Robot localization in an unknown but symmetric environment

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Section 1

Context

Motivations: underwater localization

Context

Localization problem.

Considering a known speed of sound and linear acoustic rays:

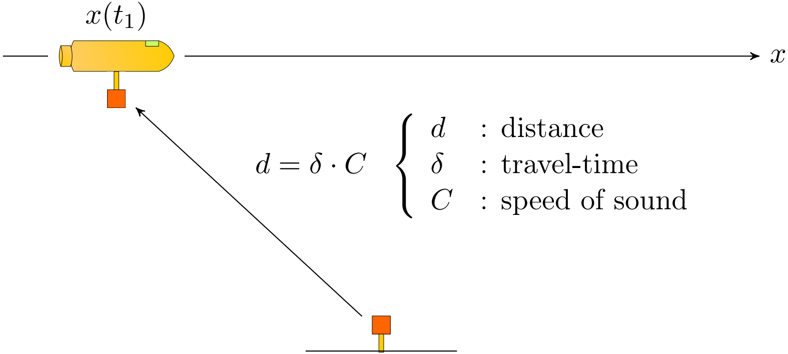
Figure : AUV's localization with an acoustic beacon on the seabed

Motivations: underwater localization

Context

Localization problem.

Considering a known speed of sound and linear acoustic rays:



$$d = \delta \cdot C \quad \begin{cases} d & : \text{distance} \\ \delta & : \text{travel-time} \\ C & : \text{speed of sound} \end{cases}$$

Figure : AUV's localization with an acoustic beacon on the seabed

Motivations: underwater localization

Context

Localization problem.

Considering refractions (Snell-Descartes) and no knowledge on C:

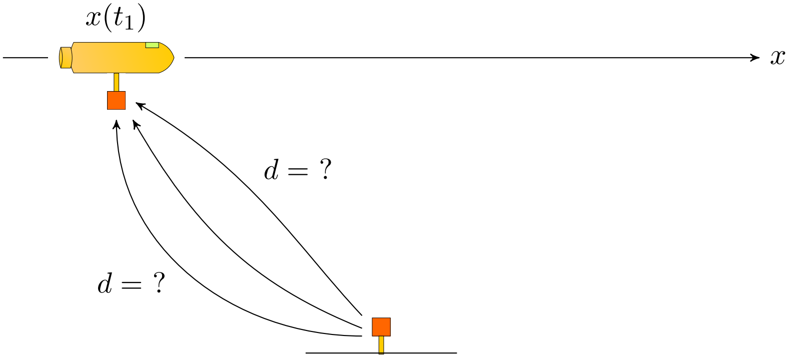


Figure : AUV's localization with an acoustic beacon on the seabed

Motivations: underwater localization

Context

Compensation of uncertainties with inter-temporal measurements.

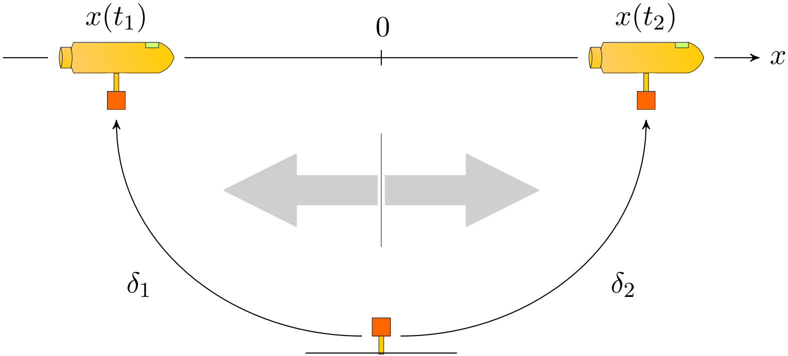


Figure : AUV's localization with an acoustic beacon on the seabed

Motivations: underwater localization

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Compensation of uncertainties with inter-temporal measurements.

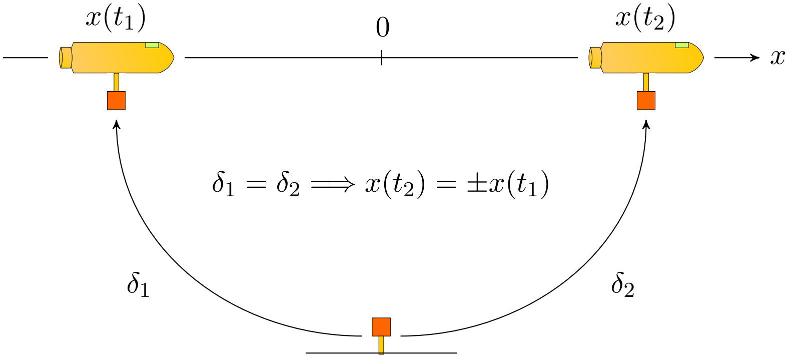

Figure : AUV's localization with an acoustic beacon on the seabed

Section 2

Formalization

System

Formalization

State estimation problem. Classically, we have:

$$\begin{cases} \dot{\mathbf{x}}(t) & = & \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \\ y(t) & = & g(\mathbf{x}(t)) \end{cases}$$

Where:

- $\mathbf{x} \in \mathbb{R}^n$ is the state vector
- $\mathbf{u} \in \mathbb{R}^m$ is the input vector
- $y \in \mathbb{R}$ is a measurement (assumed to be scalar)
- $\mathbf{f} : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is the *evolution* function
- $g : \mathbb{R}^n \to \mathbb{R}$ is the *observation* function

Distorsion function

Formalization

We introduce $h : \mathbb{R} \to \mathbb{R}$ the **distorsion function** describing the uncertainties on the environment.

$$\begin{cases} \dot{\mathbf{x}}(t) & = & \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \\ y(t) & = & h \circ g(\mathbf{x}(t)) \end{cases}$$

- the analytic expression of h is considered unknown
- we admit h is strictly increasing ($\Leftrightarrow$ *environmental gradient*)

Inter-temporality

Formalization

An inter-temporal state relation can be established by considering two identical measurements at different times:

$$\underbrace{h \circ g(\mathbf{x}(t_1))}_{y(t_1)} = \underbrace{h \circ g(\mathbf{x}(t_2))}_{y(t_2)}$$

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Since h is an injective function:

$$y(t_1) = y(t_2) \implies g(\mathbf{x}(t_1)) = g(\mathbf{x}(t_2))$$

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Since h is an injective function:

$$y(t_1) = y(t_2) \implies g(\mathbf{x}(t_1)) = g(\mathbf{x}(t_2))$$

For resolution purposes, we define:

$$\widetilde{y}(t_1, t_2) = y(t_2) - y(t_1)$$
$$\widetilde{g}(t_1, t_2) = g(\mathbf{x}(t_2)) - g(\mathbf{x}(t_1))$$

The symmetric set

Formalization

We define the **symmetric set** $\mathbb{S} \in \mathbb{T}$ such that:

$$\mathbb{S} = \left\{(t_1, t_2) \in [0, t_f]^2 \;\middle|\; \widetilde{y}(t_1, t_2) = 0 \;,\; t_1 < t_2\right\}$$

[Aub13]

- (t_1, t_2) is called a t-pair
- the set $\mathbb{T}$ of all t-pairs is called a t-plane

