Definition. *We say that* $T = (V, E)$ *is a* tournament *if it is an orientation of a complete graph. We say that* u *is* adjacent to v *and* v *is* adjacent from u *iff* $(u, v) \in E$.

Definition. *If* T *is a tournament and* $X, Y \subseteq V(T)$*, we say that* X dominates Y *if every vertex in* $Y \setminus X$ *is adjacent from some vertex in* X*. The* domination number *of* T *is the smallest cardinality of a set that dominates* $V(T)$.

Definition. *If* S, T *are tournaments, we say that* T *is* S-free *if no subtournament of* T *is isomorphic to* S.

Definition. *We say that a tournament* S *is a* rebel *if the class of all* S*-free tournaments has bounded domination.*

Definition. *A* k-colouring *of a tournament* T *is a partition of* $V(T)$ *into* k *transitive sets, and if* T *admits such a partition it is* k-colourable.

Definition. *Let us say a tournament is a* poset *tournament if its vertex set can be ordered* $\{v_1, \ldots, v_n\}$ *such that for all* $i < j < k$ *if* v_j *is adjacent from* v_i *and adjacent to* v_k*, then* v_i *is adjacent to* v_k.

Definition. *Let* H *be a hypergraph. We say that* $X \subseteq H$ *is* shattered *by* H *if for every* $Y \subseteq X$*, there exists* $A \in E(H)$ *with* $A \cap X = Y$*. The largest cardinality of a shattered set is called* Vapnik-Chervonenkis dimension *or* VC-dimension *of* H.

Definition. *Let* T *be a tournament, and for each vertex* v*, let* $N_T^-(v)$ *denote the set of all vertices of* T *that are either adjacent to* v *or equal to* v*. Thus* $\left\{N_T^-(v) \colon v \in V(T)\right\}$ *is the edge set of a hypergraph with vertex set* $V(T)$*, called the* hypergraph of in-neighbourhoods *of* T.

Definition. *If* H *is a hypergraph,* τ_H *denotes the minimum cardinality of a set which has nonempty intersection with every edge of* H*, and* τ_H^* *is a fractional relaxation of this: the minimum of* $\sum_{v \in V(H)} f(v)$ *over all functions* f *from* $V(H)$ *to the nonnegative real numbers such that* $\sum_{v \in A} f(v) \geq 1$ *for every edge* A *of* H.

Claim (Sauer-Shelah lemma)**.** *Let* H *be a hypergraph, and let* $X \subseteq V(H)$ *with* $|X| = n$*, such that no* $(d+1)$*-subset of* X *is shattered by* H*. Then there are at most* $\sum_{0 \leq i \leq d} \binom{n}{i}$ *distinct sets* $A \cap X$ *where* $A \in E(H)$.

Claim. *For every 2-colourable tournament* S*, there exists* $d \geq 0$ *with the following property. Let* $\{C, D\}$ *be a 2-colouring of* S*. Let* T *be a tournament and let* $A, B \subseteq V(T)$ *be disjoint. For each* $v \in B$*, let* $N(v)$ *denote the set of all* $u \in A$ *adjacent to* v*. Let* H *be a hypergraph with vertex set* A *and edge set* $\{N(v) \colon v \in B\}$*. Let* $X \subseteq A$ *be shattered by* H *with* $|X| \geq d$*. Then there is an embedding of* (S, C, D) *into* (T, X, B).

Lemma. *There is a tournament* R *with a 2-colouring* $\{C, I\}$*, with* $|I| = m!n$*, such that for every transitive tournament* M *with vertex set* C *there is an embedding of* (S, C, D) *into* $(R \leftarrow M, C, I)$*. Moreover no two vertices in* C *are adjacent to exactly the same vertices in* I.

Claim. *For every 2-colourable tournament* S*, there is a number* d *such that for every* S*-free tournament* T*, its hypergraph of in-neighbourhoods has VC-dimension at most* d.

Claim. *Let* $d \geq 1$ *and let* H *be a hypergraph with VC-dimension at most* d*. Then* $\tau_H \leq 2d\tau_H^* \log(11\tau_H^*)$.

Claim. *Let* T *be a tournament, let* $d \geq 1$ *and let the VC-dimension of its hypergraph of in-neighbourhoods be at most* d*. Then the domination number of* T *is at most* $18d$.

Theorem. *Every 2-colourable tournament is a rebel.*

Theorem. *Every rebel is a poset tournament.*