The QuaSEFE Problem

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Stephen Kobourov, Giuseppe Liotta, Maurizio Patrignani

27th International Symposium on Graph Drawing and
Network Visualization 2019

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Simultaneous
(Graph)
Embedding with
Fixed **E**dges

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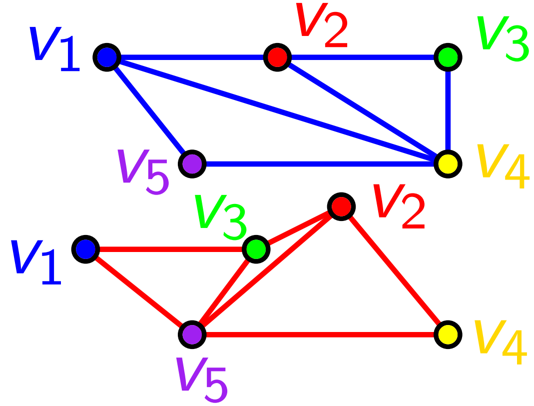
- **Input:** Set of planar graphs with shared vertex set



The QuaSEFE Problem

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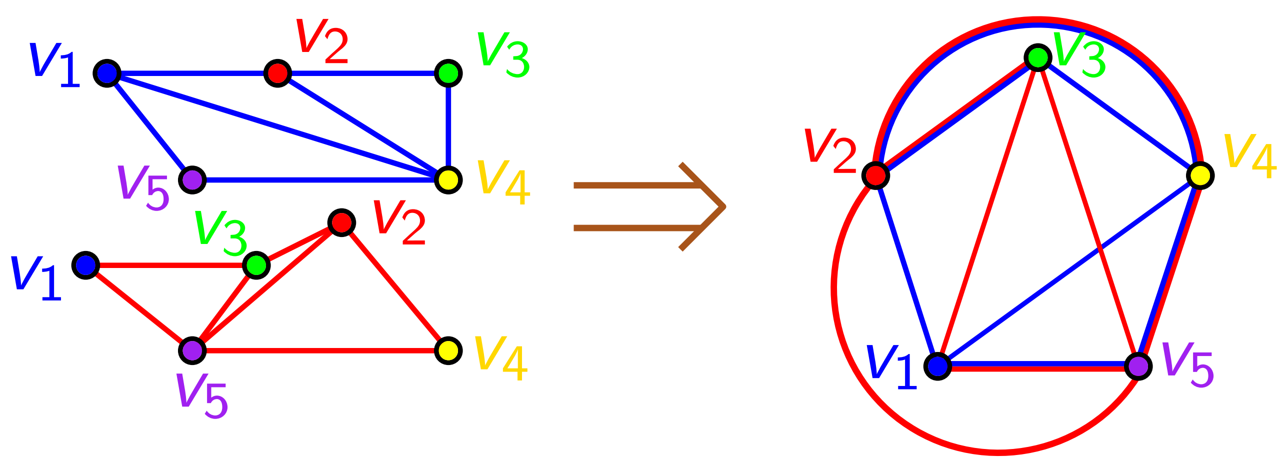
- **Input:** Set of planar graphs with shared vertex set
- **Output:** Planar drawings for all graphs such that



The QuaSEFE Problem

QuaSEFE

Simultaneous (Graph) **E**mbedding with **F**ixed **E**dges

- **Input:** Set of planar graphs with shared vertex set
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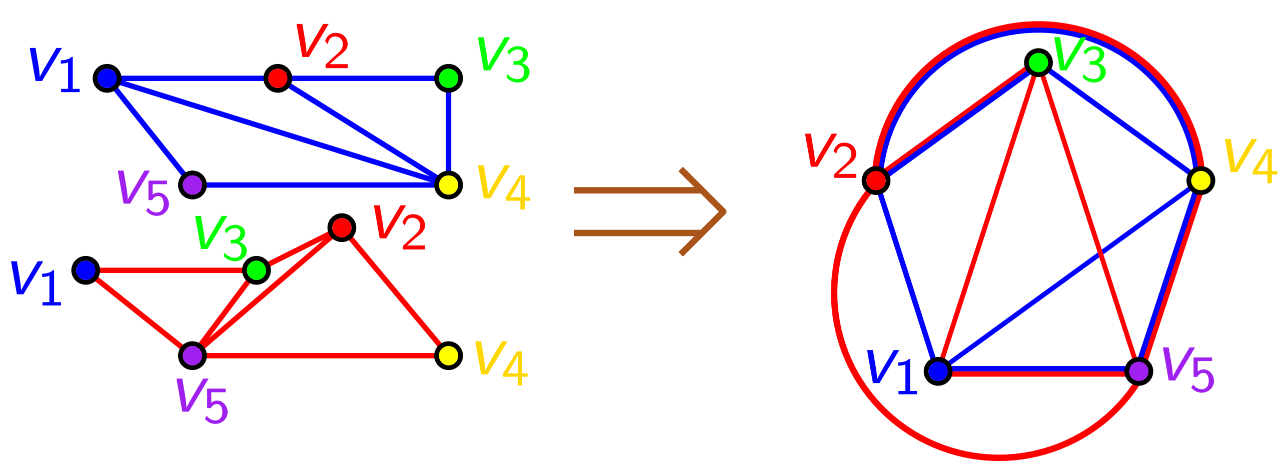
  - vertices have the same position in all drawings (*simultaneous* drawings)



The QuaSEFE Problem

QuaSEFE

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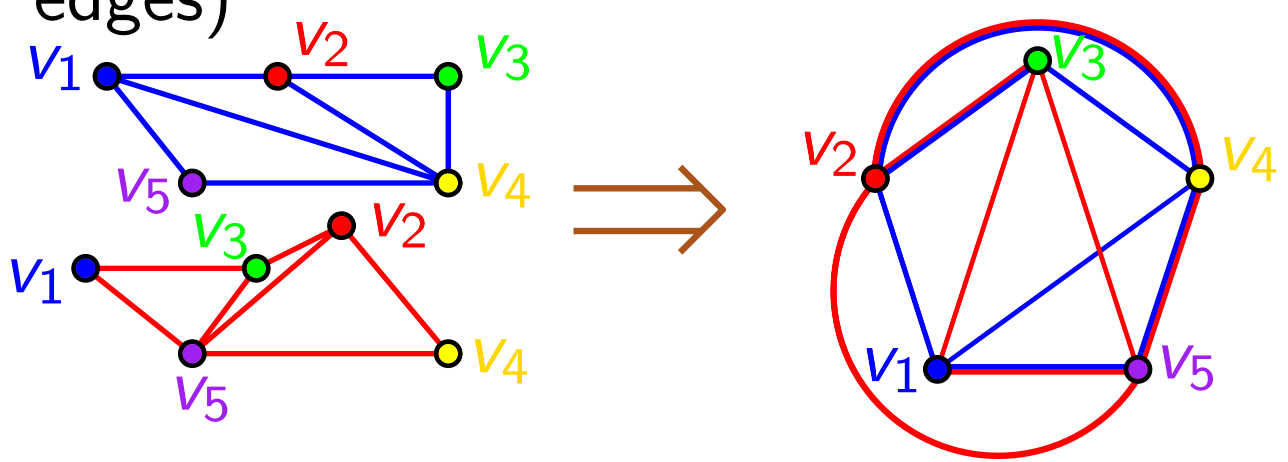
Embedding with
Fixed **E**dges