Understanding the symmetric set

Formalization

$$\mathbb{S} = \left\{(t_1, t_2) \in [0, t_f]^2 \;\middle|\; \widetilde{y}(t_1, t_2) = 0 \;,\; t_1 < t_2\right\}$$

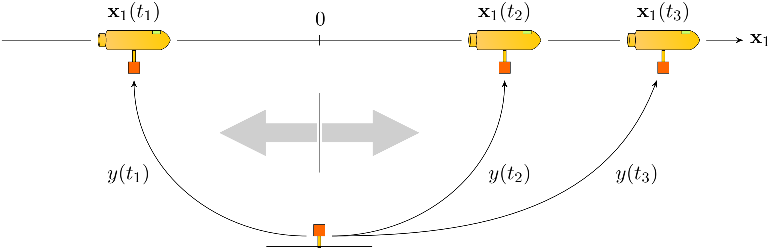

Figure : inter-temporal measurements

$(t_1, t_2) \in \mathbb{S}$ $(t_1, t_3) \notin \mathbb{S}$ $(t_2, t_3) \notin \mathbb{S}$

Section 3

Resolution with an interval method

Set-membership estimation

Resolution with an interval method

The state estimation becomes:

$$\begin{cases} \dot{\mathbf{x}}(t) & = & \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \\ y(t) & = & h \circ g(\mathbf{x}(t)) \end{cases}$$

With:

- $\mathbf{x}(t) \in [\mathbf{x}](t)$
- $\mathbf{u}(t) \in [\mathbf{u}](t)$
- $y(t) \in [y](t)$

$[\mathbf{x}](0)$ is supposed to be known.
Values evolving with time are pictured with **tubes**.

Tubes

Resolution with an interval method

Tube $[\mathbf{f}](t)$**:** interval of functions $[\mathbf{f}^-, \mathbf{f}^+]$ such that: $\forall t \in \mathbb{R}$, $\mathbf{f}^-(t) \leq \mathbf{f}^+(t)$

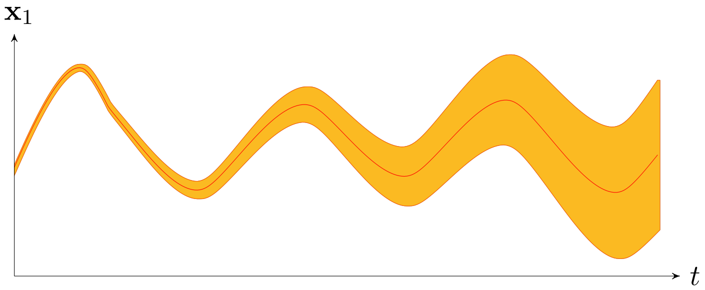

Figure : Tube $[\mathbf{x}_1](t)$ (orange) enclosing true values (red)

Proposition

Resolution with an interval method

The symmetric set $\mathbb{S}$ is:

$$\mathbb{S} = \{(t_1, t_2) \mid \widetilde{y}(t_1, t_2) = 0\}$$

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Considering: $\widetilde{y} \in [\widetilde{y}]$,
we can estimate $\mathbb{S}^+$ enclosing $\mathbb{S}$ with:

$$\mathbb{S}^+ = \{(t_1, t_2) \mid 0 \in [\widetilde{y}](t_1, t_2)\}$$

Estimation of $\mathbb{S}^+$

$$\mathbb{S} = \{(t_1, t_2) \mid \widetilde{y}(t_1, t_2) = 0\}$$
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Resolution with an interval method

A state $\mathbf{x}(t_1)$ can be associated to $\mathbf{x}(t_2)$ only if $(t_1, t_2) \in \mathbb{S}$.
For now, let us fix the value for t_2:

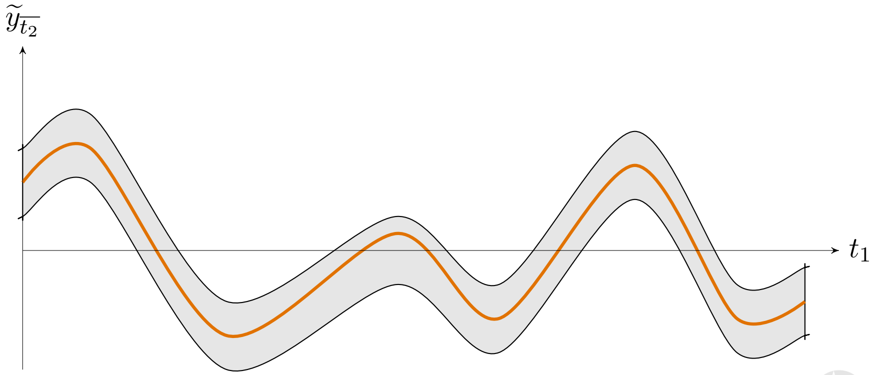

Figure : the tube $[\widetilde{y}]_{\overline{t_2}}(t_1)$

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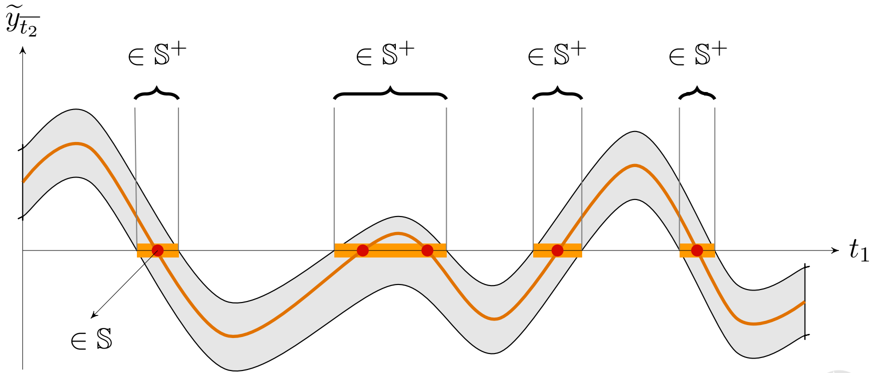

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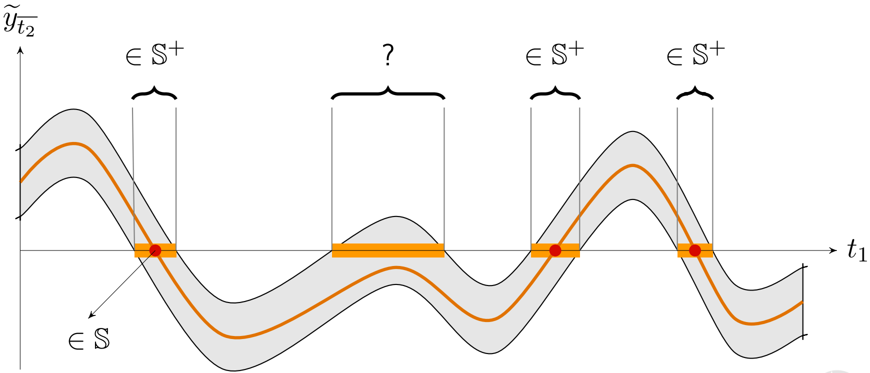

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$\mathbb{S} = \{(t_1, t_2) \mid \widetilde{y}(t_1, t_2) = 0\}$

$\mathbb{S}^+ = \{(t_1, t_2) \mid 0 \in [\widetilde{y}](t_1, t_2)\}$

We apply the intermediate value theorem.
We search solutions for $\widetilde{y}(t_1, t_2) \leq 0$: the **pre-symmetric set** $\mathbb{P}$.

