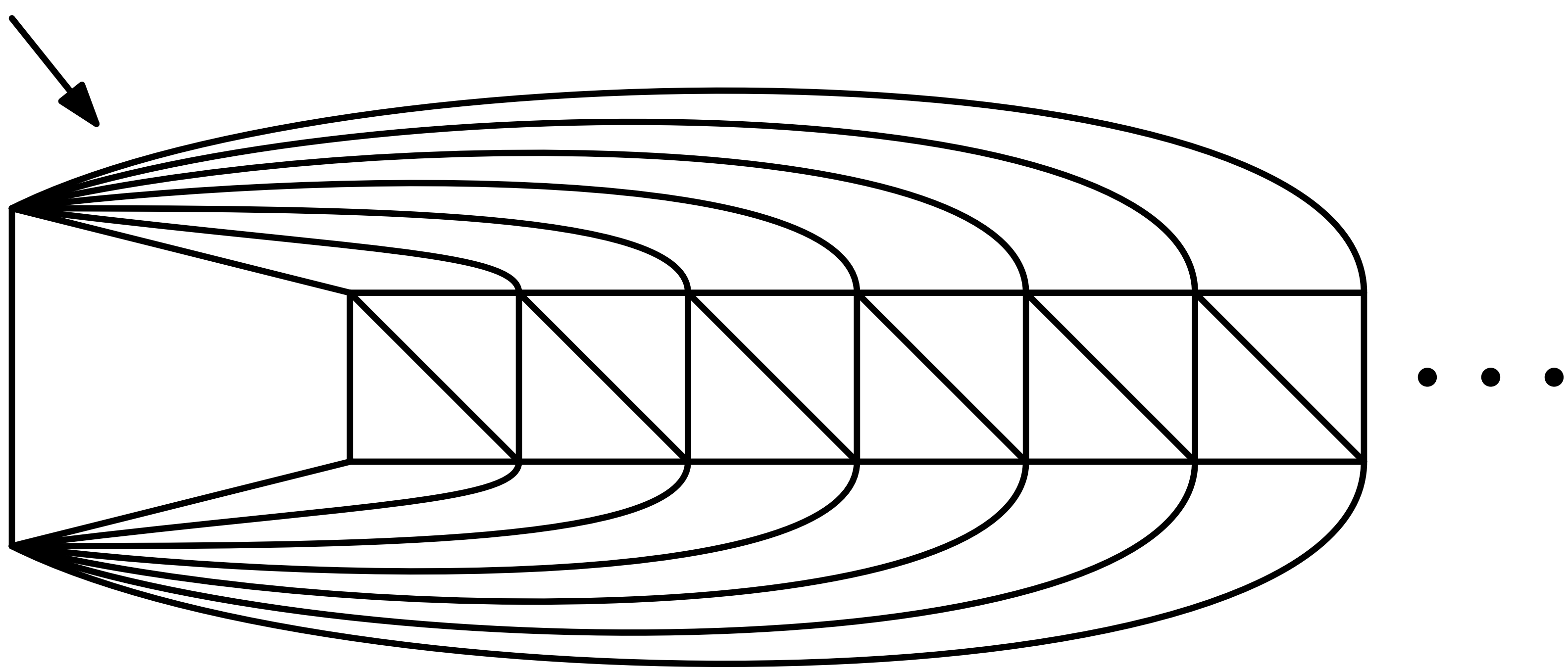
Nonsimple $\leq$ simple grid-major height



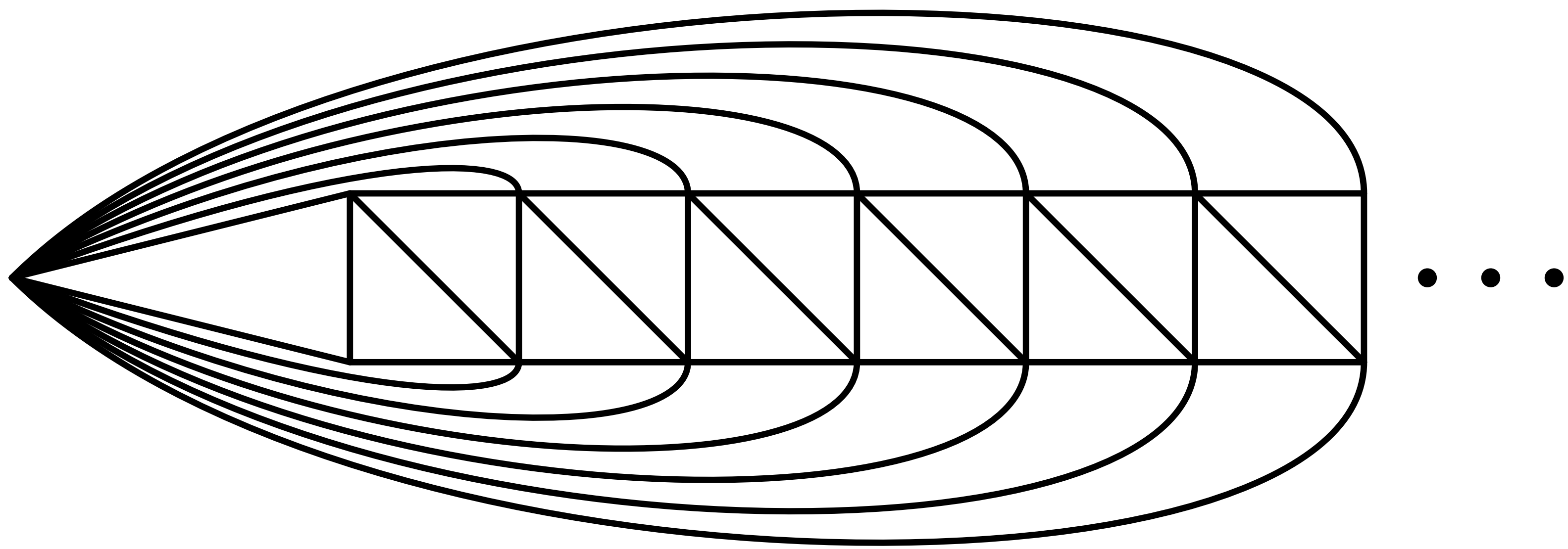
Nonsimple $\leq$ simple grid-major height