- **Input:** Set of planar graphs with shared vertex set
- **Output:** Planar drawings for all graphs such that
  - vertices have the same position in all drawings (*simultaneous* drawings)
  - edges have the same representation in all drawings (*fixed* edges)



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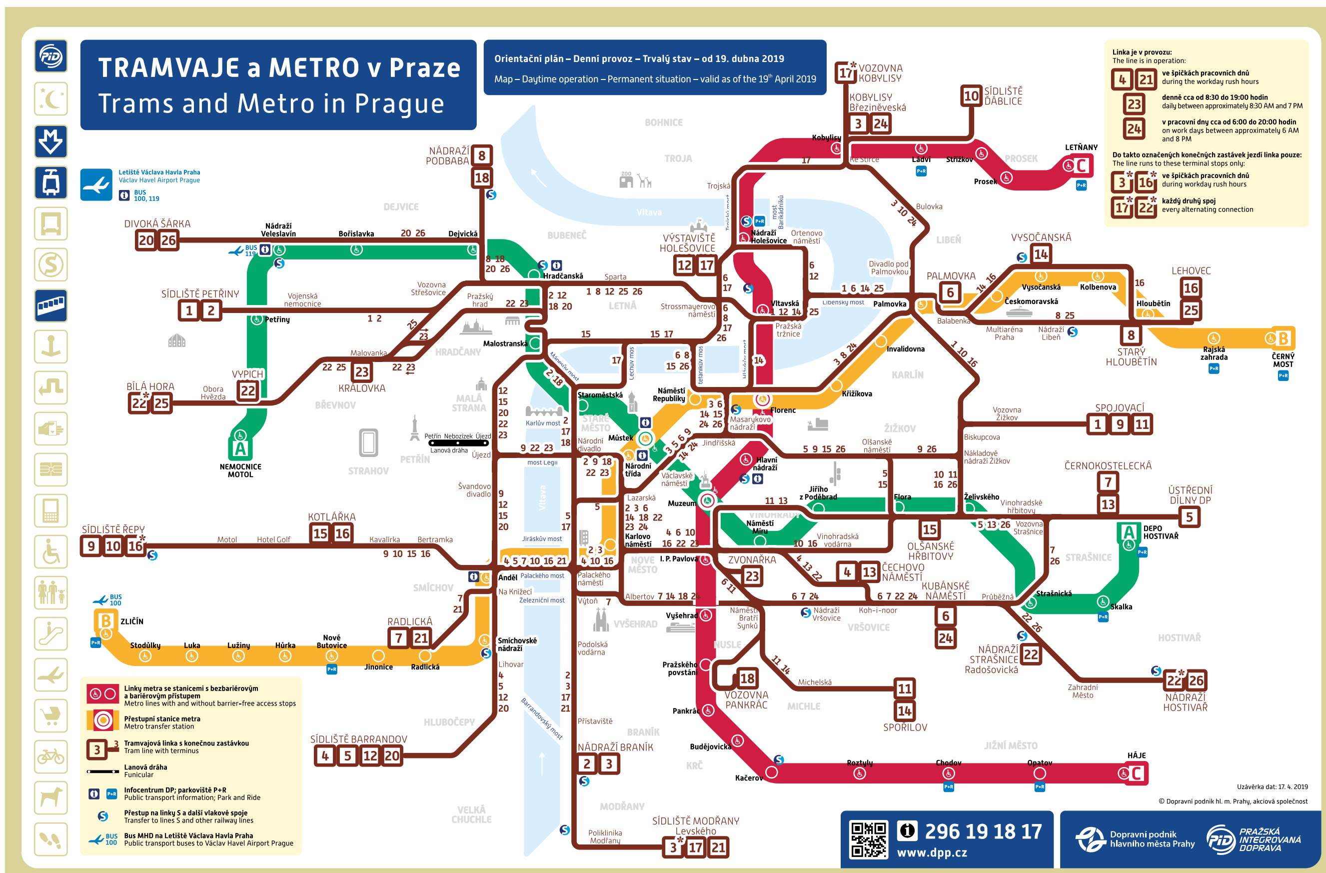

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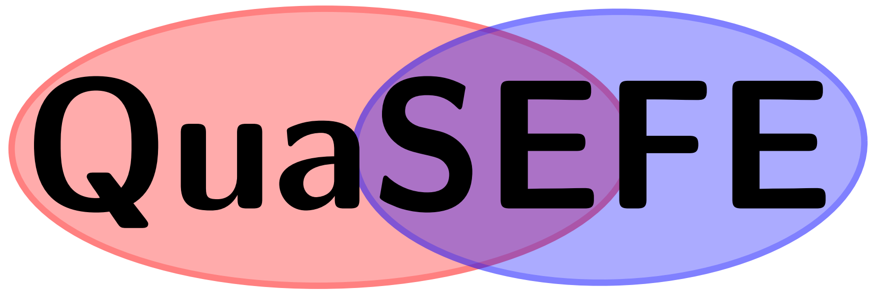

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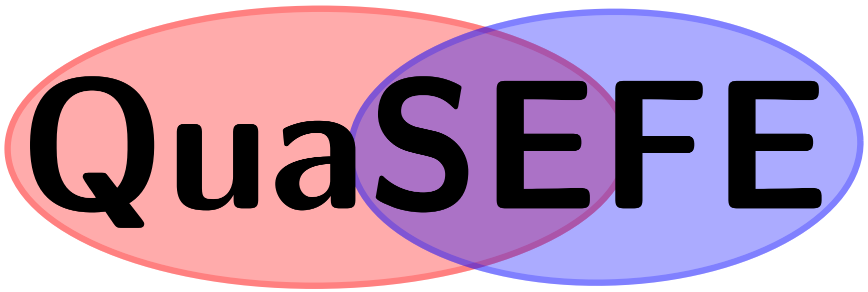
The QuaSEFE Problem

Quasiplanarity



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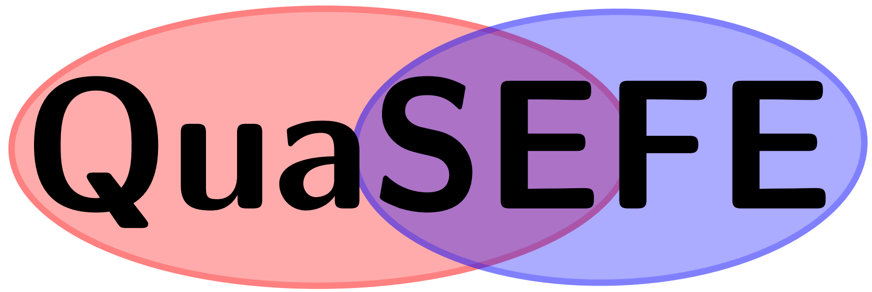

Simultaneous
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- Quasiplanar Embedding: No triple of edges crosses pairwise

