Comparison of Kalman versus Interval based loop detection problem

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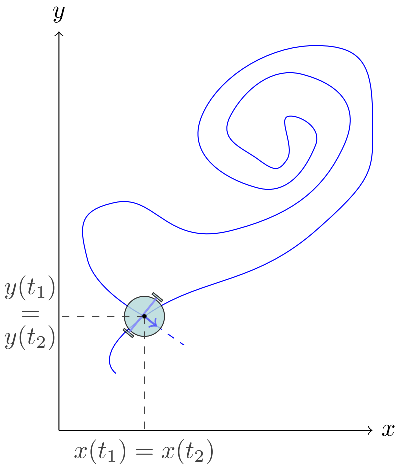
Outline

Human readable definition



A robot in a single loop trajectory

A loop is :

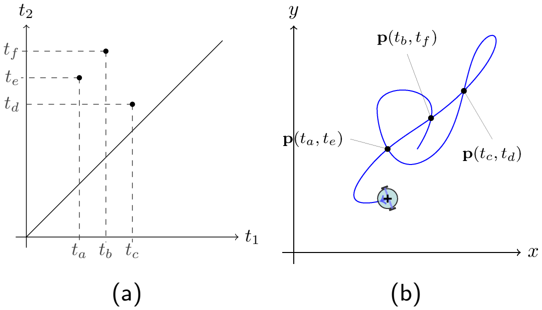
- ✓ two identical positions,
- ✓ at two different times.

We want to characterize the set of all feasible loops in the trajectory.

Formalism and representation of the loop set

Definition (Loop Set)

$$\mathbb{T}^* = \left\{(t_1, t_2) \in [0, t_{\max}]^2 \mid \mathbf{p}(t_1) = \mathbf{p}(t_2), \, t_1 < t_2\right\}.$$



(a) *t-plane* corresponding to trajectory (b).

Interval based approach

Context

- ✓ position $\mathbf{p}(t)$ unknown,
- ✓ speed $[\mathbf{v}](t)$ known with a tube,
- ✓ $[\mathbf{p}](t) = \int_0^t [\mathbf{v}](\tau)\, d\tau$

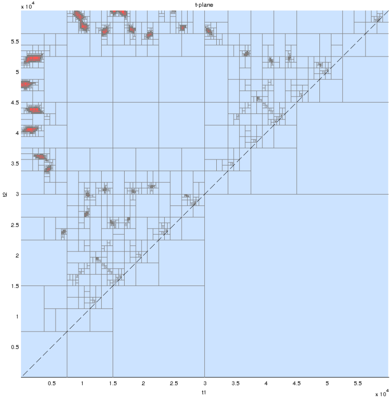
Problem

$$\mathbb{T} = \left\{ (t_1, t_2) \mid 0 \leq t_1 < t_2 \leq t_{max},\ \exists \mathbf{v} \in [\mathbf{v}],\ \int_{t_1}^{t_2} \mathbf{v}(\tau) d\tau = \mathbf{0} \right\}.$$

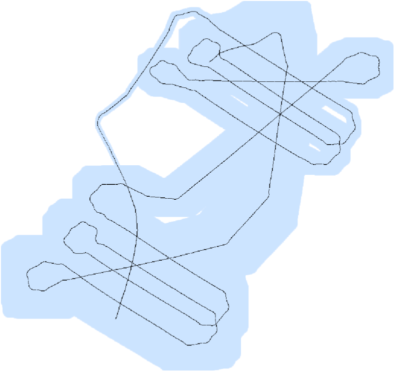
And find a subpaving approximation such that $\mathbb{T}^- \subset \mathbb{T} \subset \mathbb{T}^+$.

C. Aubry, R. Desmare, and L. Jaulin. "Loop detection of mobile robots using interval analysis". In: *Automatica* 49.2 (2013), pp. 463–470.

Resolution with interval analysis



(a)



(b)

(a) *t-plane* corresponding to trajectory (b).

Kalman predictor (1)

Robot classical state equations

$$x_{k+1} = A_k x_k + u_k, \text{ where } \begin{cases} u_k & \text{represent inputs} \\ A_k & \text{the state matrix} \\ x_k & \text{the state of the robot} \end{cases} . \quad (1)$$

In order to estimate x,

Kalman predictor (no exteroceptive measurement)

$$\begin{cases} \hat{x}_{k+1} = A_k \hat{x}_k + u_k \\ \Gamma_{k+1} = A_k \Gamma_k A_k^T + \Gamma_\alpha \end{cases} \quad (2)$$

where Γ_{k+1} is the covariance matrix representing the uncertainty and Γ_α the covariance associated with a normally distributed noise.

