Finding Cliques in Social Networks: A New Distribution-Free Model

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1 Definitions

Let $G = (V, E)$ be an undirected graph. For $v \in V$, let $N(v)$ denote the neighborhood of v and $N_2(v)$ denote the vertices at distance exactly 2 from v. Furthermore, for a set $S \subseteq V$, we denote by $N(S) = \cap_{v \in S} N(v)$ the common neighborhood of S.

Definition 1. Given a graph and a value of c, a *bad pair* is a non-adjacent pair of vertices with at least c common neighbors.

Definition 2. For a positive integer c, G is *c-closed* if it contains no bad pair.

Definition 3. A graph is *weakly c-closed* if there exists an ordering of the vertices $\{v_1, \ldots, v_n\}$ such that for all i, v_i is in no bad pairs in the graph induced by $\{v_i, v_{i+1}, \ldots v_n\}$.

2 Main results

Theorem 4. *Any weakly c-closed graph on n vertices has at most* $3^{(c-1)/3}n^2$ *maximal cliques.*

Theorem 5. *Any c-closed graph on n vertices has at most* $\min\{3^{(c-1)/3}n^2, 4^{(c+4)(c-1)/2}n^{2-2^{1-c}}\}$ *maximal cliques.*

Theorem 6. *For any positive integer c, there are c-closed graphs with n vertices and* $\Omega(c^{-3/2}2^{c/2}n^{3/2})$ *maximal cliques.*

3 Tools

Lemma 7. *For any v in c-closed graph,* $G[N(v)]$ *is a* $(c-1)$*-closed graph.*

Lemma 8. $(1-x)^k \leq 1 - \frac{xk}{2}$ *for any* $0 < x \leq 1/2$ *and* $1 \leq k \leq 2$.

Theorem 9 (Tsukiyama et. al)**.** *There is an algorithm for enumerating all maximal cliques in graph in time* $O(nm)$ *per clique.*

Theorem 10 (Moon, Moser)**.** *A graph G on n vertices has at most* $3^{k/3}$ *maximal cliques.*

Theorem 11. *There is a graph G on n vertices with girth 5 and* $\Omega(n^{3/2})$ *edges.*