Old and New Challenges in Coloring Graphs with Geometric Representations

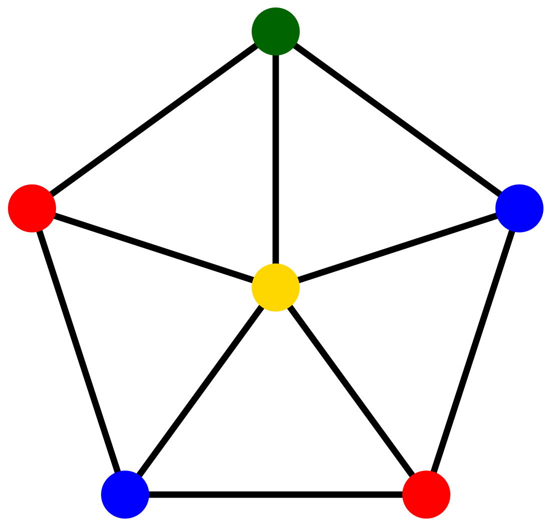
Bartosz Walczak

Jagiellonian University
Kraków, Poland

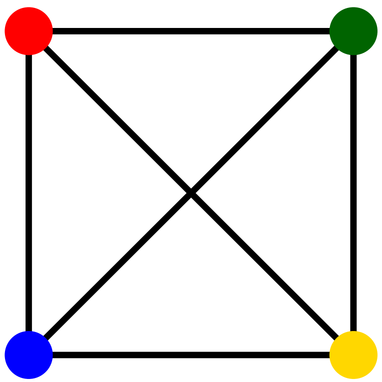
The 27th International Symposium on
Graph Drawing and Network Visualization

Chromatic number, denoted χ:

minimum number of colors in a proper coloring



What makes the chromatic number large?



Clique number, denoted ω:

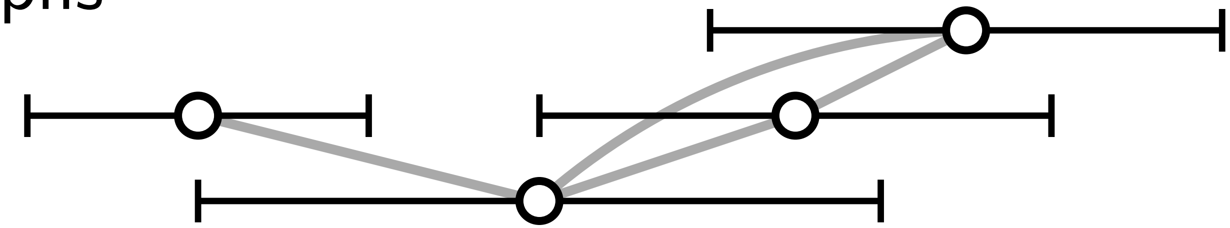
maximum size of a clique

$$\chi \geqslant \omega$$

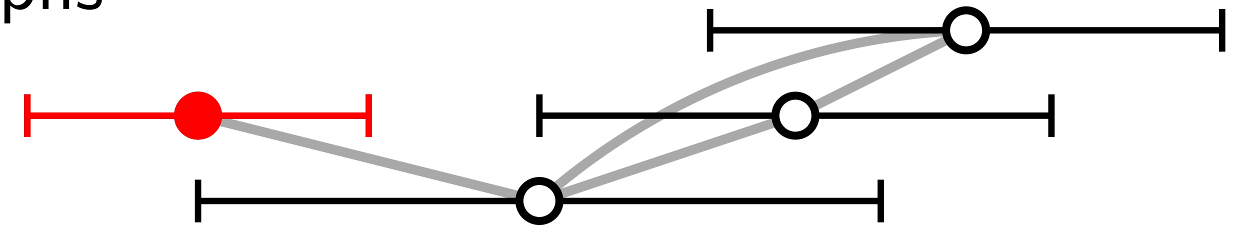
Tutte, Zykov, Mycielski... 1940/50s

There exist triangle-free graphs with arbitrarily large chromatic number.

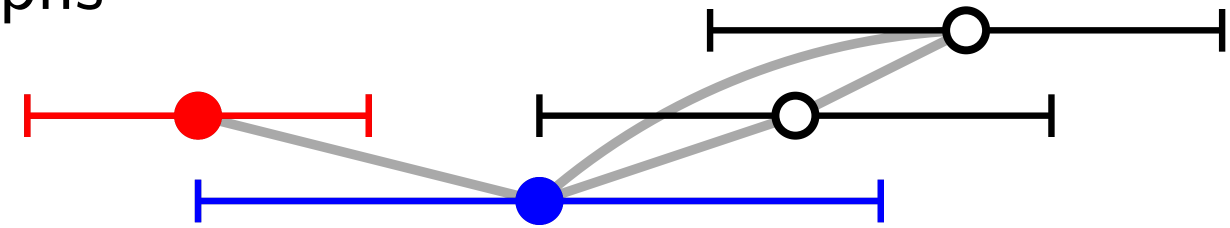
Interval graphs



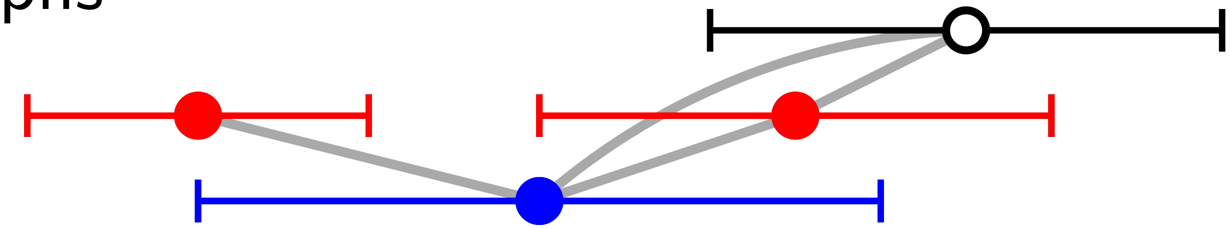
Interval graphs



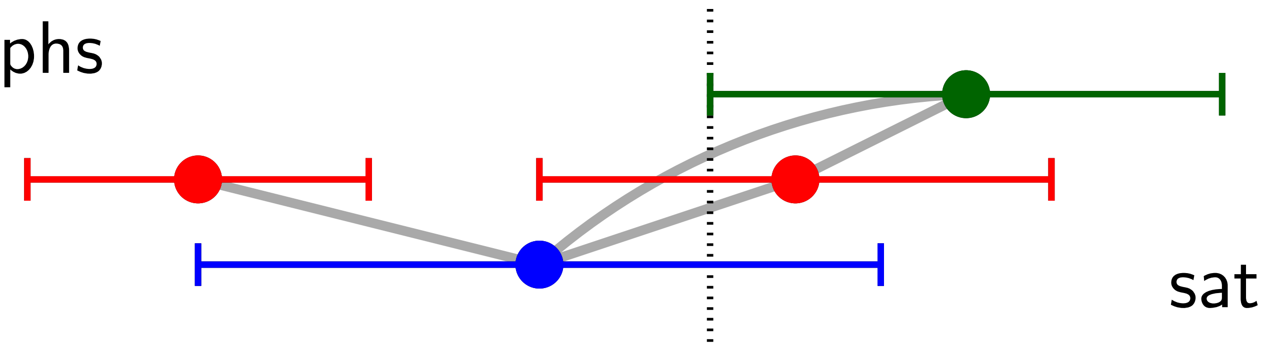
Interval graphs



Interval graphs

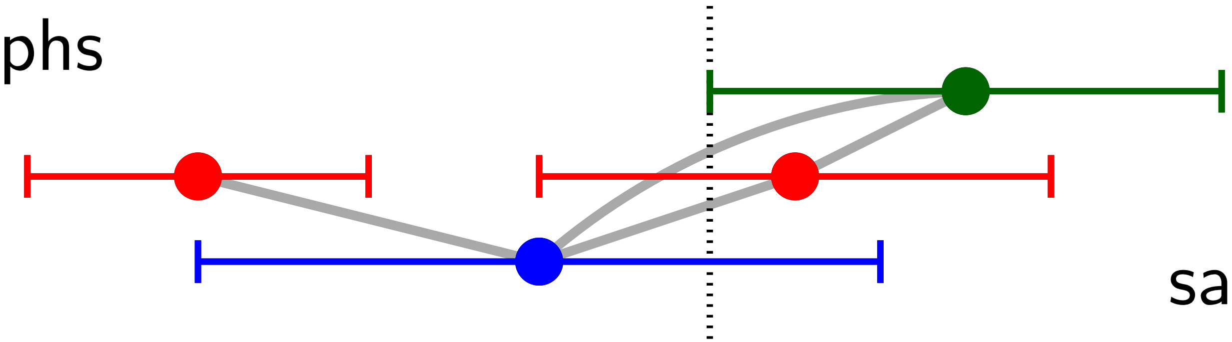


Interval graphs



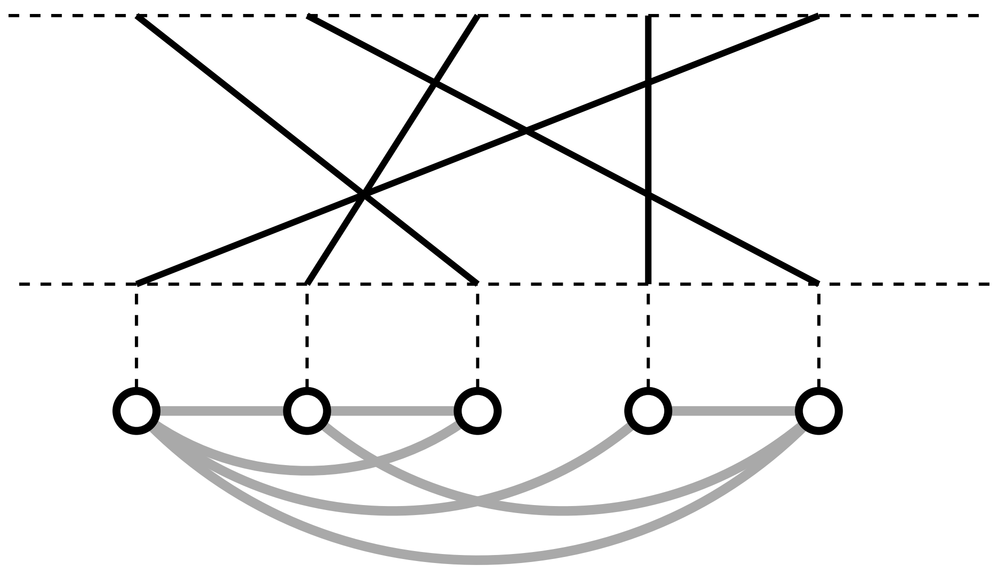
satisfy $\chi = \omega$.

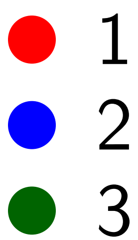
Interval graphs



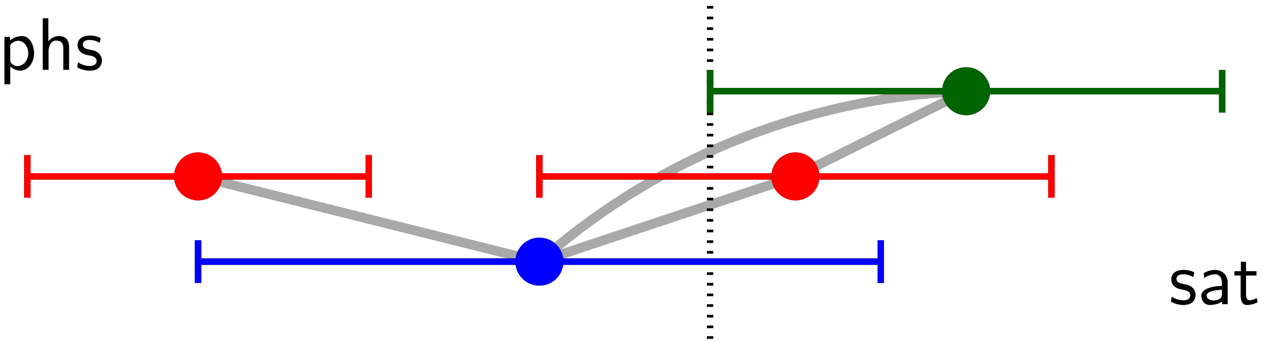
satisfy $\chi = \omega$.

Permutation graphs



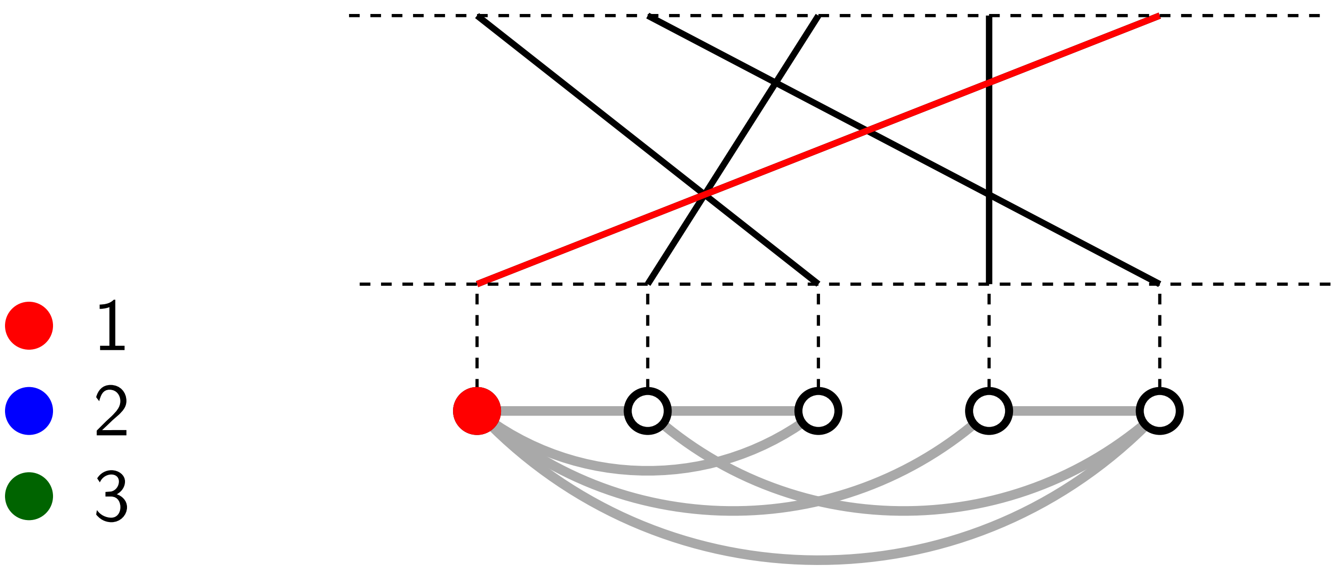


Interval graphs

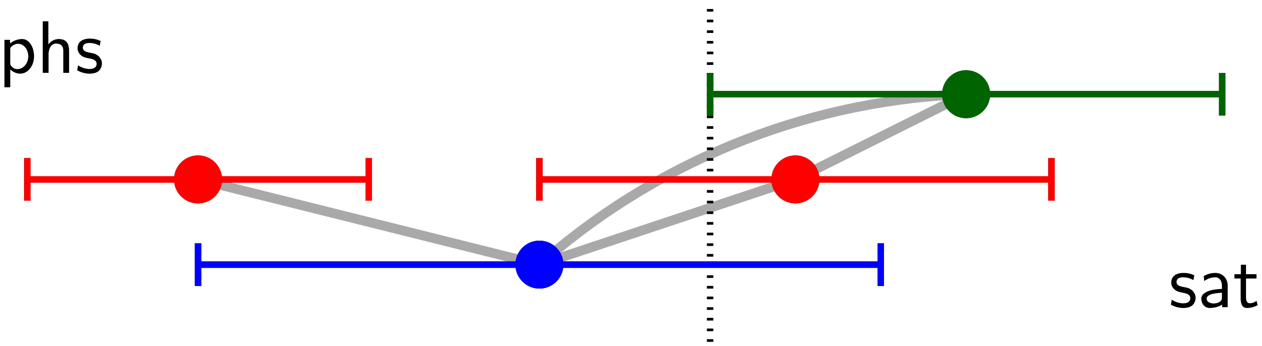


satisfy $\chi = \omega$.

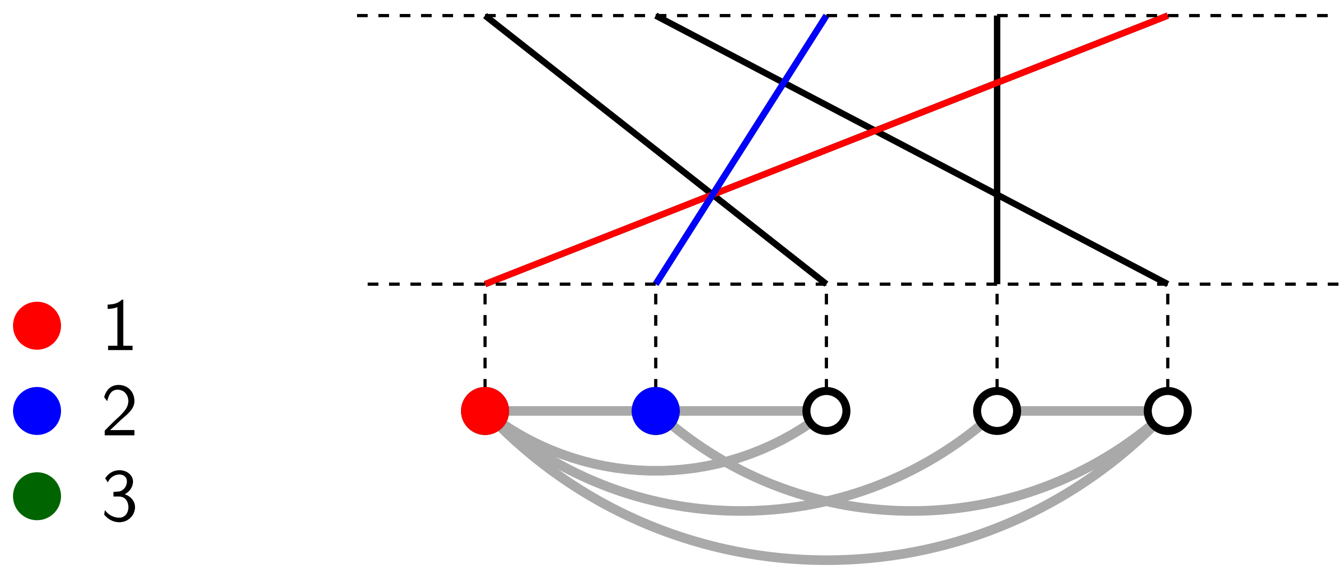
Permutation graphs



Interval graphs

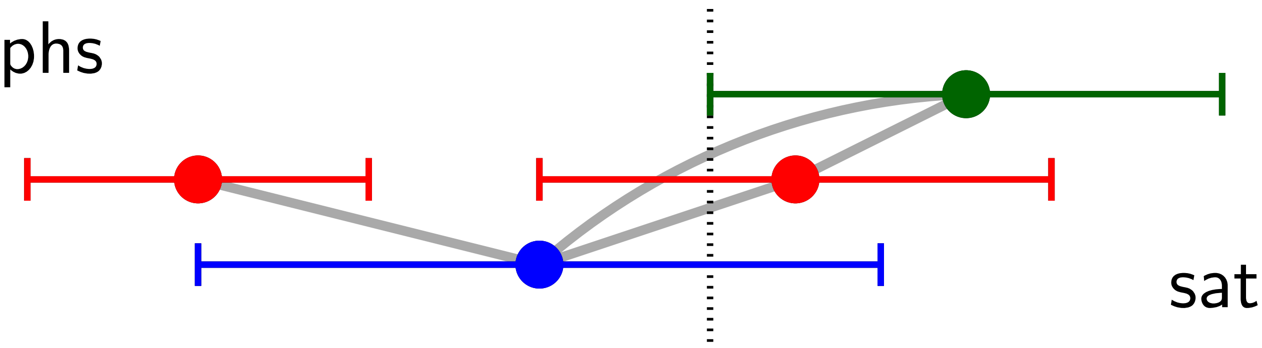


Permutation graphs



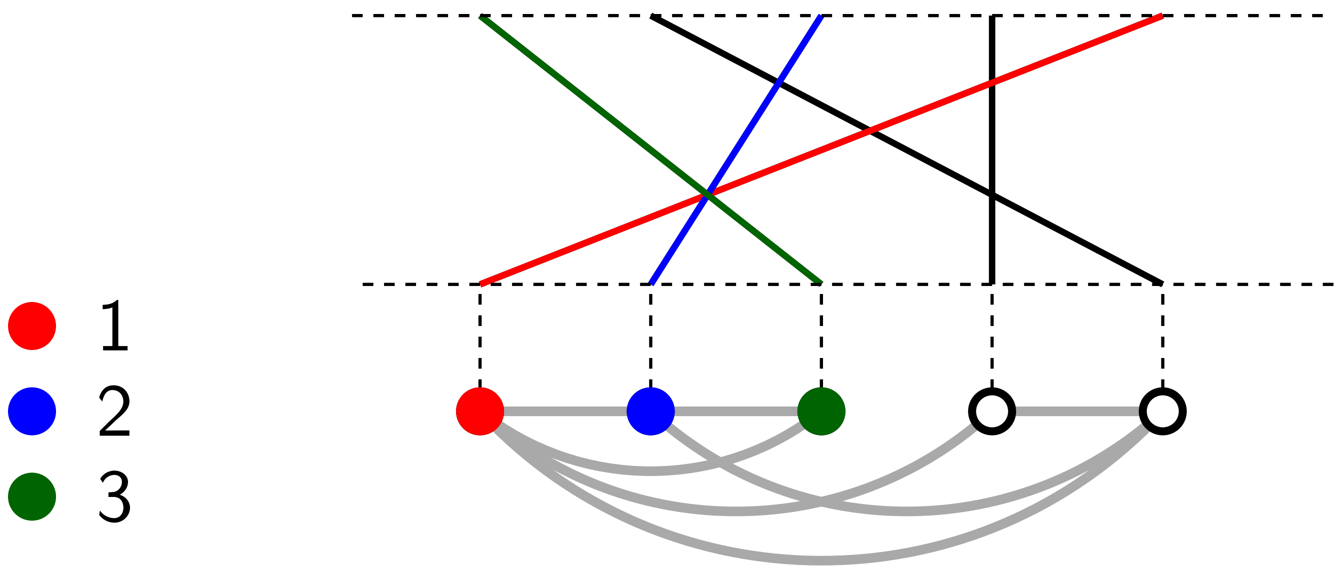
satisfy $\chi = \omega$.