The QuaSEFE Problem

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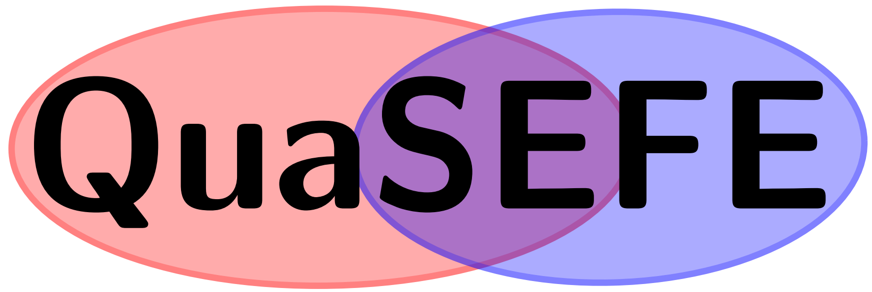

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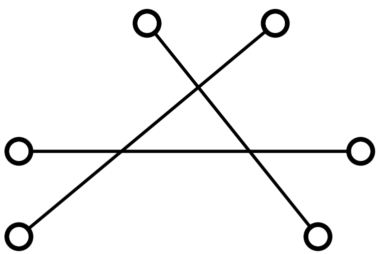

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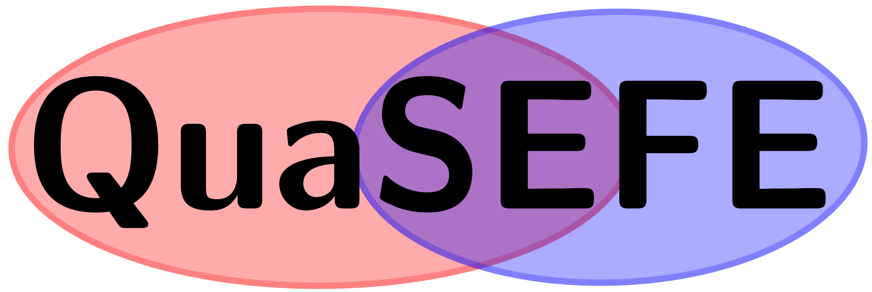

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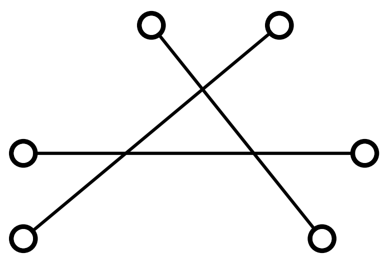

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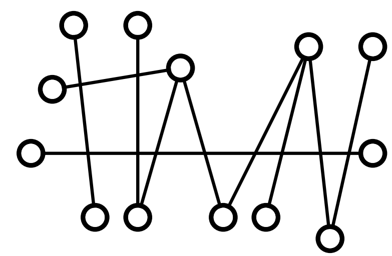

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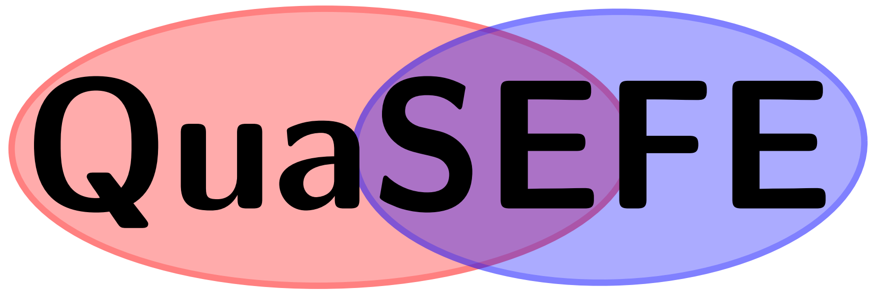

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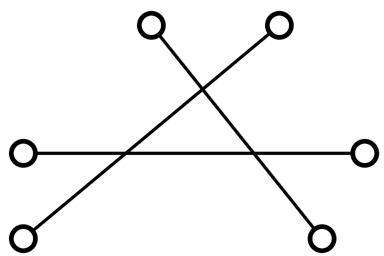

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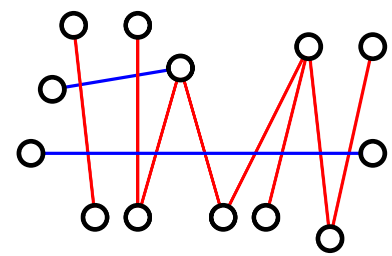

allowed
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- Thickness two drawings (i.e. two-edge colorable drawings) are quasiplanar

The QuaSEFE Problem

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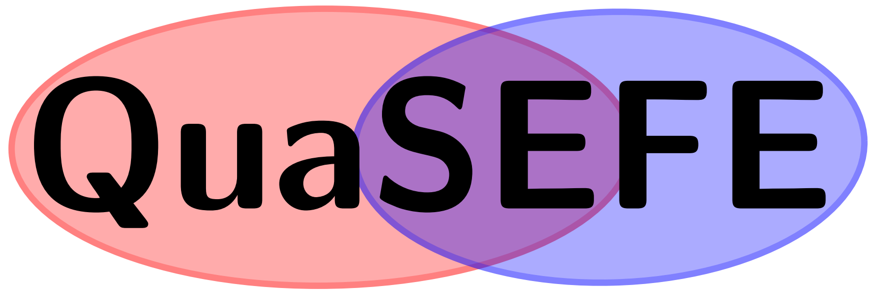

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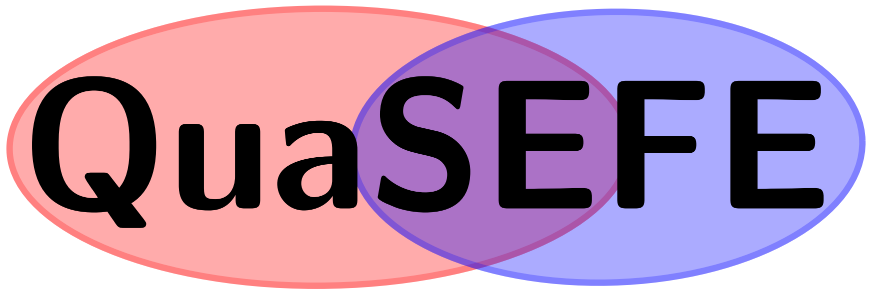

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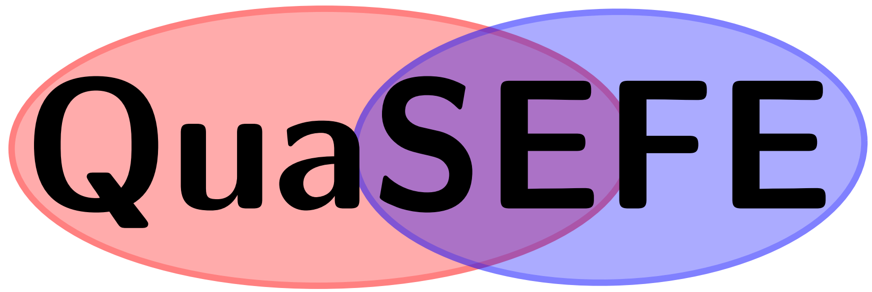

