Mixed Linear Layouts: Complexity, Heuristics, and Experiments

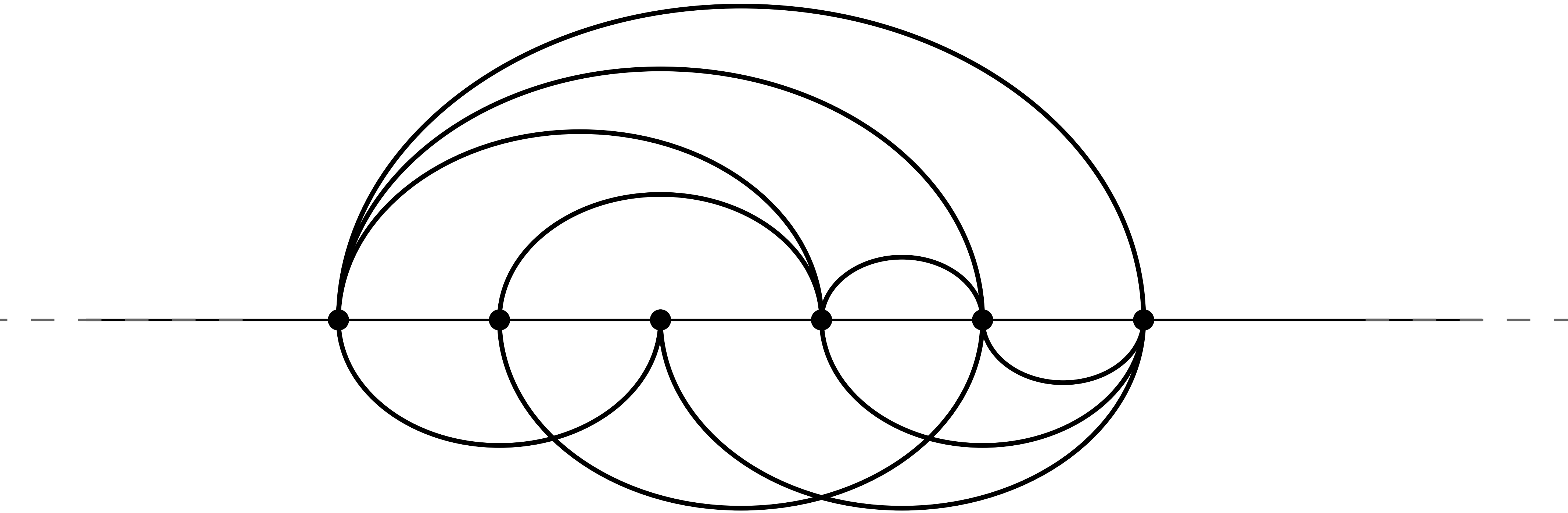
Philipp de Col, Fabian Klute, and Martin Nöllenburg

Graph Drawing 2019 · September 19, 2019

ALGORITHMS AND COMPLEXITY GROUP

Mixed Linear Layouts I

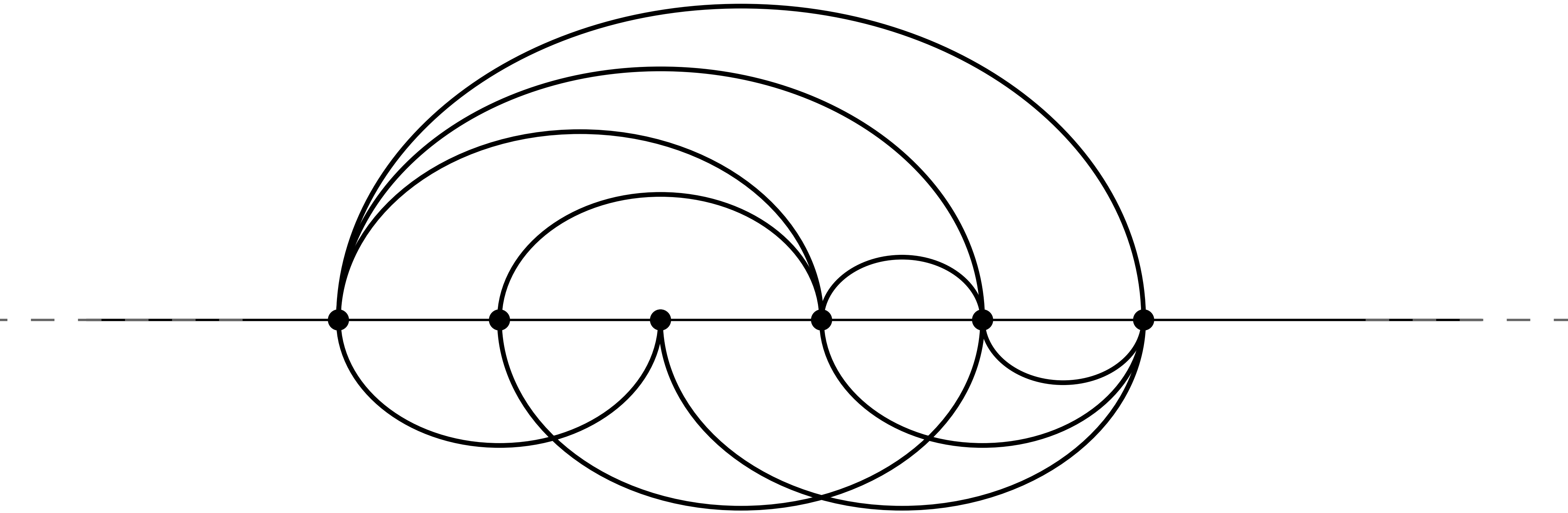
Stack



Queue

Mixed Linear Layouts I

Stack   No crossings!



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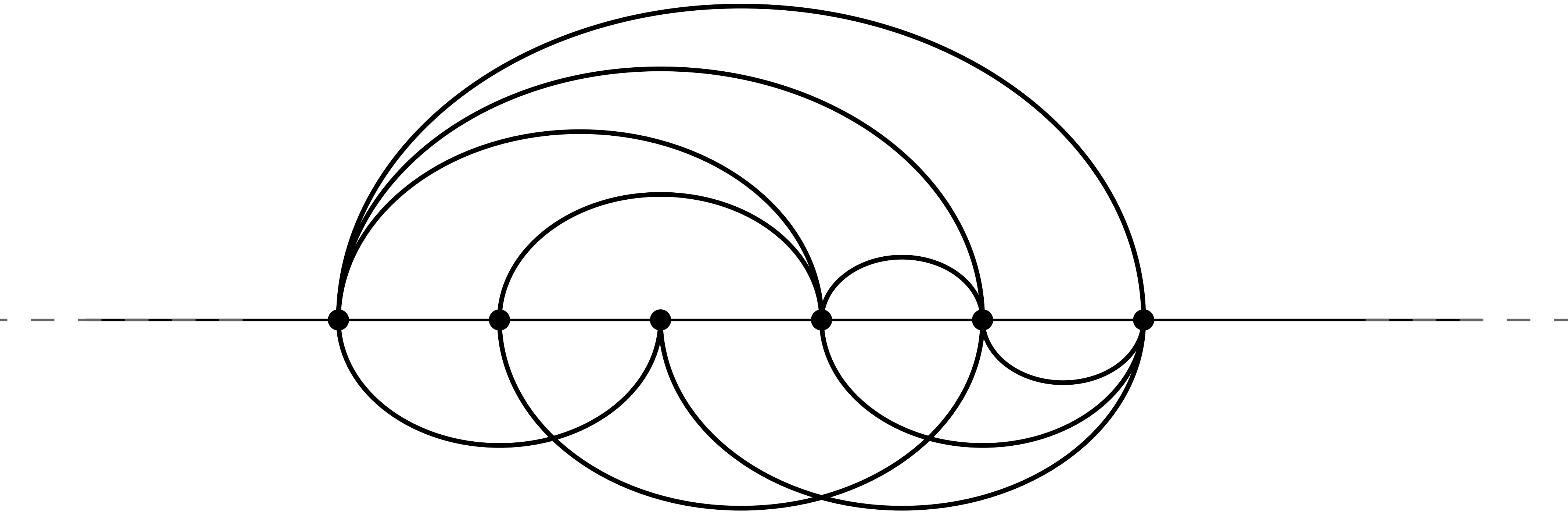

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What Kind of Questions

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Given a graph with fixed vertex order, assign the edges to pages.