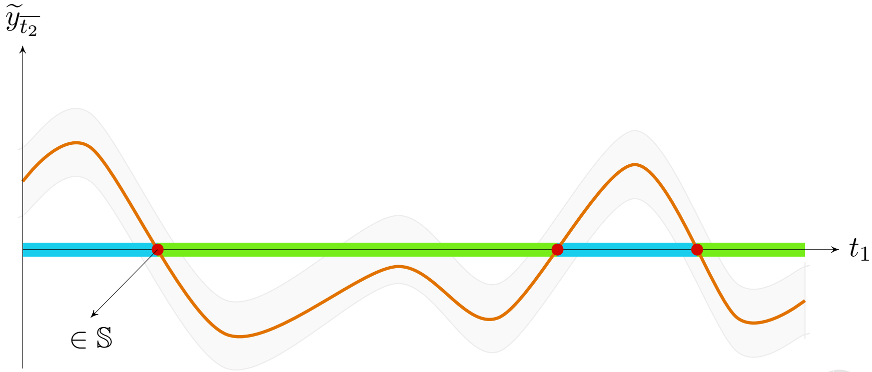

Figure : the pre-symmetric set $\mathbb{P}$:

Estimation of $\mathbb{S}^+$

Resolution with an interval method

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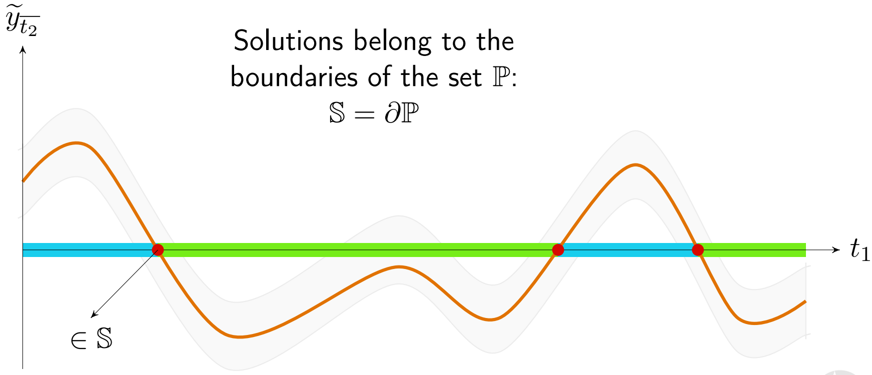

Figure : the pre-symmetric set $\mathbb{P}$:

Estimation of the pre-symmetric set $\mathbb{P}$

Resolution with an interval method

$\mathbb{P}^- =$ [green]
$\mathbb{P}^+ =$ [green] + [yellow]

The symmetric set is a boundary of the pre-symmetric set: $\mathbb{S} = \partial\mathbb{P}$

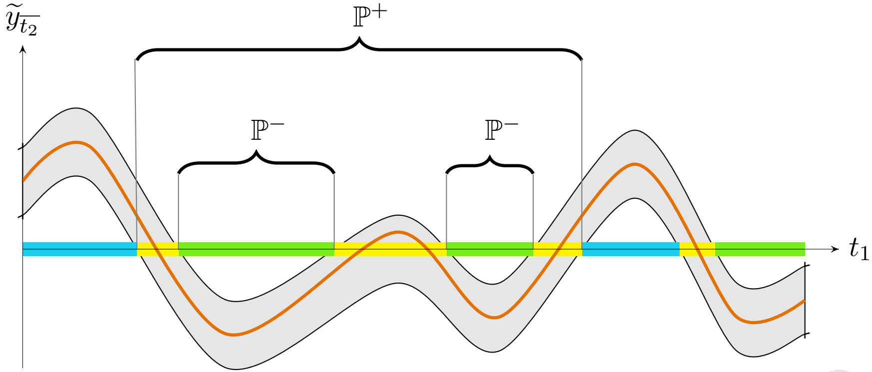

Figure : representation of the pre-symmetric set

Estimation of the pre-symmetric set $\mathbb{P}$

Resolution with an interval method

$\mathbb{P}^- =$ 

$\mathbb{P}^+ =$  $+$ $+$ 

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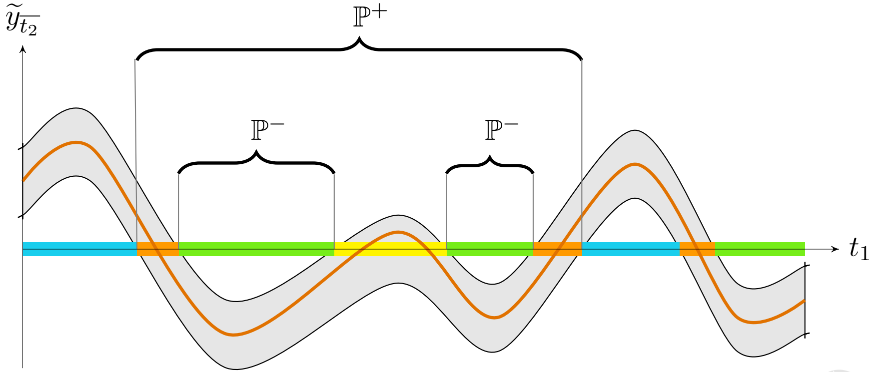
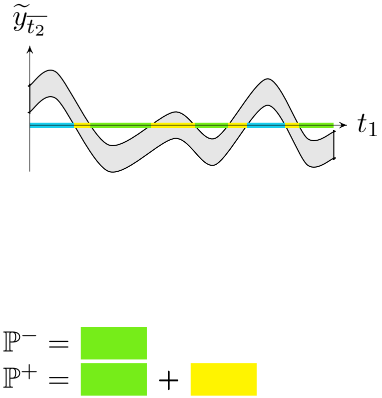


Figure : the set  = sure boundaries of $\mathbb{P}$ = true solutions for $\mathbb{S}$

