Mobile Robot Mapping using Interval Methods

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Introduction

For SLAM problem [3], building an accurate map leads to an accurate localization. We propose a guaranteed solution using interval methods for nonlinear observation model to work with holnomic robots with no rotation, where the map is proven to converge. Our approach does not require any assumptions with regard to the linearity of the observation model, nor its noise except that it needs to be bounded. We use interval methods to evaluate the domain of a function given the codomain and the function itself. This approach encapsulates all information in the current estimate, therefore, it is not necessary to keep track of all past observations. We will prove the convergence of the approach to the correct map as the robot moves in the environment over time, given that at each time step, at least one old landmark is observed, and the data association problem is assumed a solved problem.

Assumptions

In this work, we assume the following:

1. The robot is holonomic with no rotation and it is moving in a 2-D environment.

2. The robot is equipped with a LIDAR to measure the range $\rho_{m_i,t}$ and the bearing $\alpha_{m_i,t}$ of landmarks in the robot frame, where m_i is the position of the i^{th} landmark, $i = 1 : N$, and N is the total number of observed landmarks at time t.

3. Each measurements is uncertain with some known bounded noise in the range $[\omega_{\rho,m_i,t}]$ and in the bearing $[\omega_{\alpha,m_i,t}]$.

4. At least one old landmark is observed at each time step.

5. The robot can distinguish between different landmarks, i.e., data association problem is solved.

Concept

Let $d_{x,m_i,t}$ and $d_{y,m_i,t}$ be the distance between the robot and the i^{th} landmark in the x and the y directions, respectively. Then, the observation model of the sensor is in the form of:

$$z_{\rho,m_i,t} = \sqrt{d^2_{x,m_i,t} + d^2_{y,m_i,t}} \tag{1}$$

$$z_{\alpha,m_i,t} = \arctan 2(d_{y,m_i,t}, d_{x,m_i,t}) \tag{2}$$

The distance between landmark i and landmark j is defined as follows:

$$dl_{x,(i,j),t} = d_{x,m_i,t} - d_{x,m_j,t} \tag{3}$$

$$dl_{y,(i,j),t} = d_{y,m_i,t} - d_{y,m_j,t} \tag{4}$$

Since the measuremnts have some bounded uncertainty, both $z_{\rho,m_i,t}$ and $z_{\alpha,m_i,t}$ belong to *intervals*, such that $z_{\rho,m_i,t} \in [z_{\rho,m_i,t}]$ and $z_{\alpha,m_i,t} \in [z_{\alpha,m_i,t}]$. We define a constraint satisfaction problem (CPS) [1] using $dl_{x,(i,j),t}$, $dl_{y,(i,j),t}$ as *variables*, and Eq.(1-4) as *constraints*. We associate a contractor for each constraint such that: $\mathcal{C}^{d_{x,m_i,t},d_{y,m_i,t}}_{z_{\rho,m_i,t}}$ for Eq.(1), $\mathcal{C}^{d_{x,m_i,t},d_{y,m_i,t}}_{z_{\alpha,m_i,t}}$ for Eq.(2), $\mathcal{C}^{d_{x,m_i,t},d_{x,m_j,t}}_{dl_{x,(i,j),t}}$ for Eq.(3) and $\mathcal{C}^{d_{y,m_i,t},d_{y,m_j,t}}_{dl_{y,(i,j),t}}$ for Eq.(4), where $i, j = 1 : N$, and N is the number of observed landmarks at time t. For example, if the robot observes two landmarks, then, we have 6 contractors associated with 6 constraints. Next, we use contractor programming [4] to solve for the distance between landmarks i and j, which is denoted by the subpaving $\mathbb{S}_{\mathbf{dl}_{(i,j)},t}$, where $\mathbf{dl} = [dl_x, dl_y]^T$. Since the landmarks are stationary in the environment, the distance estimate $\mathbb{S}_{\mathbf{dl},(i,j),t}$ must be consistent at any time t when landmarks i and j are observed, therefore, the following equation holds:

$$\cap_{t=1}^{T} \quad \mathbb{S}_{\mathbf{dl}_{(i,j)},t} \neq \phi \tag{5}$$

As the landmarks are observed frequently, we will show that the left-hand-side of Eq.(5) converges to the true distance $\mathbf{dl}_{(i,j)}$ between the i^{th} landmark and the j^{th} landmark as $t \to \infty$. If the position of one landmark $\mathbf{m}_i$ is known exactly (anchoring landmark), then, by using the distance $\mathbb{S}_{\mathbf{dl}_{(i,j)},t}$ we can estimate the position of the second landmark $\mathbf{m}_j$ as follows:

$$\mathbf{m}_j = \mathbf{m}_i - \mathbb{S}_{\mathbf{dl}_{(i,j)},t} \tag{6}$$

where $\mathbf{m} = [m_x, m_y]^T$, and the $''-''$ operator is overloaded for subtraction of set of vectors. Since $\mathbb{S}_{\mathbf{dl}_{(i,j)},t} \to \mathbf{dl}_{(i,j)}$, the map converges to the true map. If the anchoring landmark is not available, there will be some offset between the estimated map and the true map.

References

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