Previous Work

The two most relevant papers for us

Heath and Rosenberg 1992

Introduced the mixed layout

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Needed fact

Bernhart and Kainen 1979 + Wigderson 1982

Testing if G admits 2-stack embeding is NP-complete

Our Results

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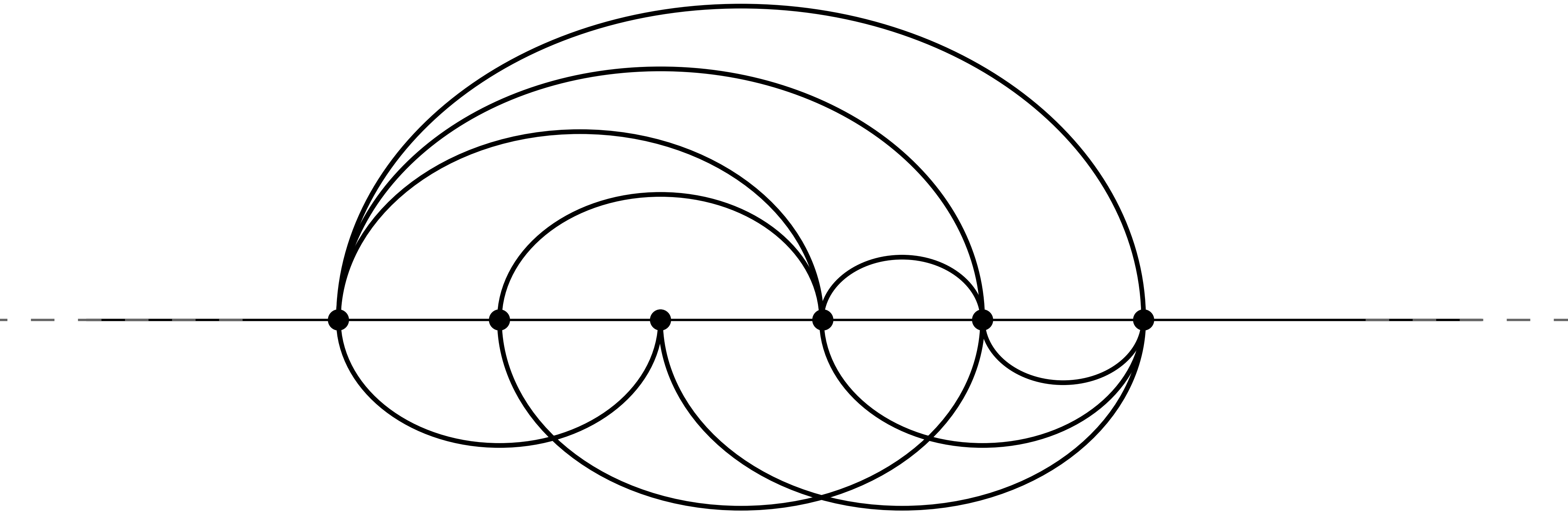
Mixed Linear Layouts II

Stack No crossings!



Length

Queue No proper nestings!

Mixed Linear Layouts II

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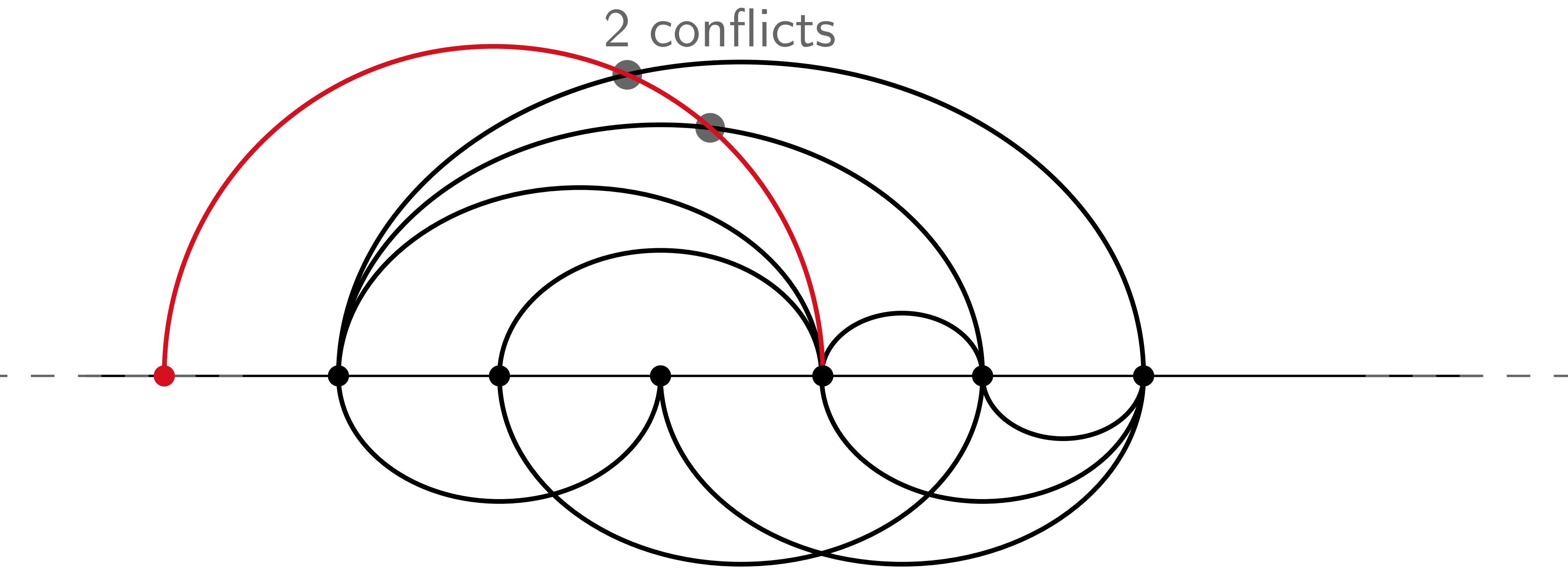

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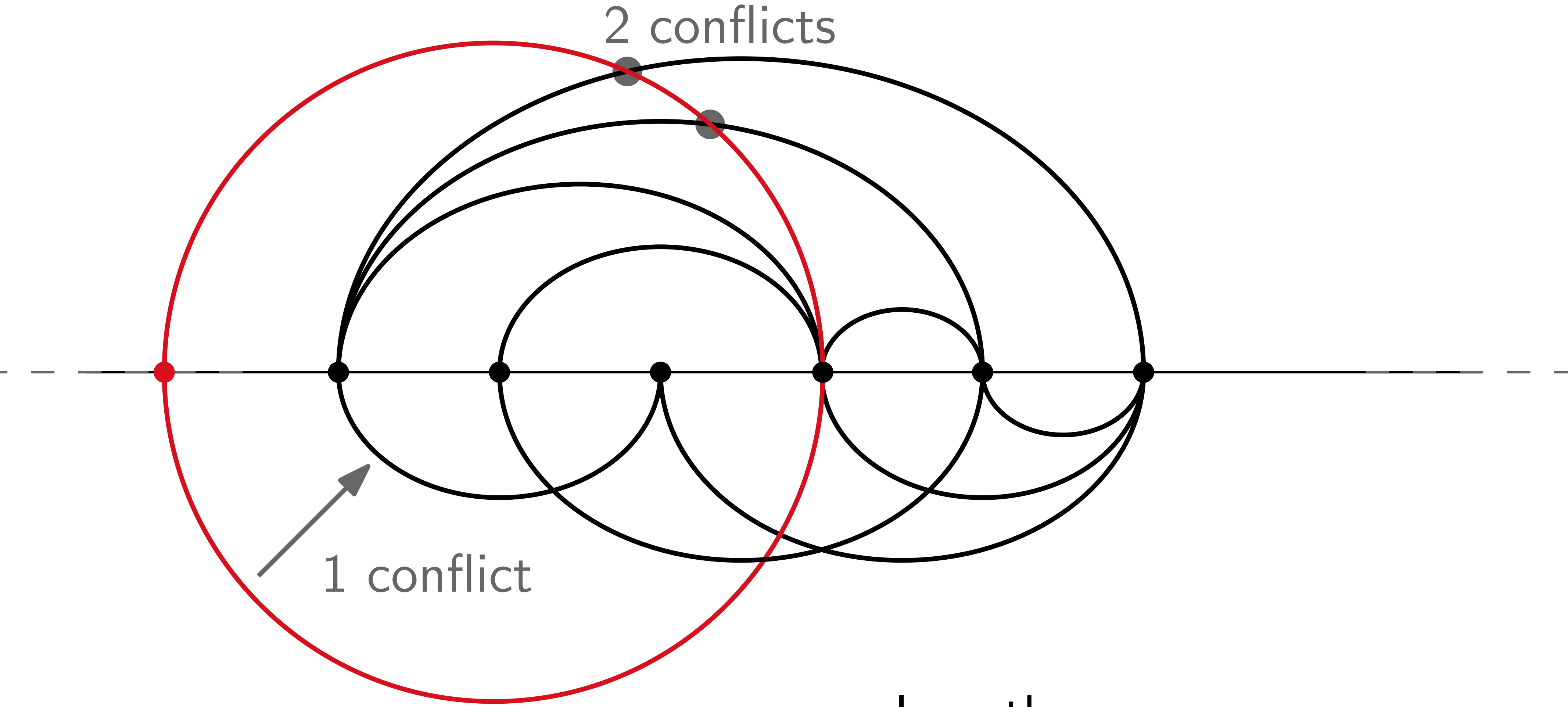