Kalman predictor (2)

From [NJP14], we know that:

$$\hat{x}_k = P_k^0 \hat{x}_0 + \sum_{i=0}^{k-1} P_{k+1}^i u_i \qquad (3)$$

$$\Gamma_k = P_k^0 \Gamma_0 \left(P_k^0\right)^T + \sum_{i=1}^{k} P_k^i \Gamma_\alpha \left(P_k^i\right)^T \qquad (4)$$

where transition matrices P_k^i are defined by

$$\begin{aligned} P_k^i &= A_{k-1} A_{k-2} \ldots A_i . I, \\ P_k^k &= I, \\ P_k^i &= P_k^l P_l^i, \\ P_k^i &= P_k^0 \left(P_i^0\right)^{-1}. \end{aligned}$$

Jeremy Nicola, Luc Jaulin, and Sébastien Pennec. “Toward the hybridization of probabilistic and set-membership methods for the localization of an underwater vehicle.” In: *7th Small Workshop on Interval Methods*. Uppsala, Sweden. 2014.

Kalman predictor (2)

Which allow us to get an evaluation of $\hat{x}_{k_1}, \Gamma_{k_1}$ and $\hat{x}_{k_2}, \Gamma_{k_2}$ in order to compute distances between uncertain position:

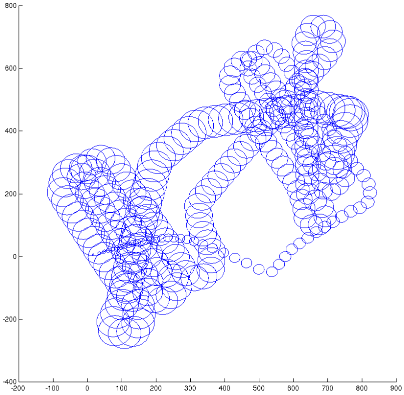
Distance operator:

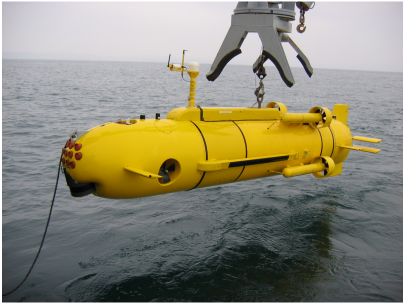
- ✓ Euclidean distances $d\left(\hat{x}_{k_1}, \hat{x}_{k_2}\right)$.
- ✓ Mahalanobis distance $D_m\left(\hat{x}_{k_1}, \hat{x}_{k_2}\right) = \sqrt{\left(\hat{x}_{k_1} - \hat{x}_{k_2}\right)^T \Gamma_{k_1,k_2}^{-1} \left(\hat{x}_{k_1} - \hat{x}_{k_2}\right)}$.

With

$$\Gamma_{k_1,k_2} = P_{k_2}^{k_1} \Gamma_{k_1} \left(P_{k_2}^{k_1}\right)^T + \sum_{i=k_1+1}^{k_2} P_{k_2}^{i} \Gamma_{\alpha} \left(P_{k_2}^{i}\right)^T \tag{5}$$

Kalman predictor (2)

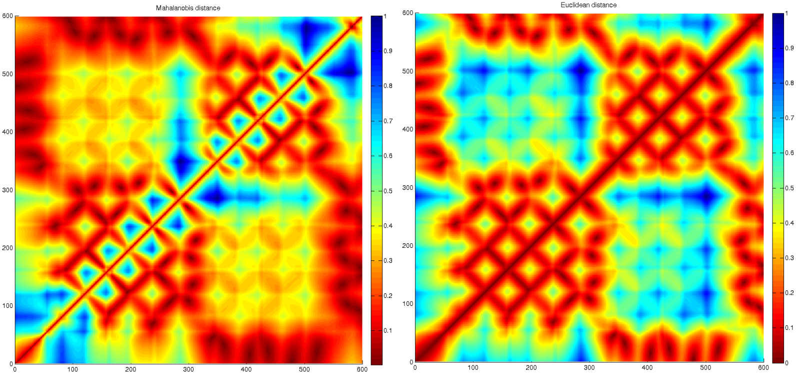




(a) (b)

(a) Positions of the underwater robot Redermor (b) from GESMA.