Estimation of the pre-symmetric set $\mathbb{P}$

Resolution with an interval method



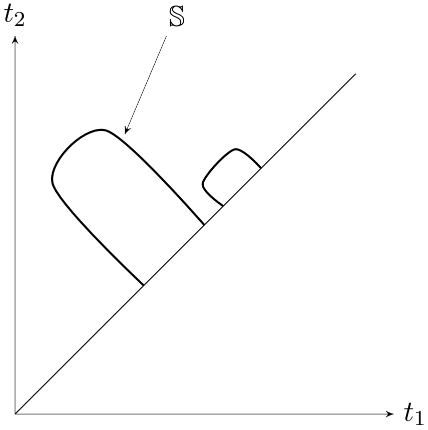
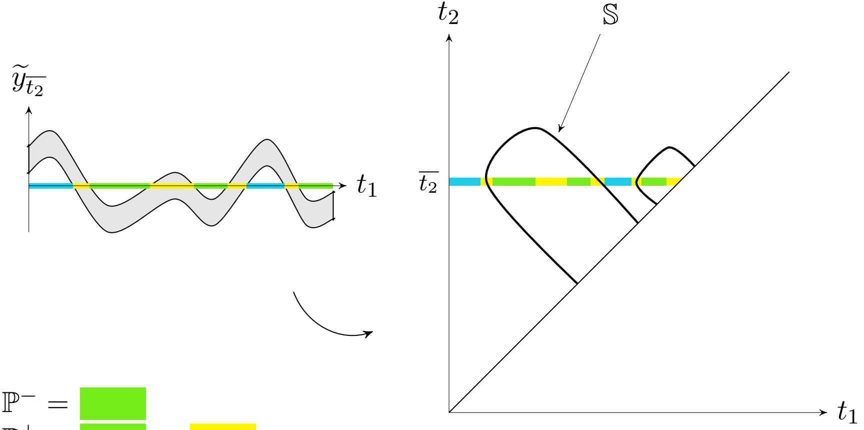


Figure : representation of $\mathbb{S}$ and $\mathbb{P}$ in a t-plane

Estimation of the pre-symmetric set $\mathbb{P}$

Resolution with an interval method



$\mathbb{P}^- =$ [green]

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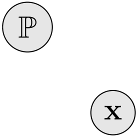
Figure : representation of $\mathbb{S}$ and $\mathbb{P}$ in a t-plane

Constraint Network

Resolution with an interval method



Variables: $\mathbf{x}$, $\mathbb{P}$

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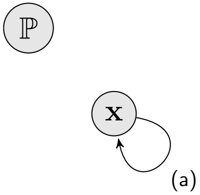
Constraints:

(a) $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u})$



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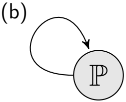
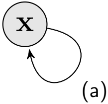


Variables: $\mathbf{x}$, $\mathbb{P}$

Constraints:

(a) $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u})$

(b) $\mathbb{P} = \{(t_1, t_2) \mid \widetilde{y}(t_1, t_2) \leq 0\}$



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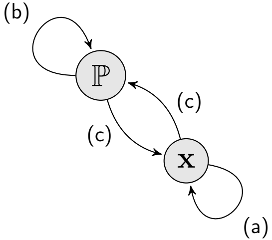

Variables: $\mathbf{x}$, $\mathbb{P}$

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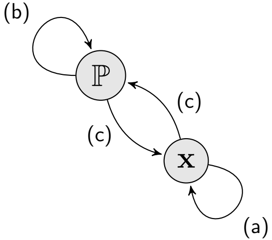

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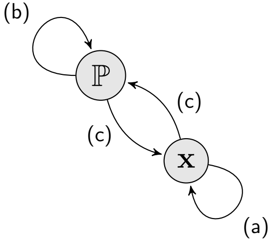
Domains: $[\mathbf{x}]$, $[\mathbb{P}]$

Constraint Network

Resolution with an interval method



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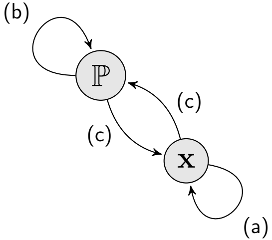
Domains: $[\mathbf{x}]$, $[\mathbb{P}]$

Initialization: $\mathbf{x} \in [-\infty, +\infty]^n$, $[\mathbb{P}] = [\varnothing, [0, t_f]^2]$

Constraint Network

Resolution with an interval method



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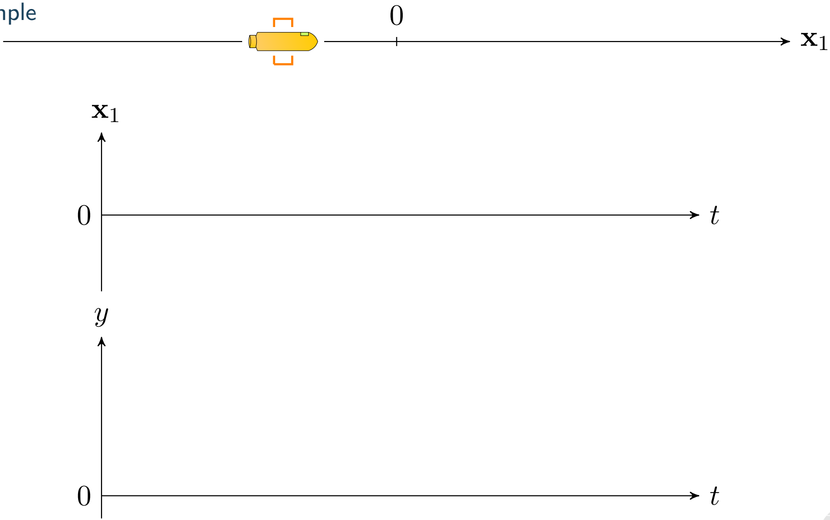
h does not appear anymore.

Section 4

Example

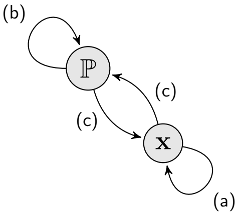
Simulation: an Autonomous Underwater Vehicle

Example



Estimation of $\mathbb{S}$

Example



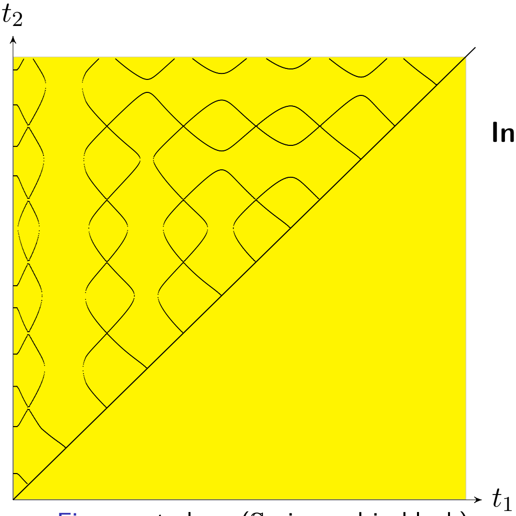


Figure : t-plane ($\mathbb{S}$ pictured in black)