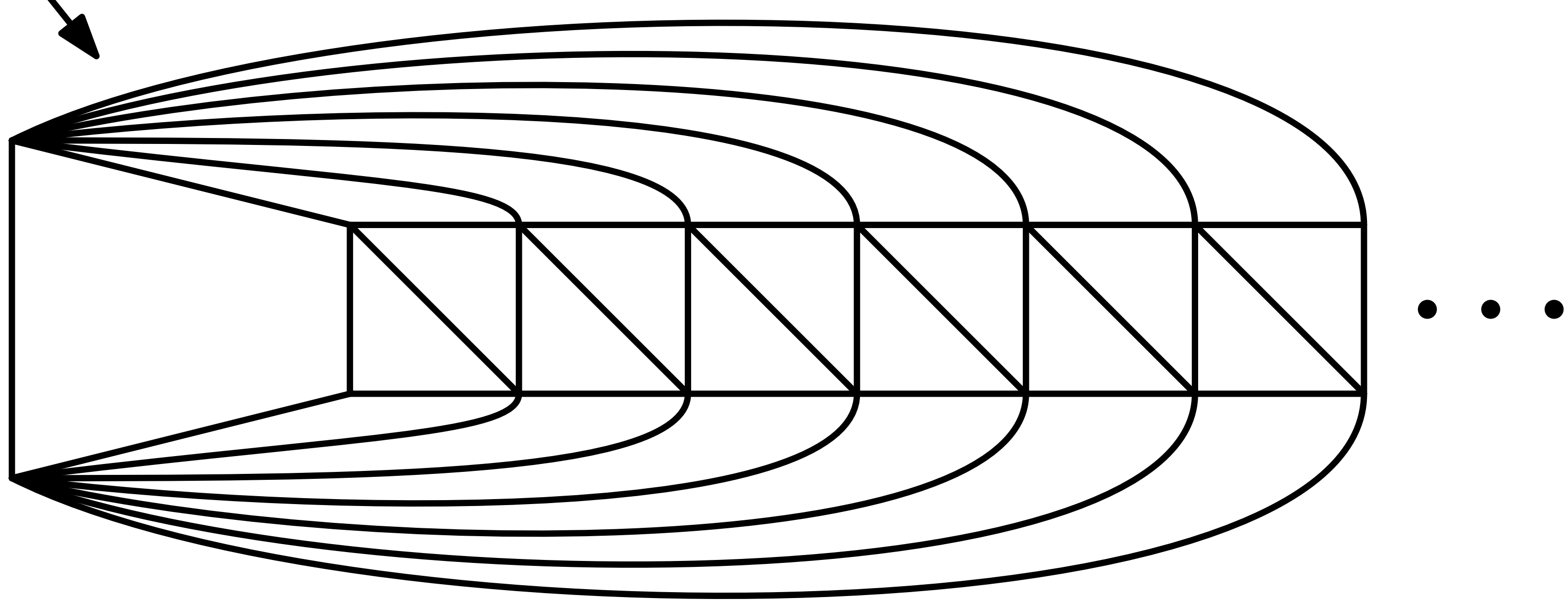
Minor of



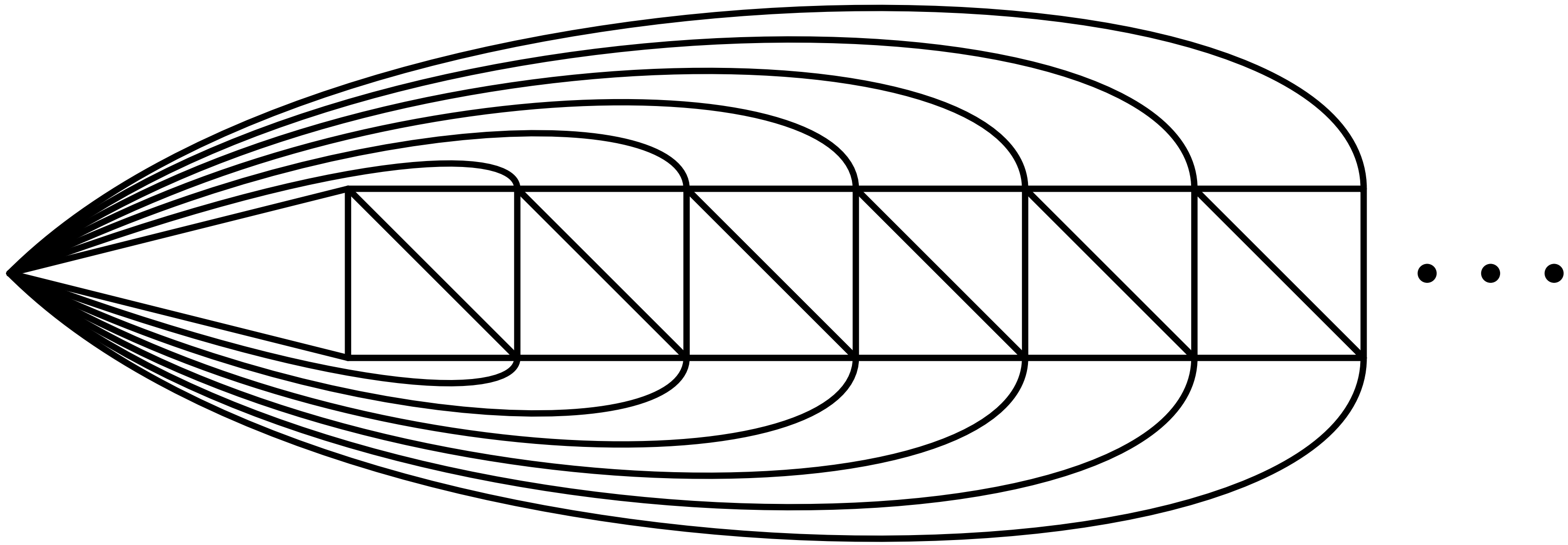
Nonsimple $\leq$ simple grid-major height