Interval graphs

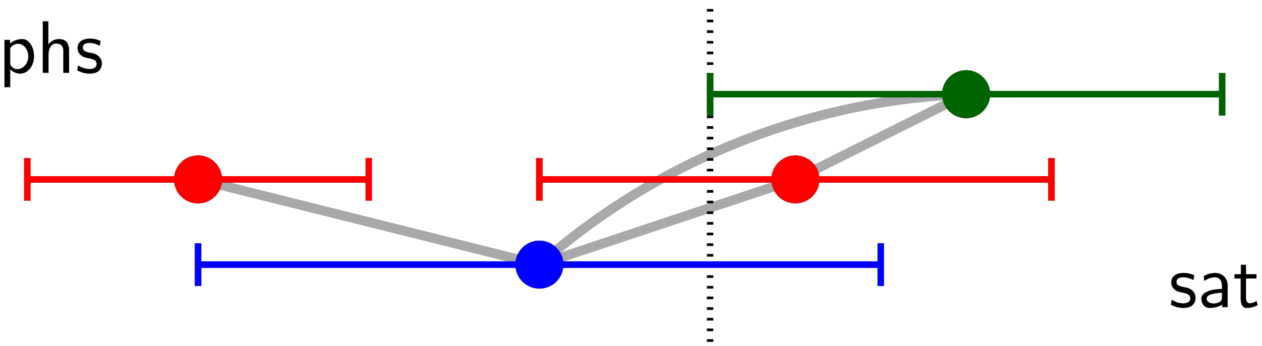


satisfy $\chi = \omega$.

Permutation graphs

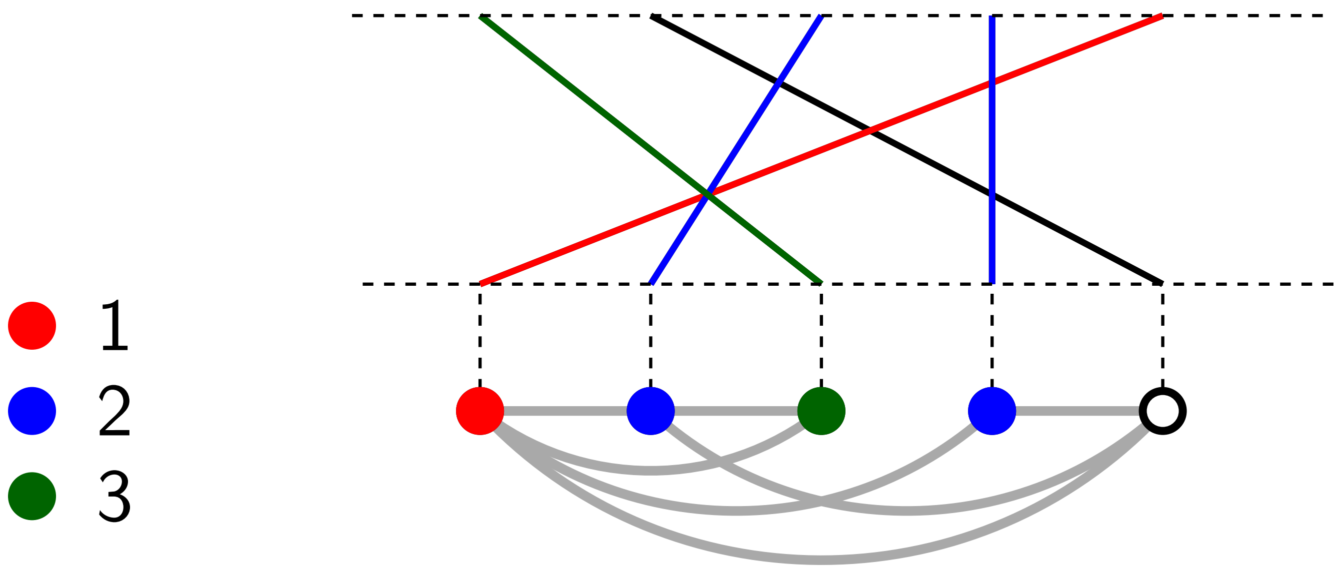


Interval graphs

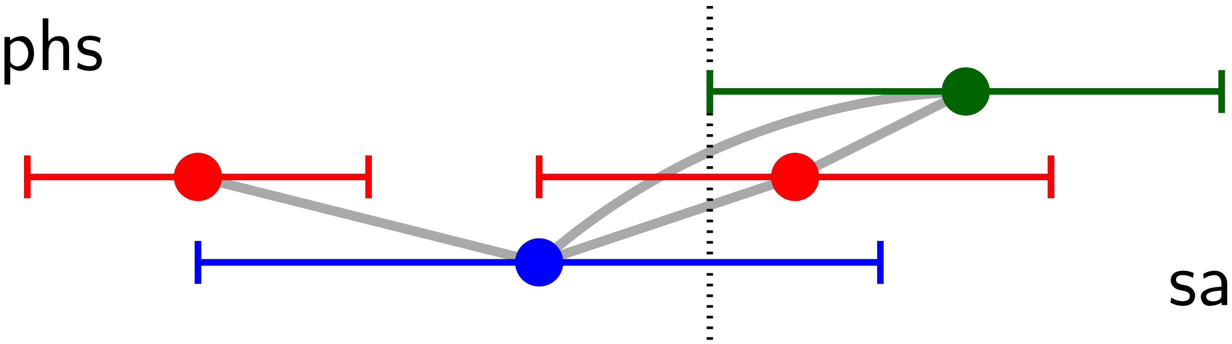


satisfy $\chi = \omega$.

Permutation graphs

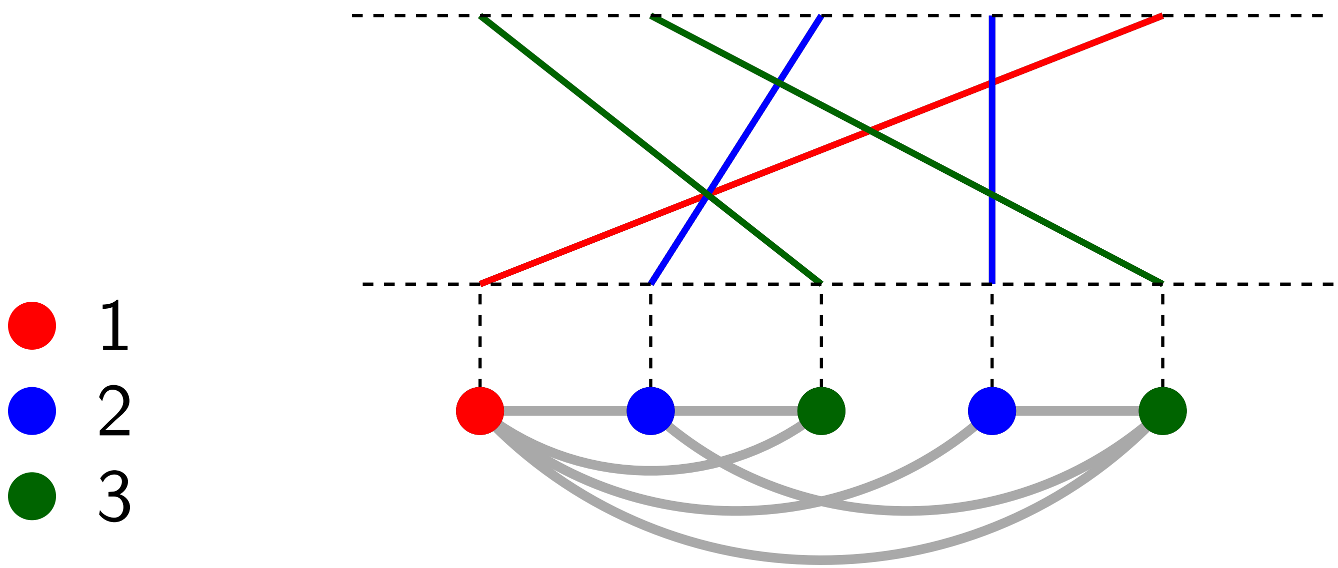


Interval graphs



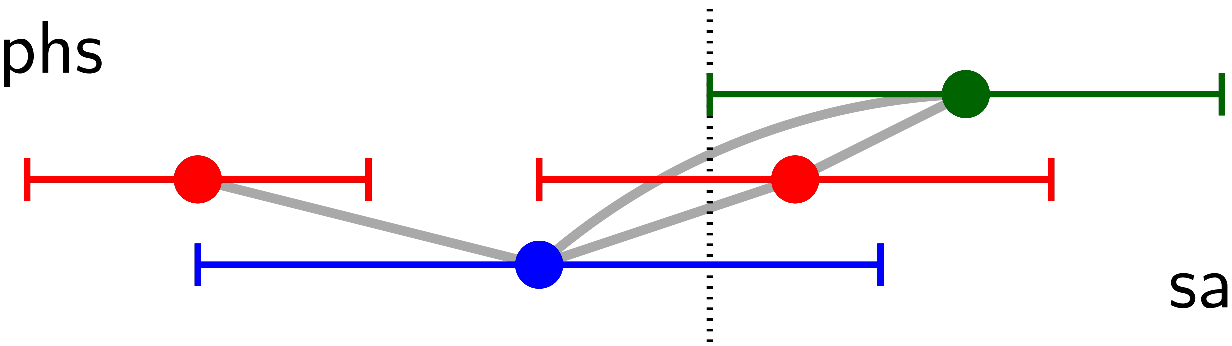
satisfy $\chi = \omega$.

Permutation graphs



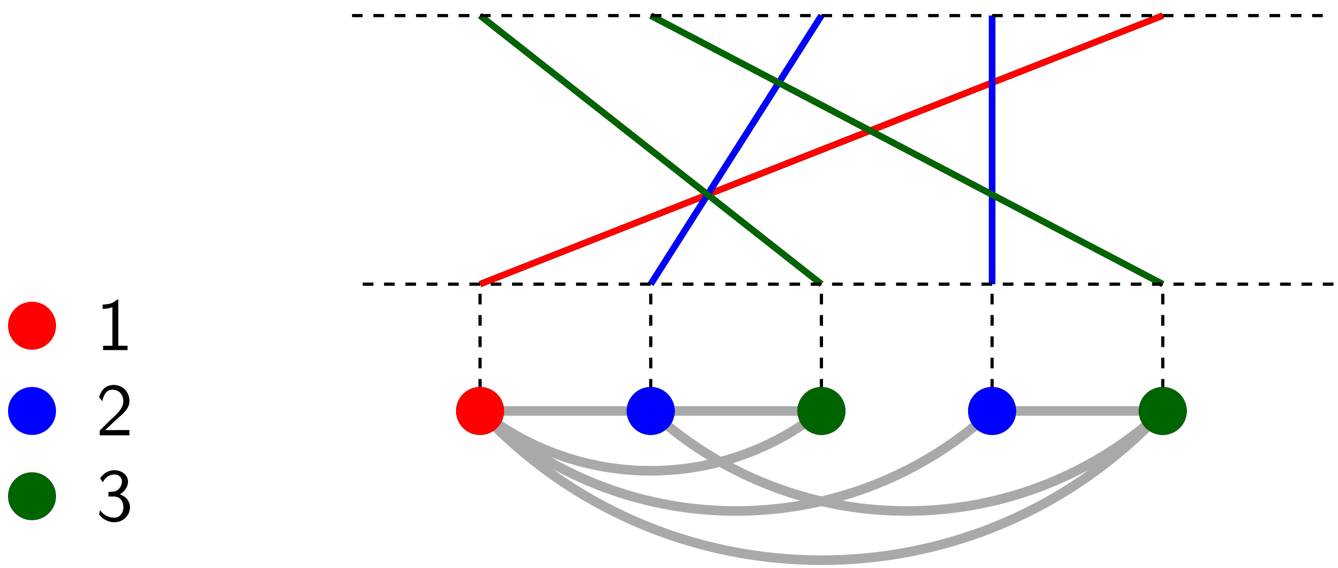
satisfy $\chi = \omega$.

Interval graphs



satisfy $\chi = \omega$.

Permutation graphs



satisfy $\chi = \omega$.

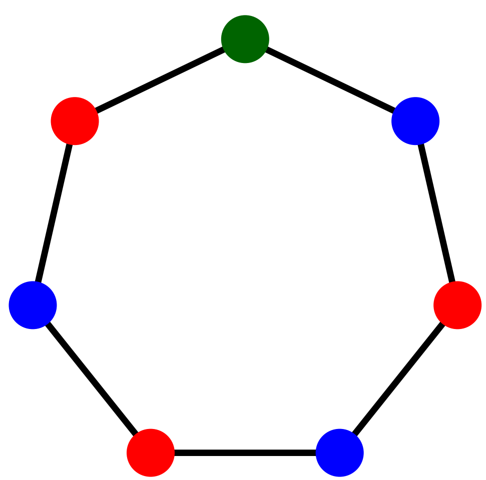
Intersection graphs: vertices – geometric objects
edges – intersecting pairs of objects

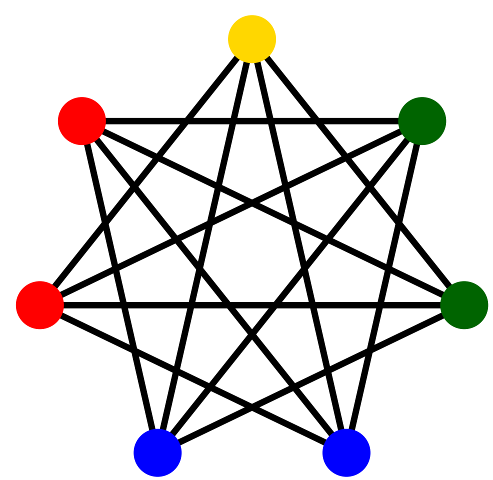
A graph is perfect if it satisfies $\chi = \omega$ and so does its every induced subgraph.

Interval graphs and permutation graphs are perfect.

Chudnovsky, Robertson, Seymour, Thomas, 2006
The Strong Perfect Graph Theorem
Every imperfect graph contains an induced subgraph that is

- a cycle of odd length $\geqslant 5$, or
- the complement of a cycle of odd length $\geqslant 5$.





A graph is perfect if it satisfies $\chi = \omega$ and so does its every induced subgraph.

Interval graphs and permutation graphs are perfect.

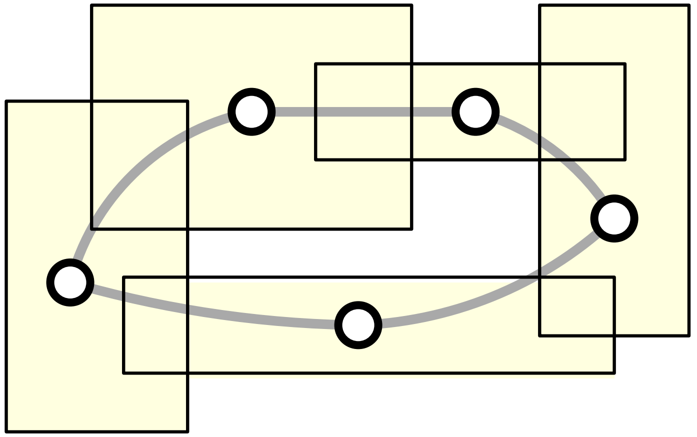
In many natural graph classes, χ is bounded by a function of ω.

Gyárfás, 1987
Problems from the world surrounding perfect graphs

A class of graphs is χ-bounded if there is a function f such that every graph in the class satisfies $\chi \leqslant f(\omega)$.

Asplund, Grünbaum, 1960

On a coloring problem

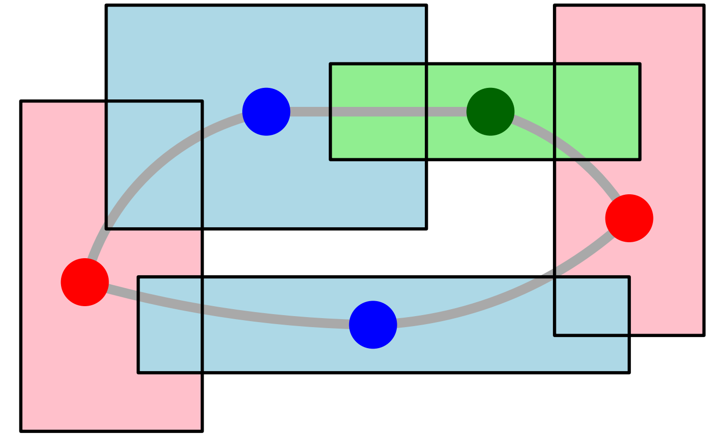


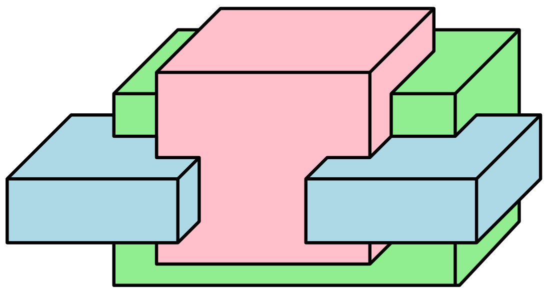
rectangle graphs

Asplund, Grünbaum, 1960

On a coloring problem

Rectangle graphs satisfy $\chi = O(\omega^2)$.





How about box graphs?

Burling, 1965

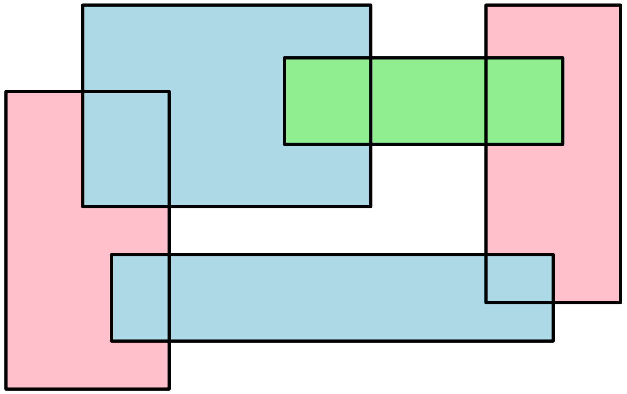
On coloring problems of families of polytopes

cited as:

On coloring problems of families of prototypes

There are triangle-free box graphs with arbitrarily large chromatic number.

Rectangle graphs



construction

$\chi = 3\omega$

Kostochka, 2004

upper bound

$\chi = O(\omega^2)$

Asplund, Grünbaum, 1960

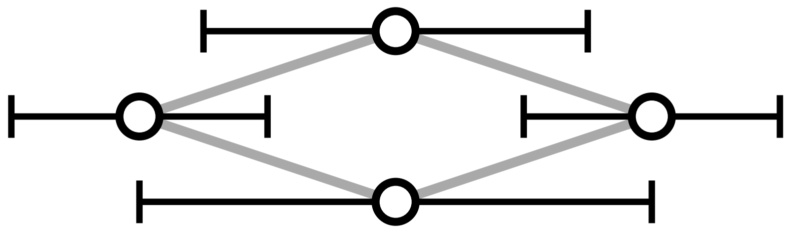
better $\chi = O(\omega^2)$

Hendler, 1998

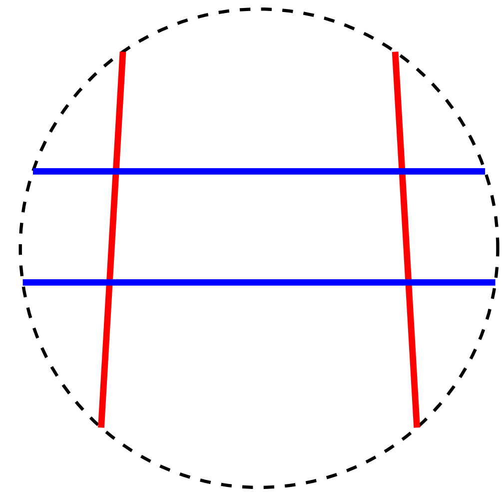
What is the truth?

Gyárfás, 1985

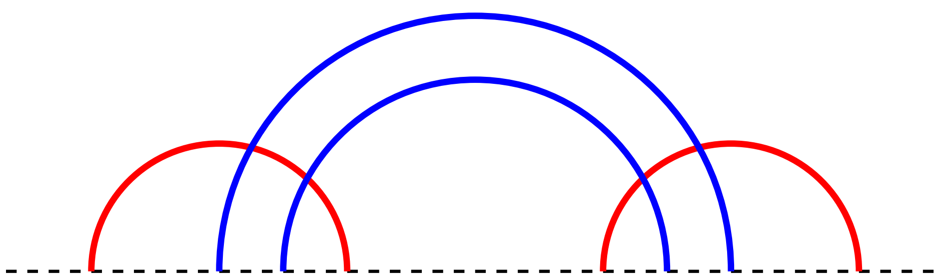
On the chromatic number of multiple interval graphs and overlap graphs



overlap graph

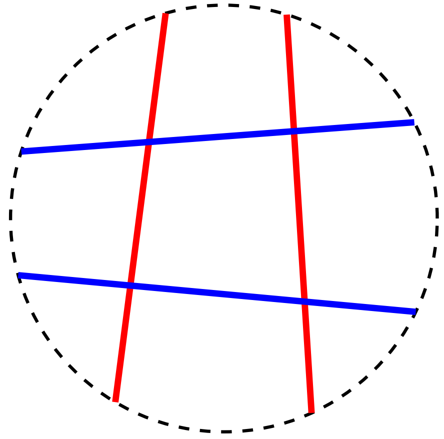


circle graph



Overlap graphs (circle graphs) are χ-bounded.