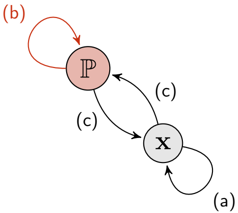
Initialization:

$$[\mathbb{P}] = [\varnothing, [0, t_f]^2]$$

$=$

Estimation of $\mathbb{P}$

Example



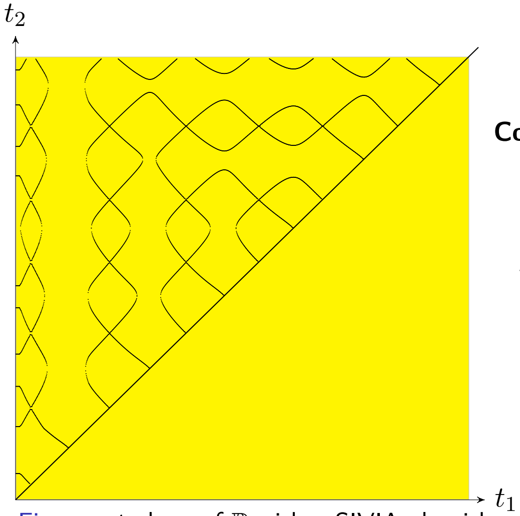


Figure : t-plane of $\mathbb{P}$ with a SIVIA algorithm

Constraint (b):

$$\mathbb{P} = \{(t_1, t_2) \mid \widetilde{y}(t_1, t_2) \leq 0\}$$

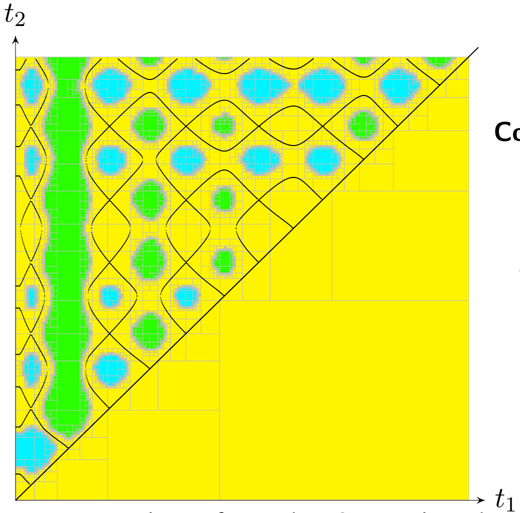
$$\underbrace{(\widetilde{y}^{+})^{-1}(\mathbb{R}^{-})}_{\mathbb{P}^{-}} \subset \mathbb{P} \subset \underbrace{(\widetilde{y}^{-})^{-1}(\mathbb{R}^{-})}_{\mathbb{P}^{+}}$$

$\mathbb{P}^{-} =$ (green)

$\mathbb{P}^{+} =$ (green) $+$ (yellow)

Estimation of $\mathbb{P}$

Example



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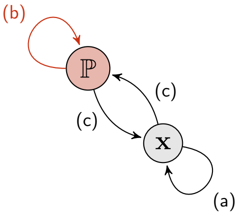

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$$\underbrace{(\widetilde{y}^+)^{-1}(\mathbb{R}^-)}_{\mathbb{P}^-} \subset \mathbb{P} \subset \underbrace{(\widetilde{y}^-)^{-1}(\mathbb{R}^-)}_{\mathbb{P}^+}$$

$\mathbb{P}^- =$ (green)

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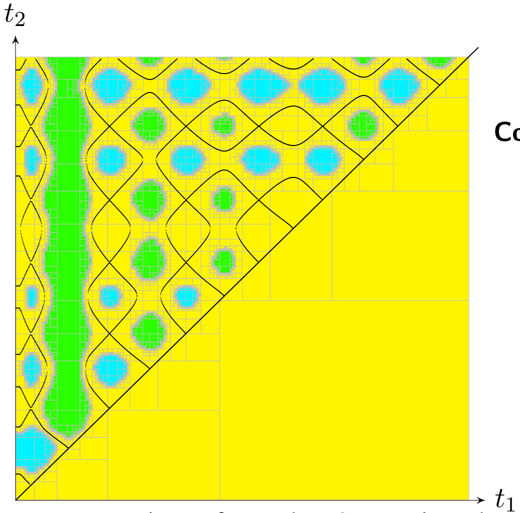
Estimation of $\mathbb{P}$

Example



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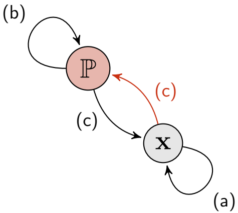


Constraint (c):

$$\mathbb{P} = \{(t_1, t_2) \mid \widetilde{g}(t_2, t_1) \leq 0\}$$

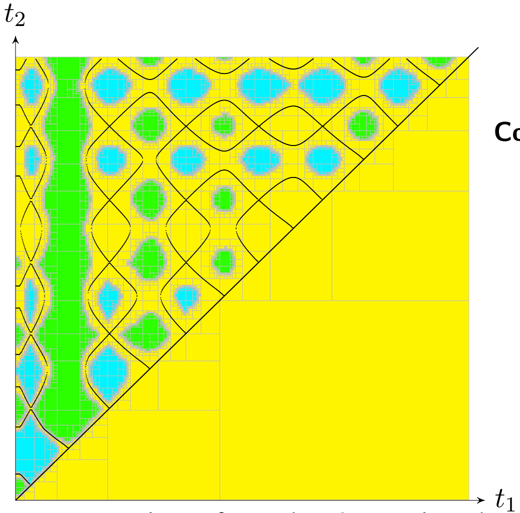
$$\underbrace{(\widetilde{g}^{+})^{-1}(\mathbb{R}^{-})}_{\mathbb{P}^{-}} \subset \mathbb{P} \subset \underbrace{(\widetilde{g}^{-})^{-1}(\mathbb{R}^{-})}_{\mathbb{P}^{+}}$$

$\mathbb{P}^{-} =$ 

$\mathbb{P}^{+} =$  $+$

Estimation of $\mathbb{P}$

Example



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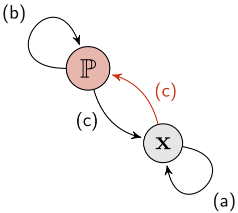


Constraint (c):

$$\mathbb{P} = \{(t_1, t_2) \mid \widetilde{g}(t_2, t_1) \leq 0\}$$

$$\underbrace{(\widetilde{g}^+)^{-1}(\mathbb{R}^-)}_{\mathbb{P}^-} \subset \mathbb{P} \subset \underbrace{(\widetilde{g}^-)^{-1}(\mathbb{R}^-)}_{\mathbb{P}^+}$$

$\mathbb{P}^- =$ [green]

$\mathbb{P}^+ =$ [green] + [yellow]

Contraction of $[\mathbf{x}]$

Example

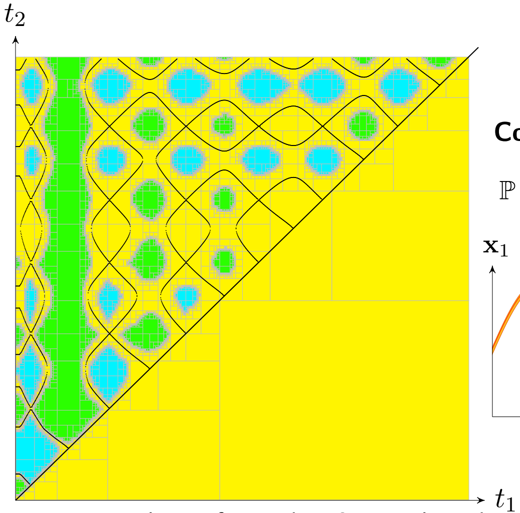
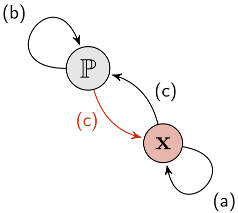


Figure : t-plane of $\mathbb{P}$ with a SIVIA algorithm



Constraint (c):

$$\mathbb{P} = \{(t_1, t_2) \mid g(\mathbf{x}(t_2)) - g(\mathbf{x}(t_1)) \leq 0\}$$

