# Distributed localization and control of a group of underwater robots using contractor programming

L. Jaulin, S. Rohou, J. Nicola,

M. Saad, F. Le Bars and B. Zerr,

ENSTA Bretagne, Lab-Sticc, rue François Verny 29200 Brest

{luc.jaulin}@ensta-bretagne.fr

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## Introduction

We consider the problem of localizing a group of underwater robots and to control the group in order to accomplish a survey. We assume here that

- 1. When a robot surfaces, it can use the GPS for its localization.
- 2. The robots can communicate with a very low symbol rate.
- 3. The robots can measure their distances with a given accuracy, but not the direction of arrival.
- 4. Some outliers on the distances could occur, but their numbers is limited.

- 5. The localization process should be fast.
- 6. The robots have to use their estimated location to control their trajectory

We propose here to use a contractor programming method [Cha09] to solve the problem.

#### Main approach

We assume that we have  $\bar{\ell}$  robots and that the motion of the  $i^{\text{th}}$  robot is described by a state equation of the form  $\mathbf{x}_{k+1}^j = \mathbf{f}(\mathbf{x}_k^j, \mathbf{u}_k^j)$  where  $\mathbf{x}_k^j$  is the state vector of the robot at time k, and  $\mathbf{u}_k^j$  is the input vector. The input vector corresponds to the proprioceptive sensors (speed, heading, actuators) and is assumed to be known with some accuracy. Moreover, the robots are able to collect some intrinsic measurements of the form  $\mathbf{y}_k^{j,\ell} = \mathbf{g}(\mathbf{x}_k^j, \mathbf{x}_k^\ell)$ . By *intrinsic*, we mean that the measurements are not related to the environment, but to the group itself. For instance,  $\mathbf{g}(\mathbf{x}_k^j, \mathbf{x}_k^\ell)$  may correspond to the distance between to the robot j and the robot  $\ell$  et time k. We also assume that each robot may also collect some *extrinsic* data which are related to the environment (distance to a landmark, for instance). This corresponds to an observation function of the form  $\mathbf{z}_k^j = \mathbf{h}(\mathbf{x}_k^j)$ . We assume that  $\mathbf{u}_k^j, \mathbf{y}_k^{j,\ell}, \mathbf{z}_k^j$  all belong to some boxes  $[\mathbf{u}_k^j], [\mathbf{y}_k^{j,\ell}], [\mathbf{z}_k^j]$ . These boxes could be small for high quality sensors and could be equal to  $\mathbb{R}^n$  when no information is available. For each robot  $\mathcal{R}_j$ , at time k, we define a distributed CSP (Constraint Satisfaction Problem) [Mou12] as follows.

**Variables**. The variables are all states on a time window of length  $\bar{h}$ , i.e.,  $\mathbf{x}_{h}^{j}$ ,  $h \in k - \bar{h}, \ldots, k + 1$ .

Constraints. The constraints are the following

$$\begin{cases} \mathbf{x}_{h+1}^{j} &= \mathbf{f}(\mathbf{x}_{h}^{j}, [\mathbf{u}_{h}^{j}]) & (E_{x}^{h,j}) \\ [\mathbf{y}_{h}^{j,\ell}] &= \mathbf{g}(\mathbf{x}_{h}^{j}, [\mathbf{x}_{h}^{\ell}]), \ \ell \neq j & (E_{y}^{h,j,\ell}) \\ [\mathbf{z}_{h}^{j}] &= \mathbf{h}(\mathbf{x}_{h}^{j}) & (E_{z}^{h,j}) \end{cases}$$