Circle graphs



upper bound

$$\chi = O(4^{\omega}\omega^2)$$

Gyárfás, 1985

$$\chi = O(2^{\omega}\omega^2)$$

Kostochka, 1988

$$\chi = O(2^{\omega})$$

Kostochka, Kratochvíl, 1997

better $\chi = O(2^{\omega})$

Černý, 2007

construction

$$\chi = \Theta(\omega \log \omega)$$

Kostochka, 1988

What is the truth?

Gyárfás, 1987

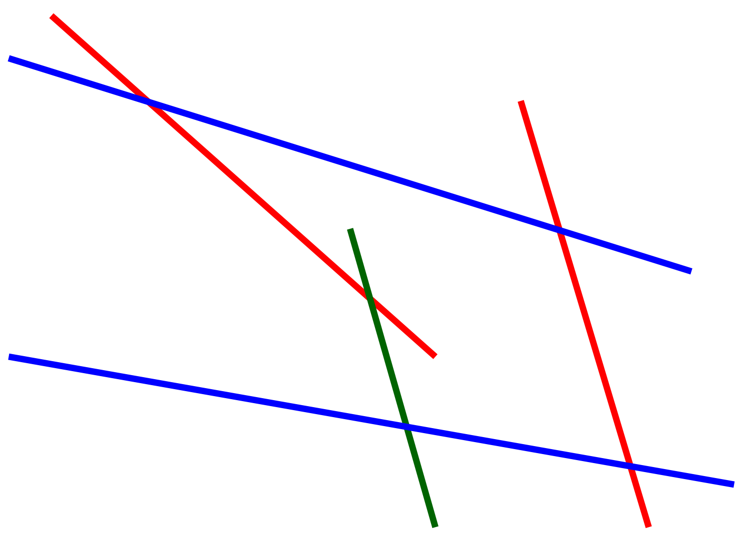
Problems from the world surrounding perfect graphs

Problems attributed to Erdős

Are segment graphs χ-bounded?

Are unit-length segment graphs χ-bounded?

Are complements of segment graphs χ-bounded?



Gyárfás, 1987

Problems from the world surrounding perfect graphs

Problems attributed to Erdős

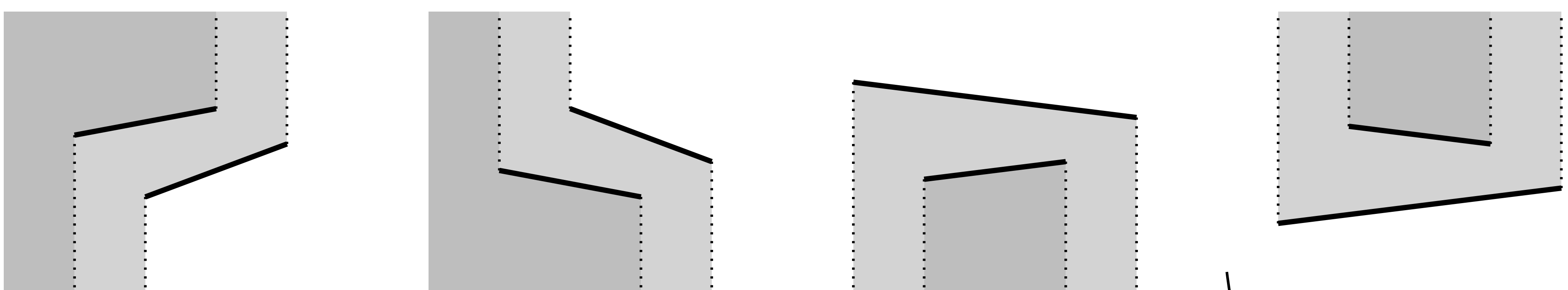
Are segment graphs χ-bounded?

Are unit-length segment graphs χ-bounded?

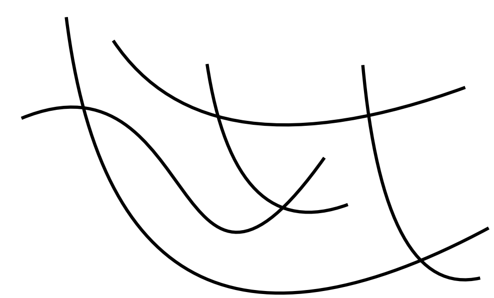
Are complements of segment graphs χ-bounded? Yes

Larman, Matoušek, Pach, Törőcsik, 1994

Complements of segment graphs satisfy $\chi \leqslant \omega^4$.



The same works for disjointness graphs of x-monotone curves.



Gyárfás, 1987

Problems from the world surrounding perfect graphs

Problems attributed to Erdős

Are segment graphs χ-bounded?

Are unit-length segment graphs χ-bounded? Yes

Are complements of segment graphs χ-bounded? Yes

Larman, Matoušek, Pach, Törőcsik, 1994

Complements of segment graphs satisfy $\chi \leqslant \omega^4$.

Suk, 2014

Unit-length segment graphs are χ-bounded.

Gyárfás, 1987
Problems from the world surrounding perfect graphs

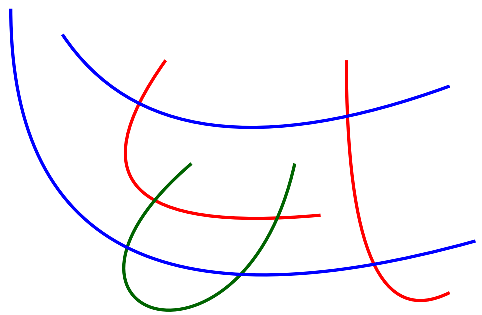
Problems attributed to Erdős

Are segment graphs χ-bounded?	No
Are unit-length segment graphs χ-bounded?	Yes
Are complements of segment graphs χ-bounded?	Yes

Larman, Matoušek, Pach, Törőcsik, 1994
Complements of segment graphs satisfy $\chi \leqslant \omega^4$.

Suk, 2014
Unit-length segment graphs are χ-bounded.

Pawlik, Kozik, Krawczyk, Lasoń, Micek, Trotter, W, 2014
There are triangle-free segment graphs with arbitrarily large chromatic number.



string graphs

Benzer, 1959
On the topology of the genetic fine structure

Sinden, 1966
Topology of thin film RC circuits

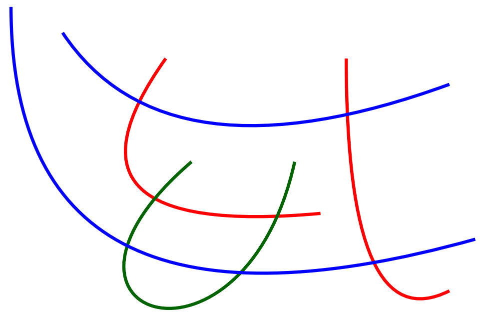
Kratochvíl, 1991
String graphs. I. The number of critical nonstring graphs is infinite
String graphs. II. Recognizing string graphs is NP-hard

Kratochvíl, Matoušek, 1991
String graphs requiring exponential representations

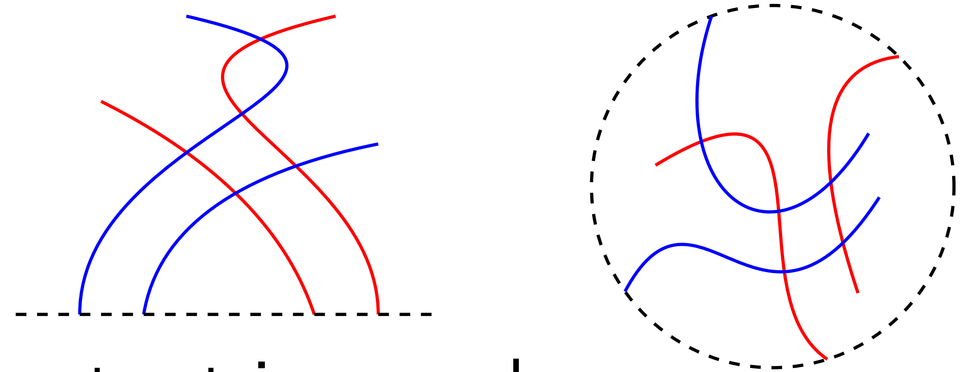
Pach, Tóth, 2001
Recognizing string graphs is decidable

Schaefer, Štefankovič, 2001
Decidability of string graphs

Schaefer, Sedgwick, Štefankovič, 2003
Recognizing string graphs in NP



string graphs



outerstring graphs

Kratochvíl, 1991

String graphs. I. The number of critical nonstring graphs is infinite

String graphs. II. Recognizing string graphs is NP-hard

Middendorf, Pfeiffer, 1993

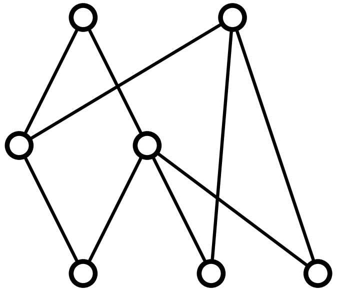
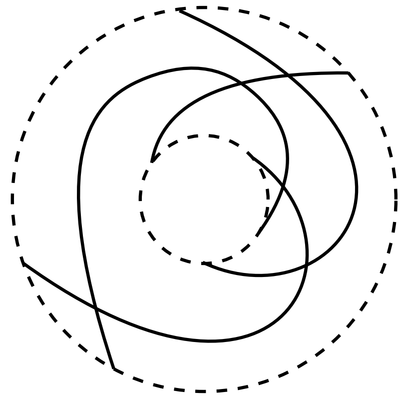
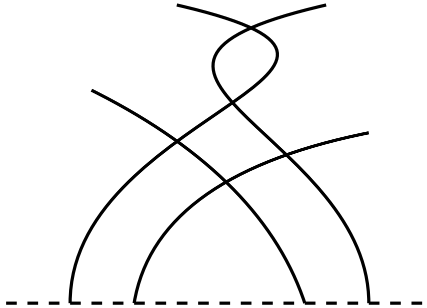
Weakly transitive orientations, Hasse diagrams and string graphs

– alternative proof that recognizing string graphs is NP-hard

Note added in proof

Though not stated there explicitly, their method can be used directly to prove that recognition of outerstring graphs is NP-hard as well.

Middendorf, Pfeiffer, 1993

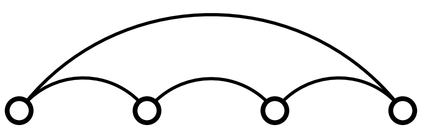
 $\subseteq$  $\subseteq$ 

Hasse diagrams co-cylinder graphs co-outerstring graphs

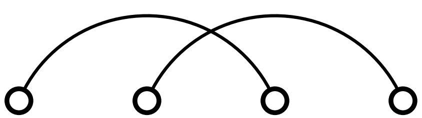
$\supseteq$ for triangle-free

Sinden, 1966

Co-outerstring graphs exclude induced ordered cycles of length $\geqslant 4$.



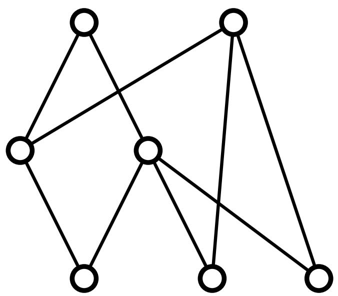
ordered 4-cycle



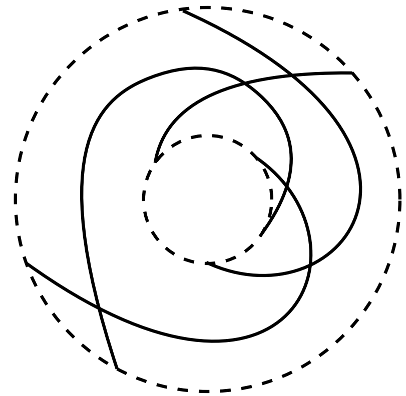
its complement

not realizable in an outerstring graph

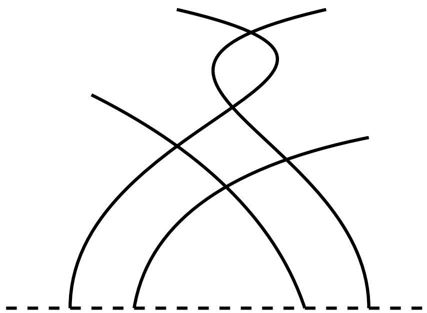
Middendorf, Pfeiffer, 1993



$\subseteq$



$\subseteq$



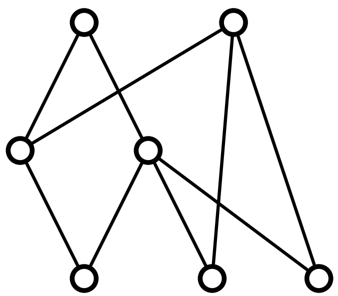
Hasse diagrams | co-cylinder graphs | co-outerstring graphs

$\supseteq$ for triangle-free

Nešetřil, Rödl, 1993; Brightwell, 1993

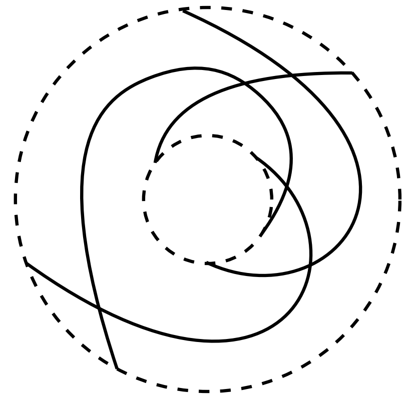
Recognition of Hasse diagrams is NP-hard.

Middendorf, Pfeiffer, 1993



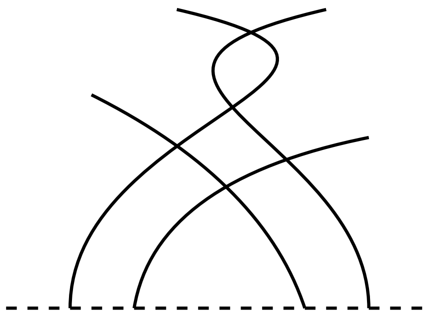
Hasse diagrams

$\subseteq$



co-cylinder graphs

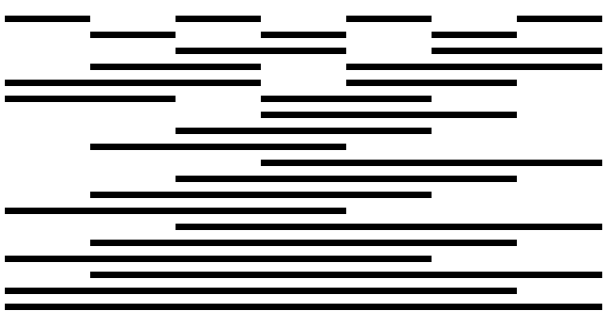
$\subseteq$



co-outerstring graphs

Erdős, Hajnal, 1964

shift graphs

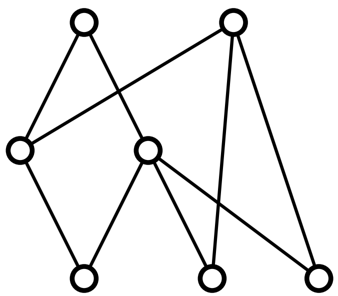


vertices: (a, b), $1 \leqslant a < b \leqslant m$

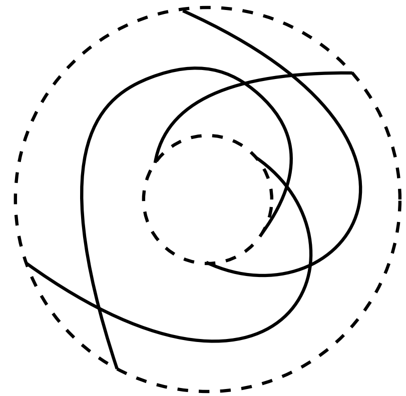
edges: touching intervals

$\omega = 2$

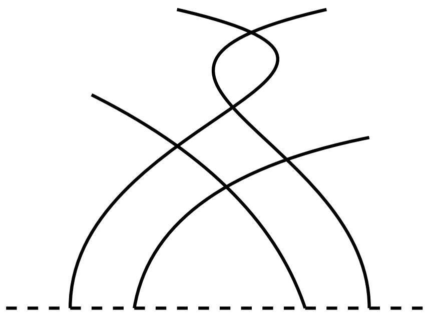
Middendorf, Pfeiffer, 1993



$\subseteq$



$\subseteq$



Hasse diagrams — co-cylinder graphs — co-outerstring graphs

Erdős, Hajnal, 1964

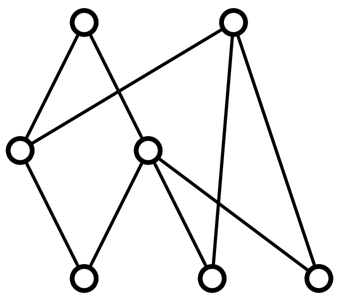
shift graphs

vertices: (a, b), $1 \leqslant a < b \leqslant m$

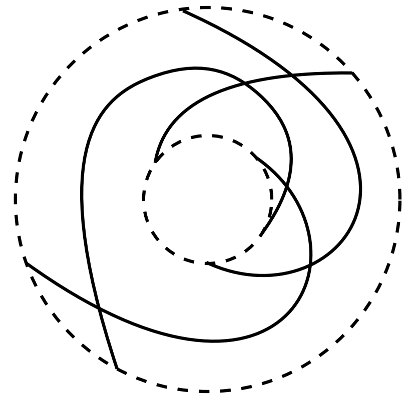
edges: touching intervals

$\omega = 2$ $\quad \chi \geqslant \lceil \log_2 m \rceil$

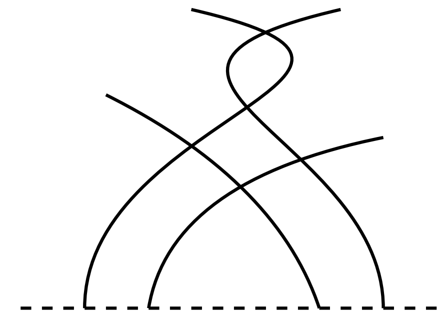
Middendorf, Pfeiffer, 1993



$\subseteq$



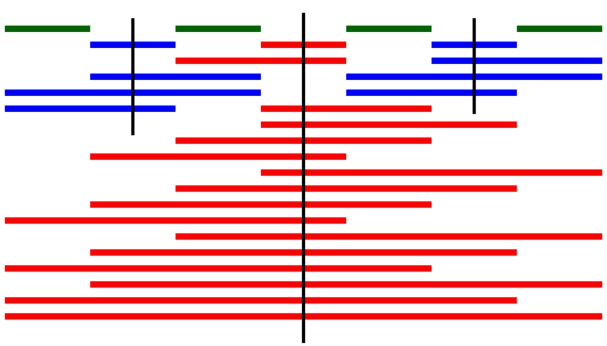
$\subseteq$



Hasse diagrams | co-cylinder graphs | co-outerstring graphs

Erdős, Hajnal, 1964

shift graphs

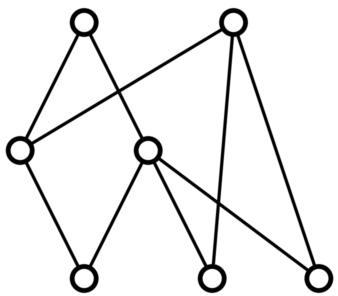
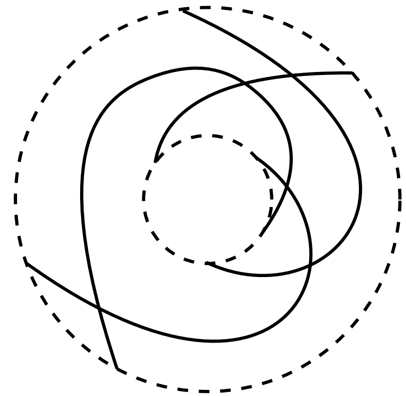
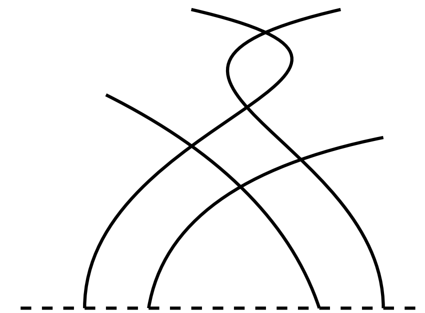


vertices: (a, b), $1 \leqslant a < b \leqslant m$

edges: touching intervals

$\omega = 2$ $\quad \chi = \lceil \log_2 m \rceil$

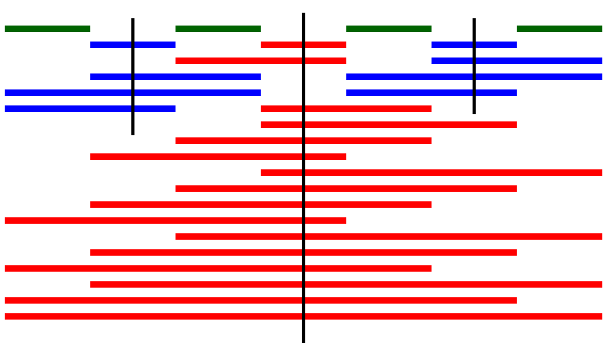
Middendorf, Pfeiffer, 1993

 $\subseteq$  $\subseteq$ 

Hasse diagrams | co-cylinder graphs | co-outerstring graphs

Erdős, Hajnal, 1964

shift graphs



vertices: (a, b), $1 \leqslant a < b \leqslant m$

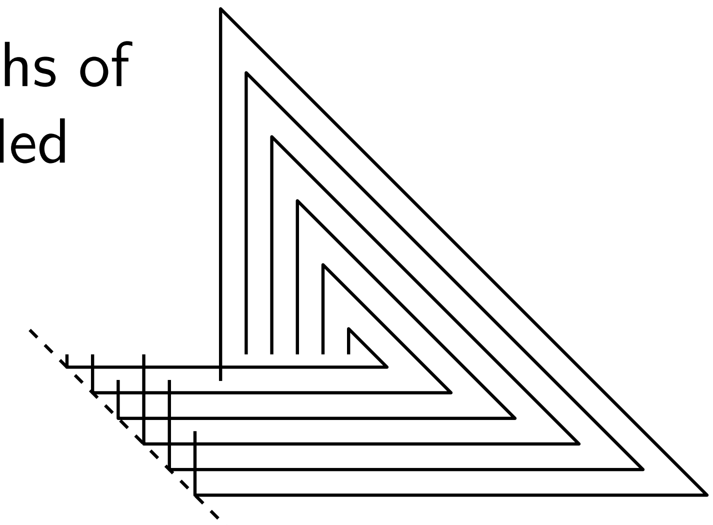
edges: touching intervals

$\omega = 2$ $\quad \chi = \lceil \log_2 m \rceil$

Pach, Tardos, Tóth, 2017; Mütze, W, Wiechert

Shift graphs are disjointness graphs of 1-intersecting curves.