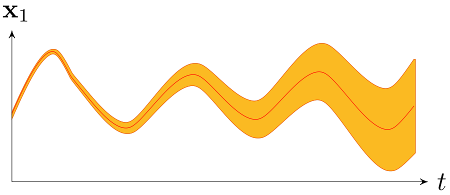
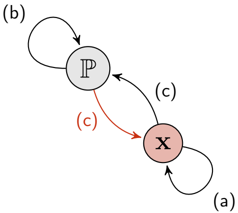


Figure : Contraction of tube $[\mathbf{x}_1](t)$

Contraction of $[\mathbf{x}]$

Example



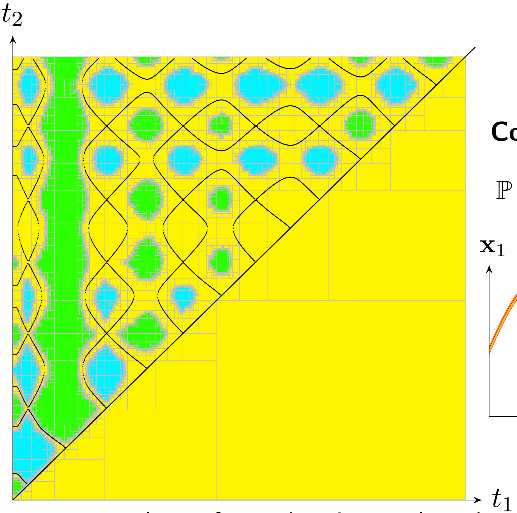


Figure : t-plane of $\mathbb{P}$ with a SIVIA algorithm

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$$\mathbb{P} = \{(t_1, t_2) \mid g(\mathbf{x}(t_2)) - g(\mathbf{x}(t_1)) \leq 0\}$$

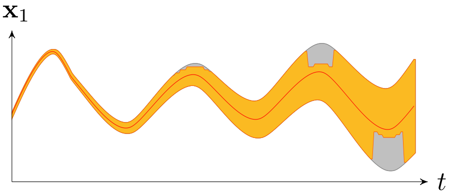


Figure : Contraction of tube $[\mathbf{x}_1](t)$

Contraction of $[\mathbf{x}]$

Example

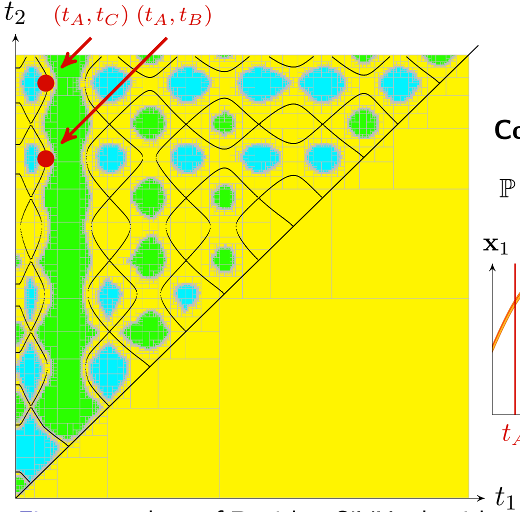
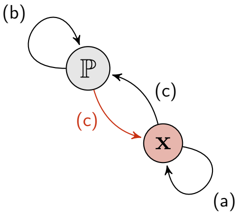


Figure : t-plane of $\mathbb{P}$ with a SIVIA algorithm



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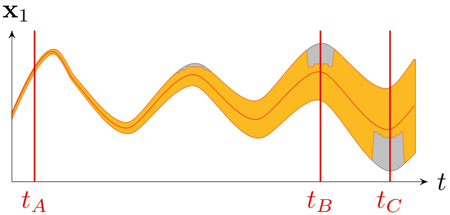
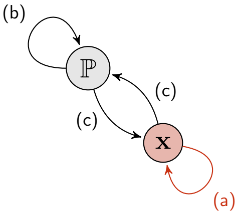


Figure : Contraction of tube $[\mathbf{x}_1](t)$

Contraction of $[\mathbf{x}]$

Example



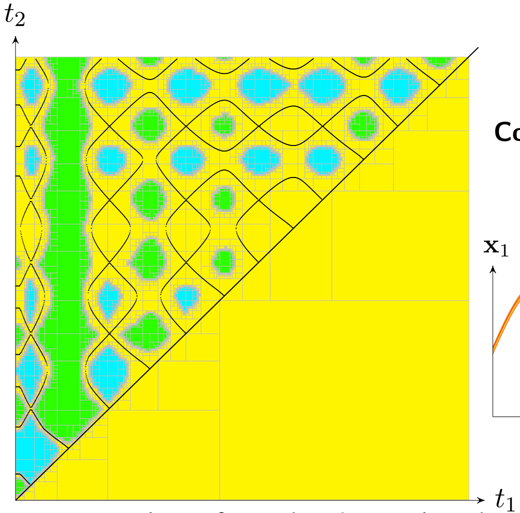


Figure : t-plane of $\mathbb{P}$ with a SIVIA algorithm

Constraint (a):

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u})$$

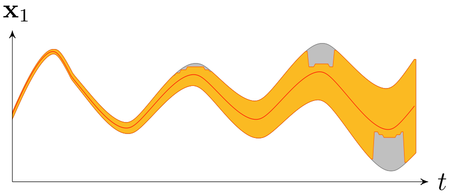
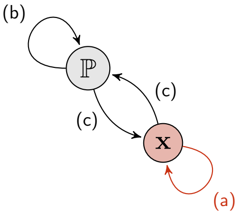
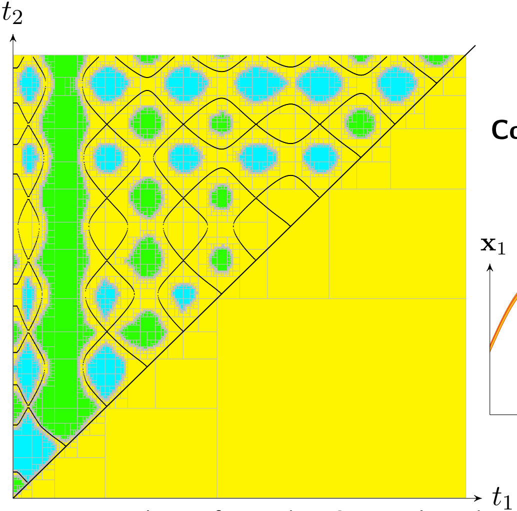


Figure : Contraction of tube $[\mathbf{x}_1](t)$

Contraction of $[\mathbf{x}]$

Example





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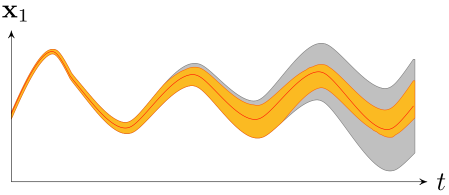