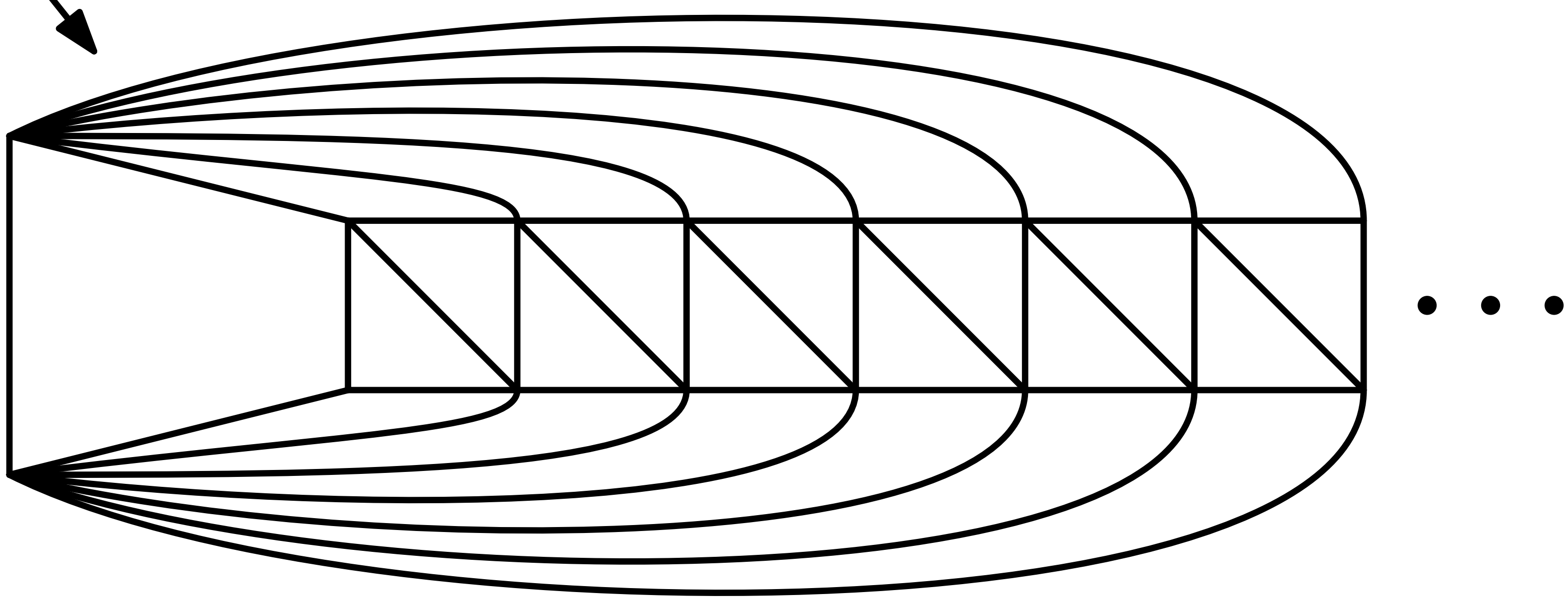
Minor of and hence of Wx4 grid



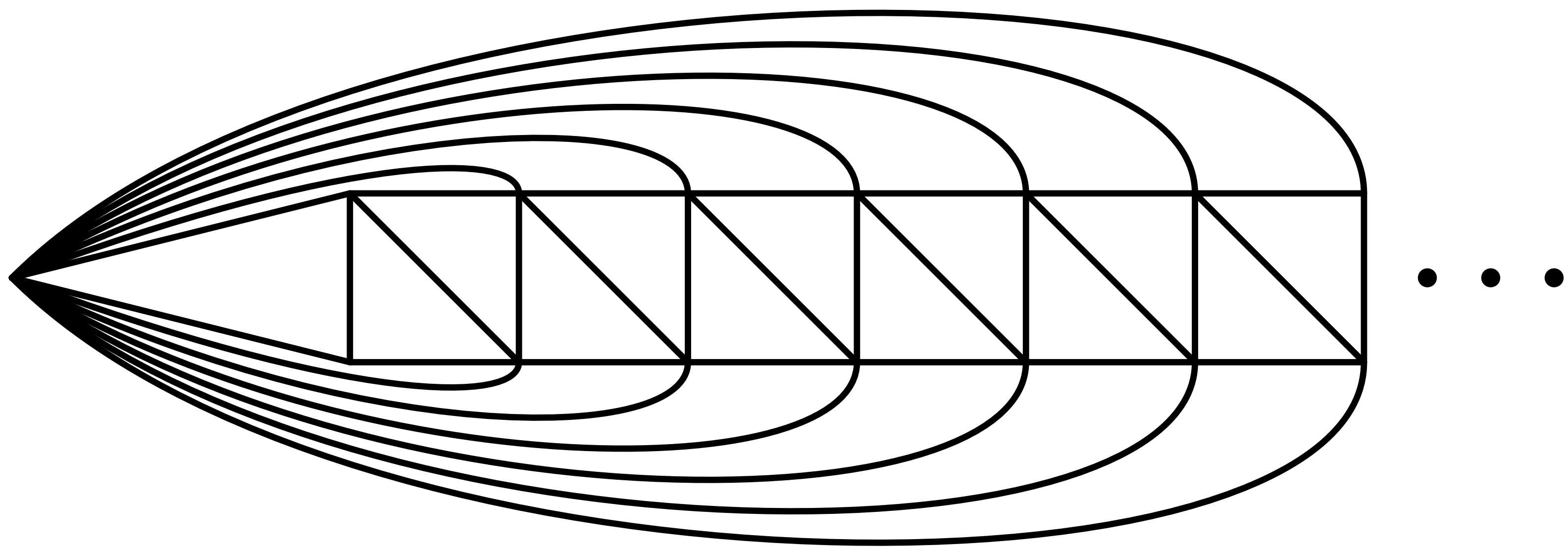
Nonsimple $\leq$ simple grid-major height



Minor of and hence of Wx4 grid $\Rightarrow$ grid-major height ≤ 4



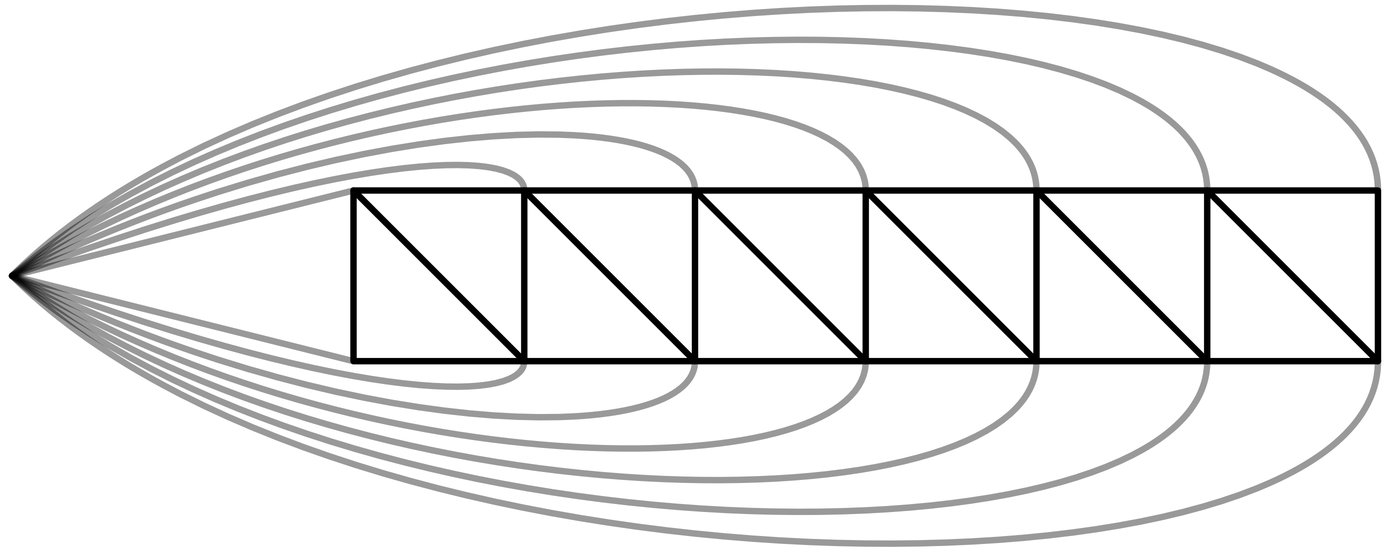
Nonsimple $\leq$ simple grid-major height



Grid-major height ≤ 4

Simple grid-major height $= \Omega(n)$:

Nonsimple $\leq$ simple grid-major height

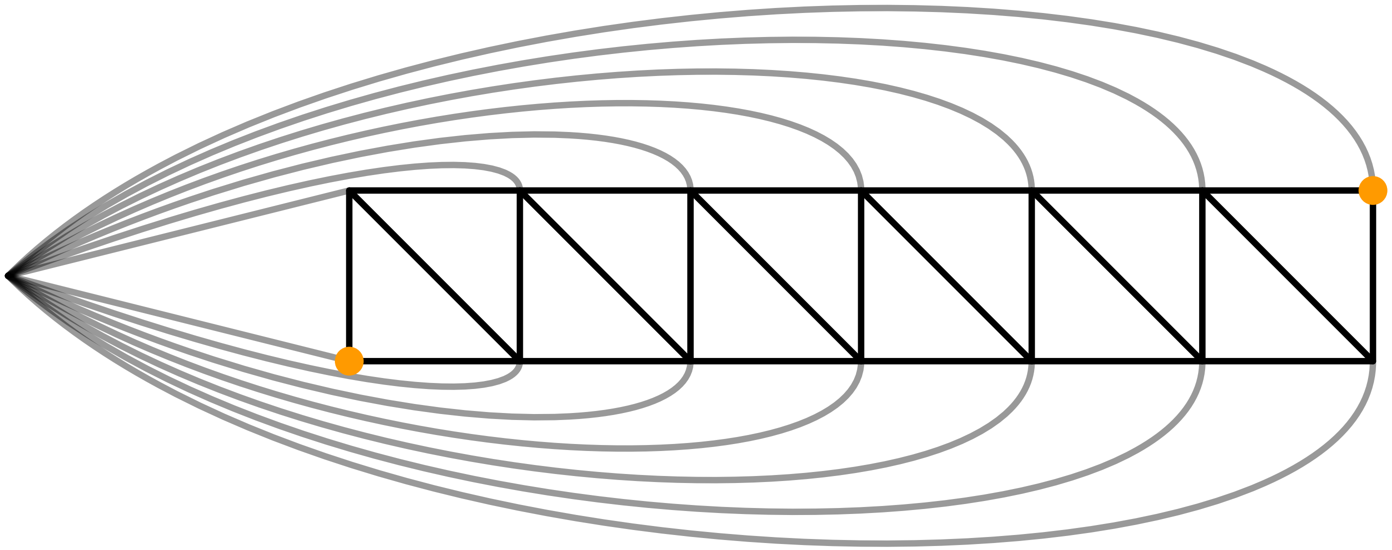


Grid-major height ≤ 4

Simple grid-major height $= \Omega(n)$:

  Diameter of subgraph is $\Omega(n)$

Nonsimple $\leq$ simple grid-major height



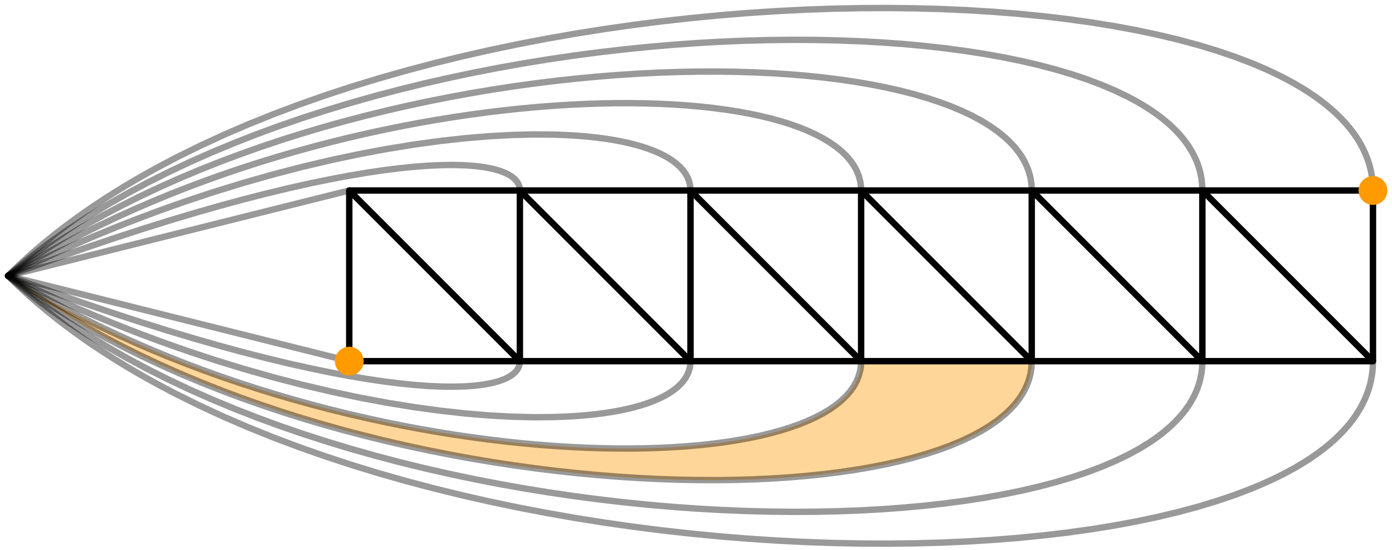
Grid-major height ≤ 4

Simple grid-major height $= \Omega(n)$:

Diameter of subgraph is $\Omega(n)$

Some vertex in subgraph is far from ‘outer face’

