Primitive Shape Characterization using Interval Methods

Salvador Pacheco and Alexandru Stancu

The University of Manchester,
School of Electrical and Electronic Engineering
Oxford Rd, Manchester, M13 9PL, UK.
{salvador.pachecogutierrez, alexandru.stancu}@manchester.ac.uk

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Introduction

In the last years, the development of laser scanning technologies has lead to the increase of investigation in areas such as computer vision, 3D modeling, scene recognition, reverse engineering, etc.

This work is focused on the recognition of primitive shapes immerse in hazardous environments for decommissioning, more specifically in nuclear ambiances. This is, the characterization of basic geometrical shapes such as lines, circles, planes, spheres, and cylinders. The information is retrieved using the structured-light scanner Kinect, developed by Microsoft.

Approach

The detection of circles on images using interval methods has been investigated by Jaulin [1], as a problem of parameter estimation under error-bounded estimation basis. Therefore,

$$\mathbb{P} = \bigcap_{i\in\{1,\ldots,m\}} \underbrace{\{\mathbf{p} \in \mathbb{R}^{n_p}, \exists[\mathbf{y}] \in [\mathbf{y}]_i, \mathbf{f}(\mathbf{p},\mathbf{y}) = 0\}}_{\mathbb{P}_i} \tag{1}$$

being $\mathbf{p}$ the parameter vector, $[\mathbf{y}]_i \subset \mathbb{R}^{n_y}$ is the ith measurement box and $\mathbf{f}$ is the model function. In this sense, the set $\mathbb{P}_i$ is the set of all parameters vector consistent with the ith measurement box.

Jaulin defined the shape extraction as a set estimation problem [3]. The above can be expressed as follows,

$$\mathbf{f} : \begin{cases} \mathbb{R}^{n_p} \times \mathbb{R}^d \rightarrow \mathbb{R}^{n_f} \\ (\mathbf{p}, \mathbf{y}) \qquad \rightarrow \mathbf{f}(\mathbf{p}, \mathbf{y}) \end{cases} \tag{2}$$

being $d \in \{2, 3\}$ the dimension of the analyzed shape. The vector $\mathbf{y} \in \mathbb{R}^d$ is the point cloud and $\mathbf{p}$ is the parameter vector that corresponds to the shape under analysis. Under this basis, the shape that corresponds to the vector $\mathbf{p}$ is defined as follows:

$$\mathcal{S}(\mathbf{p}) \overset{\text{def}}{=} \left\{ \mathbf{y} \in \mathbb{R}^d, \mathbf{f}(\mathbf{p}, \mathbf{y}) = 0 \right\} \tag{3}$$

Taking into consideration a set of boxes (measurements) in the primitive shape space dimension d, each of this boxes is assumed to touch the periphery of the considered shape. In this sense, the aim of this work is to extend the above concepts to a 3-dimensional space.

According to [3], the set of parameters for the description of each of the features of interest are the following:

Associated feature	**Parameters associated**		
	Location	Orientation	Size
Line (2D)	x_0, y_0	a, b	–
Circle (2D)	x_0, y_0	–	r
Plane	x_0, y_0, z_0	a, b, c	r
Sphere	x_0, y_0, z_0	–	r
Cylinder	x_0, y_0, z_0	a, b, c	r

These parameters are required to be accurate enough in order to have a good representation of the geometrical feature.

Results

Due to space restrictions, only the results of the plane and sphere are presented.

The shape of a sphere can be defined by the following expression:

$$\mathbf{f}(\mathbf{p}, \mathbf{y}) = (y_1 - x_0)^2 + (y_2 - y_0)^2 + (y_3 - z_0)^2 - r^2 \tag{4}$$

Using the proposed approach, the feature parameters are presented in Table 1.

Parameter	Value	Initial box	Parameters Detected
x_0	-10	$[-100, 100]$	$[-10.2597, -9.63753]$
y_0	-20	$[-100, 100]$	$[-20.3153, -19.657]$
z_0	30	$[-100, 100]$	$[29.6336, 30.3557]$
r	5	$[0, 100]$	$[4.77378, 5.28706]$

Table 1: Parameters of the sphere.

On the other hand, a plane can be defined by its general equation:

$$Ax + By + Cz + D = 0 \tag{5}$$

Fixing $B = -1$, the remaining feature parameters are presented in Table 2.

Parameter	Value	Initial box	Parameters Detected
A	2.94	$[-100, 100]$	$[2.71565, 3.20899]$
C	1.16	$[-100, 100]$	$[1.07501, 1.26956]$
D	-10.388	$[-100, 100]$	$[-11.423, -9.49593]$

Table 2: Parameters of the plane.

Moreover, a cylinder can be defined by a point lying on its central axis $\mathbf{X} = (x_0, y_0, z_0)$, the direction of such axis $\mathbf{n} = (n_x, n_y, n_z)$ and the radius r. Due to the nature of this representation, no closed form can be inferred. The procedure proposed to solve this problem lyes in transforming the 3-dimensional problem into a 2-dimensional one, by projecting all the points over a plane A in order to look for the plane parameters on which the projection of the cylinder points is a circle. Further information regarding this approach will be discussed during the workshop.

References

[**1**] L. Jaulin and S. Bazeille, Image Shape Extraction using Interval Methods, *Sysid 2009* Saint Malo, France, 2009.

[**2**] L. Jaulin and E. Walter, Guaranteed bounded-error parameter estimation for nonlinear models with uncertain experimental factors, *Automatica* 35(5):849–856, 1999.

[**3**] European Committe for Standarization, Geometrical Product Specifications (GPS) – Acceptance and reverification tests for coordinate measuring machines (CMM), ISO 10360-6:2001 *British Standards*, May 2007.