Figure : Contraction of tube $[\mathbf{x}_1](t)$

Estimation of $\mathbb{P}$

Example

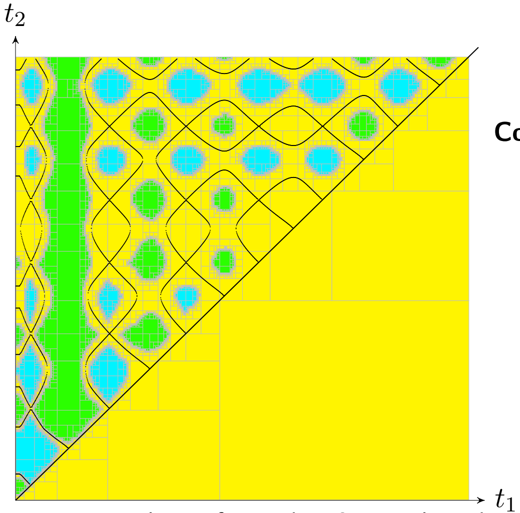
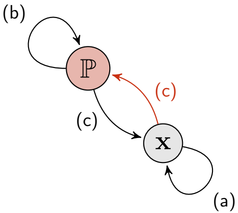


Figure : t-plane of $\mathbb{P}$ with a SIVIA algorithm



Constraint (c):

$$\mathbb{P} = \{(t_1, t_2) \mid \widetilde{g}(t_2, t_1) \leq 0\}$$

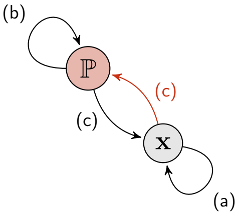
$$\underbrace{(\widetilde{g}^{+})^{-1}(\mathbb{R}^{-})}_{\mathbb{P}^{-}} \subset \mathbb{P} \subset \underbrace{(\widetilde{g}^{-})^{-1}(\mathbb{R}^{-})}_{\mathbb{P}^{+}}$$

$\mathbb{P}^{-} =$ (green)

$\mathbb{P}^{+} =$ (green) $+$ (yellow)

Estimation of $\mathbb{P}$

Example



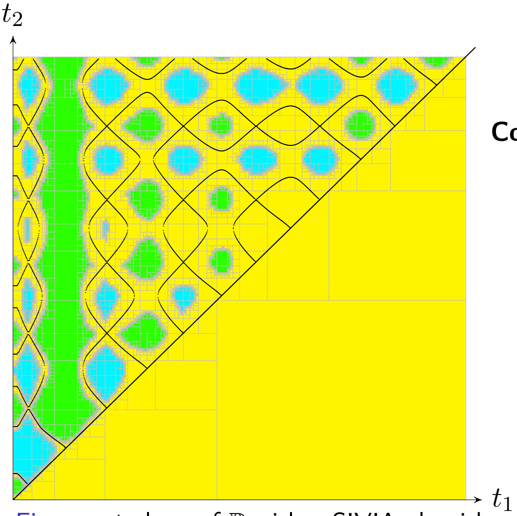


Figure : t-plane of $\mathbb{P}$ with a SIVIA algorithm

Constraint (c):

$$\mathbb{P} = \{(t_1, t_2) \mid \widetilde{g}(t_2, t_1) \leq 0\}$$

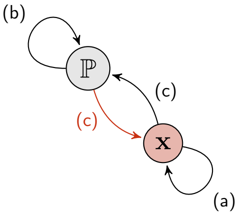
$$\underbrace{(\widetilde{g}^+)^{-1}(\mathbb{R}^-)}_{\mathbb{P}^-} \subset \mathbb{P} \subset \underbrace{(\widetilde{g}^-)^{-1}(\mathbb{R}^-)}_{\mathbb{P}^+}$$

$\mathbb{P}^- =$ (green)

$\mathbb{P}^+ =$ (green) $+$ (yellow)

Contraction of $[\mathbf{x}]$

Example



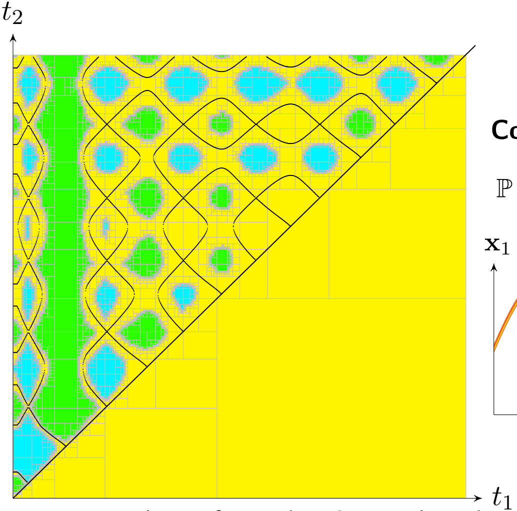


Figure : t-plane of $\mathbb{P}$ with a SIVIA algorithm

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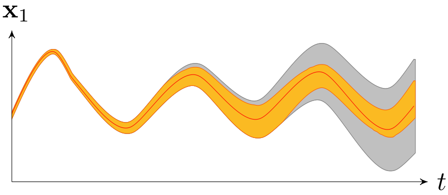
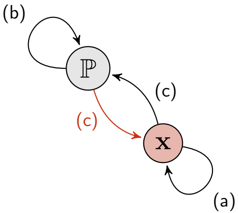


Figure : Contraction of tube $[\mathbf{x}_1](t)$

Contraction of $[\mathbf{x}]$

Example



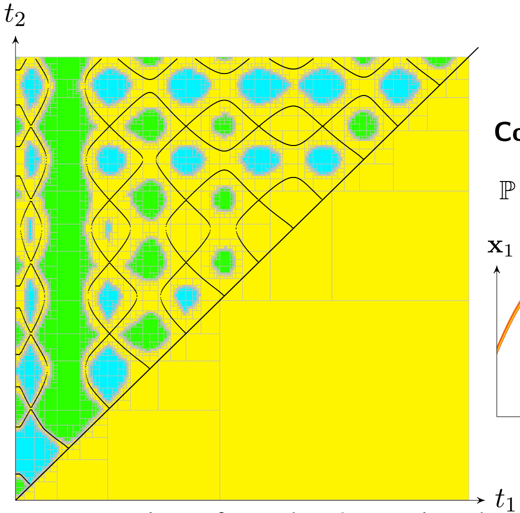


Figure : t-plane of $\mathbb{P}$ with a SIVIA algorithm

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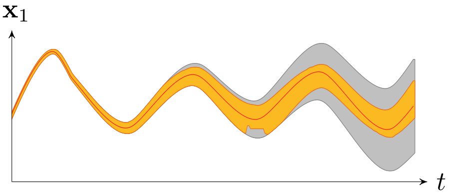
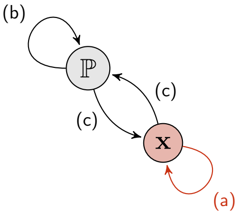