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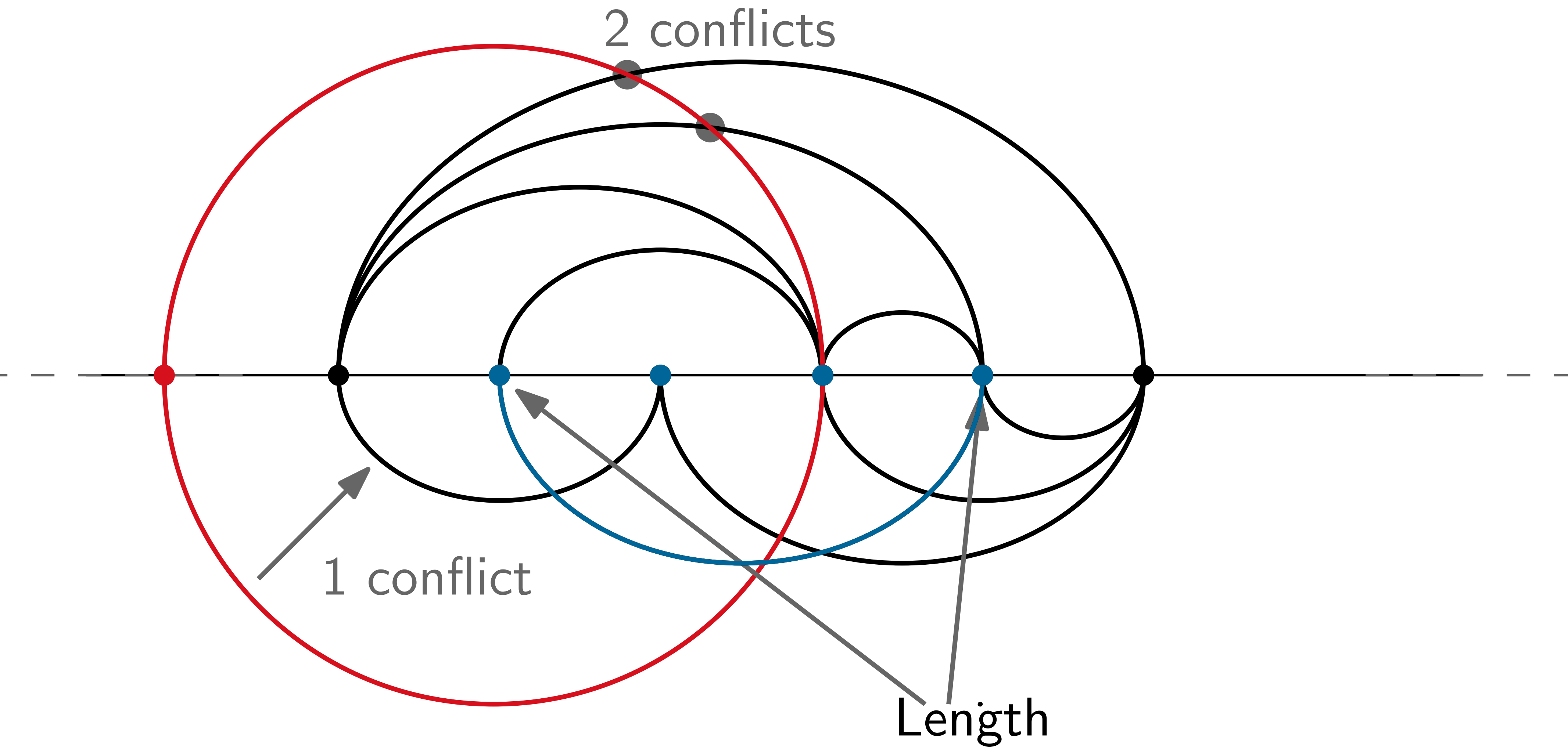

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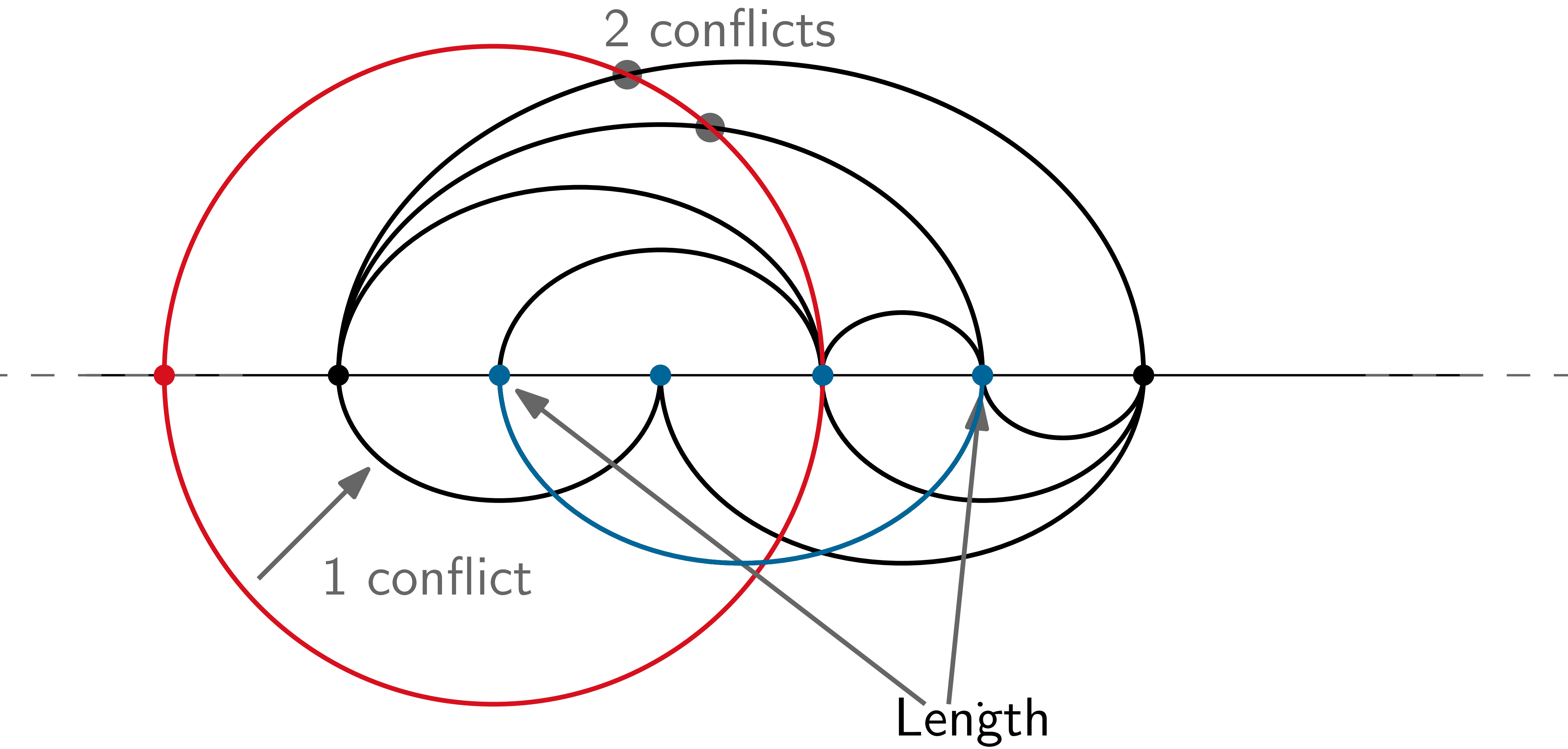
Mixed Linear Layouts II

Stack ~~Few~~ crossings!



Queue ~~Few~~ proper nestings!

Edge to Page Assignment Algorithm

Input: A graph G and a linear order of its vertices
Output: 1-stack, 1-queue layout with few crossings in the stack and few proper nestings in the queue page

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For each edge store two counter:

- Crossing counter $c(e)$
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Encountering start of an edge e

Edges are considered sorted by length

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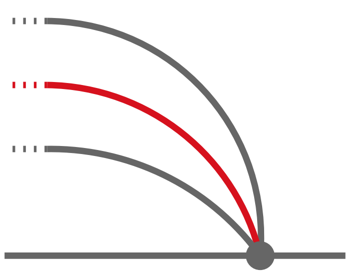

Consider how many edges are above e in $\mathcal{S}$ and $\mathcal{Q}$

$\mathcal{S} \rightarrow s_e$ $\quad$ $\mathcal{Q} \rightarrow q_e$

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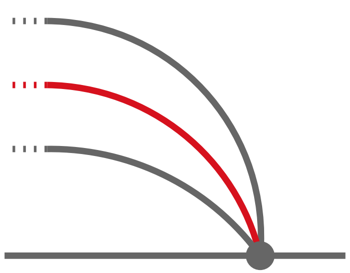


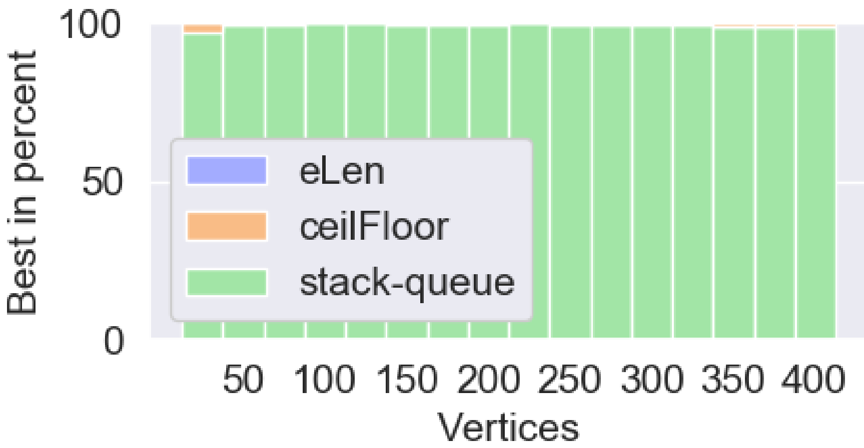
Consider how many edges are above e in $\mathcal{S}$ and $\mathcal{Q}$ → s_e, q_e

