Robot localization in an unknown but symmetric environment

Simon Rohou[1], Luc Jaulin[1], Lyudmila Mihaylova[2], Fabrice Le Bars[1], Sandor M. Veres[2]

[1] ENSTA Bretagne, Lab-STICC, rue François Verny 29200 Brest, France
[2] University of Sheffield, Western Bank Sheffield S10 2TN, UK
simon.rohou@ensta-bretagne.org

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Introduction

We consider the dynamic localization of a robot moving inside an environment which is unknown. We assume the environment has some symmetry properties (*e.g.*, cylindrical, axial, etc.). This type of localization can be met in an underwater context where the celerity of the sound is unknown but depth-dependent. In such a context, a classical state estimation approach such as [Gni10] cannot be applied without some modifications based on the formalization presented in this paper.

We assume that the robot is described by:

$$\begin{cases} \dot{\mathbf{x}}(t) & = & \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \\ y(t) & = & h \circ g(\mathbf{x}(t)) \end{cases}$$

Where $\mathbf{x} \in \mathbb{R}^n$ is the state vector, $\mathbf{u} \in \mathbb{R}^m$ is the input vector, $\mathbf{f} : \mathbb{R}^n \to \mathbb{R}^n$ is the *evolution* function, $g : \mathbb{R}^n \to \mathbb{R}$ the *observation* function and $y \in \mathbb{R}$ is a measurement which is assumed to be scalar. We call $h : \mathbb{R} \to \mathbb{R}$ the *distortion* function which pictures the uncertainties of the environment.

Here, h is considered unknown but strictly increasing and we will assume that $\mathbf{u}(t)$ and $y(t)$ are not known precisely. Instead, we have two tubes $[\mathbf{u}](t)$ and $[y](t)$ containing $\mathbf{u}(t)$ and $y(t)$.

The approach to be considered here extends the double weighing principle of Borda. More precisely, we will introduce some intertemporal relations in order to cancel the unknown effects of the environment. A contractor programming method [Cha09] will then be applied to enclose the trajectory of the robot.

Main approach

To solve this problem, we define the function:

$$\varphi(t_1, t_2) = h \circ g(\mathbf{x}(t_2)) - h \circ g(\mathbf{x}(t_1)).$$

Since h is injective, we have: $\varphi(t_1, t_2) = 0 \quad \Rightarrow \quad g(\mathbf{x}(t_1)) = g(\mathbf{x}(t_2))$. As a consequence if $\varphi(t_1, t_2) = 0$, the two states $\mathbf{x}(t_1)$ and $\mathbf{x}(t_2)$ are symmetrical with respect to the environment. A known relation between these two states (here $g(\mathbf{x}(t_1)) = g(\mathbf{x}(t_2))$) can be derived. This relation can then be used for state estimation. The main problem we now have is to find these t-pairs (t_1, t_2) [Aub13] such that $\varphi(t_1, t_2) = 0$.

Proposition 1. If we define:

$$[\varphi](t_1, t_2) = [y](t_2) - [y](t_1) = [y^-(t_2) - y^+(t_1), y^+(t_2) - y^-(t_1)]$$

then we have $\varphi(t_1, t_2) \in [\varphi](t_1, t_2)$.

Proof. Since for all t, $h \circ g(\mathbf{x}(t)) = y(t)$ and since $y(t) \in [y](t)$, we have $\varphi(t_1, t_2) \in [\varphi](t_1, t_2) = [y](t_2) - [y](t_1)$.

Definition. We define the *presymmetric* set $\mathbb{S}$ as:

$$\mathbb{S} = \{(t_1, t_2) \mid g(\mathbf{x}(t_2)) - g(\mathbf{x}(t_1)) \leq 0\}$$

Since h is increasing, we have:

$$\mathbb{S} = \{(t_1, t_2) \mid \varphi(t_1, t_2) \leq 0\} = \varphi^{-1}(\mathbb{R}^-)$$

Now, φ is not known exactly, *i.e.*, we only have $\varphi \in [\varphi] = [\varphi^-, \varphi^+]$. As a consequence, we have:

$$\underbrace{(\varphi^+)^{-1}(\mathbb{R}^-)}_{\mathbb{S}^-} \;\subset\; \mathbb{S} \;\subset\; \underbrace{(\varphi^-)^{-1}(\mathbb{R}^-)}_{\mathbb{S}^+}$$