Figure : Contraction of tube $[\mathbf{x}_1](t)$

Contraction of $[\mathbf{x}]$

Example



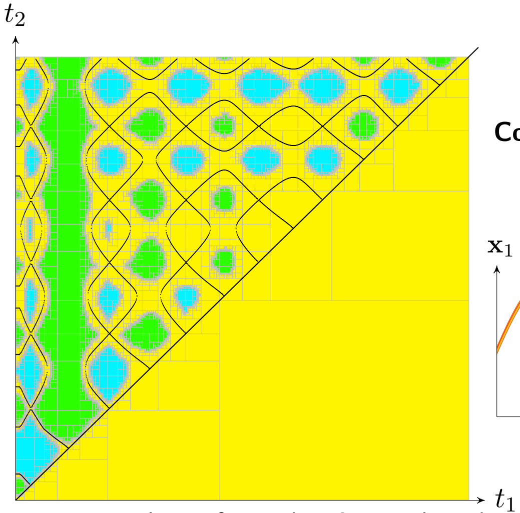


Figure : t-plane of $\mathbb{P}$ with a SIVIA algorithm

Constraint (a):

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u})$$

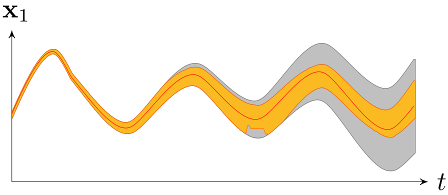
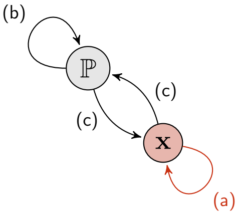


Figure : Contraction of tube $[\mathbf{x}_1](t)$

Contraction of $[\mathbf{x}]$

Example



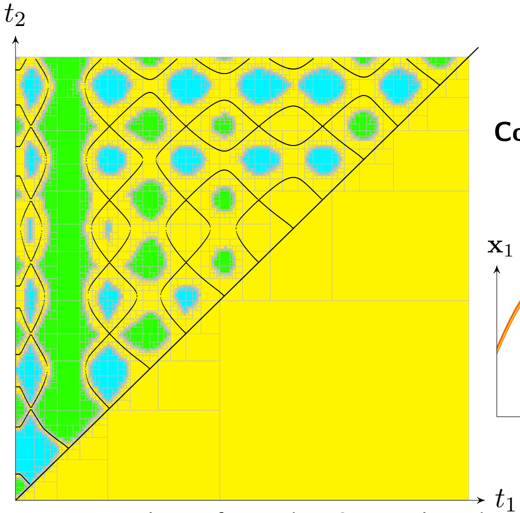


Figure : t-plane of $\mathbb{P}$ with a SIVIA algorithm

Constraint (a):

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u})$$

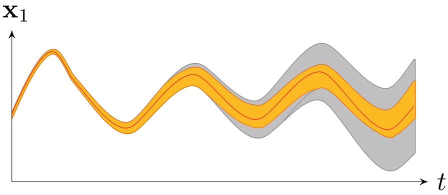


Figure : Contraction of tube $[\mathbf{x}_1](t)$

Estimation of $\mathbb{P}$

Example

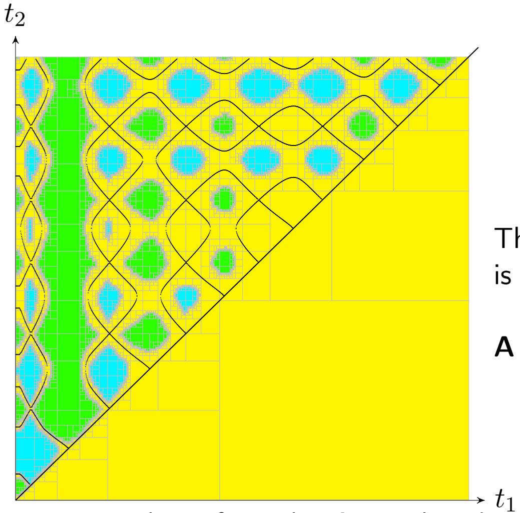
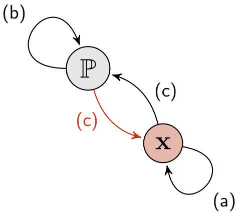


Figure : t-plane of $\mathbb{P}$ with a SIVIA algorithm



The simulation is run until there is no more contraction for $\mathbf{x}$.

A fixpoint has been reached.

Conclusion

Robot localization in an unknown but symmetric environment:

- use of inter-temporal measurements $y(t_1, t_2)$, $y(t_3, t_4)$, ...
- uncertainties compensated
- state estimation

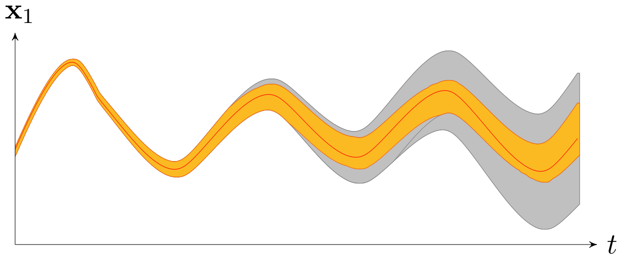


Figure : final contraction of robot's position

Support: *Direction Générale de l'Armement* (DGA - FR)

References:

[Aub13] C. Aubry, R. Desmare and L. Jaulin,
Loop detection of mobile robots using interval analysis,
Automatica, 2013.

[Bar12] F. LeBars, J. Sliwka, O. Reynet and L. Jaulin,
State estimation with fleeting data,
Automatica, 2012.

Tools:

IBEX library
used for interval arithmetic

VIBES
used for rendering

Appendix: estimation of the pre-symmetric set $\mathbb{P}$

Definition of the **pre-symmetric set** $\mathbb{P}$ with the measurements y:

$$\begin{aligned}\mathbb{P} &= \{(t_1,t_2) \mid \overbrace{\widetilde{y}(t_1,t_2)}^{y(t_2)-y(t_1)} \leq 0\} \\ &= \widetilde{y}^{-1}(\mathbb{R}^-)\end{aligned}$$

$\widetilde{y}$ is not known exactly, *i.e.*:

$$\widetilde{y} \in [\widetilde{y}] = [\widetilde{y}^-, \widetilde{y}^+]$$

As a consequence, we have:

$$\underbrace{(\widetilde{y}^+)^{-1}(\mathbb{R}^-)}_{\mathbb{P}^-} \subset \mathbb{P} \subset \underbrace{(\widetilde{y}^-)^{-1}(\mathbb{R}^-)}_{\mathbb{P}^+}$$

Appendix: estimation of the pre-symmetric set $\mathbb{P}$

Equivalently, $\mathbb{P}$ can be estimated with the states $\mathbf{x}$:

$$\begin{aligned}\mathbb{P} &= \{(t_1,t_2) \mid \overbrace{\widetilde{g}(t_1,t_2)}^{g(\mathbf{x}(t_2))-g(\mathbf{x}(t_1))} \leq 0\} \\ &= \widetilde{g}^{-1}(\mathbb{R}^-)\end{aligned}$$

$\widetilde{g}$ is not known exactly, *i.e.*:

$$\widetilde{g} \in [\widetilde{g}] = [\widetilde{g}^-, \widetilde{g}^+]$$

As a consequence, we have:

$$\underbrace{(\widetilde{g}^+)^{-1}(\mathbb{R}^-)}_{\mathbb{P}^-} \subset \mathbb{P} \subset \underbrace{(\widetilde{g}^-)^{-1}(\mathbb{R}^-)}_{\mathbb{P}^+}$$