Are disjointness graphs of 1-intersecting grounded curves χ-bounded?



Rok, W, 2014

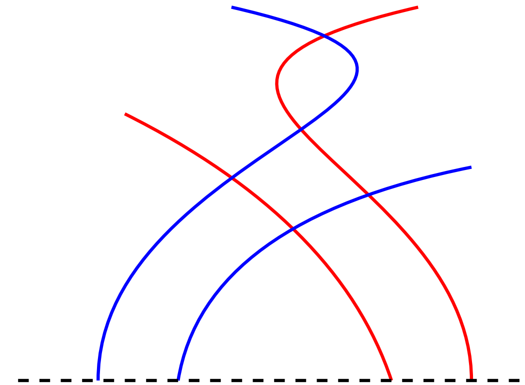
Outerstring graphs are χ-bounded

builds on earlier work:

McGuinness, 1996 and 2000

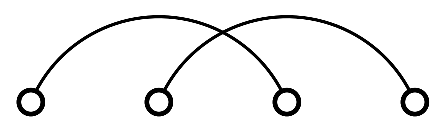
Suk, 2014

Lasoń, Micek, Pawlik, W, 2014



Sinden, 1966

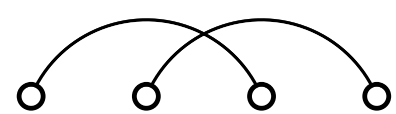
Outerstring graphs exclude complements of induced ordered cycles of length $\geqslant 4$.

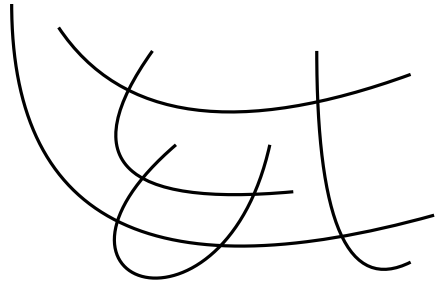


not realizable in an outerstring graph

Tomon, unpublished

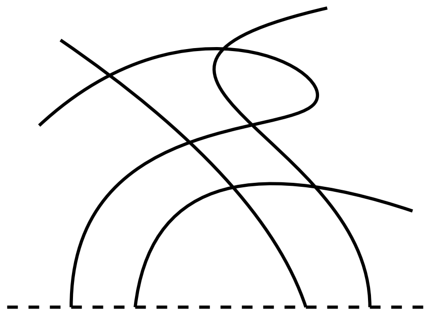
Ordered graphs excluding a fixed non-crossing ordered matching are χ-bounded.

Are ordered graphs excluding induced  χ-bounded?



intersection graphs: not χ-bounded

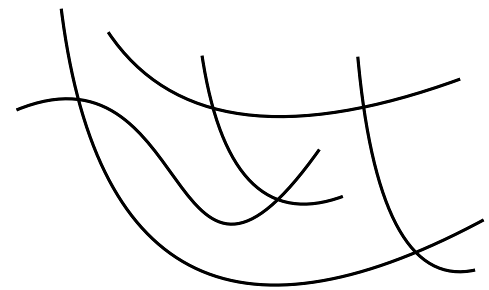
disjointness graphs: not χ-bounded



intersection graphs: χ-bounded
Rok, W, 2014

disjointness graphs: not χ-bounded
Middendorf, Pfeiffer, 1993

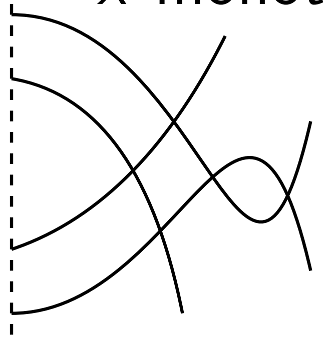
x-monotone



intersection graphs: not χ-bounded
Pawlik et al., 2014

disjointness graphs: χ-bounded
Larman et al., 1994

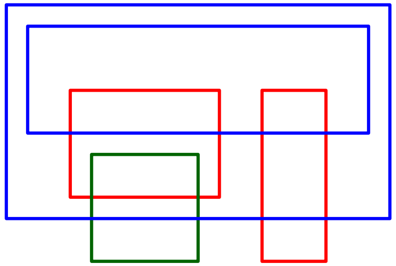
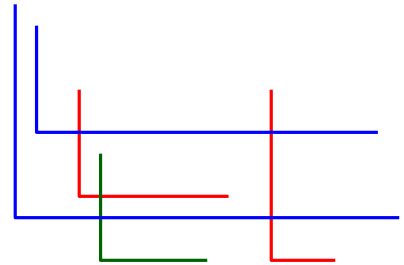
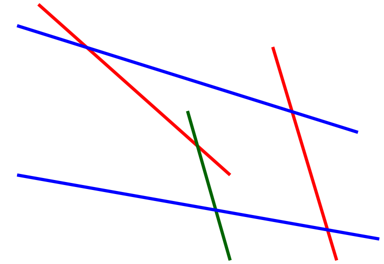
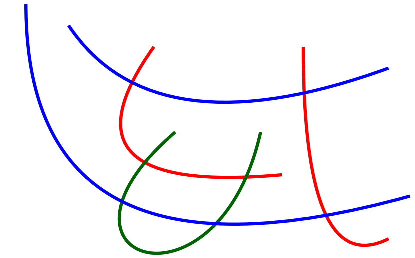
x-monotone



intersection graphs: χ-bounded

disjointness graphs: χ-bounded

Pach, Tomon, 2019
$\chi \leqslant \binom{\omega+1}{2}$ tight!

$\underbrace{\text{downward frame graphs} \subseteq \text{L-graphs} \subseteq \text{segment graphs}}_{}$ $\subseteq$ string graphs

$\Omega(\log \log n)$ for $\omega = 2$
Pawlik et al. 2013

$\Omega_\omega((\log \log n)^{\omega-1})$
Krawczyk, W 2017

$O_\omega((\log \log n)^{\omega-1})$
Krawczyk, W 2017

$O_\omega(\log n)$
McGuinness 1996

$O_\omega(\log n)$
Suk 2014

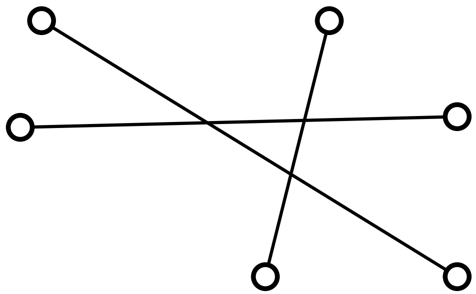
$(\log n)^{O(\log \omega)}$
Fox, Pach 2014

$O(\log \log n)$ for $\omega = 2$
W 2019

$\Theta(\sqrt{n/\log n})$ for general triangle-free graphs
Ajtai, Komlós, Szemerédi, 1980; Kim, 1995

Are there triangle-free co-string graphs with $\chi = \Omega(n^\varepsilon)$?

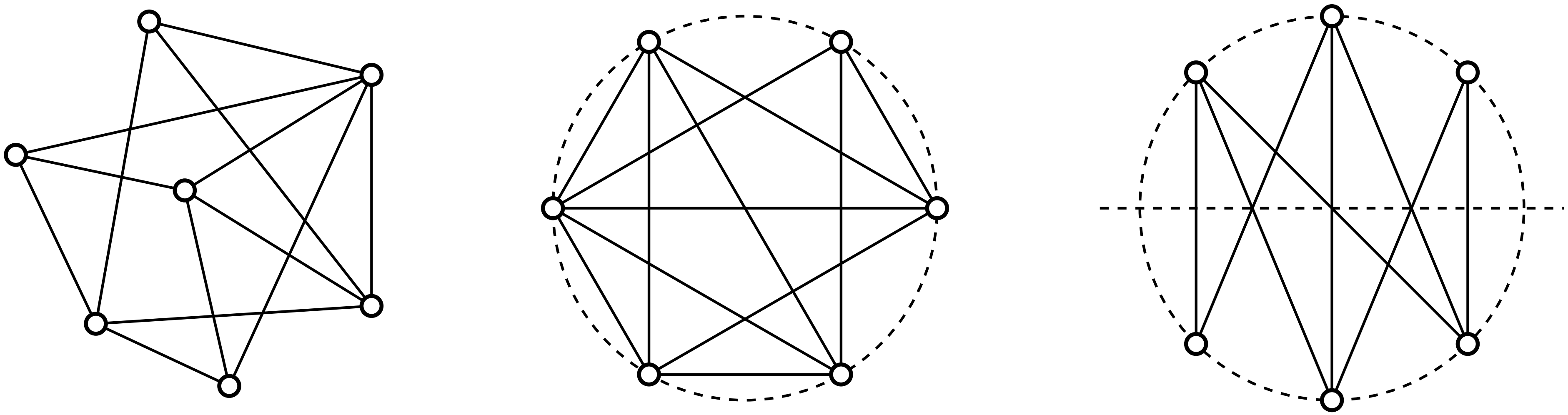
A graph drawn in the plane is k-quasi-planar if no k of its edges pairwise cross.



not 3-quasi-planar

A graph drawn in the plane is k-quasi-planar if no k of its edges pairwise cross.

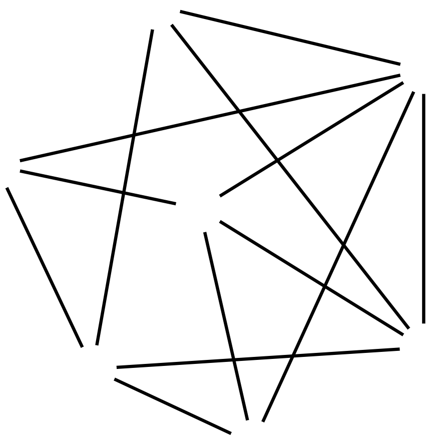
geometric graphs | convex geometric graphs | convex bipartite geometric graphs



How many edges can an n-vertex k-quasi-planar graph have?

A graph drawn in the plane is k-quasi-planar if no k of its edges pairwise cross. $\Rightarrow \omega \leqslant k-1$

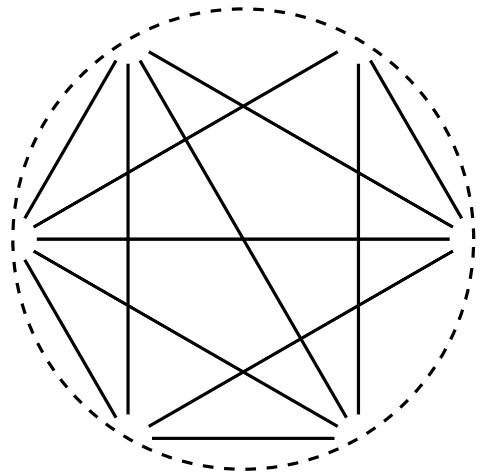
geometric graphs



segment graph

$\chi = O_k(\log n)$

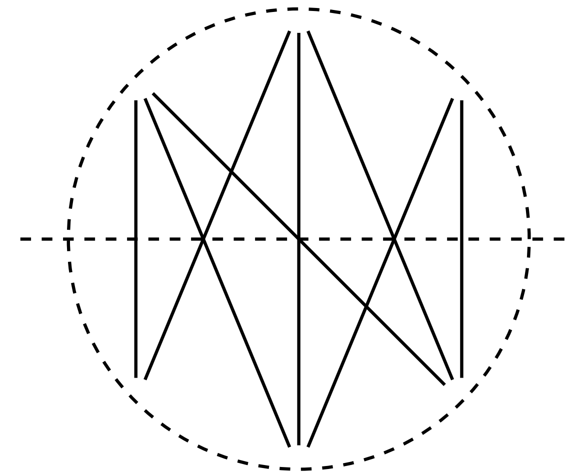
convex geometric graphs



circle graph

$\chi = O_k(1)$

convex bipartite geometric graphs

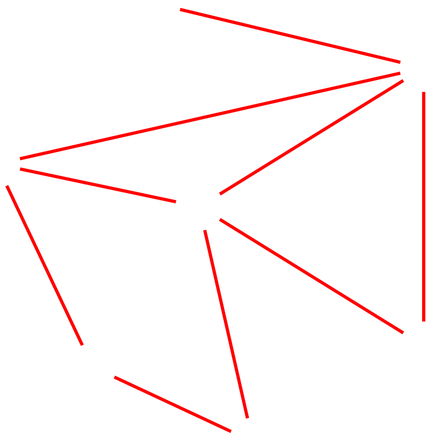


permutation graph

$\chi \leqslant k-1$

A graph drawn in the plane is *k-quasi-planar* $\Rightarrow \omega \leqslant k-1$
if no k of its edges pairwise cross.

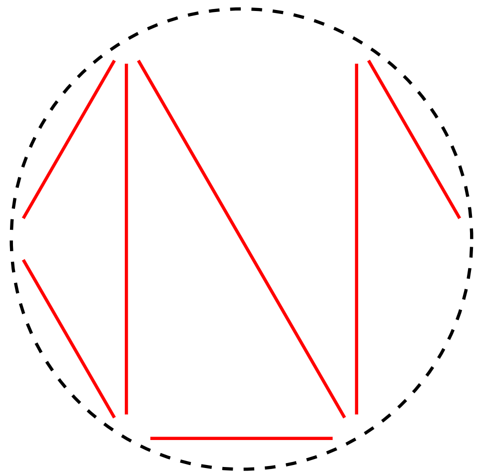
geometric graphs



segment graph

$\chi = O_k(\log n)$

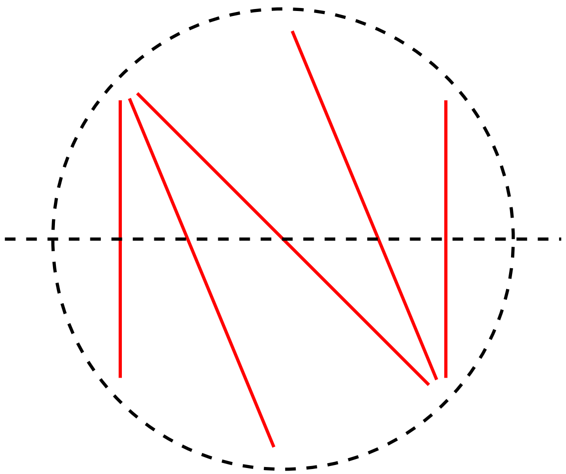
convex geometric graphs



circle graph

$\chi = O_k(1)$

convex bipartite geometric graphs

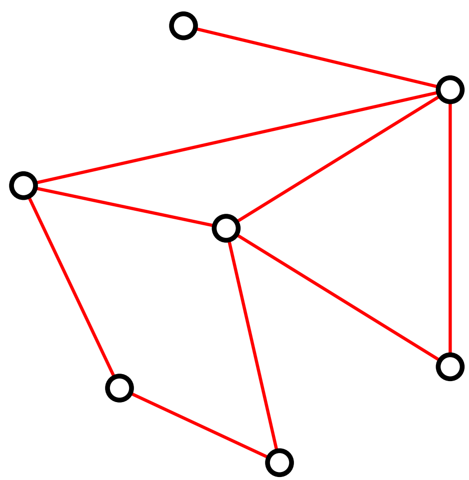


permutation graph

$\chi \leqslant k-1$

A graph drawn in the plane is *k-quasi-planar* if no k of its edges pairwise cross.

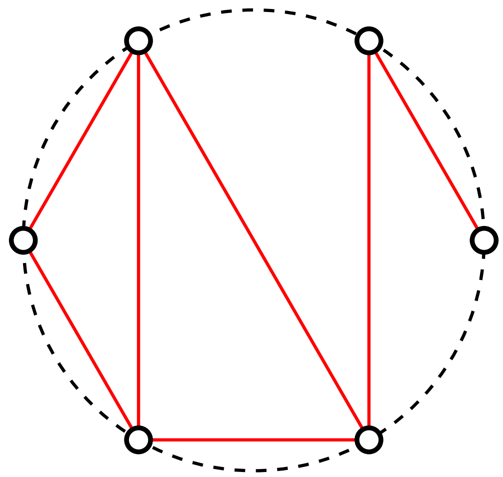
geometric graphs



$\chi = O_k(\log n)$

$O(n)$ edges of one color

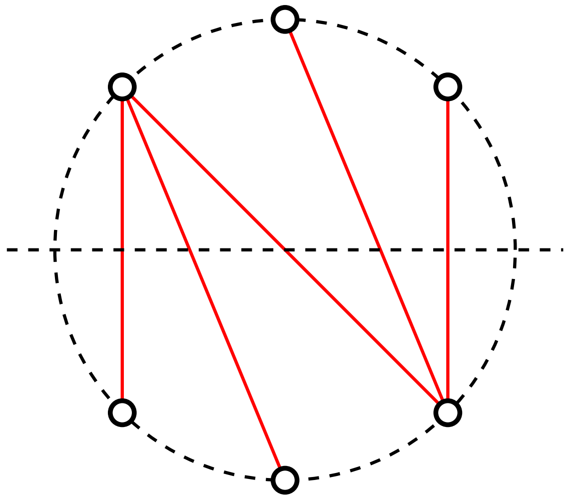
convex geometric graphs



$\chi = O_k(1)$

$O(n)$ edges of one color

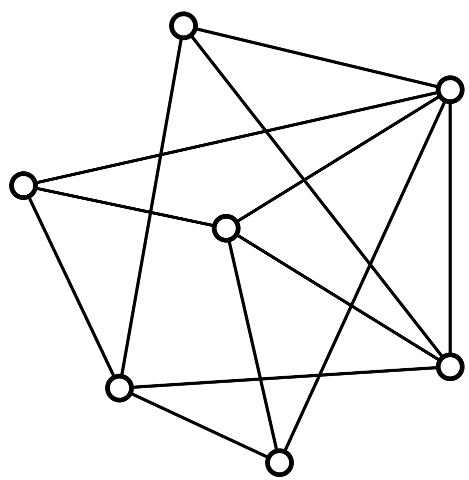
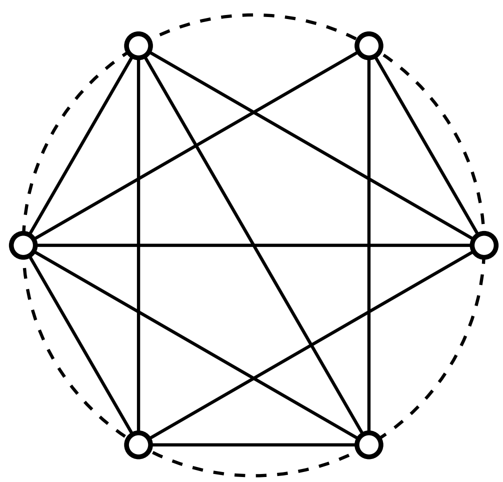
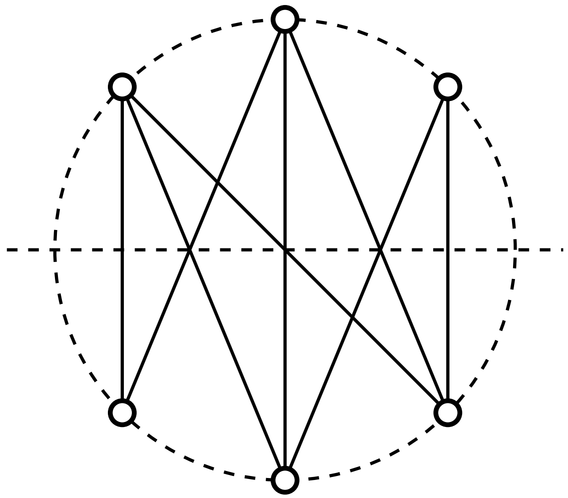
convex bipartite geometric graphs



$\chi \leqslant k - 1$

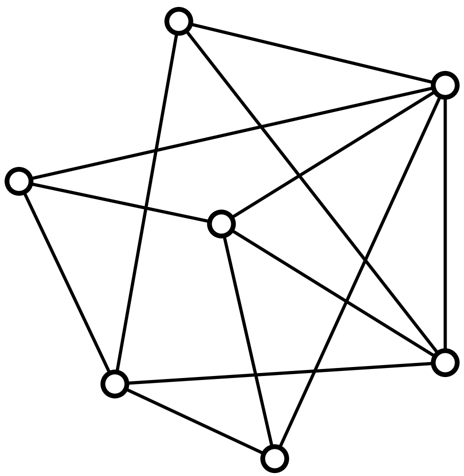
$\leqslant n - 1$ edges of one color

A graph drawn in the plane is *k-quasi-planar* if no k of its edges pairwise cross.

geometric graphs	convex geometric graphs	convex bipartite geometric graphs
		
$O_k(n \log n)$ edges	$O_k(n)$ edges	$\leqslant (k-1)(n-1)$ edges
$\chi = O_k(\log n)$	$\chi = O_k(1)$	$\chi \leqslant k-1$
$O(n)$ edges of one color	$O(n)$ edges of one color	$\leqslant n-1$ edges of one color

A graph drawn in the plane is k-quasi-planar if no k of its edges pairwise cross.