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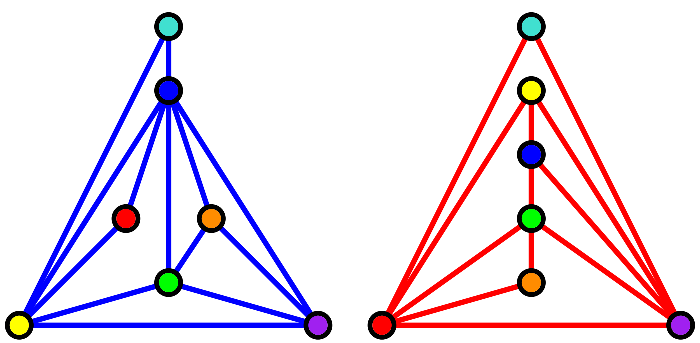
  - **Input:** Set of *quasiplanar* graphs with shared vertex set



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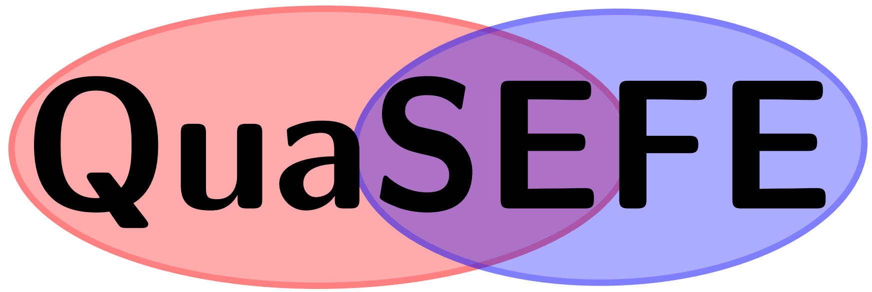

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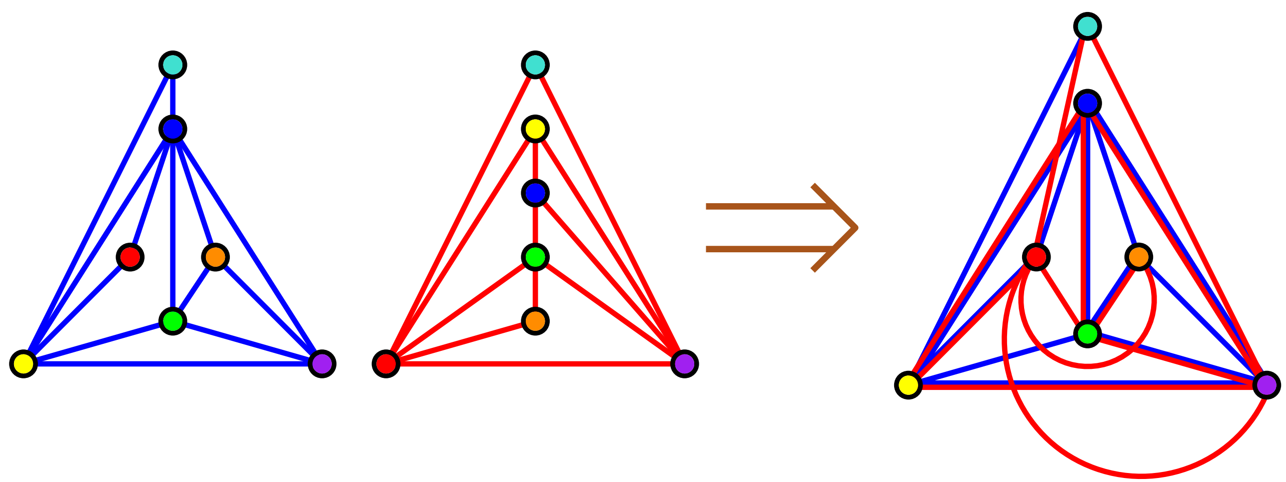
  - **Output:** Simultaneous *quasiplanar* drawings for all graphs with fixed edges



Related Work

- always positive instances for SEFE

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  - two caterpillars (in polynomial area) [Brass et al. '06]

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  - geometric simultaneous embedding (GSE) [Angelini et al. '11, Di Giacomo et al. '15]

Related Work - SEFE and Beyond Planarity

- quasiplanar GSE

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  - not every two quasiplanar graphs [Di Giacomo et al. '15]
- simultaneous RAC drawings [Argyriou et al. '13, Bekos et al. '16, Evans et al. '16, Grilli '18]

Our Results

- always positive instances for QuaSEFE

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  - two planar graphs and a tree

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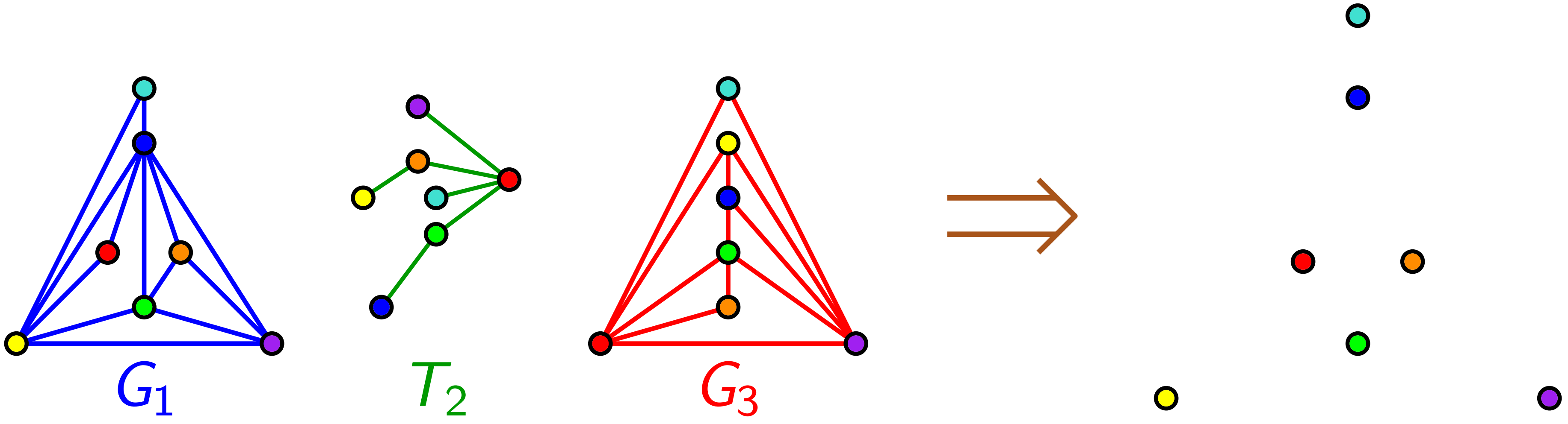
  - a 1-planar graph and a planar graph
  - planar graphs with restrictions on their intersection graphs
- counterexamples for QuaSEFE in two special settings

