Recent Results on Cooperative Interval Games

Jan Bok[1] and Milan Hladik[2]

[1] Computer Science Institute, Charles University in Prague, Malostranské náměstí 25, 11800, Prague, Czech Republic
bok@iuuk.mff.cuni.cz
[2] Department of Applied Mathematics, Faculty of Mathematics and Physics, Charles University in Prague, Malostranské náměstí 25, 11800, Prague, Czech Republic
hladik@kam.mff.cuni.cz

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Introduction

Cooperative interval games are a special model of cooperative transferable utility games in coalition form in which characteristic function maps to the set of closed real intervals instead of real numbers. This can be viewed as an approach to deal with inexact data since all numbers from interval are equally likely to occur and therefore, bounds of interval can be interpreted as the worst and the best possible outcome that can happen if some coalition cooperates.

There is an existing research on cooperative interval games, mainly done by Alparslan-Gök and Branzei. However, none of the existing papers focuses on selections, that is on possible outcomes of interval games (in other words, all possible classical cooperative games in which values of coalitions are contained in respective intervals of given cooperative interval game). There are some results on selections of interval games (for example analogous theorem to Bondareva-Shapley theorem), but that is almost all what is known about selections. Large portion of results is focused on examining interval valued characteristic function only, using a weakly better operator $\succ$. Interval (a, b) is weakly better than (c, d) if $a \geq c$ and $b \geq d$. Solutions (such as core

and the Shapley value) and classes obtained in this way have a big flaw that not all of their selection have the corresponding properties (for example, not all selections of a convex interval game is a classical convex game).

Basic properties

Definition 1. (Cooperative interval game) *A cooperative game is an ordered pair* (N, w)*, where* $N = \{1, 2, \ldots, n\}$ *is a set of players and* $w : 2^N \to \mathbb{IR}$ *is a characteristic function of the cooperative game. We further assume that* $w(\emptyset) = [0, 0]$*.*

The set of all interval cooperative games on player set N *is denoted by* IG^N*.*

Main results

We examine a problem of core coincidence. Core coincidence problem asks for characterization of games in which set of vectors generated by interval core coincides with the set of vectors of selection core. We found and proved such characterization. Furthermore, we introduced a concept of *strong core* - an universal stable payoff which is included in every selection and therefore we can depend on it, since it has to occur. Consequently, characterization of games with nonempty strong core is given.

We also introduced a new classes of interval games - selection monotonic, selection superadditive and selection convex games. These classes have an important and desirable property – all selections of selection monotonic games are monotonic classical game. Similarly for selection convex and selection superadditive games. We found and proved characterization theorems of all three classes. For convex selection games, characterization are inspired by Shapley's characterization of convex games. Furthermore, we show that classes based on selections and classes based on weakly better operator are incomparable for nontrivial player set (more than one player).

Selections of an interval game are useful, since they do not contain any additional uncertainty. On the top of that, selection-based classes, strong core and strong imputation have crucial property that although we deal with uncertain data, all possible outcomes preserve important properties. In case of selection classes it is preserving superadditivity, supermodularity etc. In case of strong core it is an invariant of having particular stable payoffs in each selection. Furthermore, concepts like selection core are important as well since if selection core is empty, no selection has stable payoff.

The importance of studying selection-based classes instead of the existing classes using weakly better operator can be further illustrated by the following two facts:

- Classes based on weakly better operator may contain games with selections that do not have any link with the properties of their border games and consequently no link with the name of the class. For example, superadditive interval games may contain a selection that is not superadditive.

- Selection-based classes are not contained in corresponding classes based on weakly better operator. Therefore, the results on existing classes are not directly extendable to selection-based classes.

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References

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