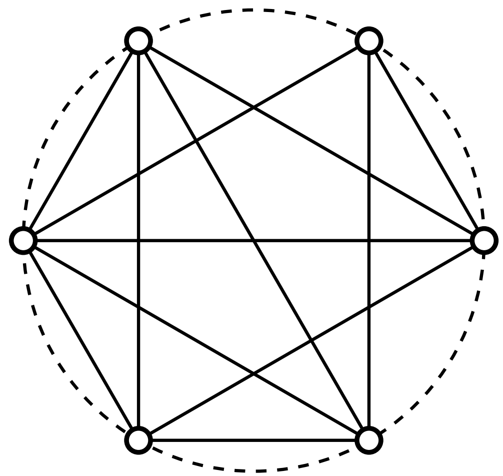
geometric graphs



$O_k(n \log n)$ edges

Valtr, 1997

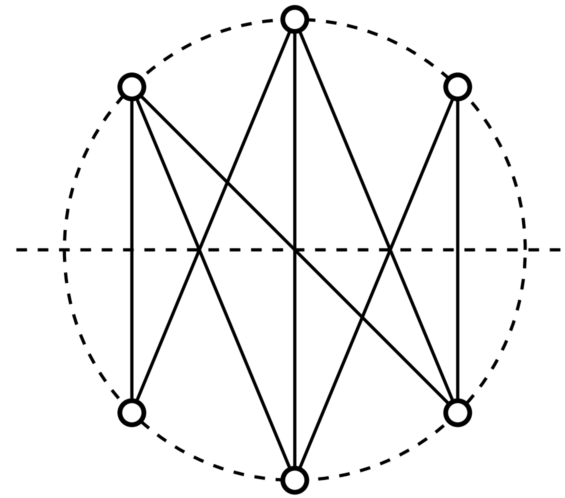
convex geometric graphs



$O_k(n)$ edges

Capoyleas, Pach, 1992

convex bipartite geometric graphs



$\leqslant (k-1)(n-1)$ edges

topological graphs

$n \log^{O(k)} n$ edges

Fox, Pach, 2012

1-intersecting topological graphs

$O_k(n \log n)$ edges

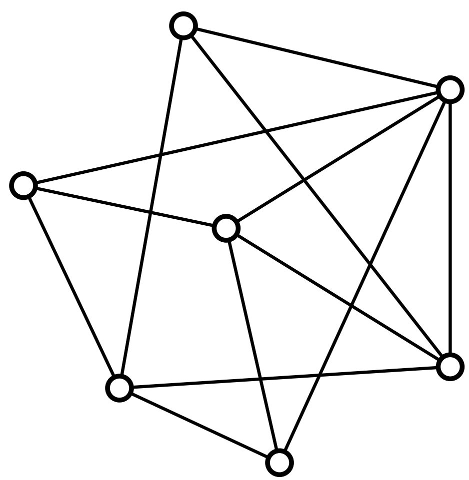
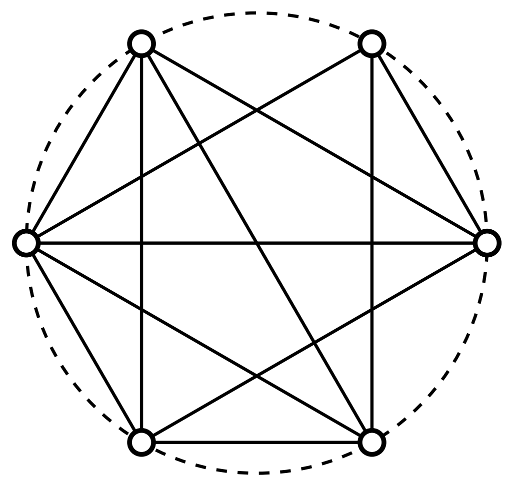
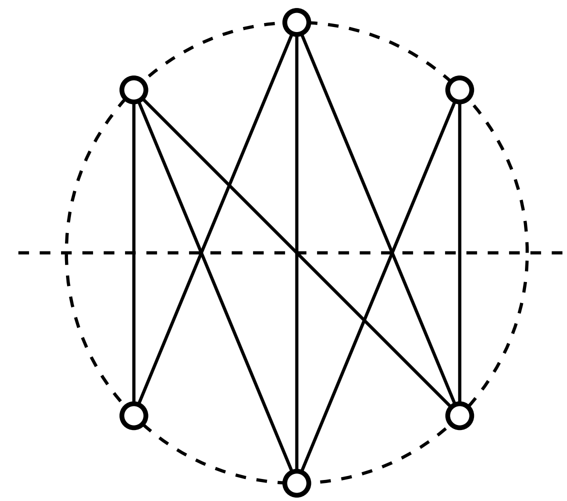
Suk, W, 2015

k-intersecting topological graphs

$O_k(n \log n)$ edges

Rok, W, 2017

A graph drawn in the plane is *k-quasi-planar* if no k of its edges pairwise cross.

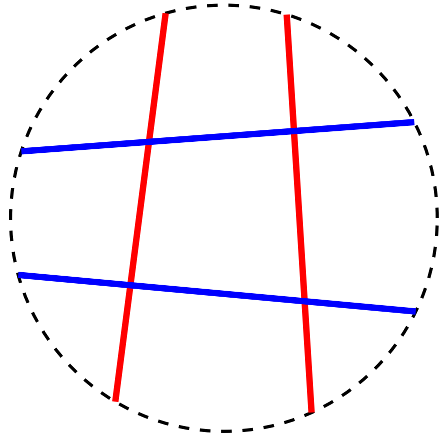
geometric graphs	convex geometric graphs	convex bipartite geometric graphs
		
$O_k(n \log n)$ edges	$O_k(n)$ edges	$\leqslant (k-1)(n-1)$ edges
Valtr, 1997	Capoyleas, Pach, 1992	

Conjecture – Pach, Shahrokhi, Szegedy, 1996

For every k, k-quasi-planar graphs have linearly many edges.

Ackerman, 2009: True up to $k = 4$.

Circle graphs



upper bound

$\chi = O(4^\omega \omega^2)$
Gyárfás, 1985

$\chi = O(2^\omega \omega^2)$
Kostochka, 1988

construction

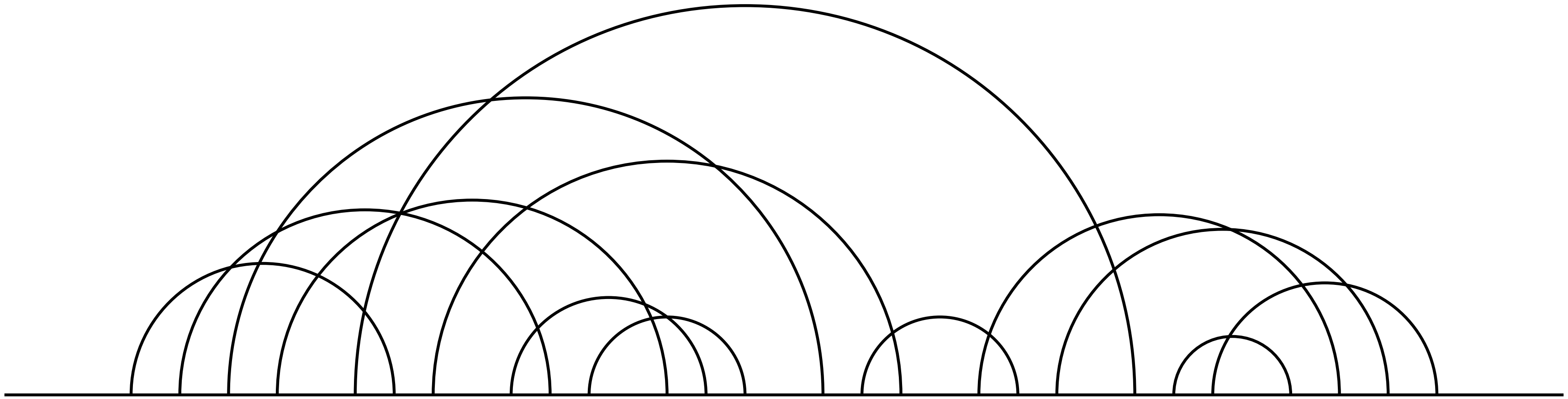
$\chi = \Theta(\omega \log \omega)$
Kostochka, 1988

$\chi = O(2^\omega)$
Kostochka, Kratochvíl, 1997

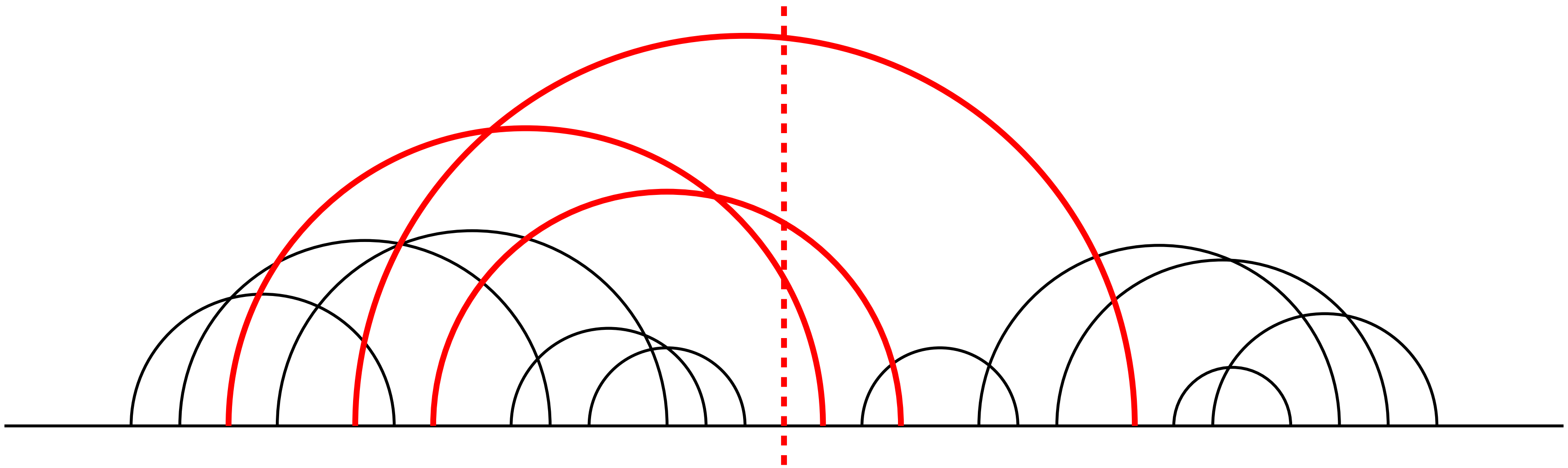
better $\chi = O(2^\omega)$
Černý, 2007

Davies, McCarty, 2019+
Circle graphs are quadratically χ-bounded

Divide-and-conquer?



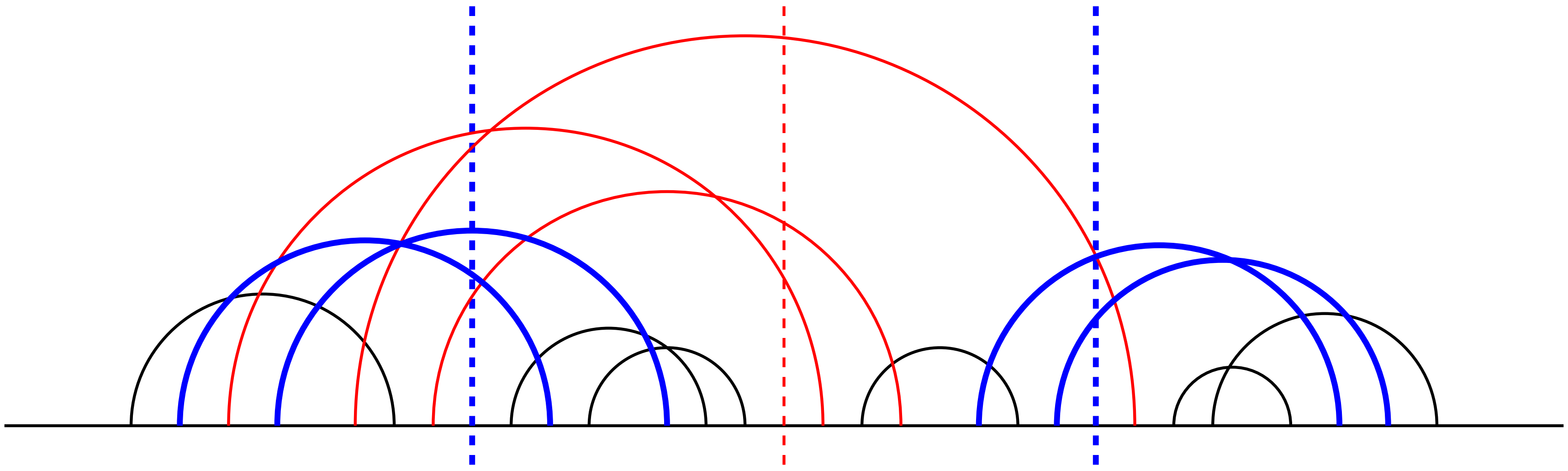
Divide-and-conquer?



permutation graph

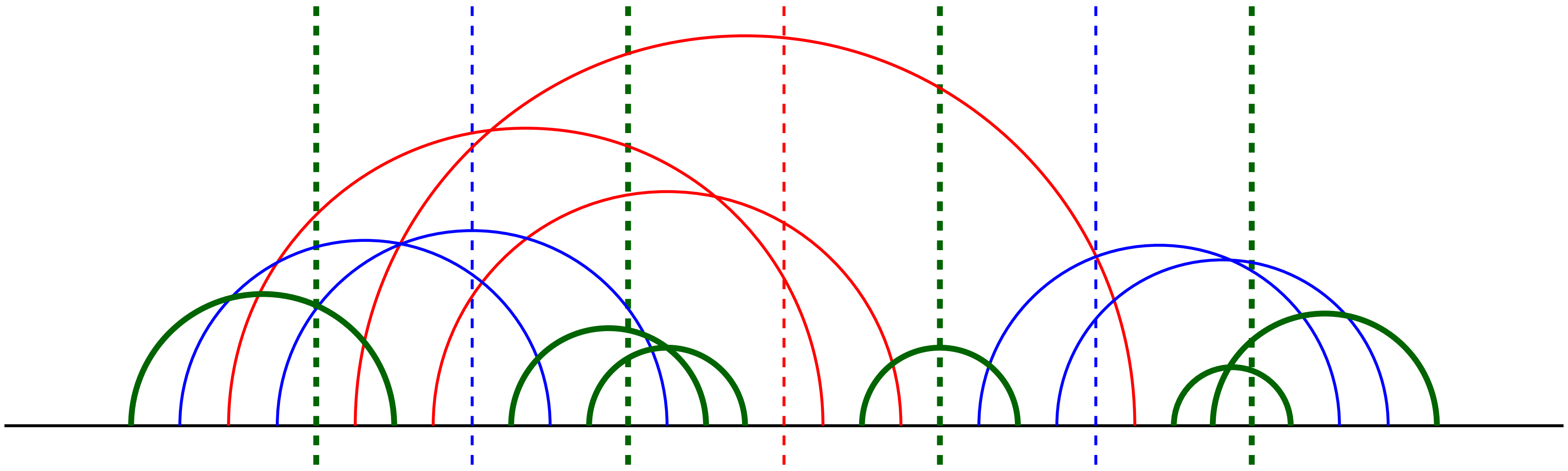
a group of ω colors

Divide-and-conquer?



permutation graphs
a group of ω colors

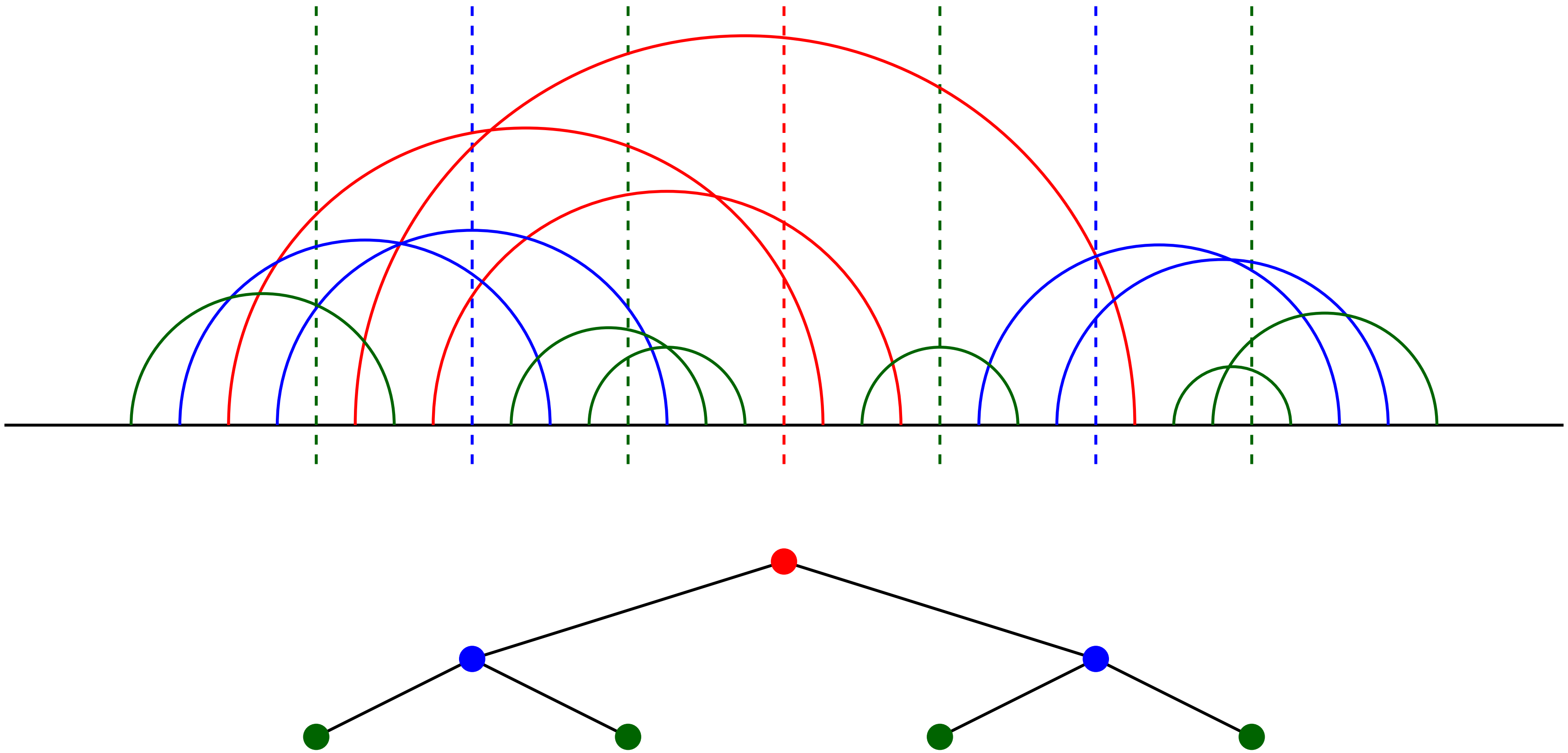
Divide-and-conquer?



permutation graphs

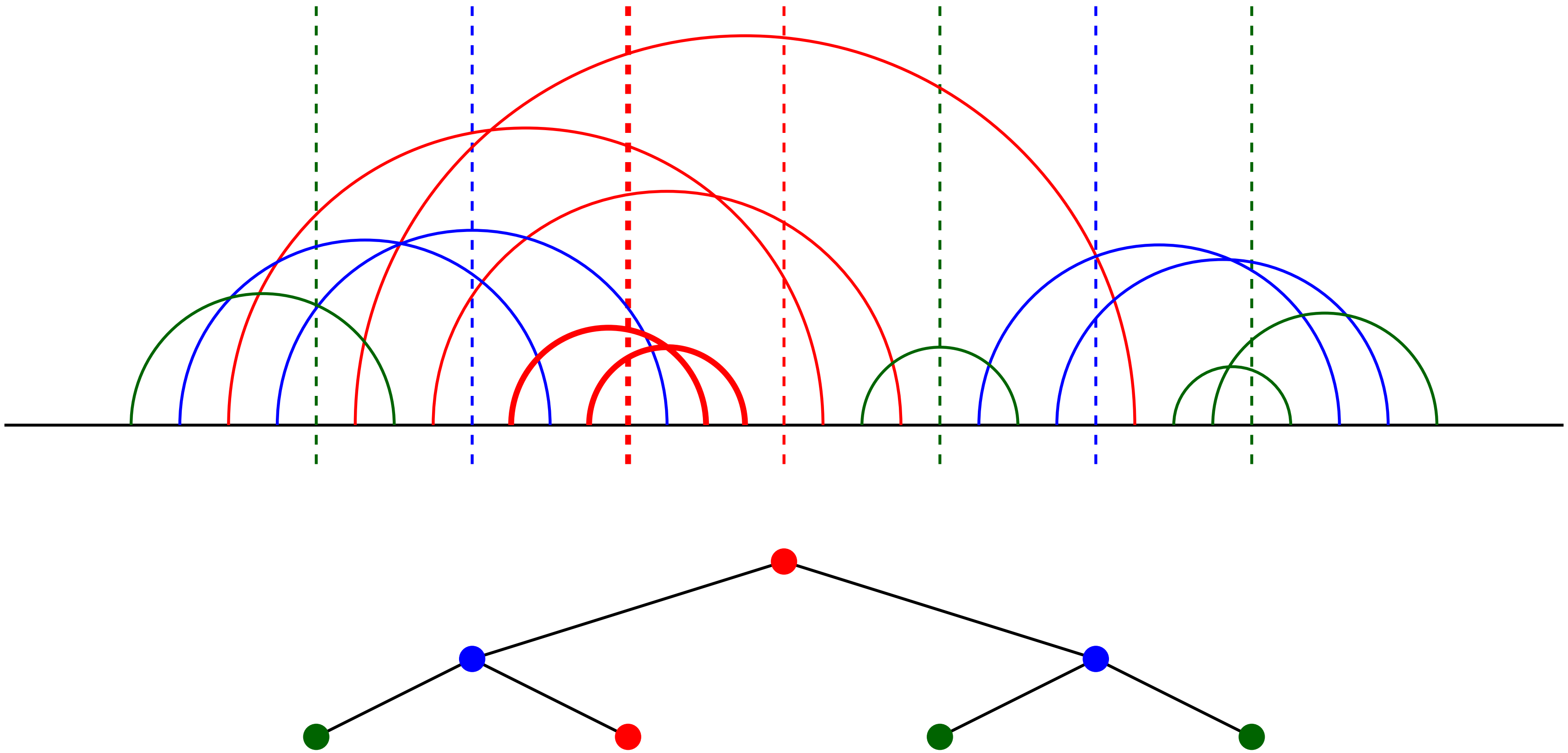
a group of ω colors

Divide-and-conquer?