Two Planar Graphs and a Tree ✓

- 1. Draw G_1 planar



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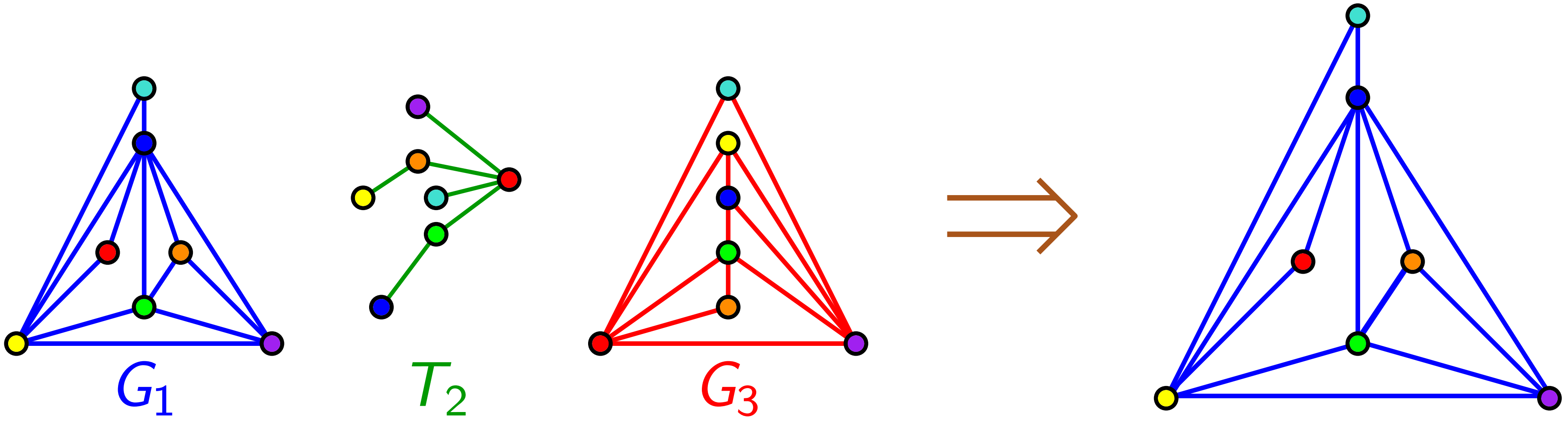

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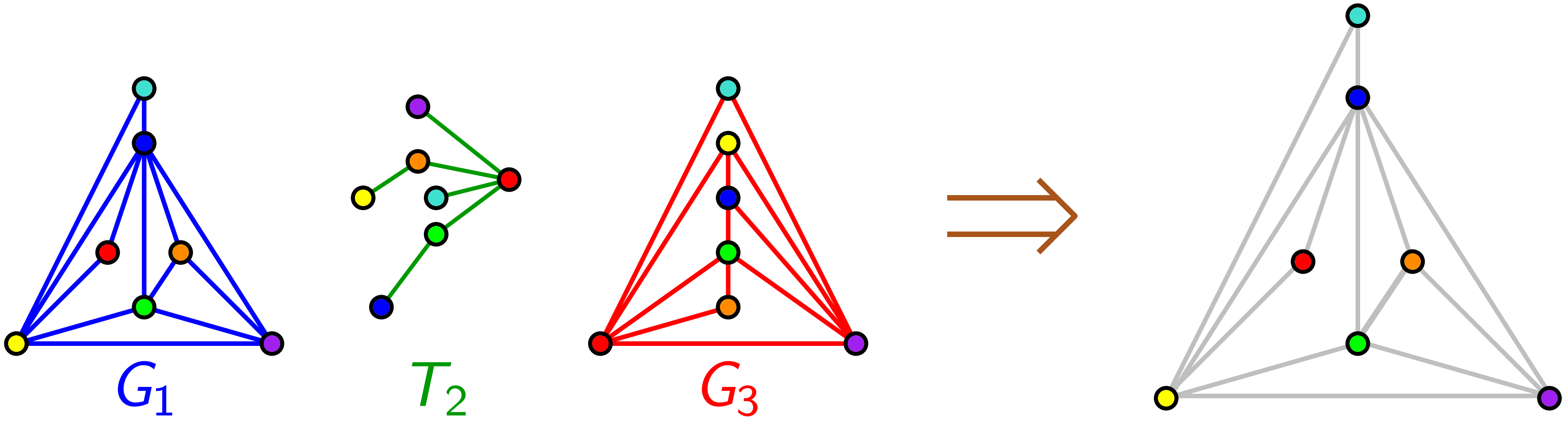

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- 1. Draw G_1 planar
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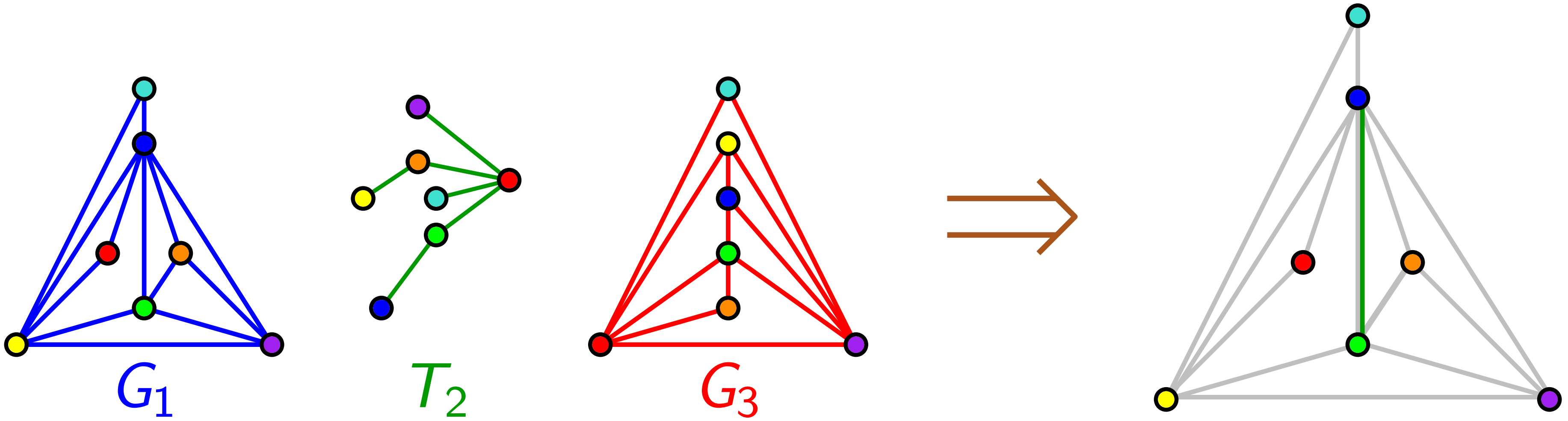
  - some edges fixed by G_1