Add e to stack if $c(e) + 0.5 s_e \leq n(e) + 0.5 q_e$

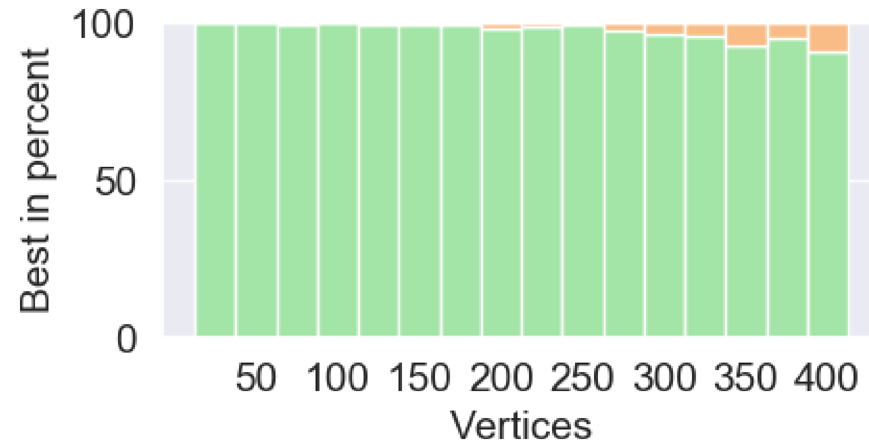
Increase $c(f)$ and $n(f)$ for all edges f above e in $\mathcal{S}/\mathcal{Q}$

Experimental Results – Random Graphs

Fully random graphs



$|\text{edges}| = 3|\text{vertices}|$



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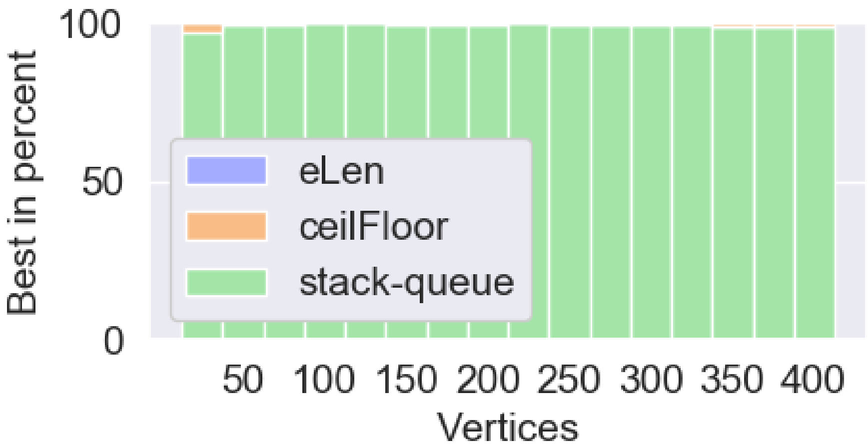

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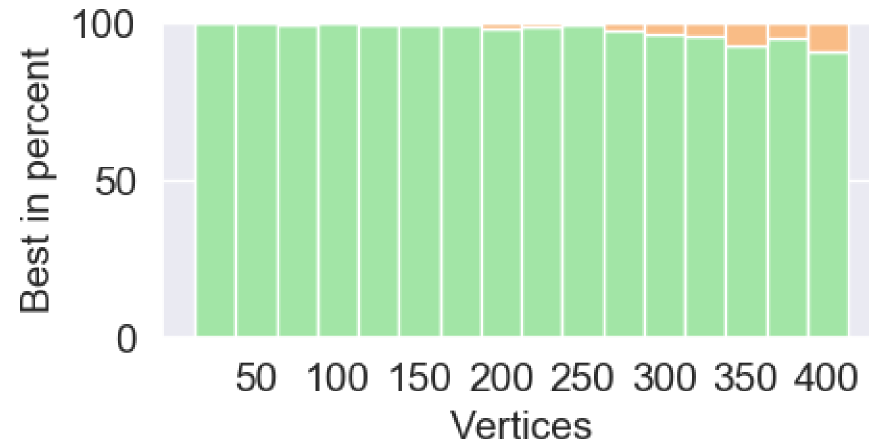

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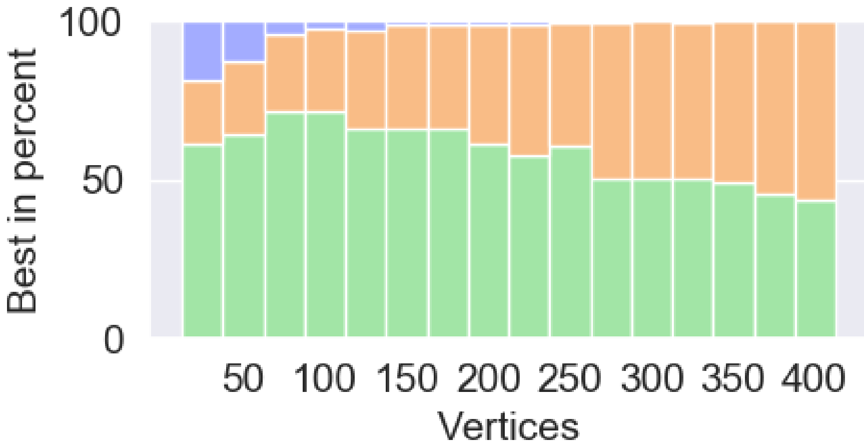
Result

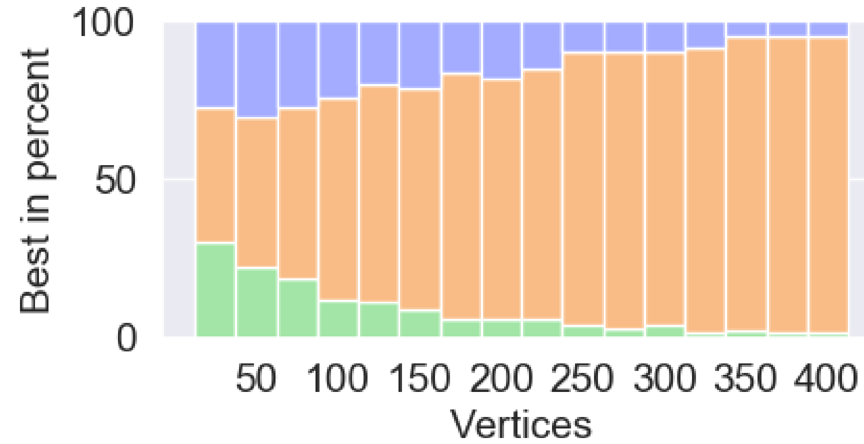
- We produce the best results for the majority of instances
- Difference is narrow

Experimental Results – Planar Graphs

Two more general cases



Planar Bipartite



Triangulations

Experimental Results – Planar Graphs

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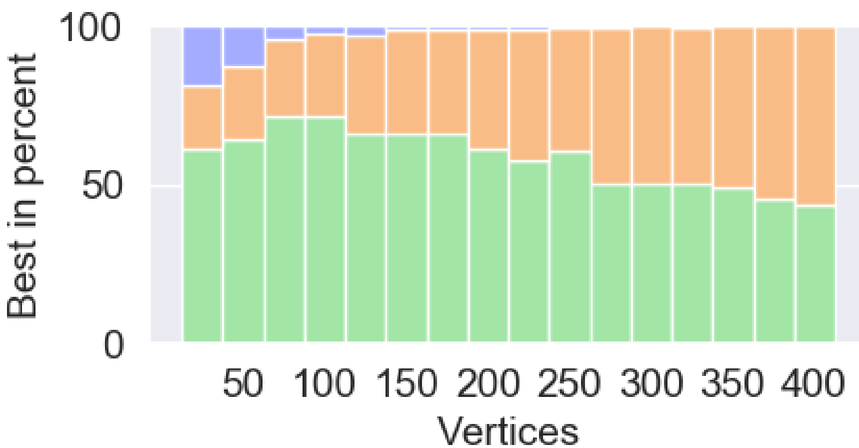

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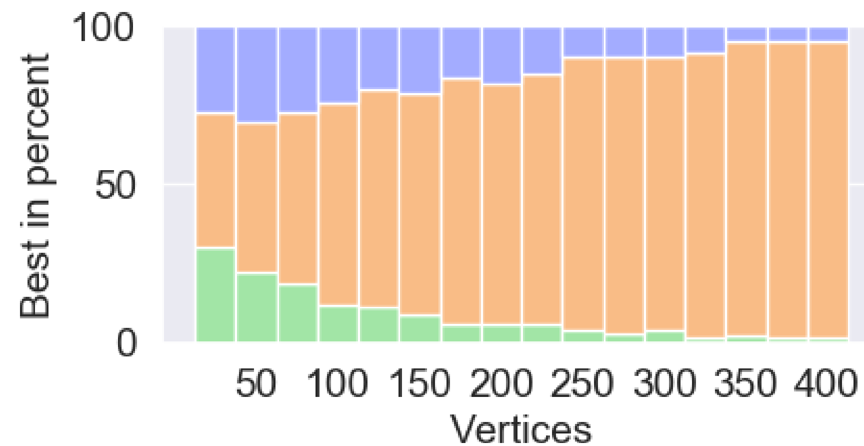