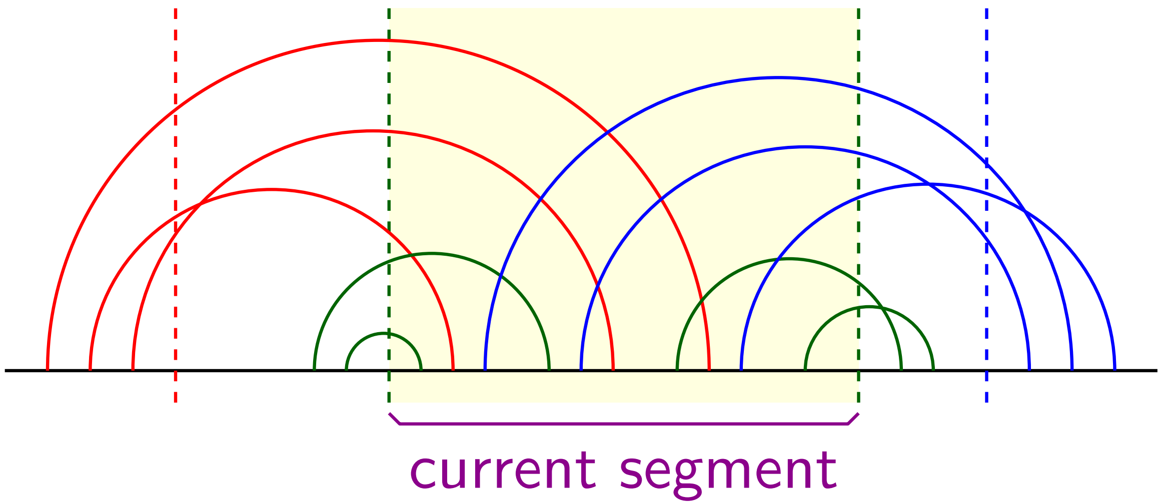


$O(\omega \log n)$ colors ☹

Divide-and-conquer?



Idea: Reuse colors in a smart way



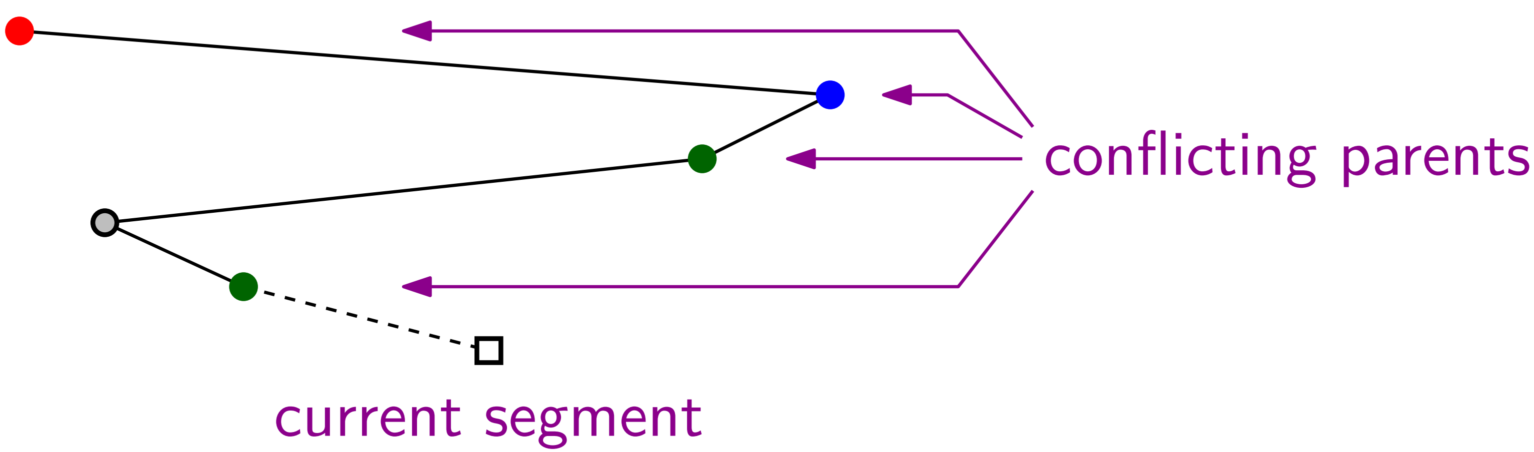
max # color groups:

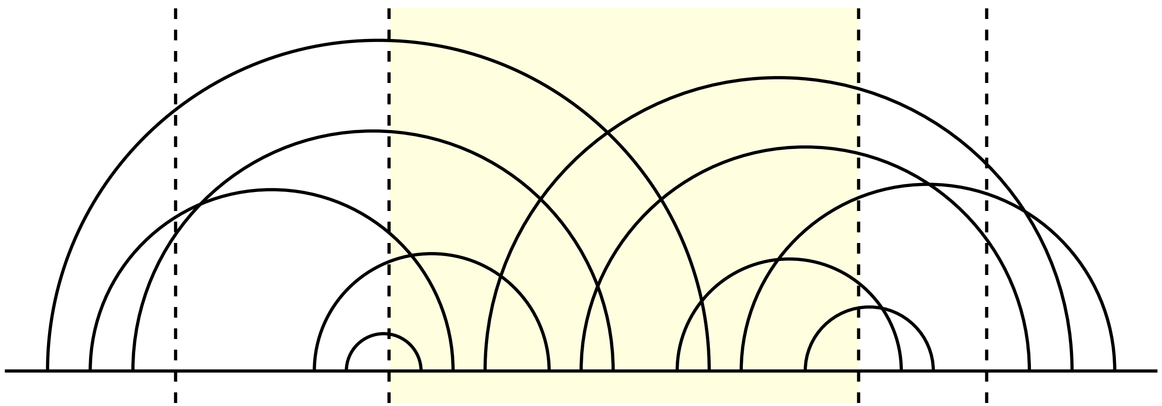
$$\omega + 2m + 1$$

max # conflicting parents:

$$\omega + m + 1$$

m to be determined later





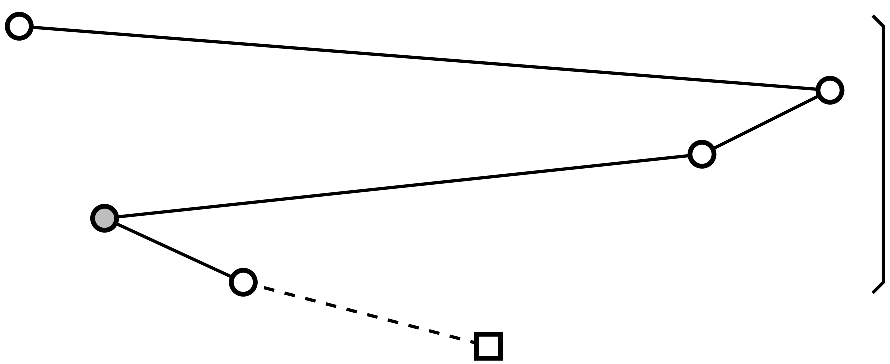
max # color groups:

$$\omega + 2m + 1$$

max # conflicting parents:

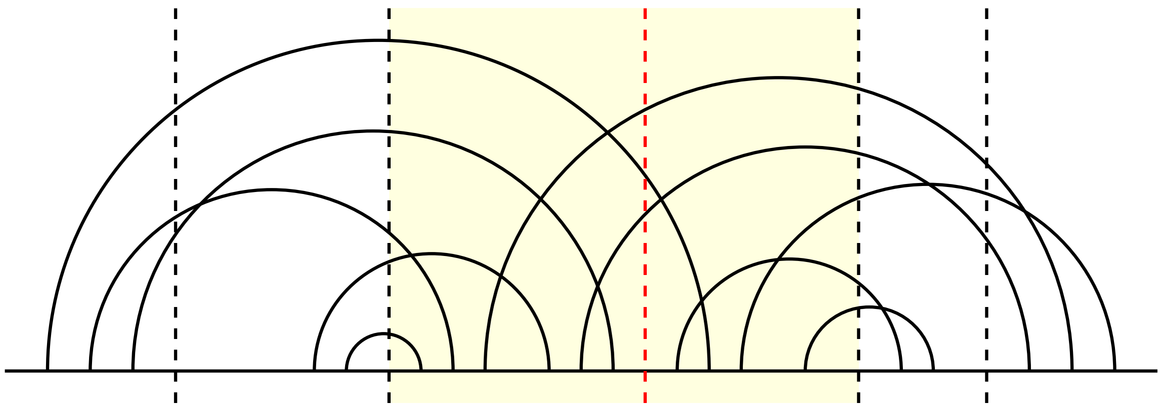
$$\omega + m + 1$$

m to be determined later



Easy case:

$\leqslant \omega + m$ conflicting parents



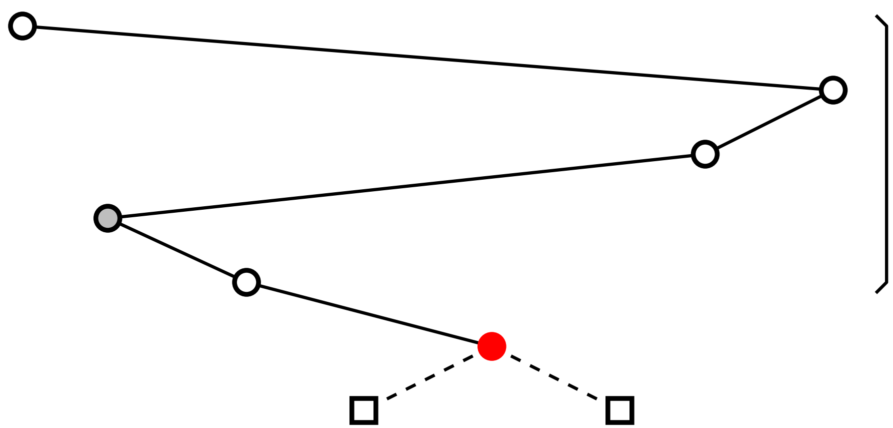
max # color groups:

$$\omega + 2m + 1$$

max # conflicting parents:

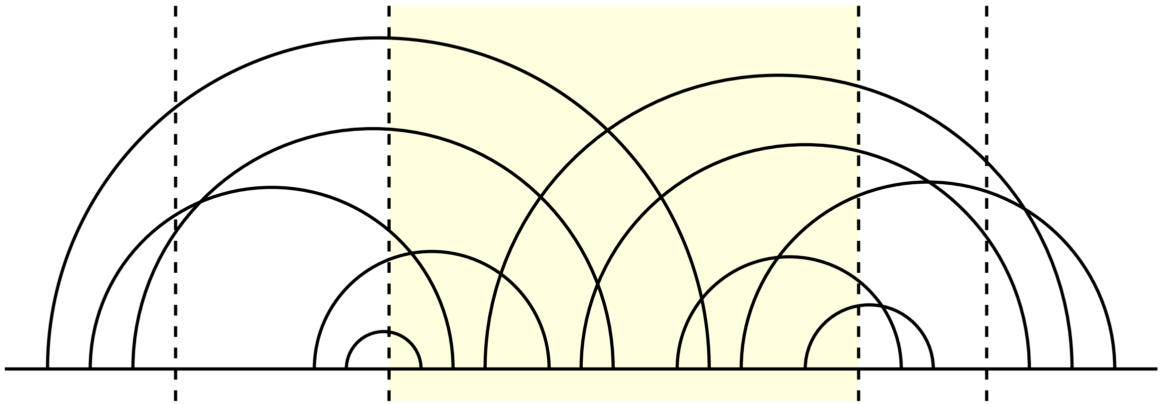
$$\omega + m + 1$$

m to be determined later



Easy case:

$\leqslant \omega + m$ conflicting parents



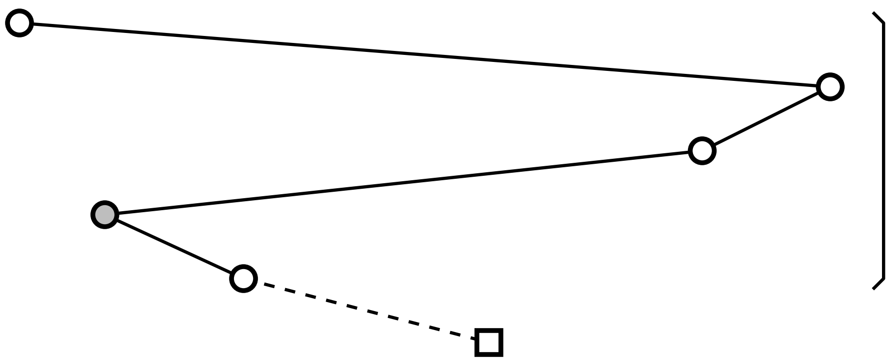
max # color groups:

$$\omega + 2m + 1$$

max # conflicting parents:

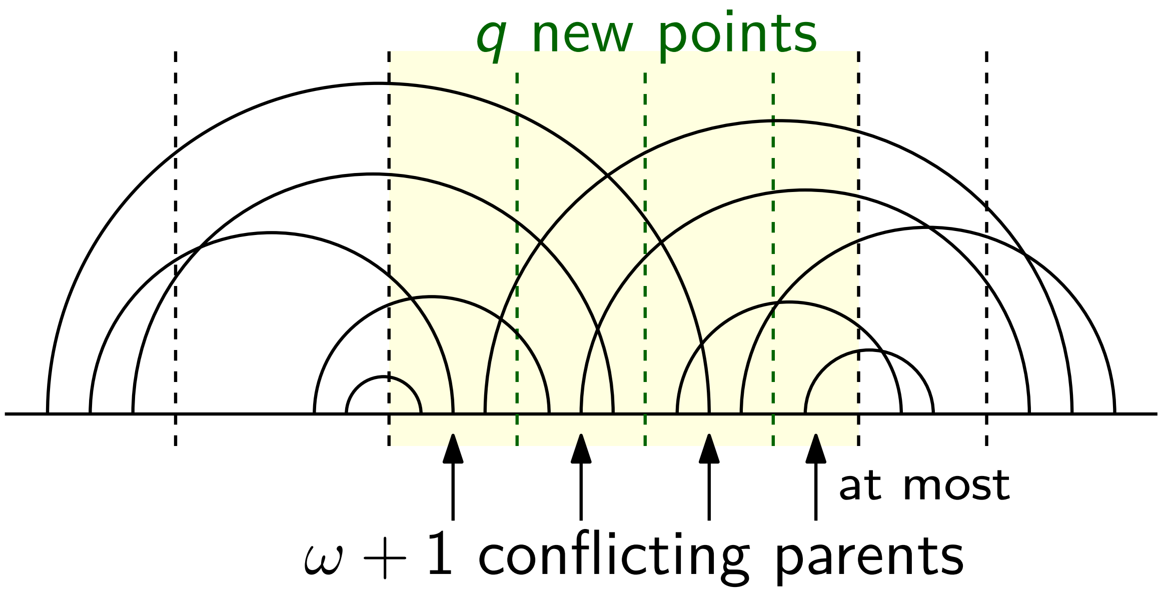
$$\omega + m + 1$$

m to be determined later



Difficult case:

$\omega + m + 1$ conflicting parents



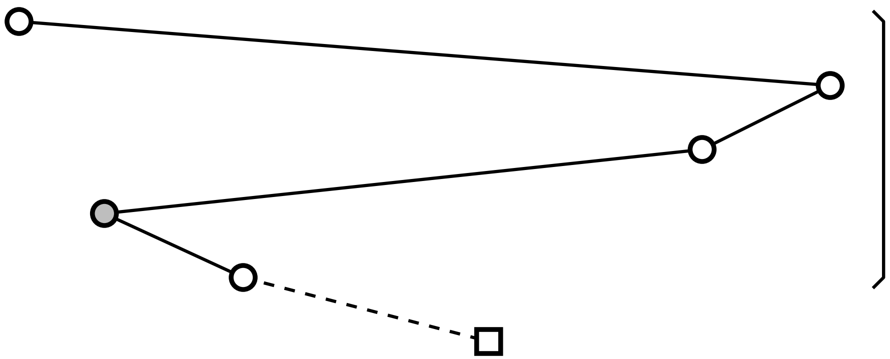
max # color groups:

$$\omega + 2m + 1$$

max # conflicting parents:

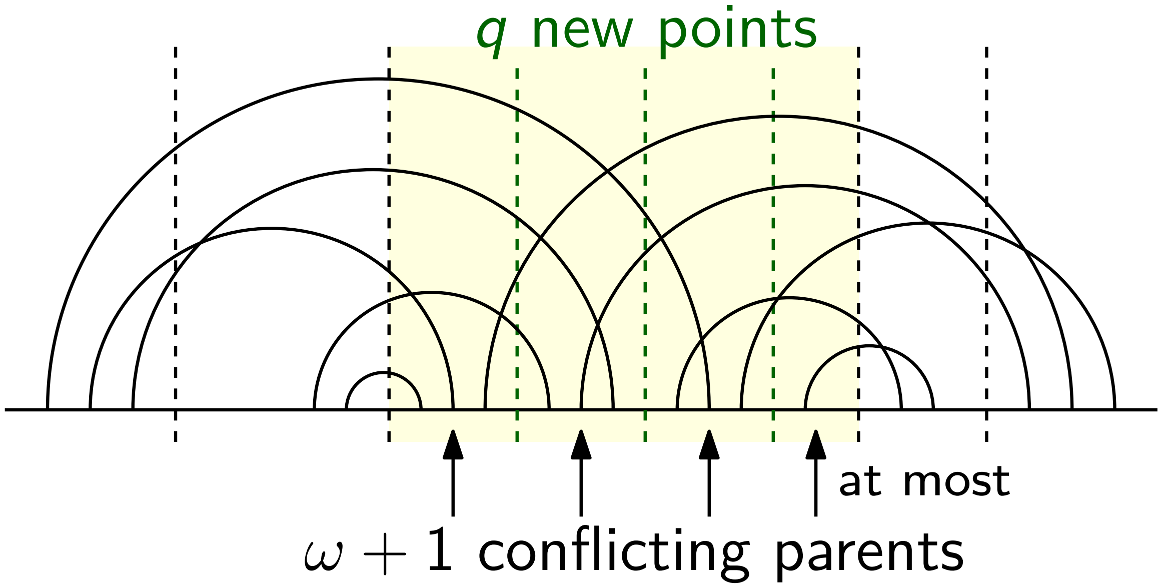
$$\omega + m + 1$$

m to be determined later



Difficult case:

$\omega + m + 1$ conflicting parents



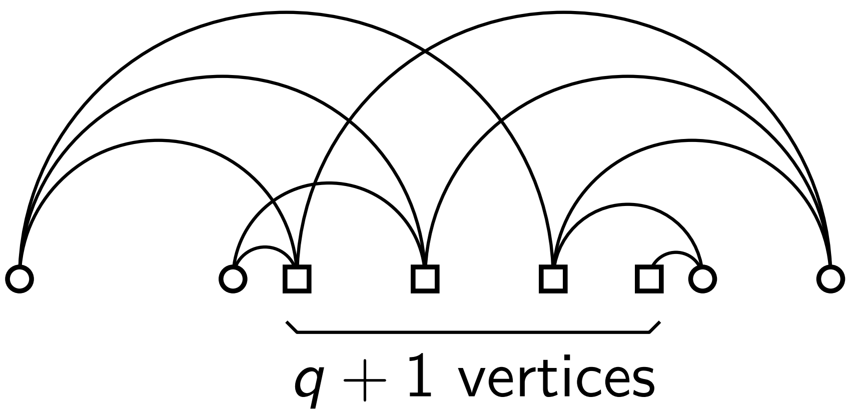
max # color groups:

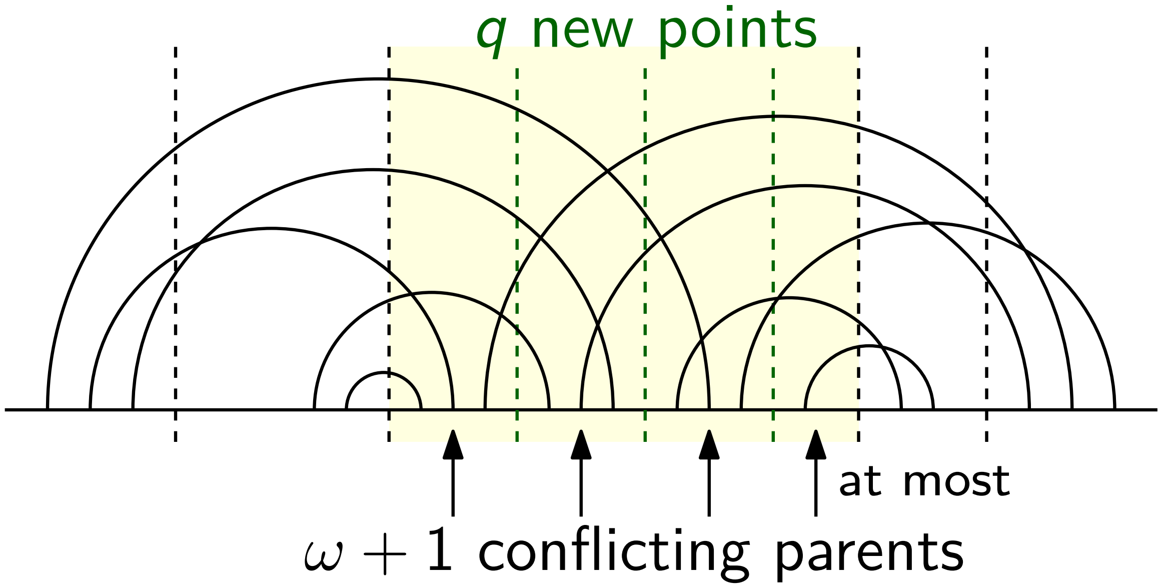
$$\omega + 2m + 1$$

max # conflicting parents:

$$\omega + m + 1$$

m to be determined later





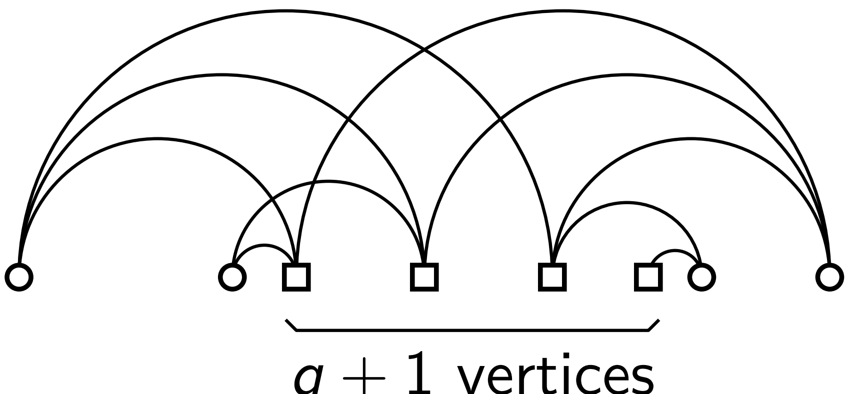
max # color groups:

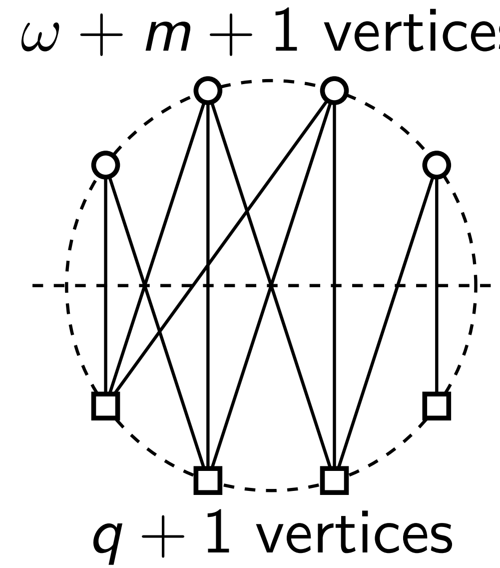
$$\omega + 2m + 1$$

max # conflicting parents:

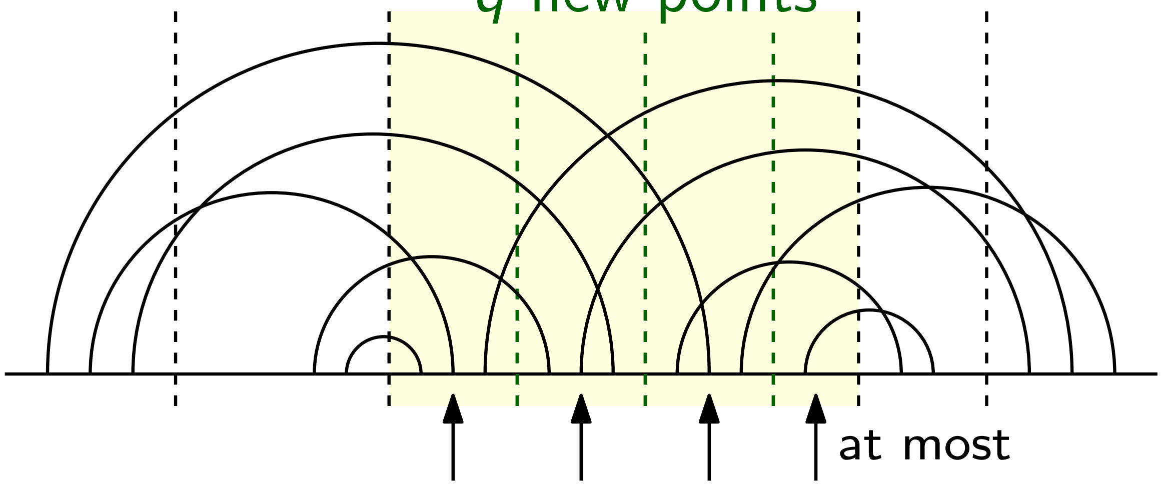
$$\omega + m + 1$$

m to be determined later





convex bipartite $(\omega + 1)$-quasi-planar graph on $q + \omega + m + 2$ vertices q new points



$\omega + 1$ conflicting parents

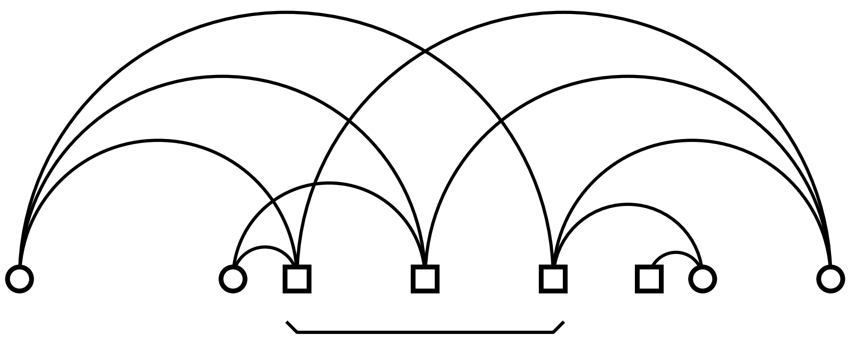
max # color groups:

$$\omega + 2m + 1$$

max # conflicting parents:

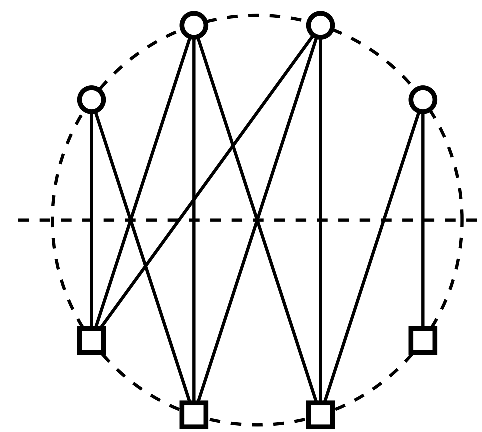
$$\omega + m + 1$$

m to be determined later



q vertices of degree $\omega + 1$

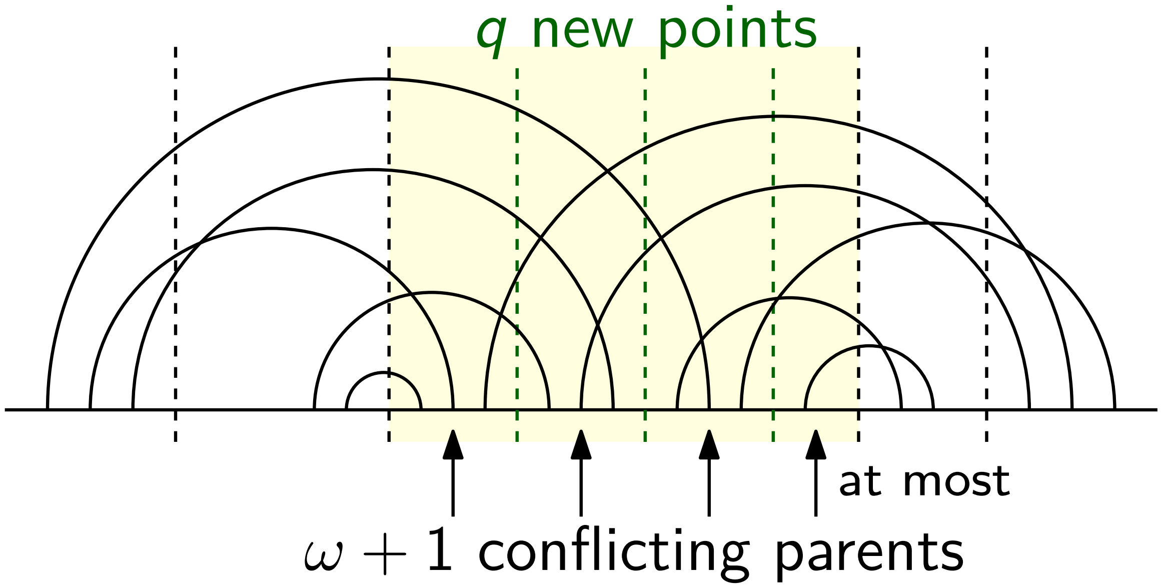
$\omega + m + 1$ vertices



$q + 1$ vertices

convex bipartite $(\omega + 1)$-quasi-planar graph on $q + \omega + m + 2$ vertices

$$q\omega + q = q(\omega + 1) \leqslant \#\,\text{edges} \leqslant (q + \omega + m + 1)\omega = q\omega + \omega^2 + m\omega + \omega$$



max # color groups:

$$\omega + 2m + 1$$

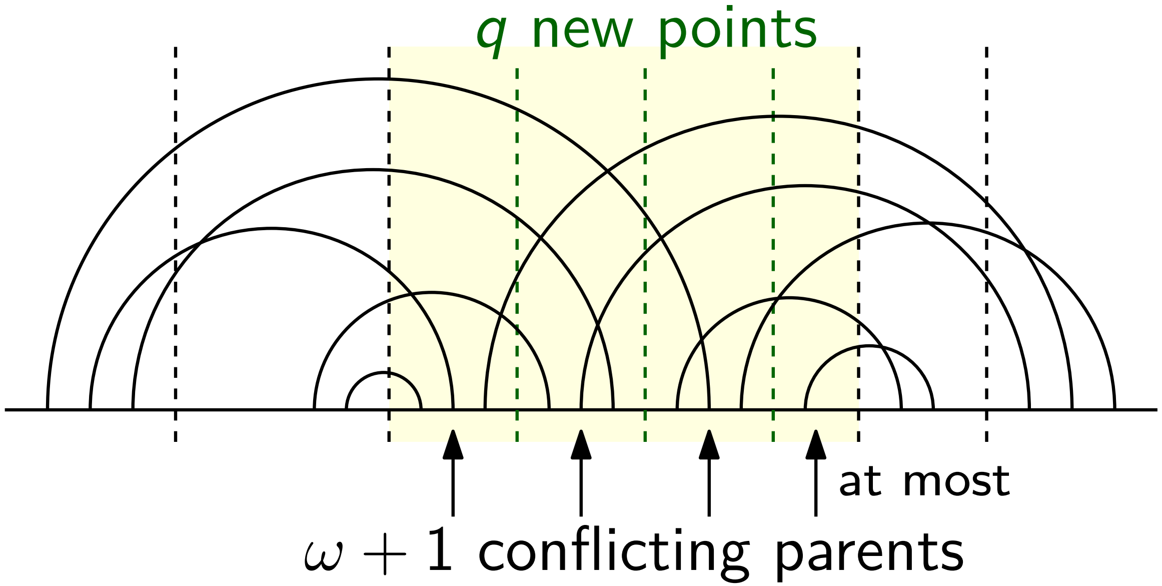
max # conflicting parents:

$$\omega + m + 1$$

m to be determined later

$q \leqslant \omega^2 + m\omega + \omega$

$$\cancel{q\omega} + q = q(\omega + 1) \leqslant \#\,\text{edges} \leqslant (q + \omega + m + 1)\omega = \cancel{q\omega} + \omega^2 + m\omega + \omega$$



max # color groups:

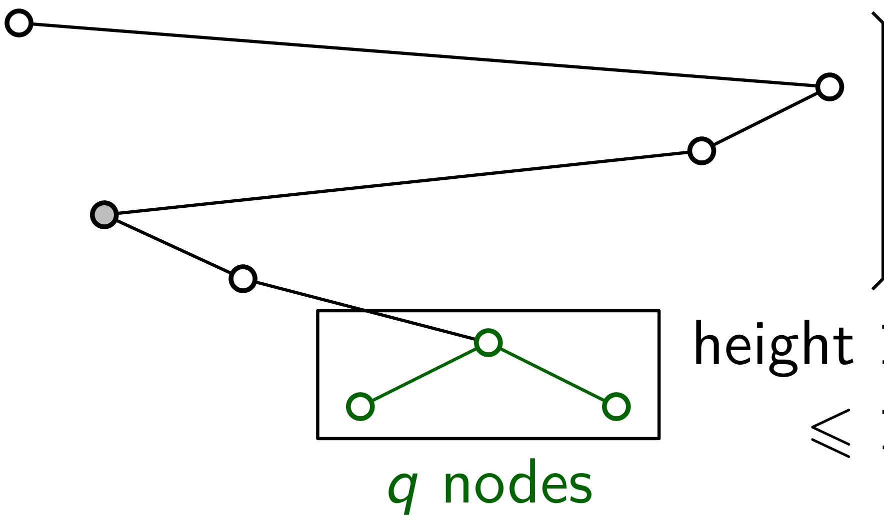
$$\omega+2m+1$$

max # conflicting parents:

$$\omega+m+1$$

m to be determined later

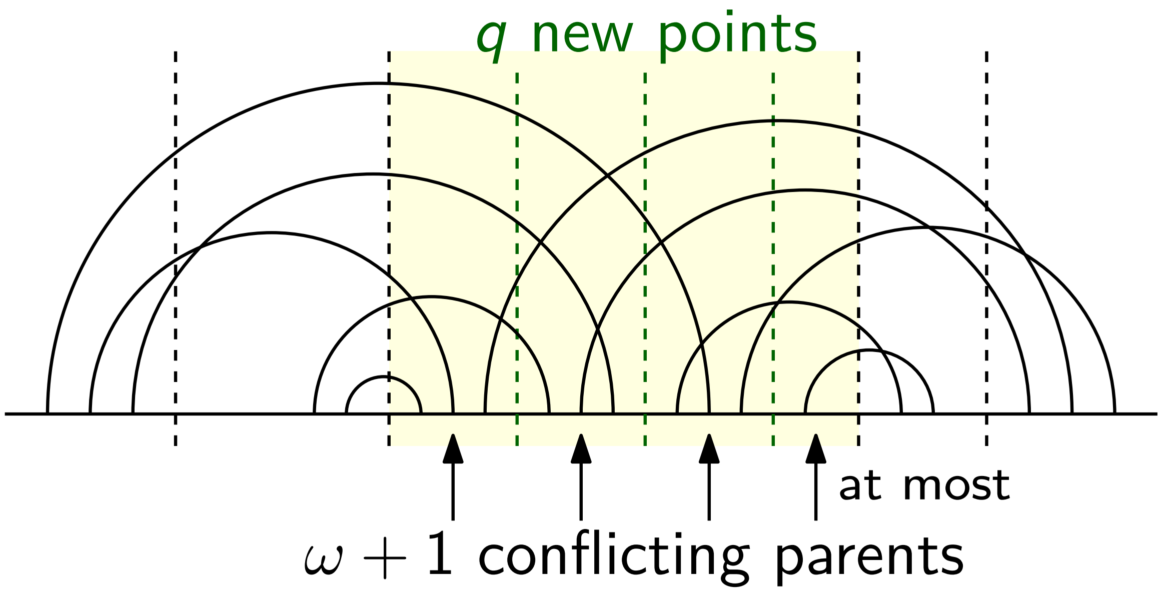
$q \leqslant \omega^2+m\omega+\omega$



$\omega+m+1$ conflicting parents

height $1+\lfloor \log_2 q \rfloor$

$$\leqslant 1+\lfloor \log_2(\omega^2+m\omega+1) \rfloor = m$$



max # color groups:

$$\omega + 2m + 1$$

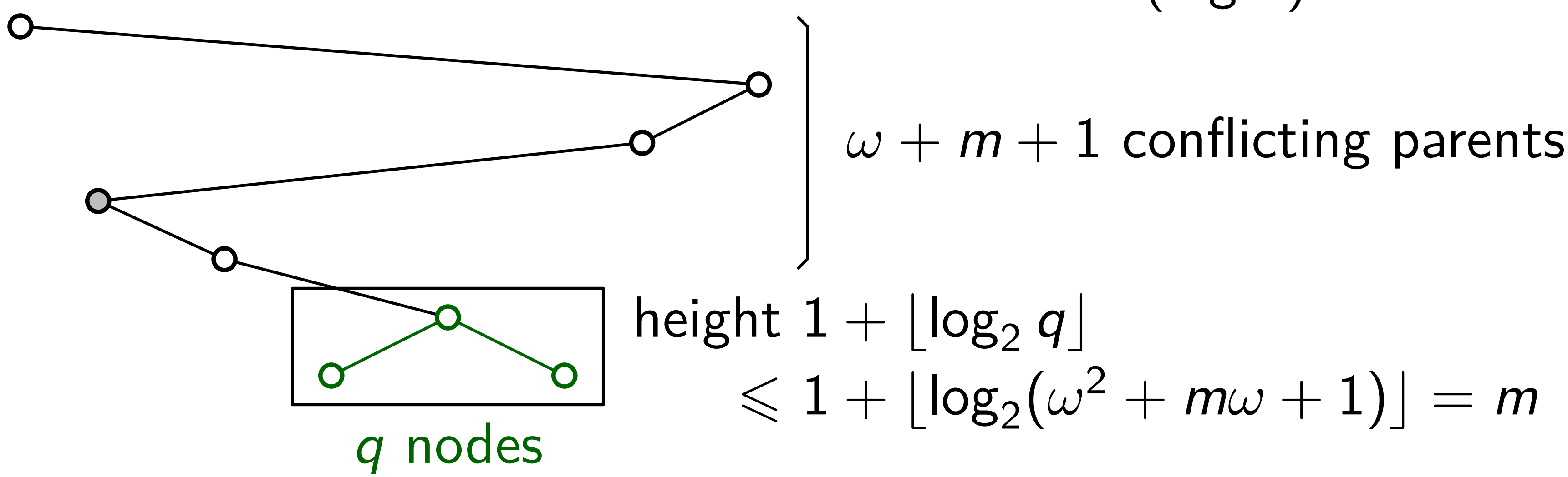
max # conflicting parents:

$$\omega + m + 1$$

~~m to be determined later~~

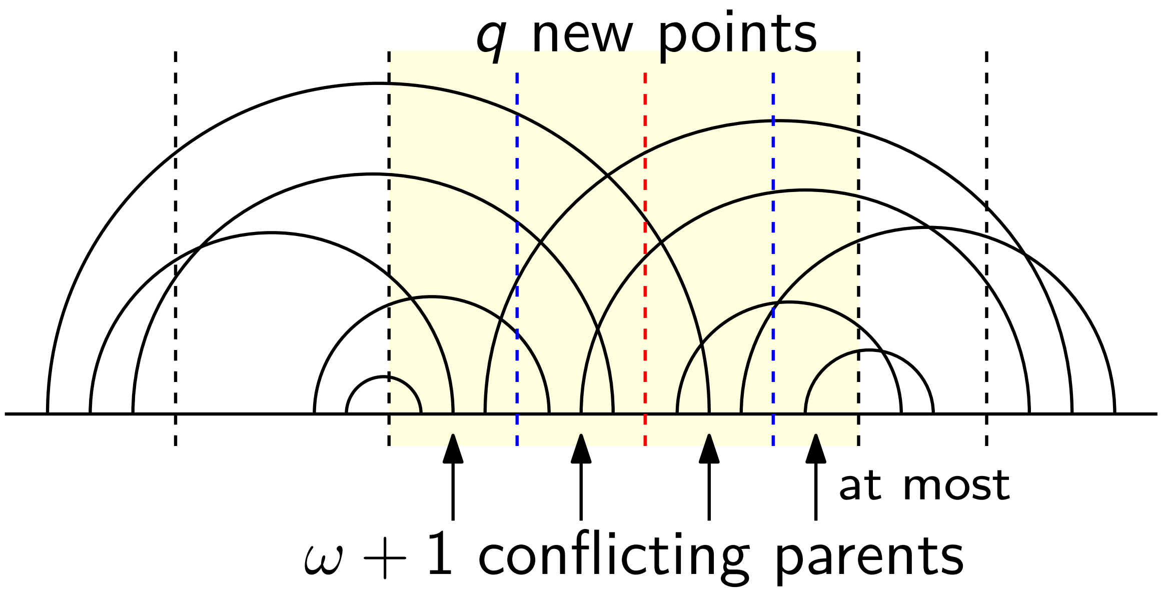
$q \leqslant \omega^2 + m\omega + \omega$