Two Planar Graphs and a Tree ✓

- 1. Draw G_1 planar
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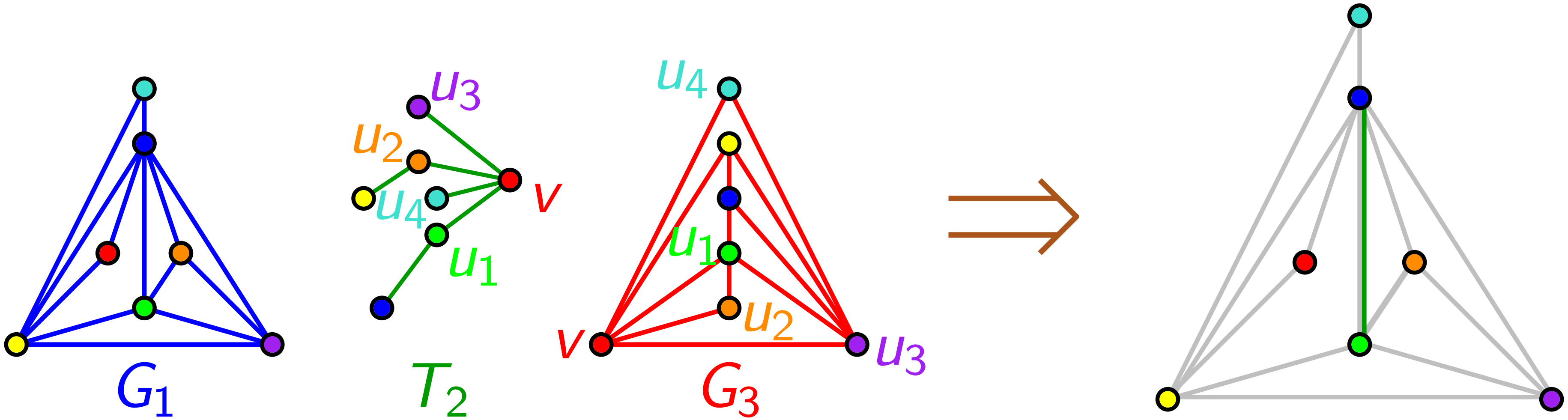
  - choose planar rotation system from G_3 for edges in $G_3 \setminus G_1$



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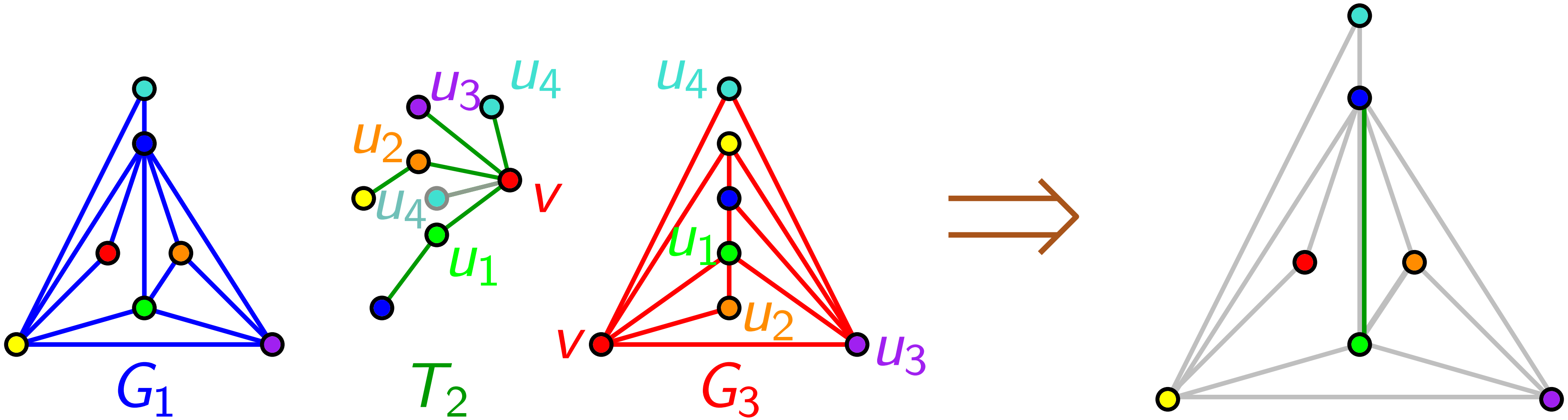

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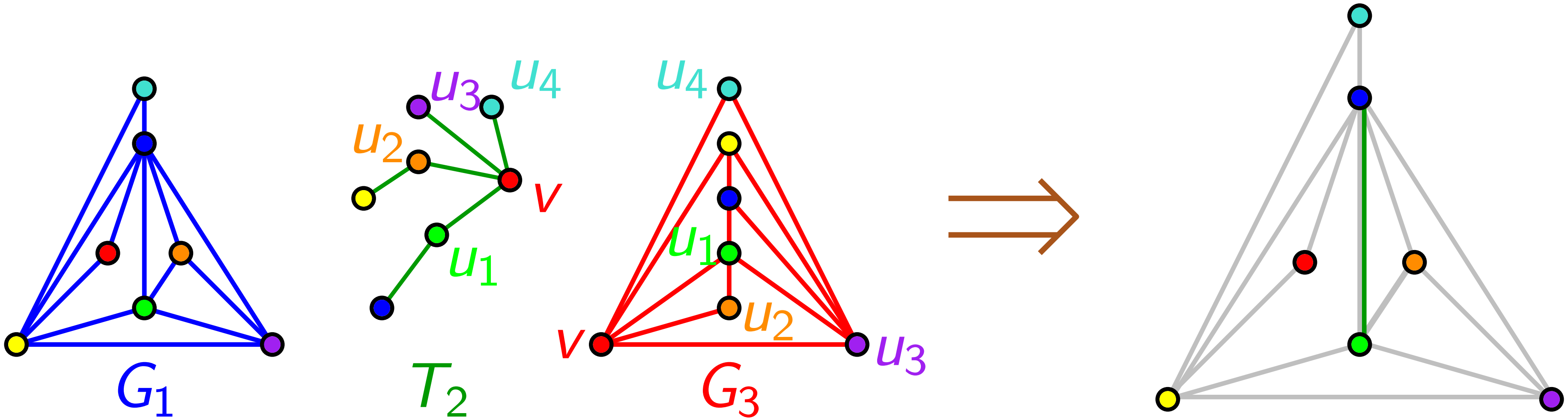

