A weaker version of Lovász' path removal conjecture

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Conjecture 1 (Lovász)**.** *There exists a function $f = f(k)$ such that the following holds. For every $f(k)$-connected graph G and two wertices s and t of G, there exists a path P with endpoints s and t such that $G - V(P)$ is k-connected.*

Conjecture 2 (Kriessel)**.** *There exists a function $f = f(k)$ such that the following holds. For every $f(k)$-connected graph G and two wertices s and t of G, there exists an induced path P with endpoints s and t such that $G - E(P)$ is k-connected.*

Theorem 1. *There exists a function $f(k) = O(k)^4$ such that the following holds: for any two vertices s and t of an $f(k)$-connected graph G, there exists an induced s-t path P such that $G - E(P)$ is k-connected.*

Theorem 2 (Mader)**.** *Every graph of minimum degree $4k$ contains a k-connected subgraph.*

Theorem 3 (Thomassen)**.** *Let k be any natural number, and G be any graph of minimum degree $> 4k^2$. Then G contains a k-connected subgraph with more than $4k^2$ vertices whose boundary has at most $2k^2$ vertices.*

Definition (Separation)**.** *A* separation *of a graph is a pair (A, B) of subsets of vertices of G such that $A \cup B$ is equal to $V(G)$, and for every edge $e = uv$ of G, either both u and v are contained in A or both are contained in B. The* order *of a separation (A, B) is $|A \cap B|$.*

Definition (Linkage)**.** *A* linkage *is a graph where every connected component is a path. A* linkage problem *in a graph G is a set of pairs of vertices of G. A* solution *to a linkage problem $\mathcal{L} = \{\{s_1, t_1\}, \ldots, \{s_k, t_k\}\}$ is a set of pairwise internally disjoint paths $P_1, \ldots, P_k$ such that ends of P_i are s_i and t_i, and furthermore, if $x \in V(P_i) \cap V(P_j)$ for some distinct indices i and j, then $x = s_i$ or $x = t_i$.*

A graph is strongly k-linked *if every linkage problem $\mathcal{L} = \{\{s_1, t_1\}, \ldots, \{s_k, t_k\}\}$ consisting of k pairs in G has a solution.*

Theorem 4. *Every $10k$-connected graph is strongly k-linked.*