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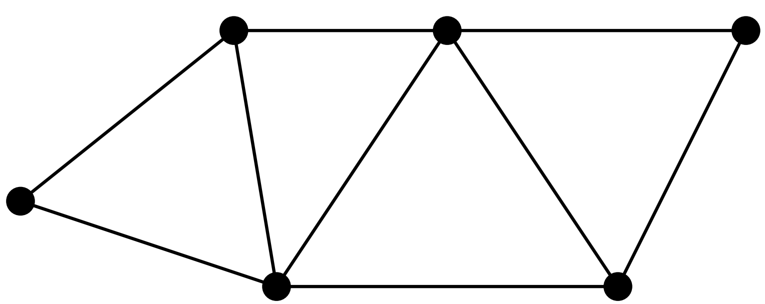
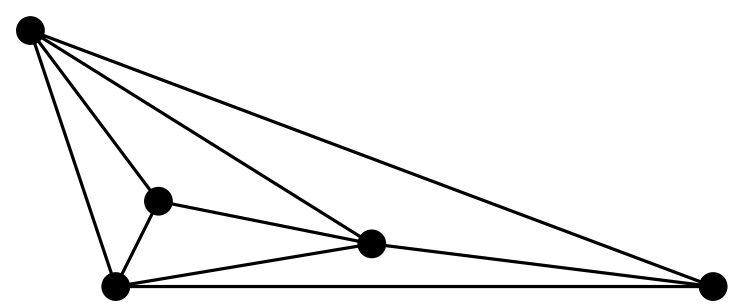
Planar 2- and 3-trees



Planar 2-trees



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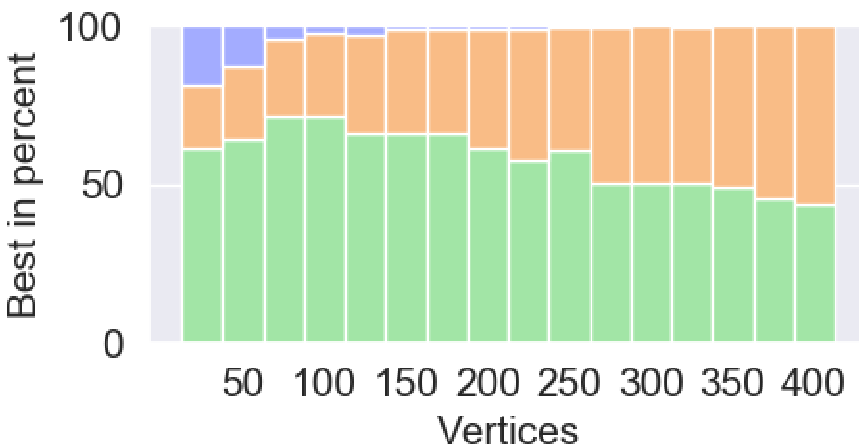

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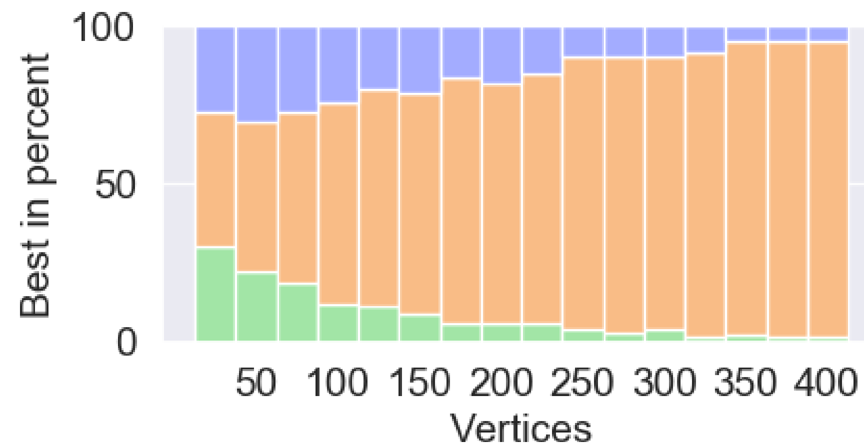

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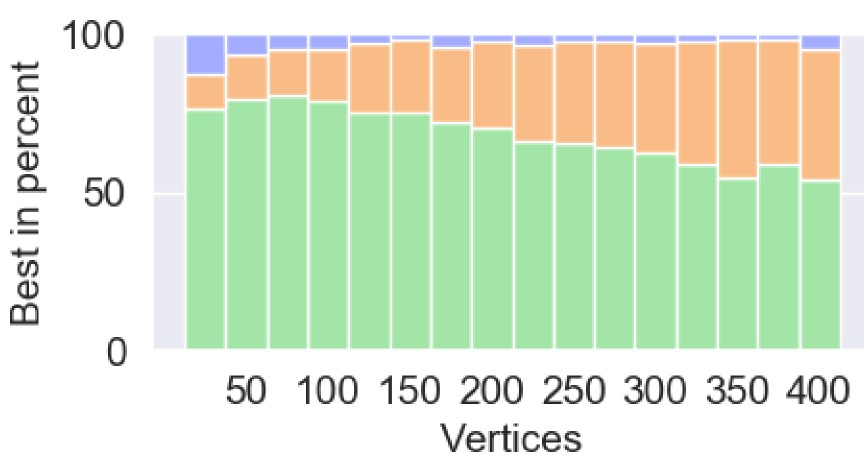

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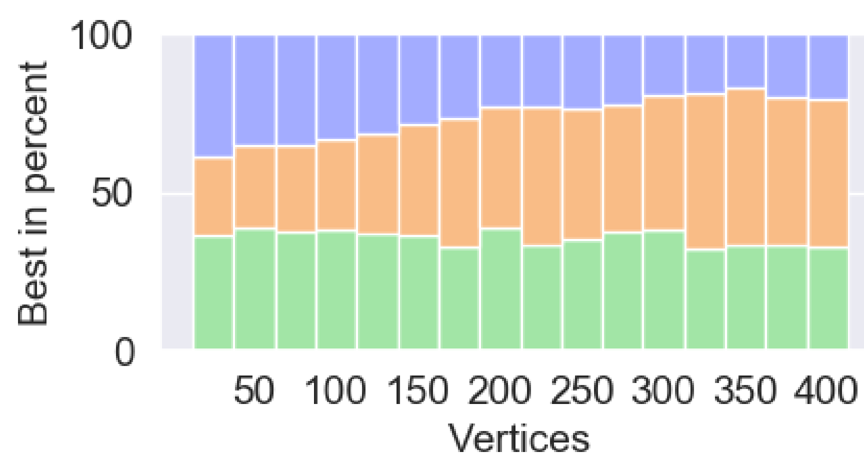

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Reduction from testing 2-stack

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- K_8, largest complete graph that has 2-stack, 1-queue layout

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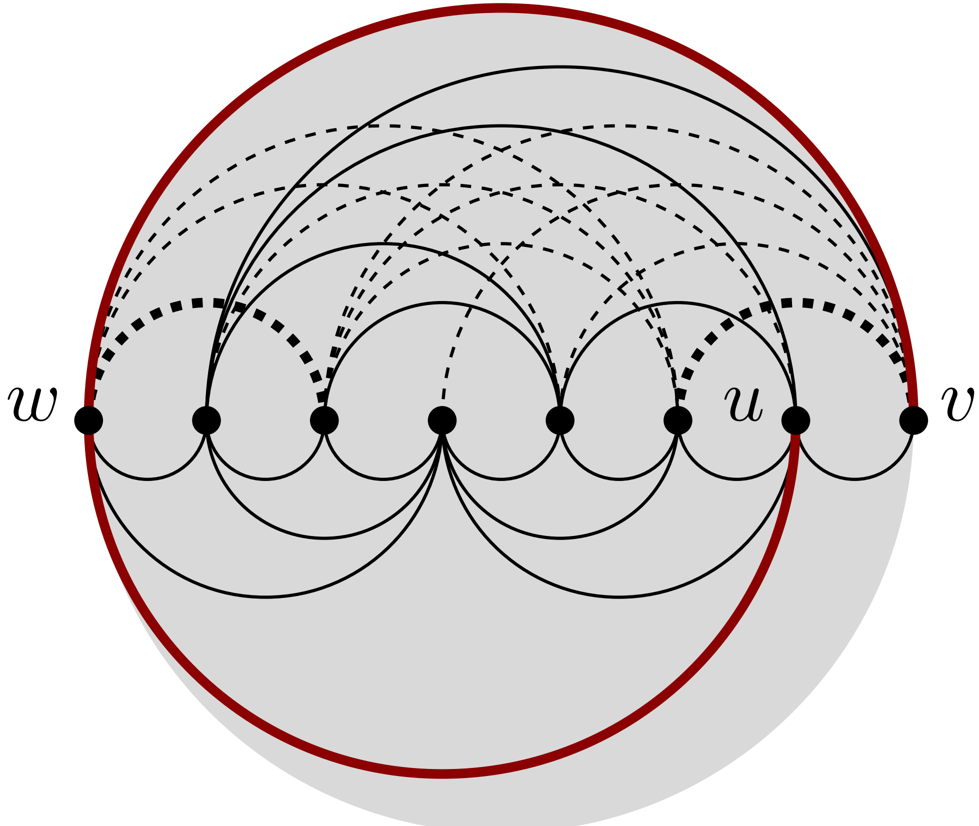