We can reformulate the problem into the following *Constraint Network*:

$$\left\{\begin{array}{l}
\quad \textbf{Variables: } \mathbf{x}, \mathbb{S} \\
\quad \textbf{Constraints:} \\
\qquad \text{(a) } \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}) \\
\qquad \text{(b) } \mathbb{S} = \{(t_1, t_2) \mid g(\mathbf{x}(t_2)) - g(\mathbf{x}(t_1)) \leq 0\} \\
\qquad \text{(c) } (\varphi^+)^{-1}(\mathbb{R}^-) \;\subset\; \mathbb{S} \;\subset\; (\varphi^-)^{-1}(\mathbb{R}^-) \\
\qquad\quad \text{where } \varphi^-(t_1, t_2) = y^-(t_2) - y^+(t_1) \text{ and } \varphi^+(t_1, t_2) = y^+(t_2) - y^-(t_1) \\
\textbf{Domains: } [\mathbf{x}], [\mathbb{S}] \\
\textbf{Initialization}\text{: } \mathbf{x} \in [-\infty, +\infty]^2, \; [\mathbb{S}] = [\emptyset, [0, t_{max}]^2].
\end{array}\right.$$

In this network, the unknow function h does not appear anymore. This is due to the fact that inter-temporal constraints compensate the unknown influence of h.

Test case

We consider a boat moving along a line. A static beacon stands on the seabed just below the origin of the frame. We do know neither the depth of the beacon nor the celerity of the sound. We assume that the environment is symmetric with respect to any horizontal translation. The robot is able to measure signals time of flight between its position and the beacon. The state vector of the system is $\mathbf{x} = \{s, \dot{s}\}^\intercal$ where s is the position of the robot. At first, the robot is completely lost ($s \in [-\infty, +\infty]$). Whith relevant inter-temporal measurements, the robot is able to contract $[\mathbf{x}]$ in *forward-backward*. This will be done computing an approximation of the set $\mathbb{S}$ as illustrated by Fig 1.

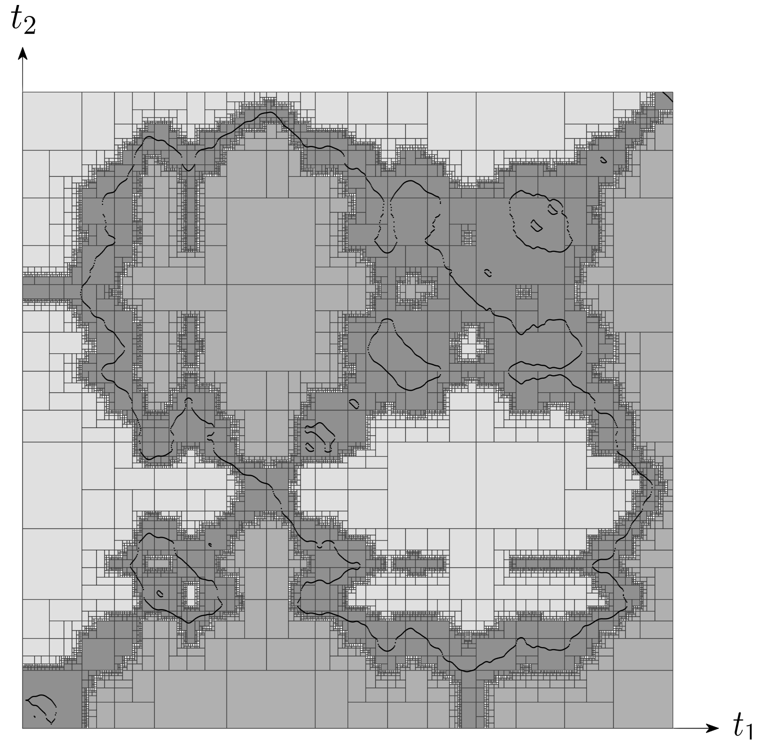

Figure 1: a t-plane obtained with a Sivia algorithm. Middle-gray boxes are inside the *presymmetric* set $\mathbb{S}$ whereas light-gray ones are outside. We can guarantee true symmetries (pictured with black lines) belong to dark-gray areas.

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