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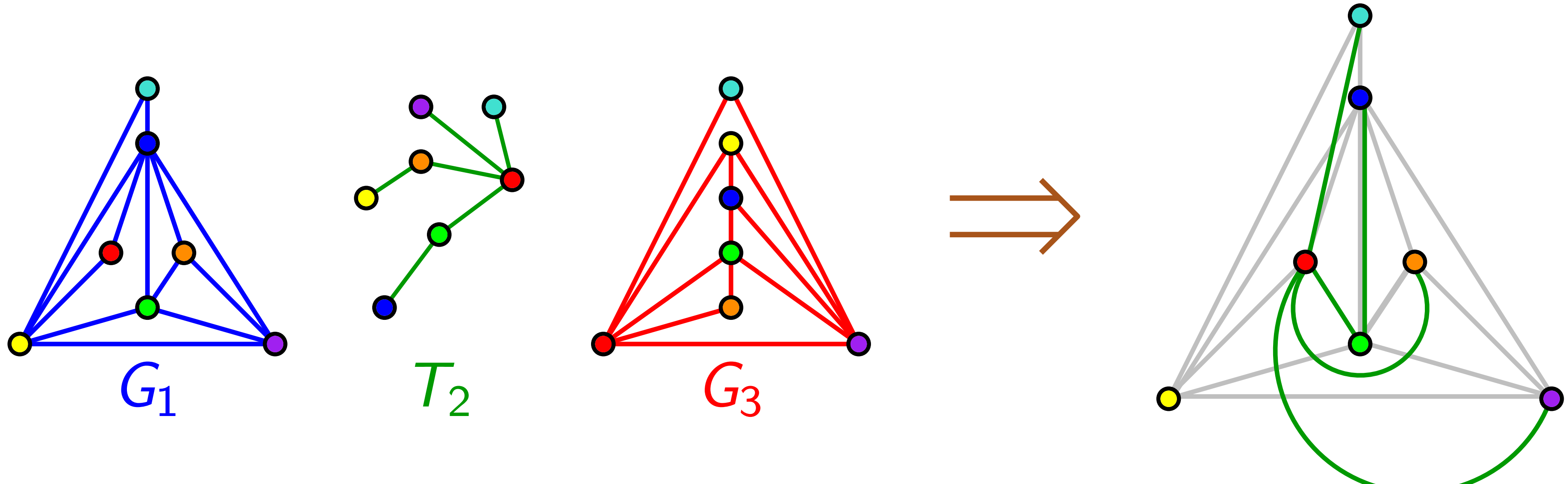

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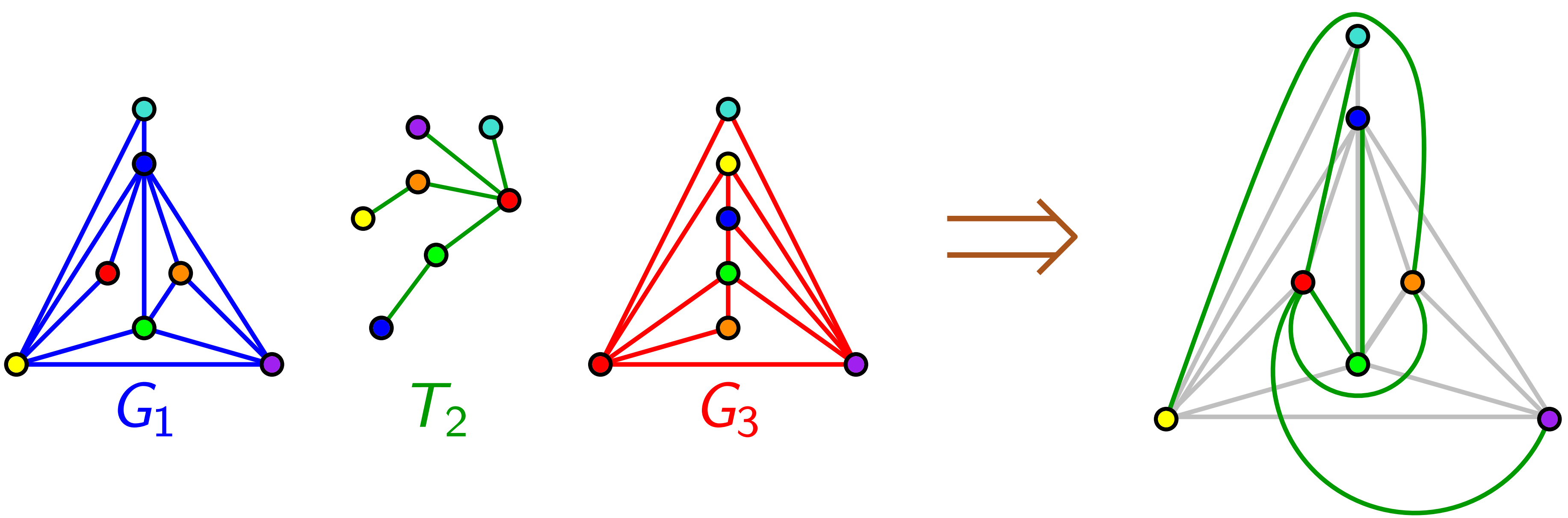
  - remaining edges embedded planar



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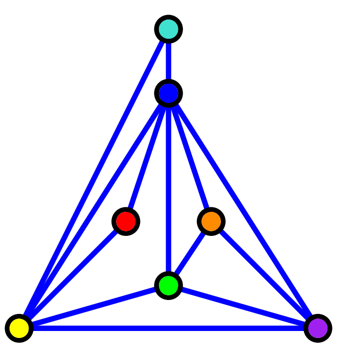

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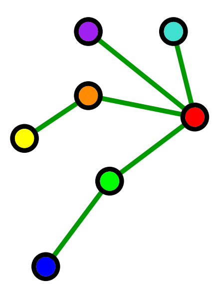
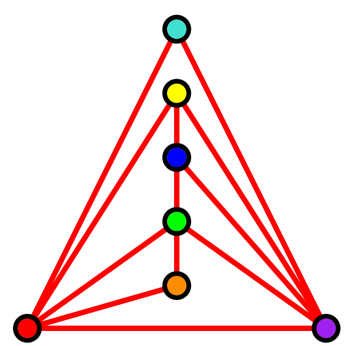
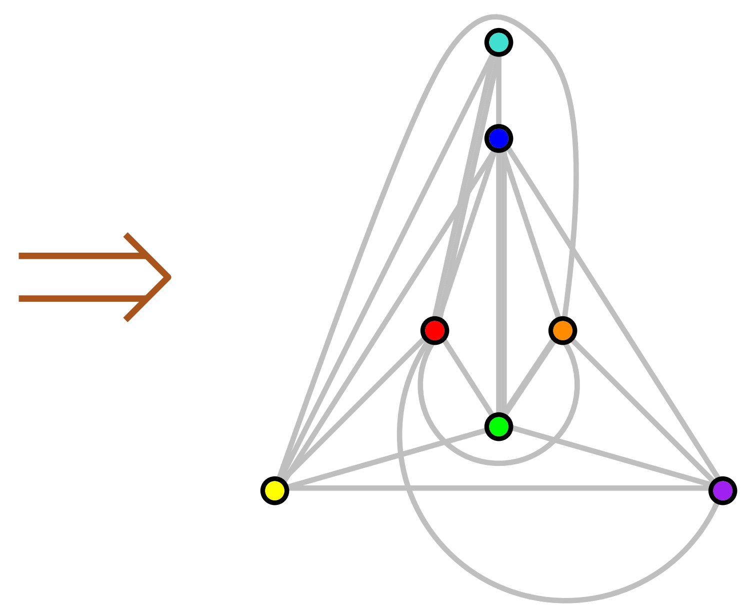