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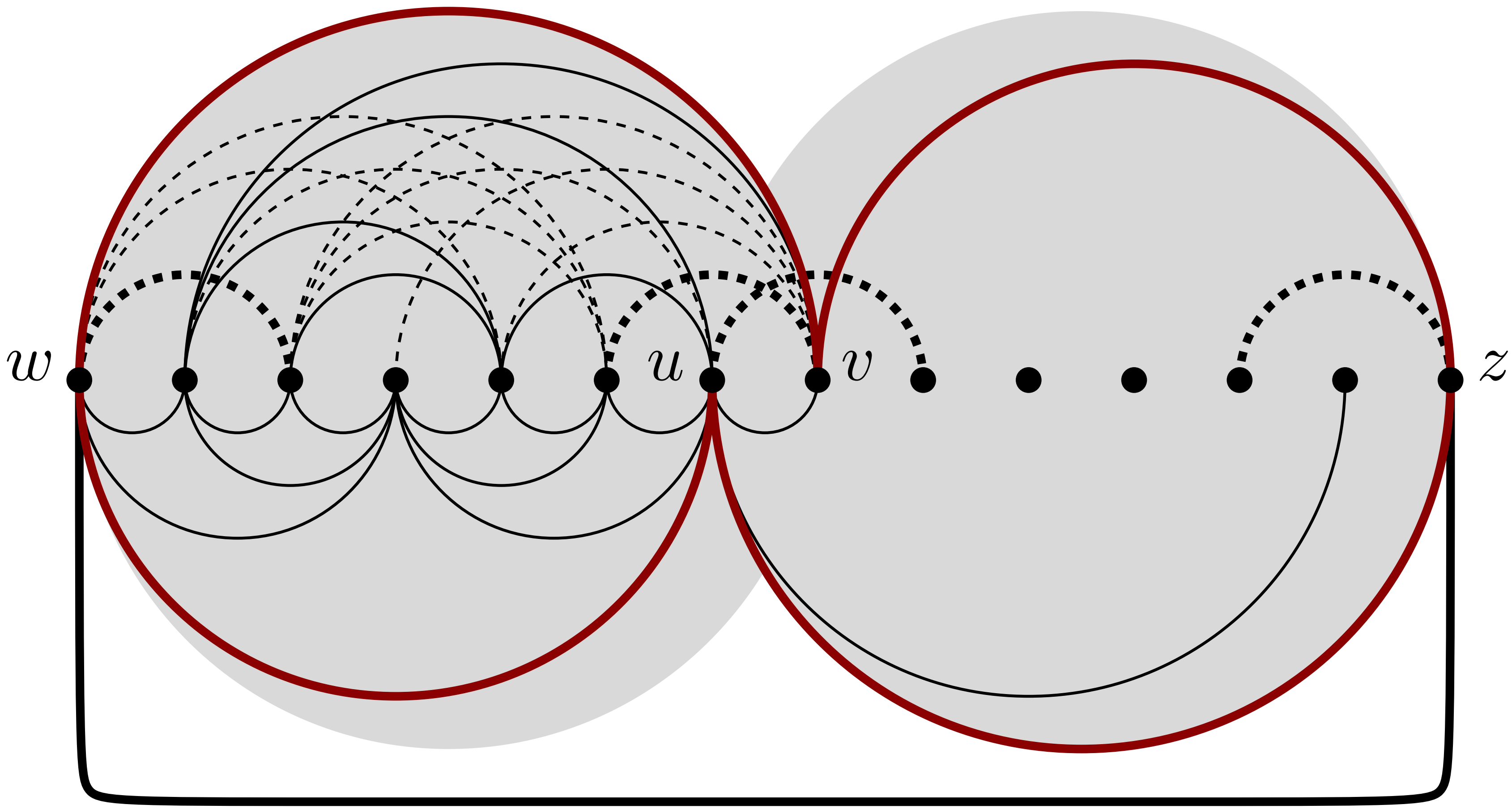

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- Form Double K_8 by identifying two vertices + add edge wz

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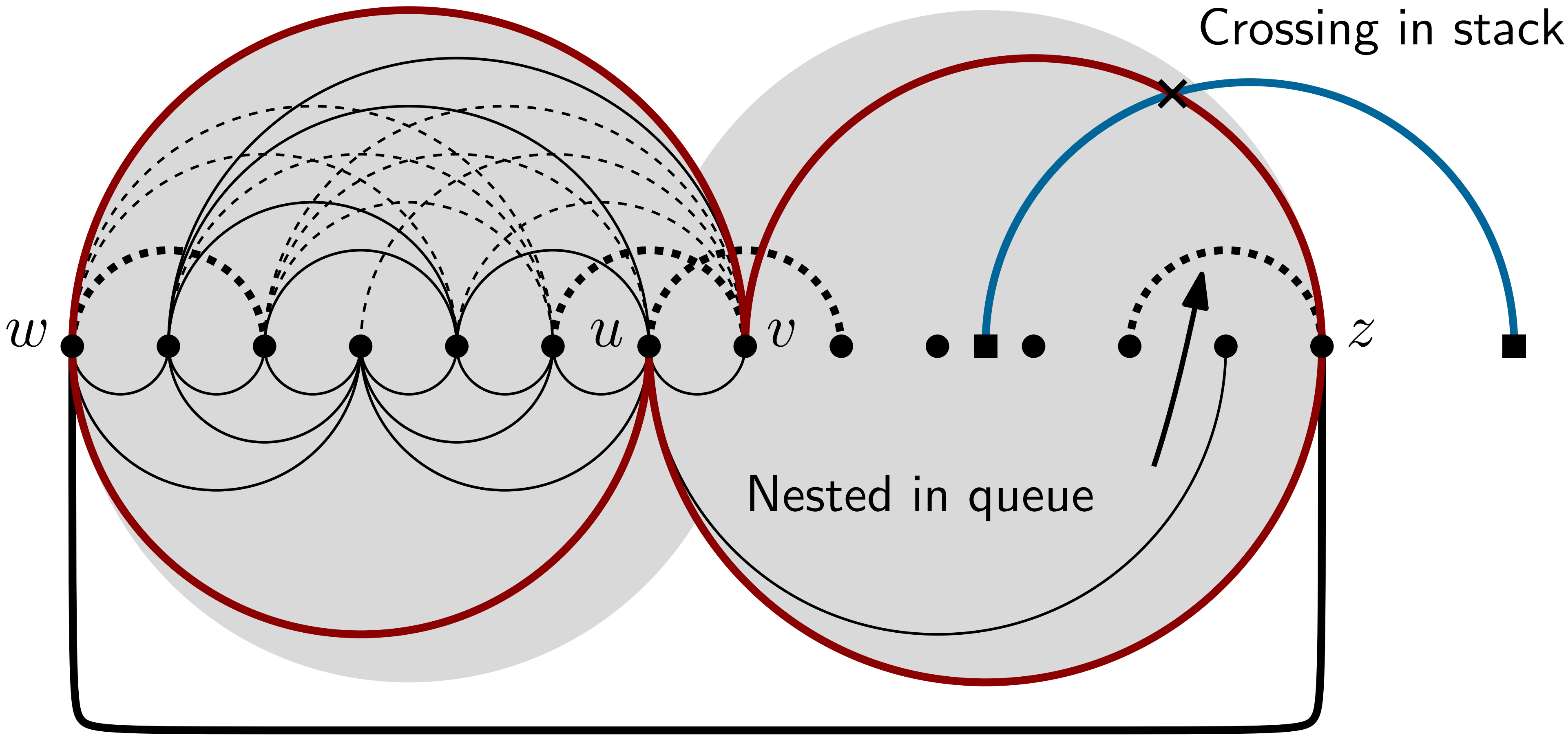

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- Double K_8 has very limited interaction with rest of graph

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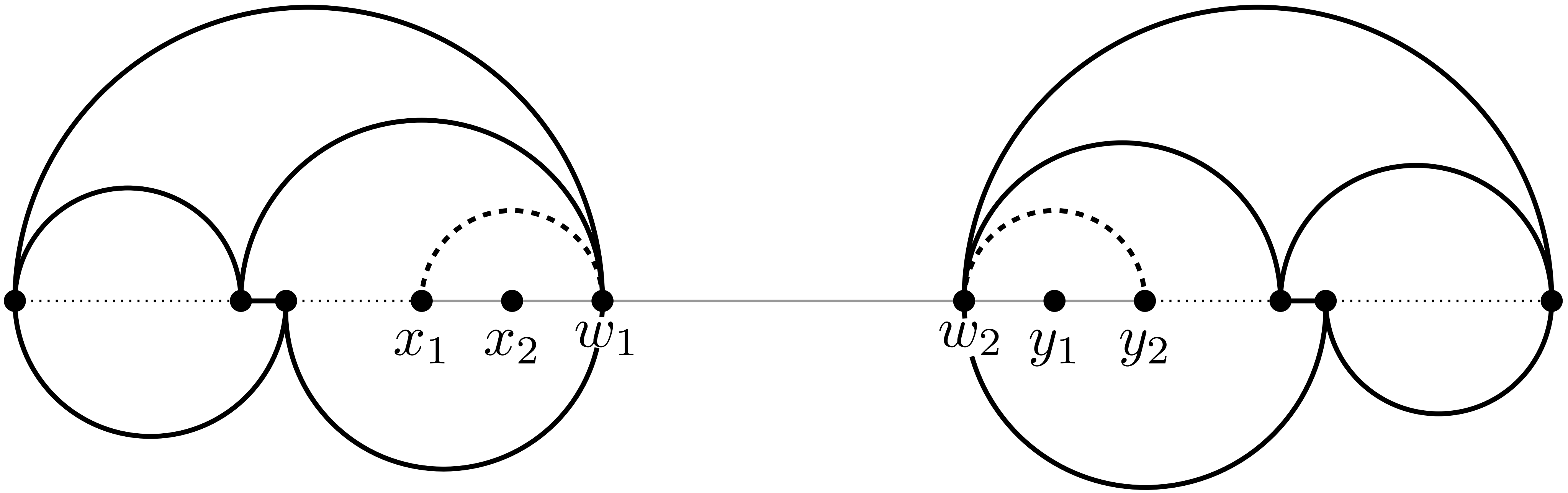