Results: normalized distances

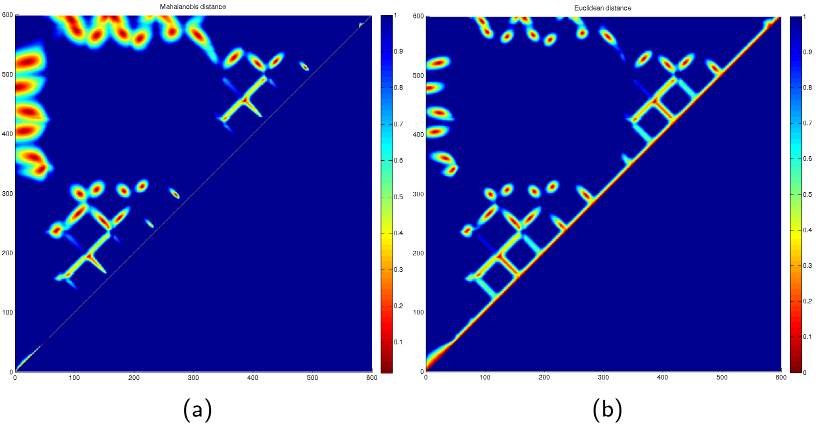


(a) (b)

T-planes: Normalized Mahalnobis distance (a), Normalized Euclidean distance (b).

Maximum distances: 1148.02(euclidean); 1105.79(Mahalanobis).

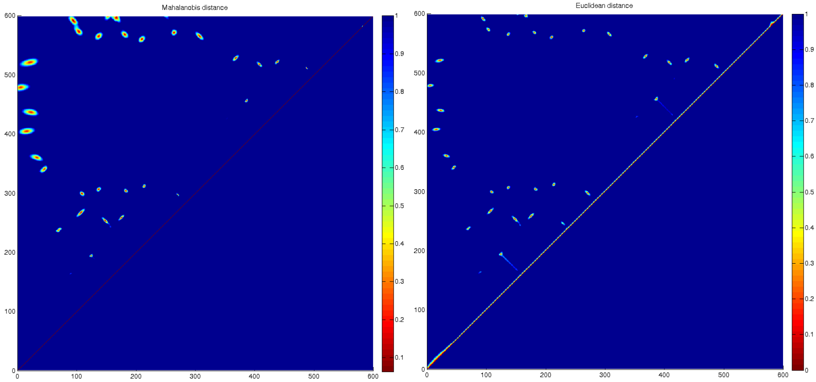
Results: thresholded normalized distances



t-planes: (a) Normalized Mahalnobis distance, (b) Normalized Euclidean distance.

With a threshold at 200

Results: thresholded normalized distances



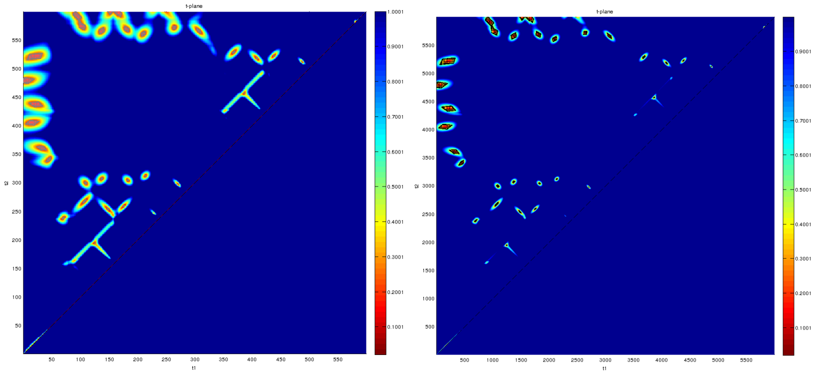
(a) (b)

t-planes: (a) Normalized Mahalnobis distance, (b) Normalized Euclidean distance.

With a threshold at 50

Comparison: decreasing threshold (1)

T-planes from Kalman + Mahalanobis in the background and from inner test of interval analysis on the foreground.