where  $h \in \{k - \bar{h}, ..., k\}$  and  $\ell \in \{1, ..., \bar{\ell}\}$ . In these equation, when we write " $[\mathbf{y}_{h}^{j,\ell}] = \mathbf{g}(\mathbf{x}_{h}^{j}, [\mathbf{x}_{h}^{\ell}])$ ", we mean " $\exists \mathbf{y}_{h}^{j,\ell} \in [\mathbf{y}_{h}^{j,\ell}], \exists \mathbf{x}_{h}^{\ell} \in [\mathbf{x}_{h}^{\ell}], \mathbf{y}_{h}^{j,\ell}\mathbf{g}(\mathbf{x}_{h}^{j}, [\mathbf{x}_{h}^{\ell}])$ ". For each constraint  $E_{x}^{h,j}, E_{y}^{h,j,\ell}, E_{z}^{h,j,\ell}$ , we associate a contractor  $C_{x}^{h,j}, C_{y}^{h,j,\ell}, C_{z}^{h,j}$ . A contractor programming approach [Cha09] can then be used in order to perform the localization. In our CSP, we have two types of domains: the *internal domains* of the CSP:  $[\mathbf{x}_{h}^{j}], h \in$  $\{k - \bar{h}, ..., k+1\}$  and the *external domains*  $[\mathbf{u}_{h}^{j}], [\mathbf{y}_{h}^{j,\ell}], [\mathbf{z}_{h}^{j}], [\mathbf{x}_{h}^{\ell}], \ell \neq j, h \in$  $\{k - \bar{h}, ..., k\}$ . There is no need to contract these external domains, and the robot  $\mathcal{R}_{j}$  will not try to contract them.

**Communication**. Each robot  $\mathcal{R}_j$  contracts its own domain  $[\mathbf{x}_k^j]$  and broadcasts this information through the network. Since under the water, the communication rate is very low, each robot  $\mathcal{R}_j$  only broadcasts the box  $[\mathbf{x}_k^j]$  at time k.

**Initialization.** At the initial time k = 0, all variables are initialized with  $\mathbb{R}^n$ . When we switch from k to k+1, we remove the variable  $\mathbf{x}_{k-\bar{h}}^j$  from the CSP of each robot and we add the variable  $\mathbf{x}_{k+2}^j$  with the domain  $[\mathbf{x}_{k+2}^j] = \mathbb{R}^n$ .

#### Range-only distributed localization and control

As an example, we will consider some underwater robots moving in the ocean [Dre13]. The intrinsic observations correspond to the distances between robots and the extrinsic observations correspond to the GPS available when a robot surfaces. We assume that at most q outliers in the intrinsic measurements could occur within a time window of length  $\bar{h}$  [Leg10]. The following contractor, is implemented in each robot  $\mathcal{R}_{j}$ :

$$C^{k,j} = C^{k,j}_{x,z} \bigcap \ C^{k,j}_{x,y}$$

where

$$C^{k,j}_{x,z} = \bigcap_{h \in \{k-\bar{h},\dots,k\}} \left( C^{h,j}_x \circ C^{h,j}_z \right)$$

and

$$C_{x,y}^{k,j} = \bigcap_{\substack{h \in \{k - \bar{h}, \dots, k\} \\ \ell \neq j}}^{\{q\}} \left( C_x^{h,j} \circ C_y^{h,j,\ell} \right).$$

We will show that  $C^{k,j}$  will not remove the true position for the robot. For the control, a vector field approach will be considered. A test case will also be presented in order to illustrate the efficiency of the approach.

### References

- [Mou12] F. MOURAD, H. SNOUSSI, M. KIEFFER AND C. RICHARD, Robust interval-based localization algorithms for mobile sensor networks, *International Journal of Distributed Sensor Networks*, 1-7, 2012.
- [Leg10] J. LEGER AND M. KIEFFER, Guaranteed robust distributed estimation in a network of sensors, in *Proc. ICASSP'10*, 2010.
- [Dre13] V. DREVELLE, L. JAULIN AND B. ZERR, Guaranteed Characterization of the Explored Space of a Mobile Robot by using Subpavings, NOLCOS, 2013.
- [Cha09] G. CHABERT AND L. JAULIN, Contractor programming. Artificial Intelligence, 2009.