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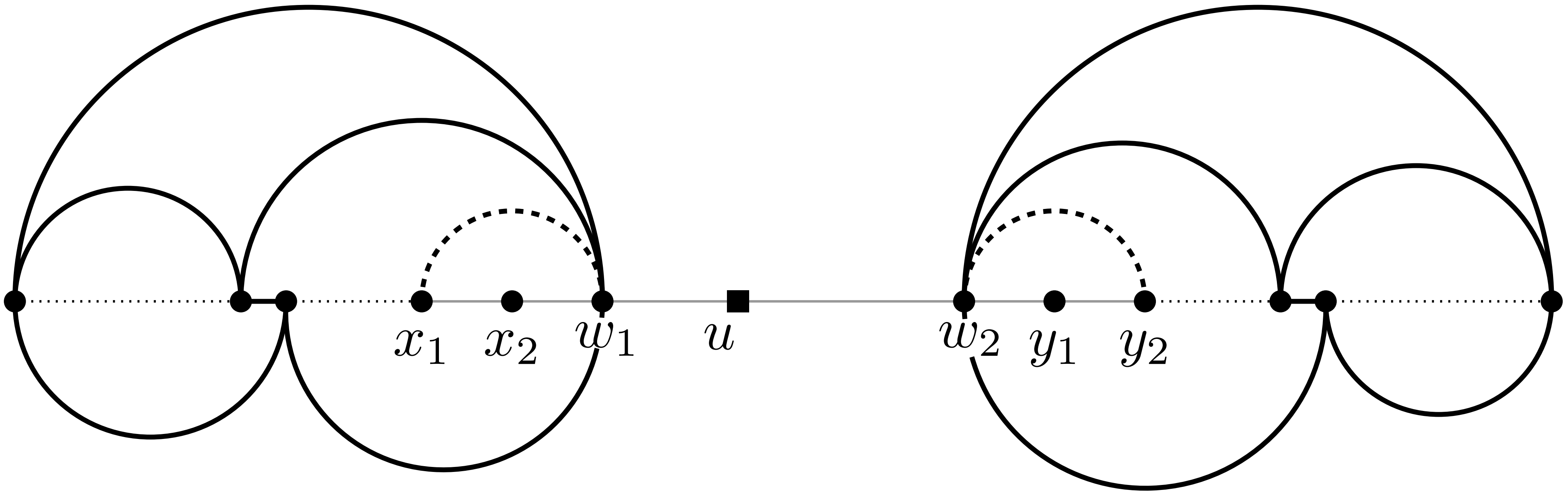

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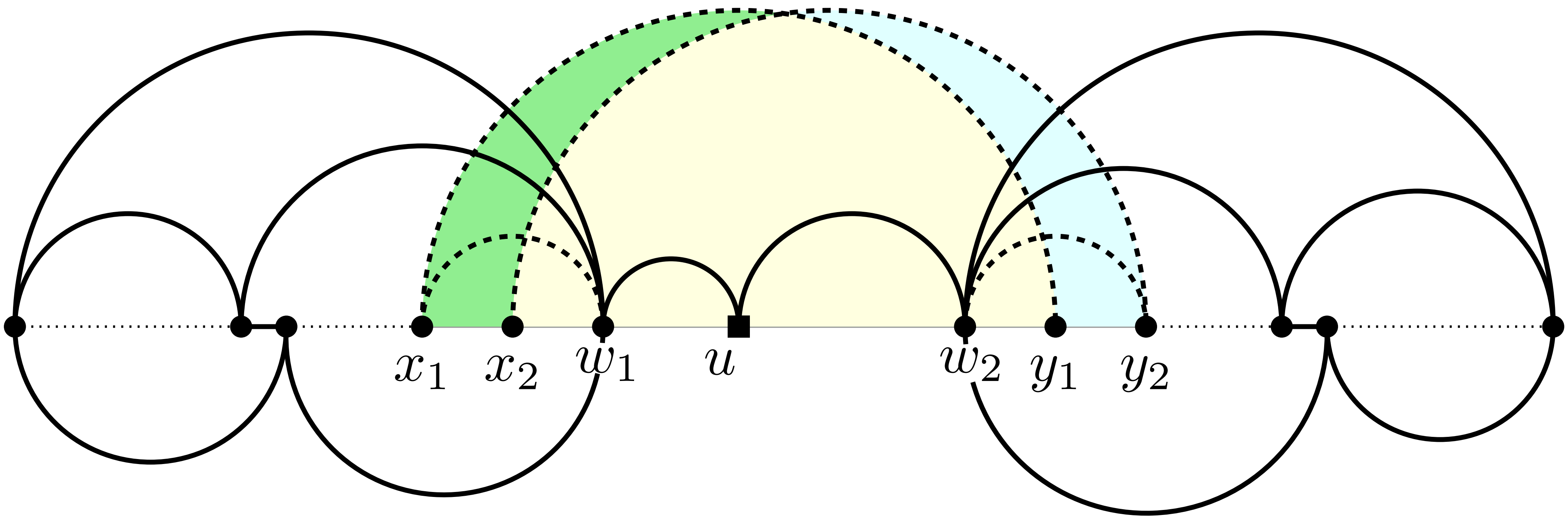

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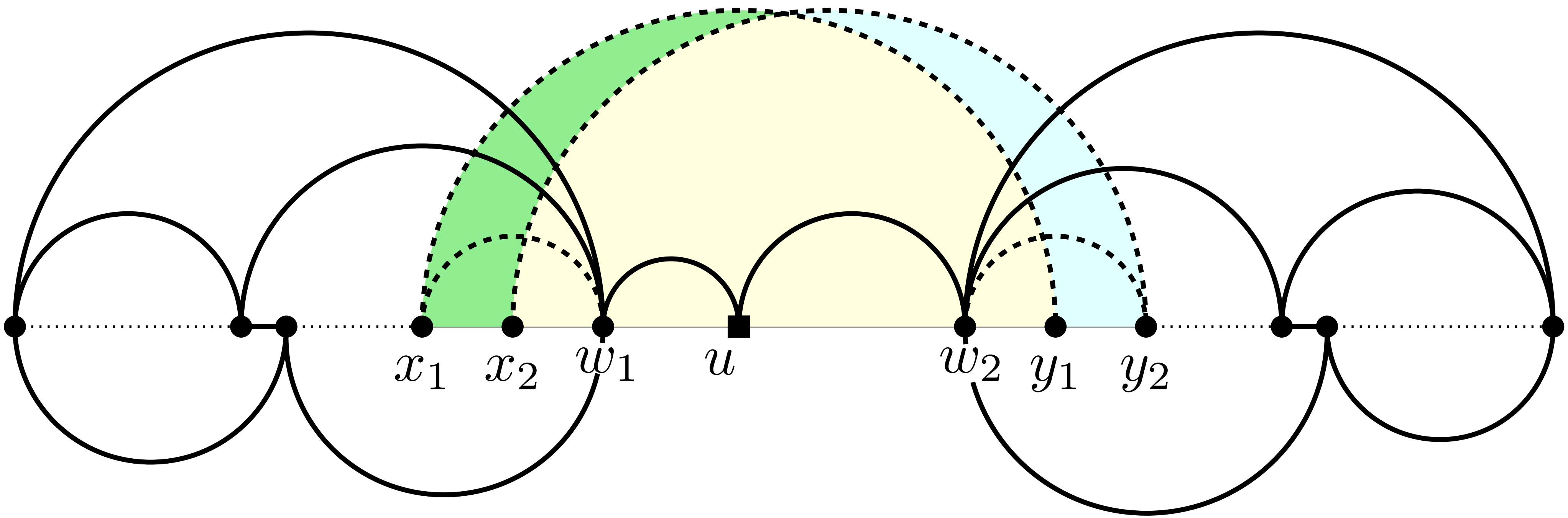
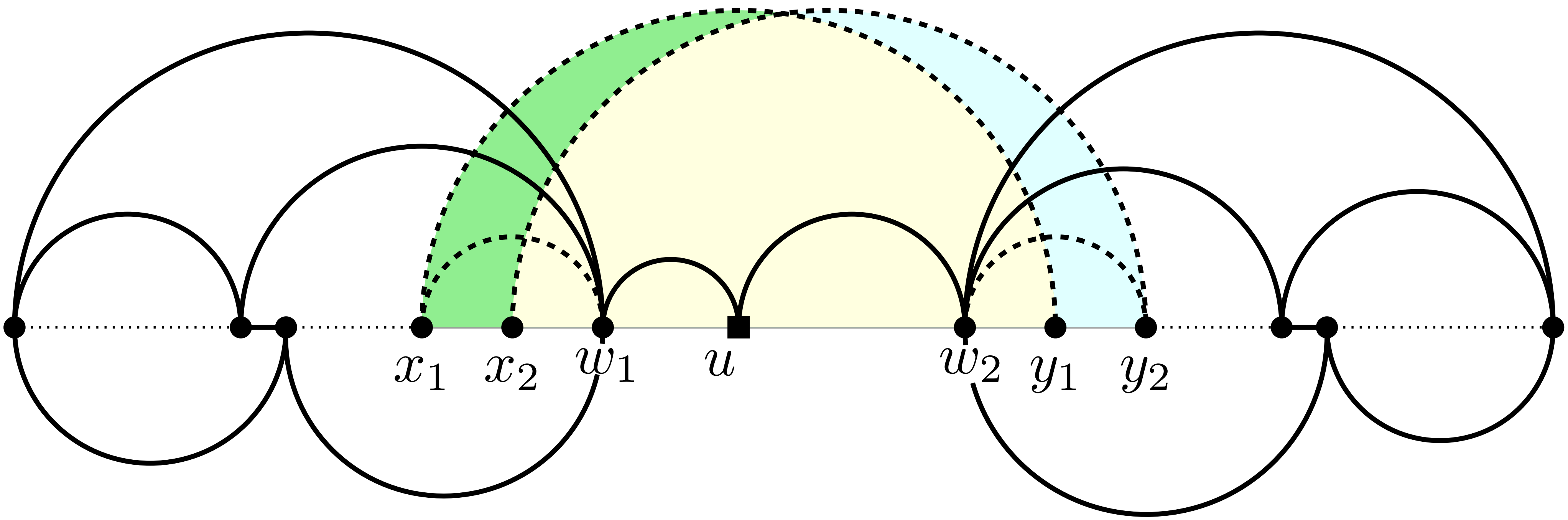