Nonsimple $\leq$ simple grid-major height

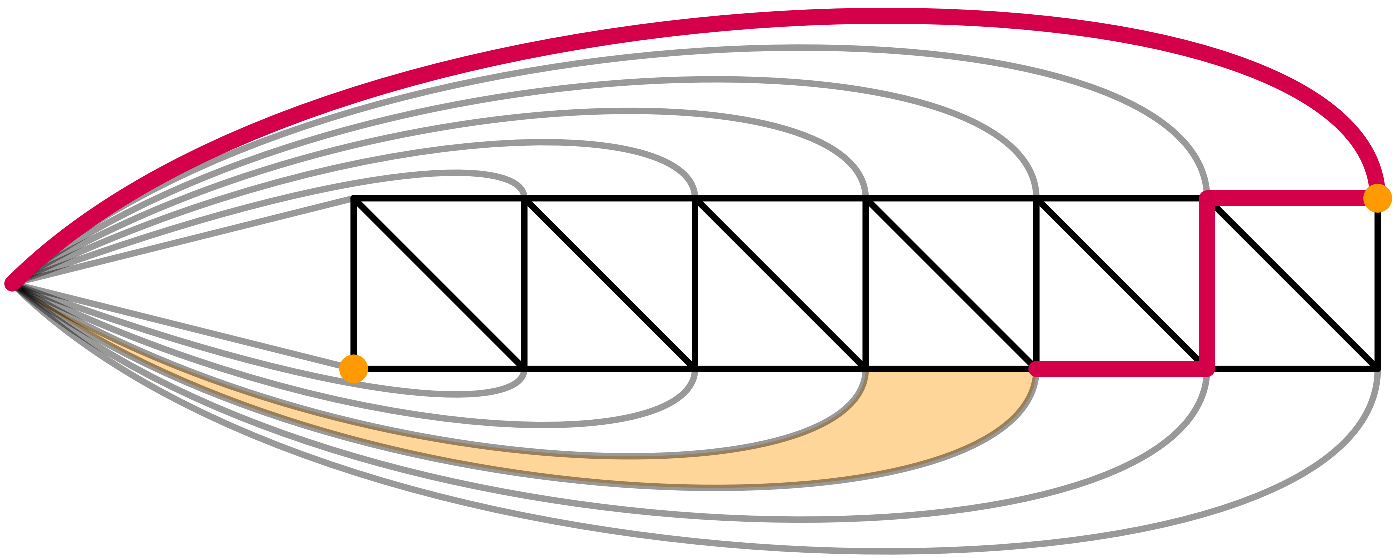


Grid-major height ≤ 4

Simple grid-major height $= \Omega(n)$:

- Diameter of subgraph is $\Omega(n)$
- Some vertex in subgraph is far from 'outer face'

Nonsimple $\leq$ simple grid-major height

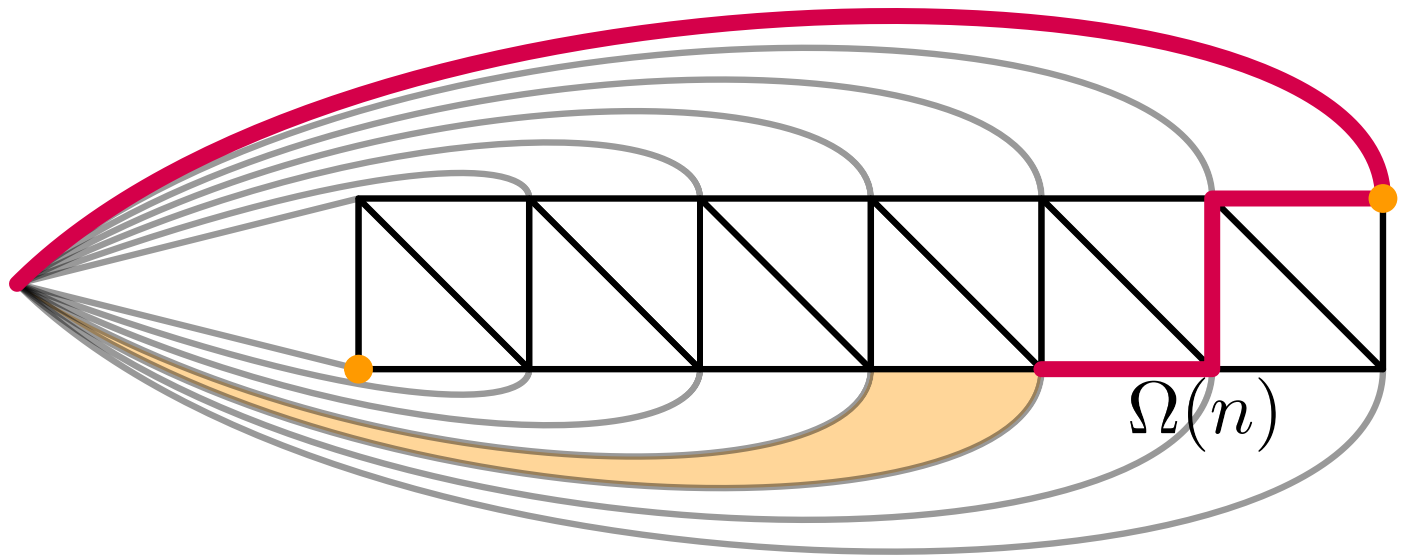


Grid-major height ≤ 4

Simple grid-major height $= \Omega(n)$:

- Diameter of subgraph is $\Omega(n)$
- Some vertex in subgraph is far from ‘outer face’
- That vertex splits some path in sweep in two pieces

Nonsimple $\leq$ simple grid-major height



Grid-major height ≤ 4

Simple grid-major height $= \Omega(n)$:

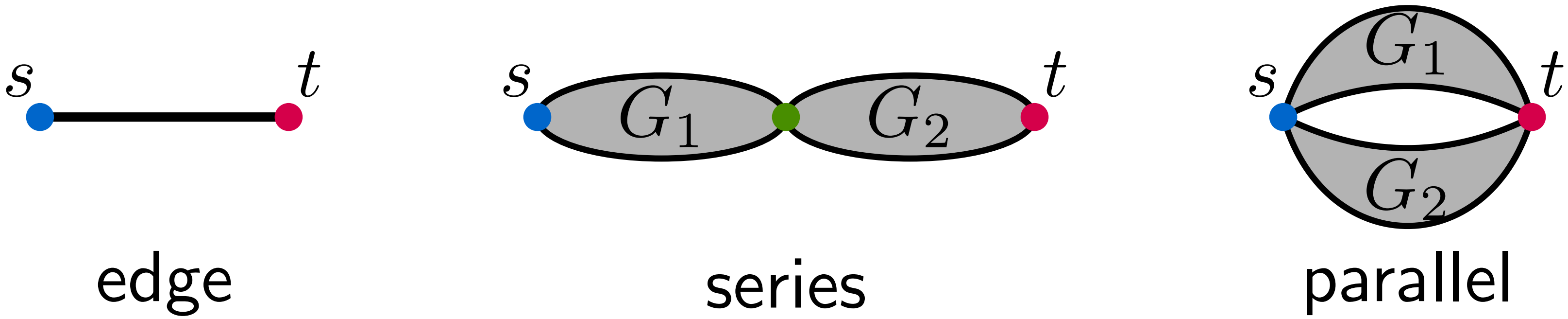
- Diameter of subgraph is $\Omega(n)$
- Some vertex in subgraph is far from 'outer face'
- That vertex splits some path in sweep in two pieces
- At least one piece lies in subgraph, and is therefore long

Simple grid-major height $\leq$ graph-drawing height

For series-parallel graphs, simple grid-major height is $O(\log n)$

Simple grid-major height $\leq$ graph-drawing height

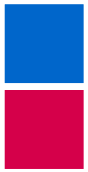
For series-parallel graphs, simple grid-major height is $O(\log n)$



Simple grid-major height $\leq$ graph-drawing height

For series-parallel graphs, simple grid-major height is $O(\log n)$

Contact-representation with source/target in top/bottom-right

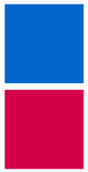


edge

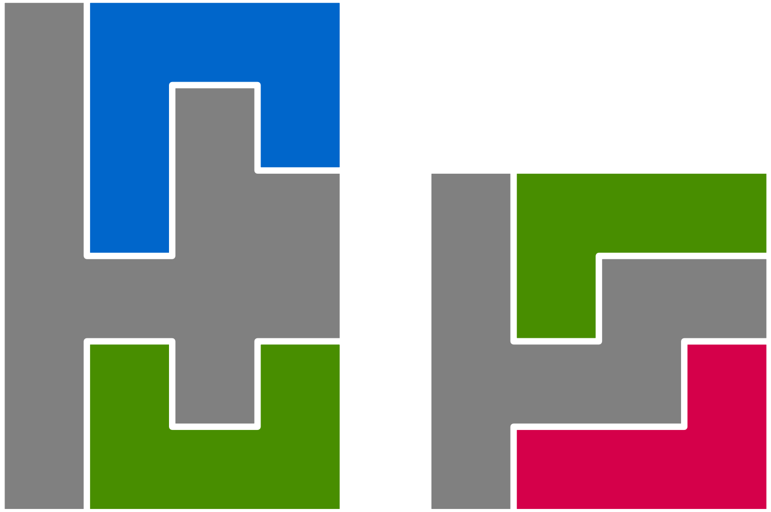
Simple grid-major height $\leq$ graph-drawing height

For series-parallel graphs, simple grid-major height is $O(\log n)$

Contact-representation with source/target in top/bottom-right



edge

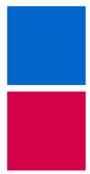


series

Simple grid-major height $\leq$ graph-drawing height

For series-parallel graphs, simple grid-major height is $O(\log n)$

Contact-representation with source/target in top/bottom-right



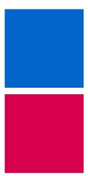
edge

series

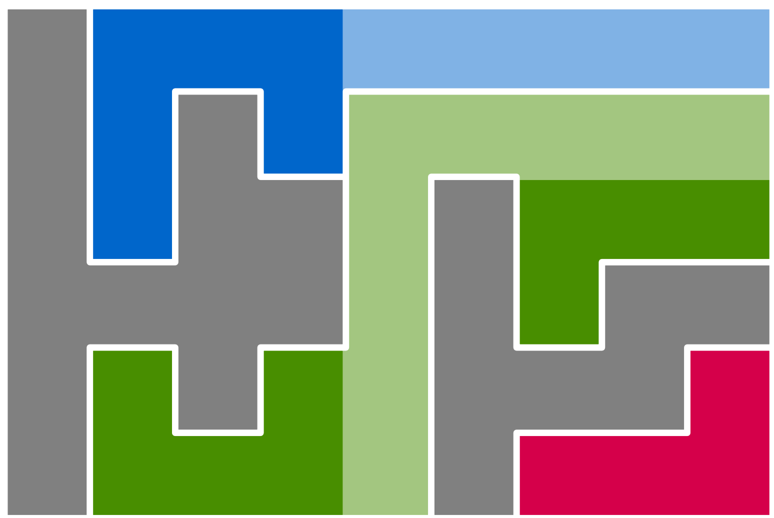
Simple grid-major height $\leq$ graph-drawing height