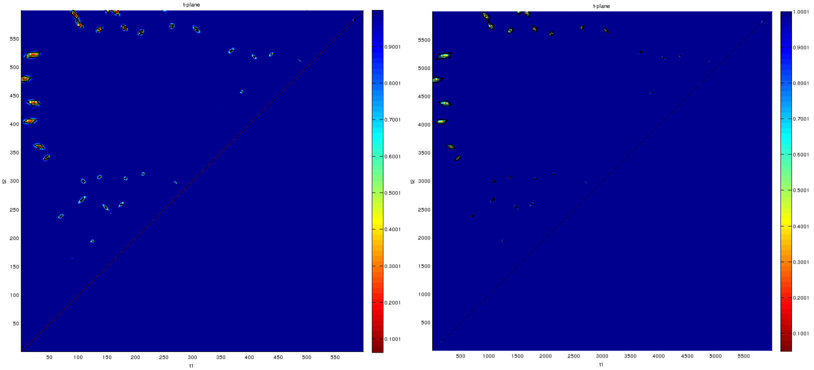


(a) (b)

t-planes results of loop detection problem solved by both methods with Mahalanobis distance and a threshold at 150 (a), 50 (b).

Comparison: decreasing threshold (2)

T-planes from Kalman + Mahalanobis in the background and from inner test of interval analysis on the foreground.

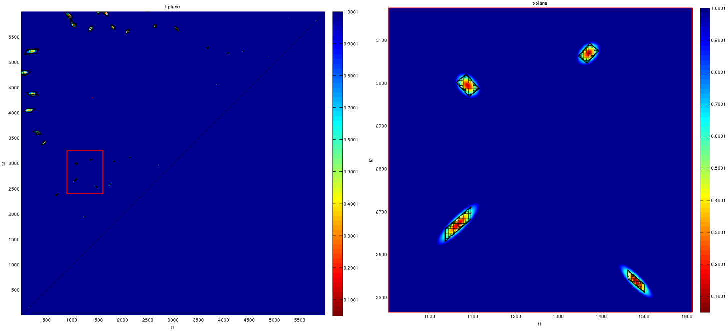


(a) (b)

t-planes results of loop detection problem solved by both methods with Mahalanobis distance and a threshold at 25 (a) and 10 (b).

"Fusion" of Kalman and Interval method (1)

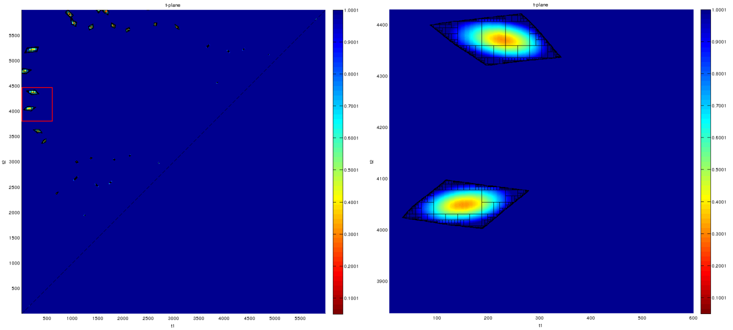
On each figures, t-planes from Kalman with thresholded (value=10) Mahalanobis distance in the background and boxes that pass inner test of interval based method. Zoom on some parts of the loop set.



(left) zoom on the red box on t-plane from (right).

"Fusion" of Kalman and Interval method (2)

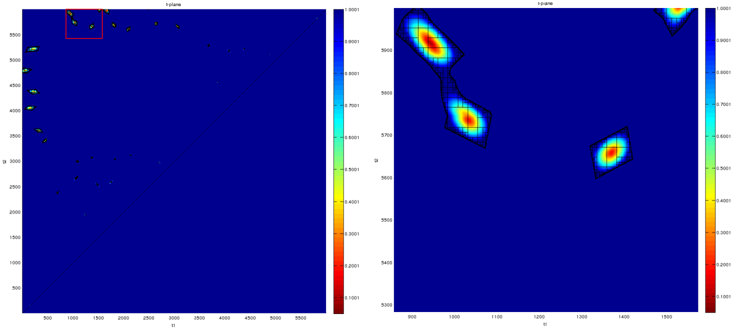
On each figures, t-planes from Kalman with thresholded (value=10) Mahalanobis distance in the background and boxes that pass inner test of interval based method. Zoom on some parts of the loop set.



(left) zoom on the red box on t-plane from (right).

"Fusion" of Kalman and Interval method (2)

On each figures, t-planes from Kalman with thresholded (value=10) Mahalanobis distance in the background and boxes that pass inner test of interval based method. Zoom on some parts of the loop set.



(left) zoom on the red box on t-plane from (right).

Conclusion and discussion

1. State model of the robot helps us to compute Kalman estimates $(A_k = I)$.
2. Kalman bring us an information: where is (probably!) the loop in a subpavement that compose the loop set.
3. The further we are from $t_1 = t_2$ line, the better is the quality of Kalman information compared to Interval method.

Thanks for your attention.
Questions?