Lemma: u must be between w_1 and w_2 for any vertex-ordering of the above graph

Testing 2-stack, 1-queue is NP- complete III

Given graph G, task find 2-stack layout of G



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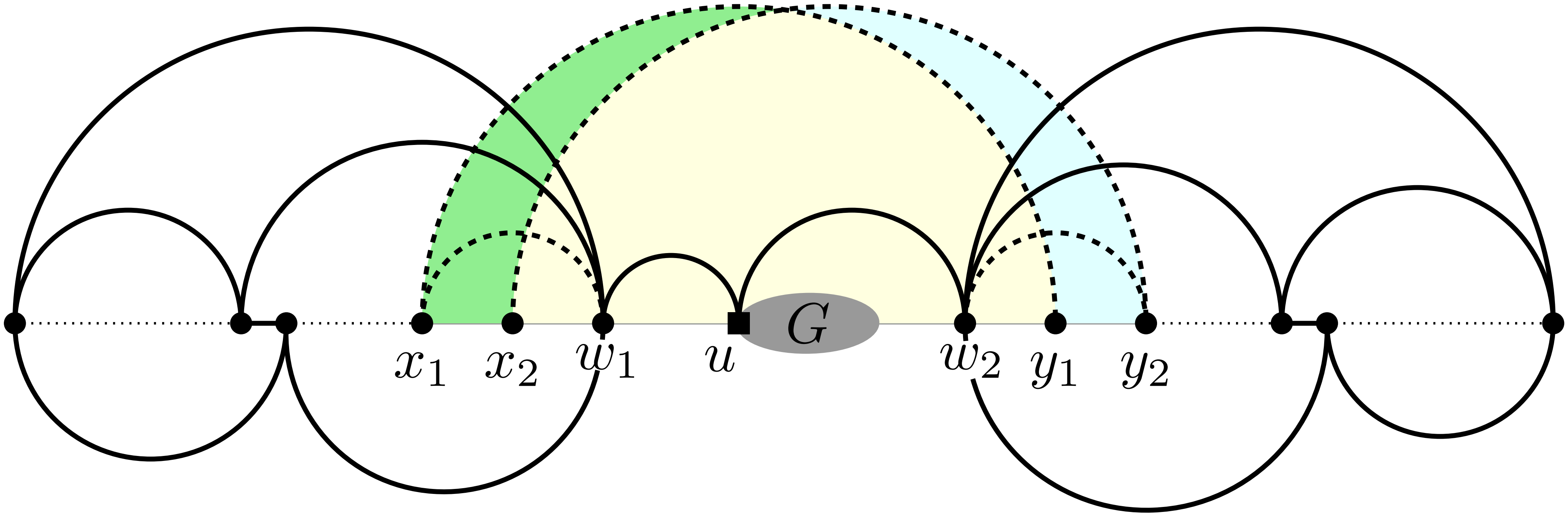

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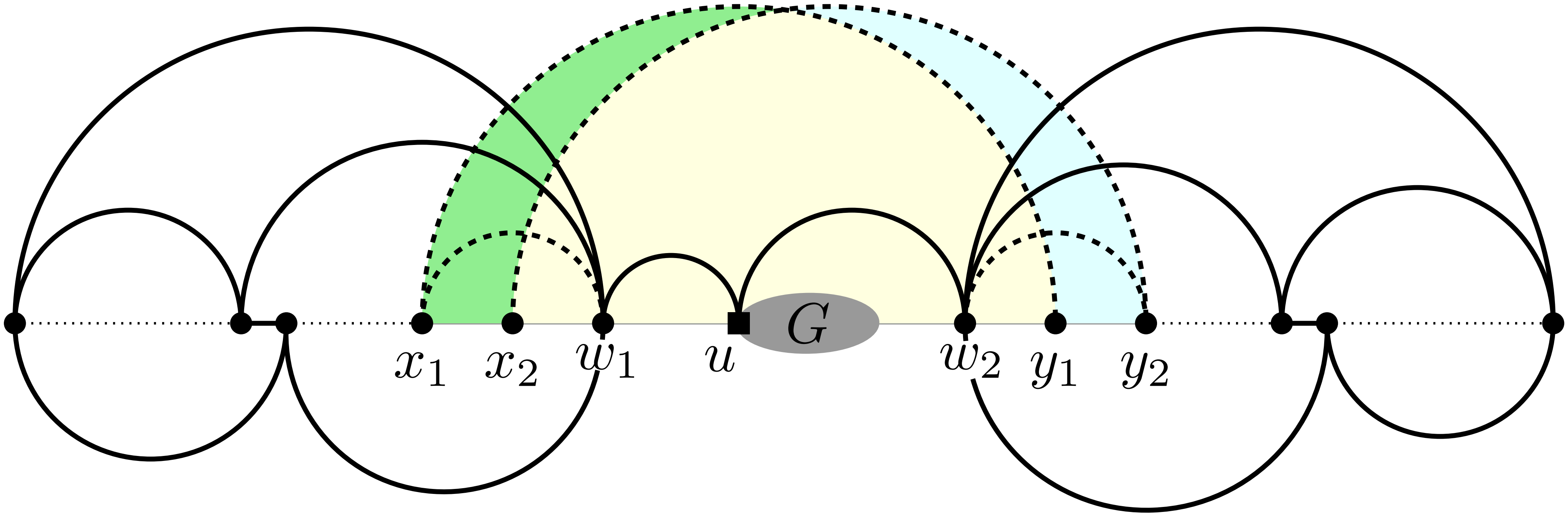

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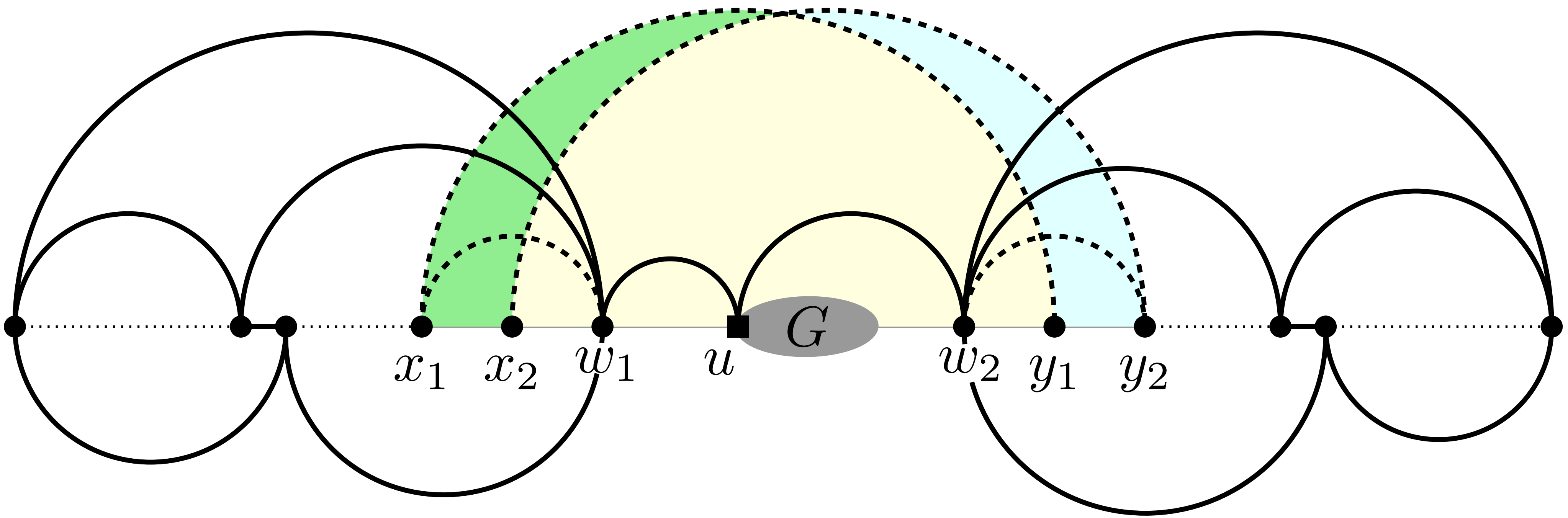

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For other direction:

Previous lemma holds for the neighbors of u

$\Rightarrow$ Induction gives the result

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Open: Does every planar bipartite graph admit a 1-stack, 1-queue layout? [Pupyrev 2017]