For series-parallel graphs, simple grid-major height is $O(\log n)$

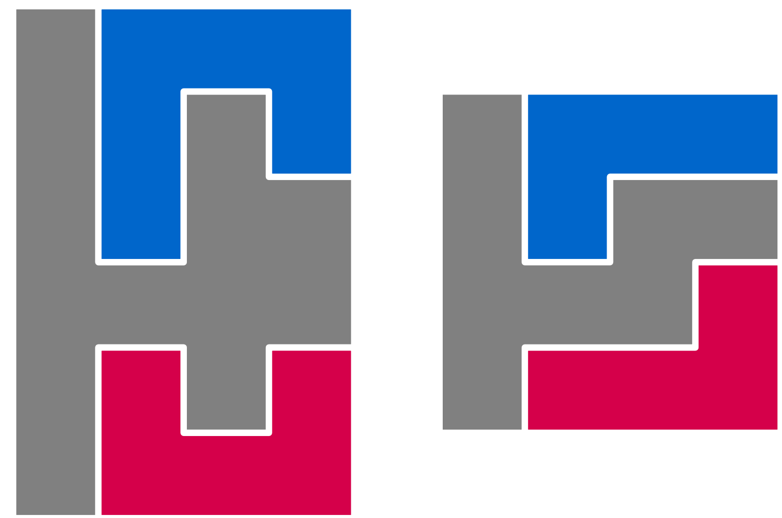
Contact-representation with source/target in top/bottom-right



edge



series

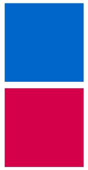


parallel

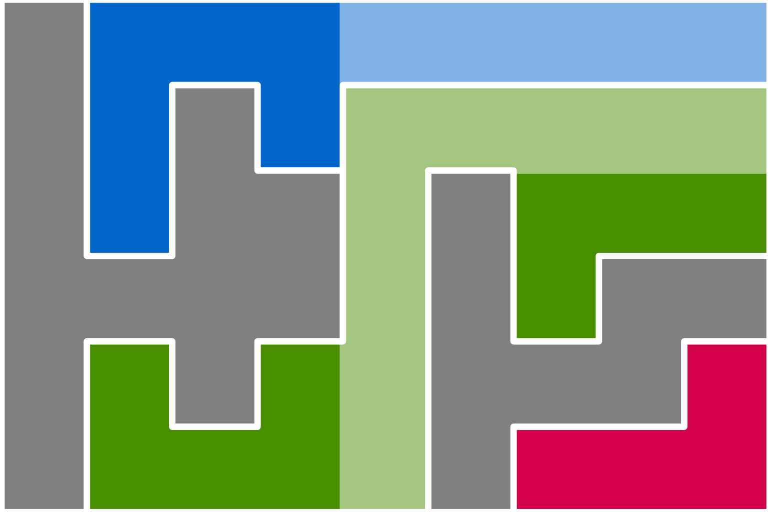
Simple grid-major height $\leq$ graph-drawing height

For series-parallel graphs, simple grid-major height is $O(\log n)$

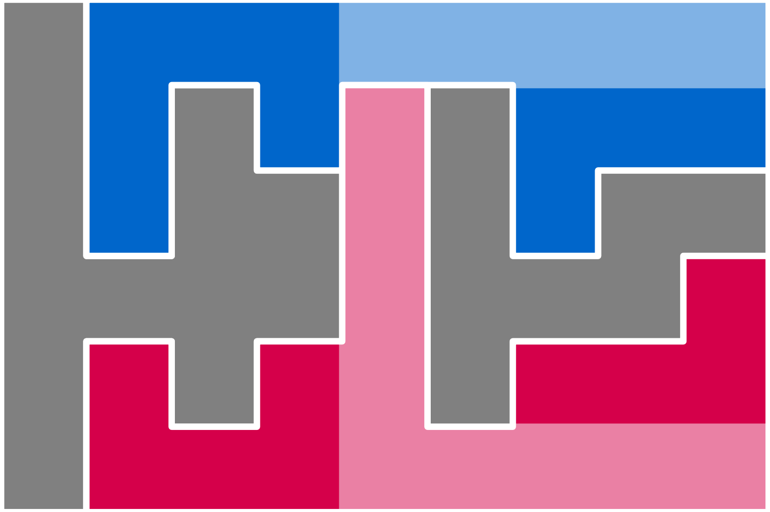
Contact-representation with source/target in top/bottom-right



edge



series



parallel

Simple grid-major height $\leq$ graph-drawing height

For series-parallel graphs, simple grid-major height is $O(\log n)$

Contact-representation with source/target in top/bottom-right

Height increases (by 2) only if combined grids are similar height

$\Rightarrow$ grid-major height $= O(\log n)$

Simple grid-major height $\leq$ graph-drawing height

For series-parallel graphs, simple grid-major height is $O(\log n)$

There exist series-parallel graphs with
graph-drawing height $= \Omega\big(2^{\sqrt{\log n}}\big)$ [Frati10]

Simple grid-major height $\leq$ graph-drawing height

For series-parallel graphs, simple grid-major height is $O(\log n)$

There exist series-parallel graphs with
graph-drawing height $= \Omega\left(2^{\sqrt{\log n}}\right)$ [Frati10]

Triangulating them cannot decrease height

Overview of results

2 outerplanarity -1 and pathwidth

$\leq$

grid-major height

$=$

contact representation height $=$ homotopy height

$\leq$

simple grid-major height

$=$

simple contact representation height $=$ simple homotopy height

$\leq$

visibility representation height

$=$

straight-line drawing height

inequalities are strict

gaps are nonconstant

Overview of results

Can we efficiently compute these parameters?

2 outerplanarity -1 and pathwidth (they are FPT in height)

$\leq$

grid-major height

$=$

contact representation height $=$ homotopy height

$\leq$

simple grid-major height

$=$

simple contact representation height $=$ simple homotopy height

$\leq$

visibility representation height

$=$

straight-line drawing height

inequalities are strict

gaps are nonconstant