$m = O(\log \omega)$



height $1 + \lfloor \log_2 q \rfloor$

$$\leqslant 1 + \lfloor \log_2(\omega^2 + m\omega + 1) \rfloor = m$$



max # color groups:

$$\omega + 2m + 1$$

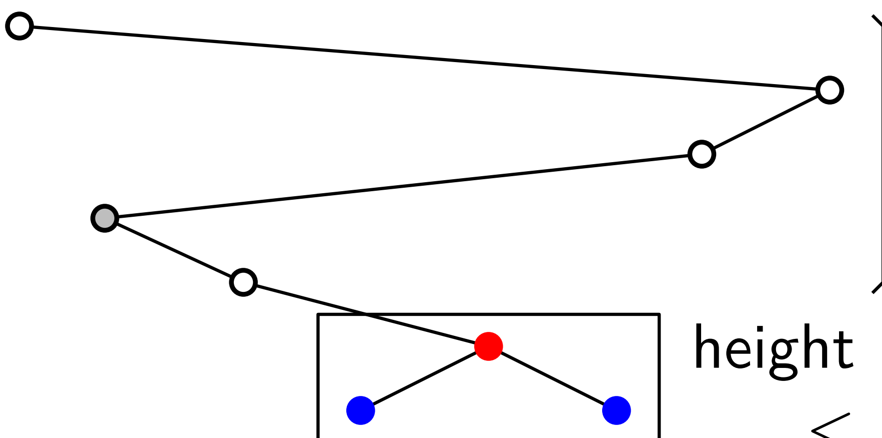
max # conflicting parents:

$$\omega + m + 1$$

~~m to be determined later~~

$q \leqslant \omega^2 + m\omega + \omega$

$m = O(\log \omega)$

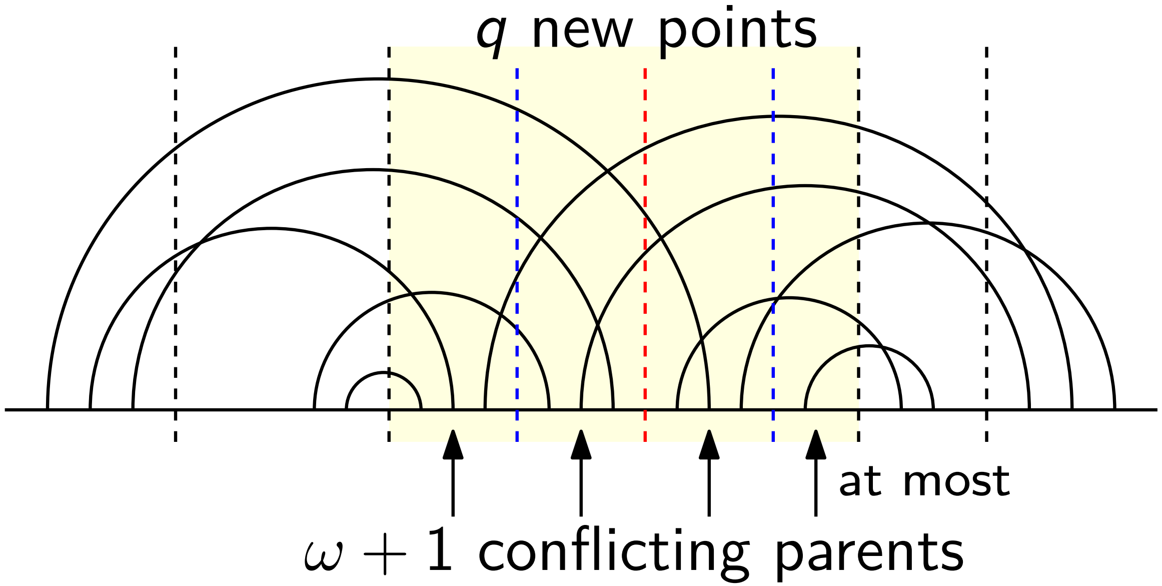


$\omega + m + 1$ conflicting parents

height $1 + \lfloor \log_2 q \rfloor$

$$\leqslant 1 + \lfloor \log_2(\omega^2 + m\omega + 1) \rfloor = m$$

m color groups available



max # color groups:

$$\omega + 2m + 1$$

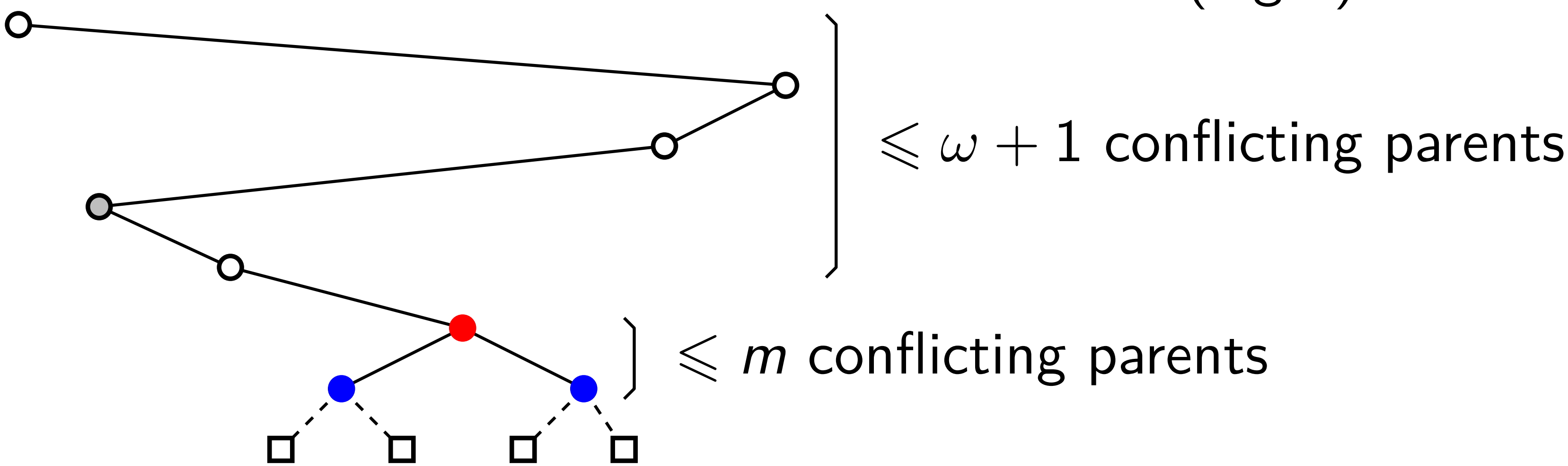
max # conflicting parents:

$$\omega + m + 1$$

~~m to be determined later~~

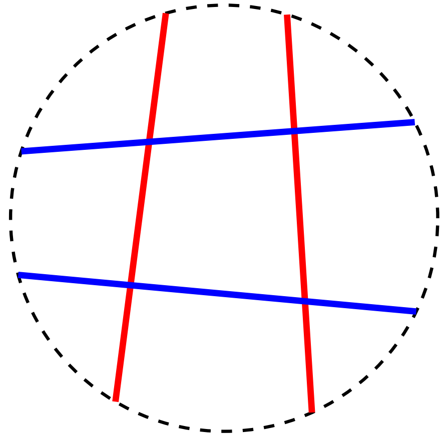
$q \leqslant \omega^2 + m\omega + \omega$

$m = O(\log \omega)$



$$\#\,\text{colors} = (\#\,\text{color groups}) \cdot \omega \leqslant \omega^2 + 2m\omega + \omega = \omega^2 + O(\omega \log \omega)$$

Circle graphs



upper bound

$\chi = O(4^\omega \omega^2)$
Gyárfás, 1985

$\chi = O(2^\omega \omega^2)$
Kostochka, 1988

$\chi = O(2^\omega)$
Kostochka, Kratochvíl, 1997

better $\chi = O(2^\omega)$
Černý, 2007

$\chi = O(\omega^2)$
Davies, McCarty, 2019+

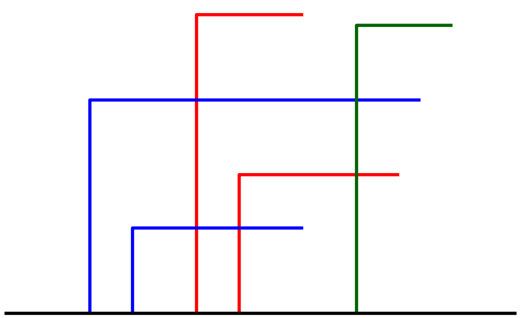
construction

$\chi = \Theta(\omega \log \omega)$
Kostochka, 1988

What is the truth?

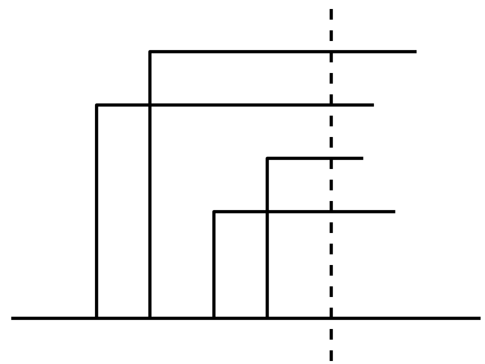
Davies, Krawczyk, McCarty, W, 2019+

Grounded L-graphs are polynomially χ-bounded.

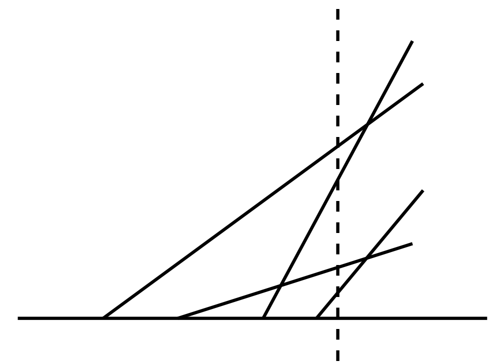


χ-bounded

McGuinness 1996



permutation graph



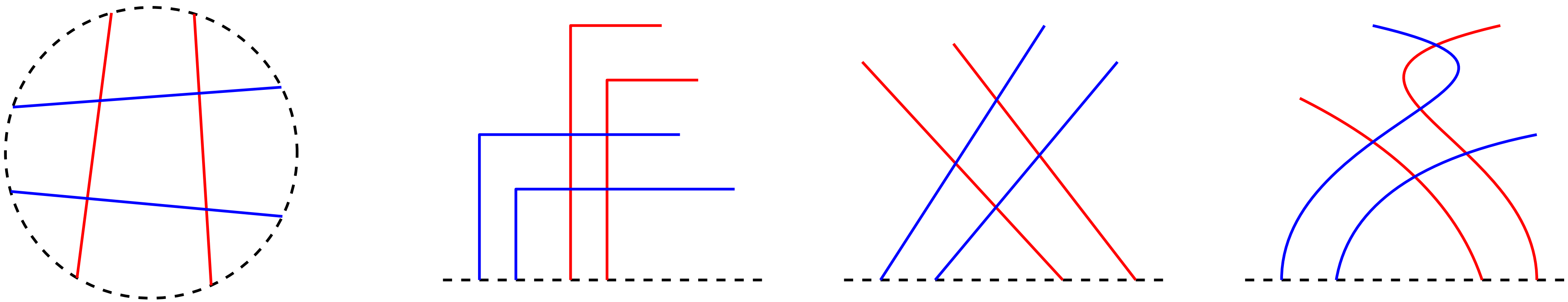
?

Are grounded segment graphs polynomially χ-bounded?

Are outerstring graphs polynomially χ-bounded?

Esperet, 2017

Is every χ-bounded class of graphs polynomially χ-bounded?



circle graphs $\subseteq$ grounded L-graphs $\subseteq$ grd. segment graphs $\subseteq$ outerstring graphs

$\Omega(\omega \log \omega)$
Kostochka 1988

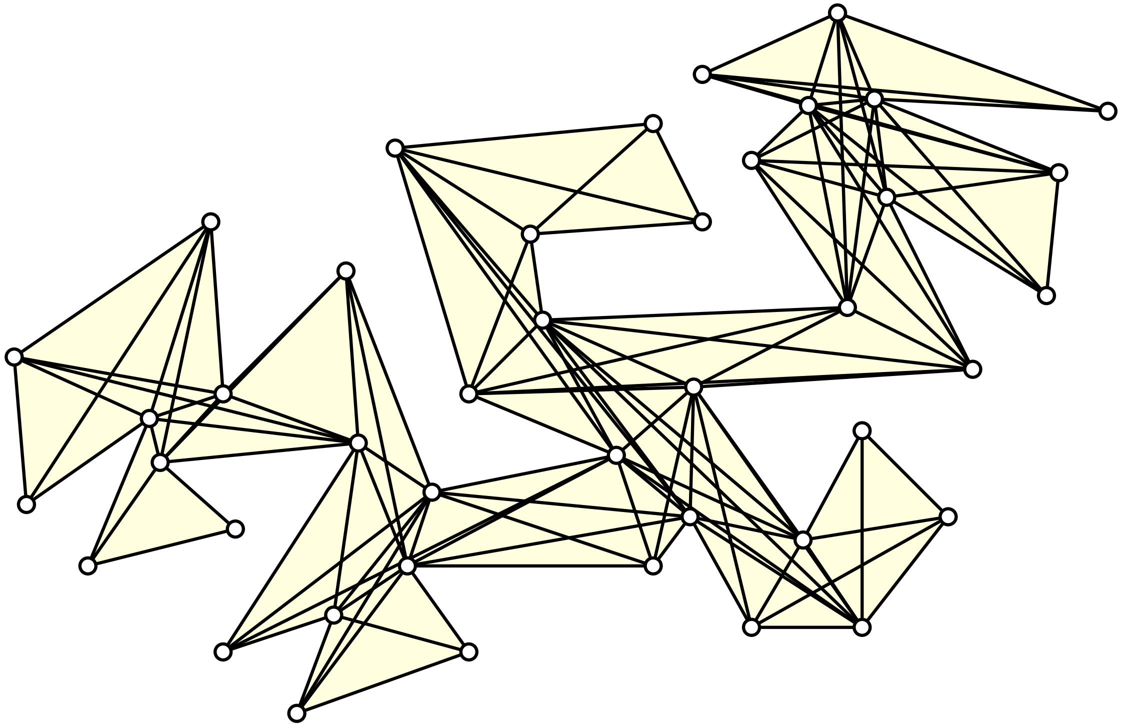
$\Omega(\omega^2)$
Krawczyk, W 2017

$O(\omega^2)$
Davies, McCarty 2019

$O(\omega^4)$
Davies, Krawczyk, McCarty, W 2019+

χ-bounded
Suk 2014

χ-bounded
Rok, W 2014



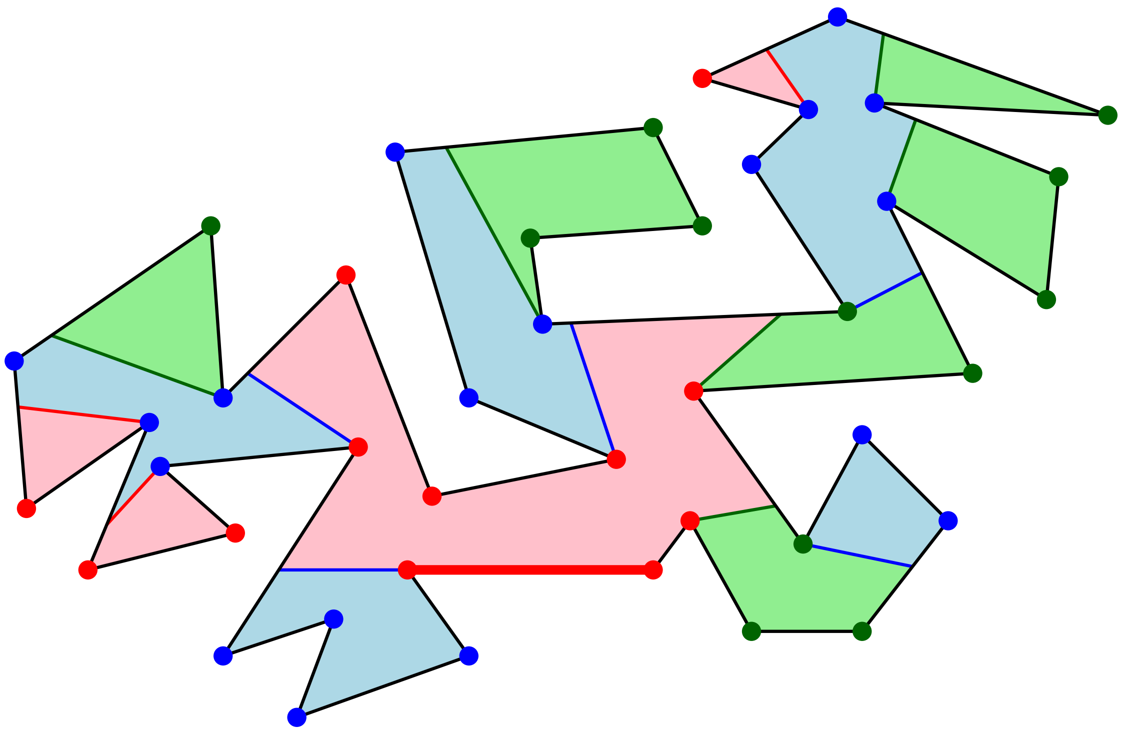
polygon visibility graph

Kára, Pór, Wood, 2005

Are polygon visibility graphs χ-bounded?

Davies, Krawczyk, McCarty, W, 2019+

Yes, they are.

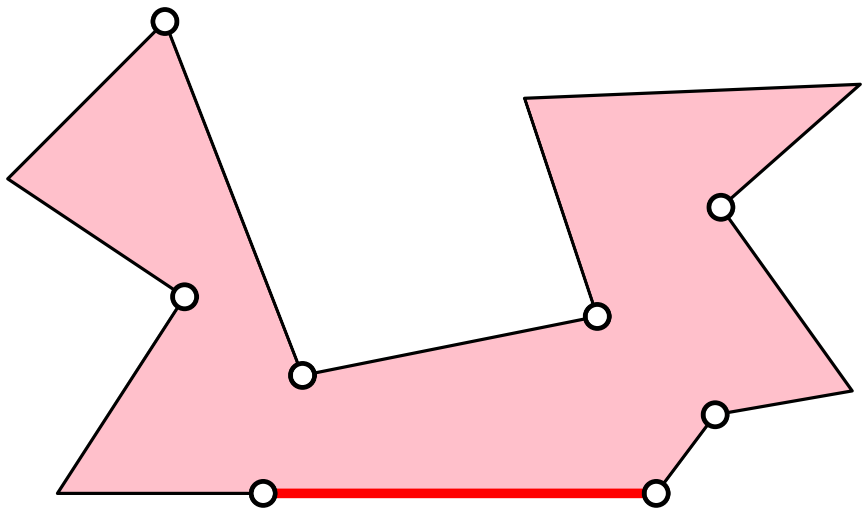


Kára, Pór, Wood, 2005

Are polygon visibility graphs χ-bounded?

Davies, Krawczyk, McCarty, W, 2019+

Yes, they are.



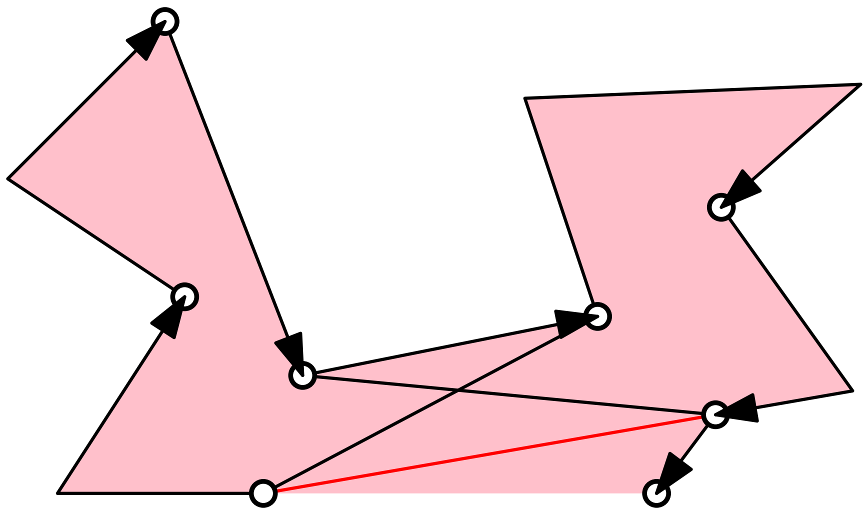
all vertices visible from a common boundary segment

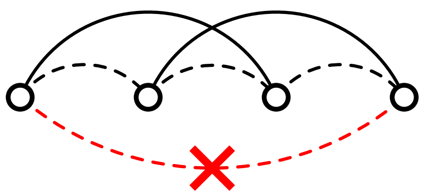
Kára, Pór, Wood, 2005

Are polygon visibility graphs χ-bounded?

Davies, Krawczyk, McCarty, W, 2019+

Yes, they are.



 excluded!

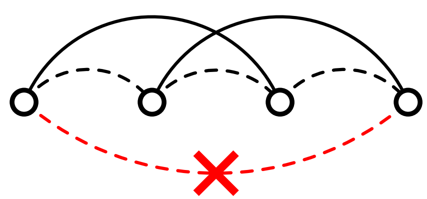
Kára, Pór, Wood, 2005

Are polygon visibility graphs χ-bounded?

Davies, Krawczyk, McCarty, W, 2019+

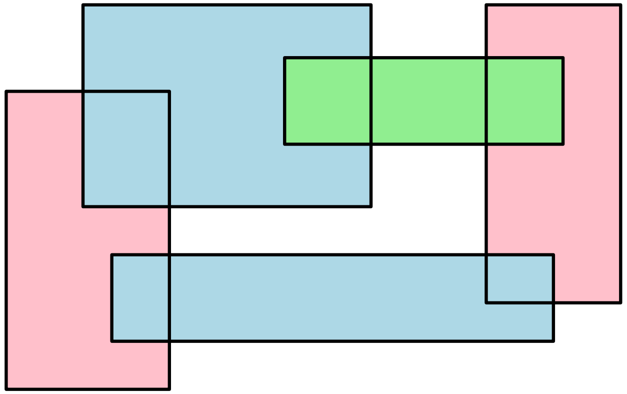
Yes, they are.

Davies, Krawczyk, McCarty, W, 2019+

Ordered graphs with  excluded are χ-bounded.

Are these graphs (or polygon visibility graphs) polynomially χ-bounded?

Rectangle graphs



construction

$\chi = 3\omega$

Kostochka, 2004

upper bound

$\chi = O(\omega^2)$

Asplund, Grünbaum, 1960

better $\chi = O(\omega^2)$

Hendler, 1998

Chalermsook, W, 2019+

Rectangle graphs satisfy $\chi = O(\omega \log \omega)$.