T_2



G_3



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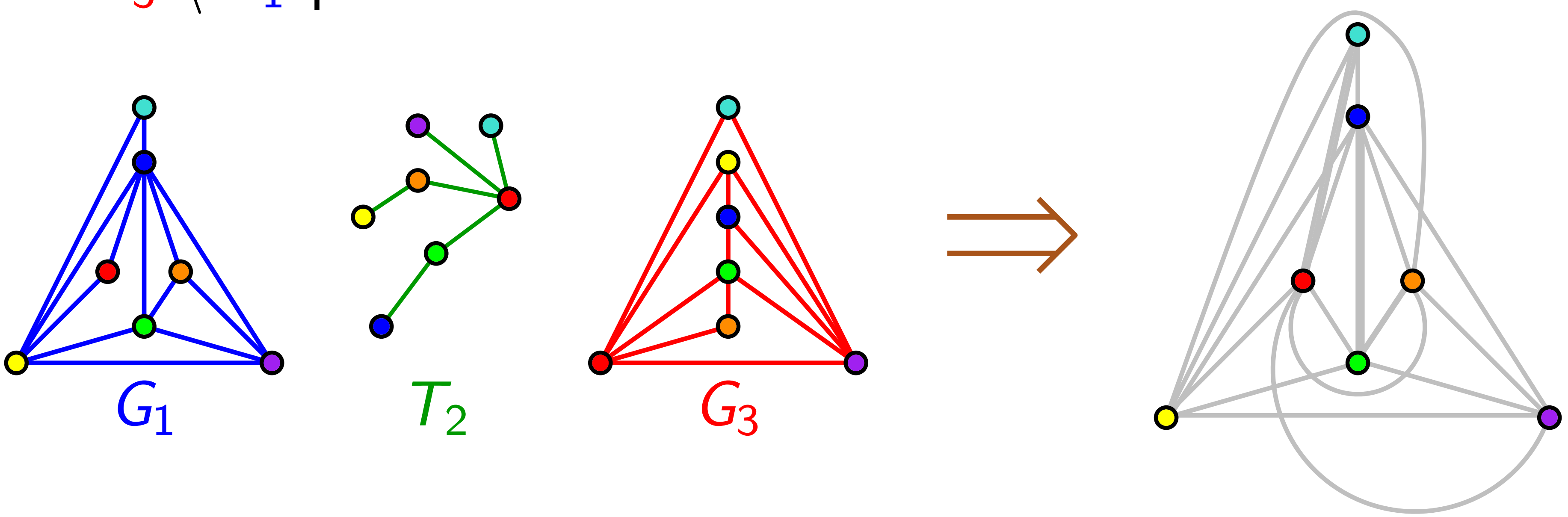
- 3. Draw G_3 quasiplanar
  - $G_3 \setminus G_1$ planar



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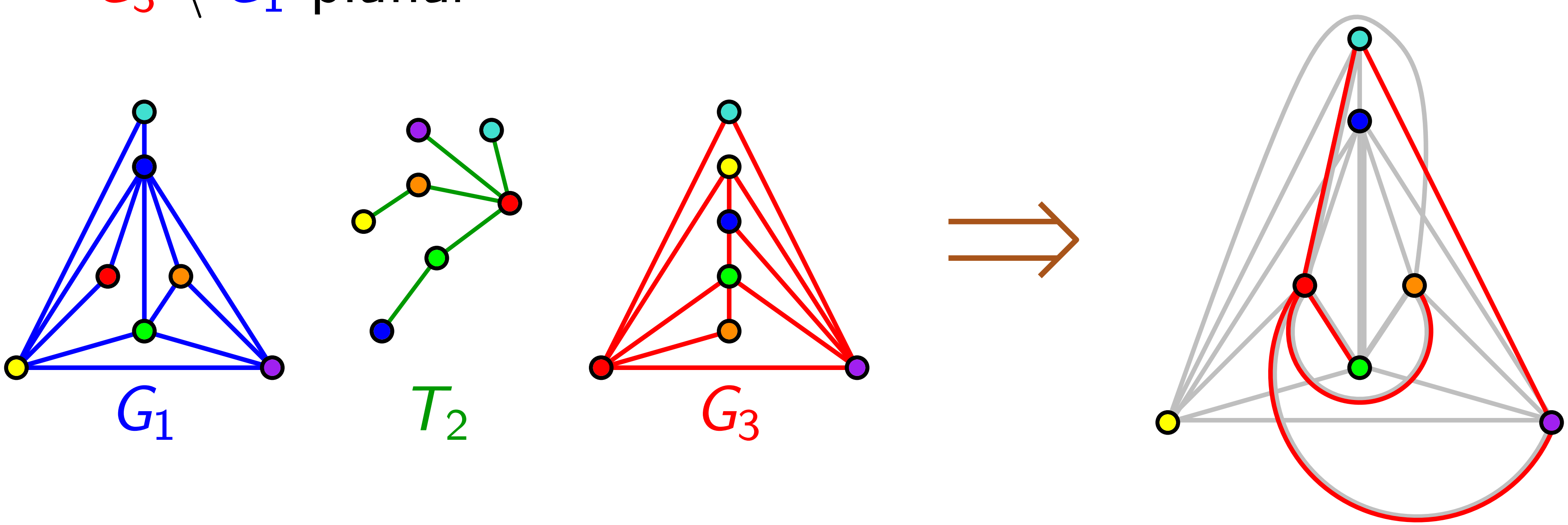

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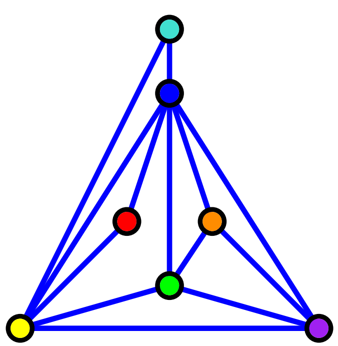

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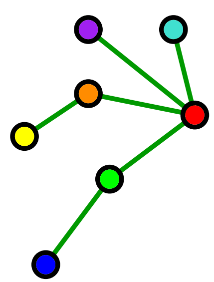
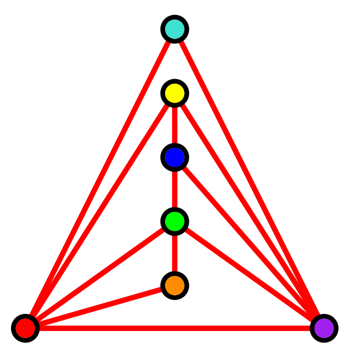
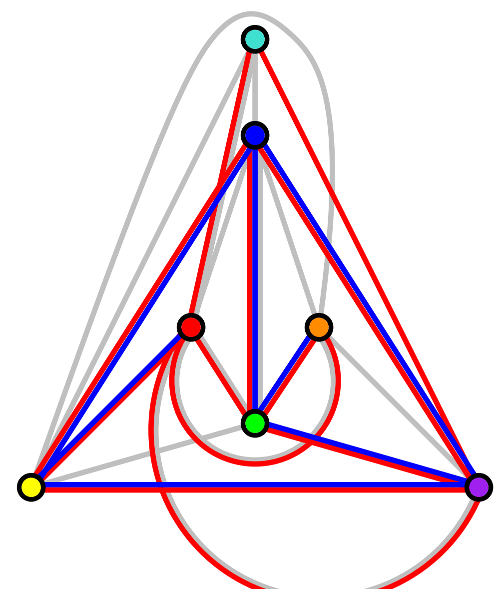


T_2



G_3

$\Longrightarrow$



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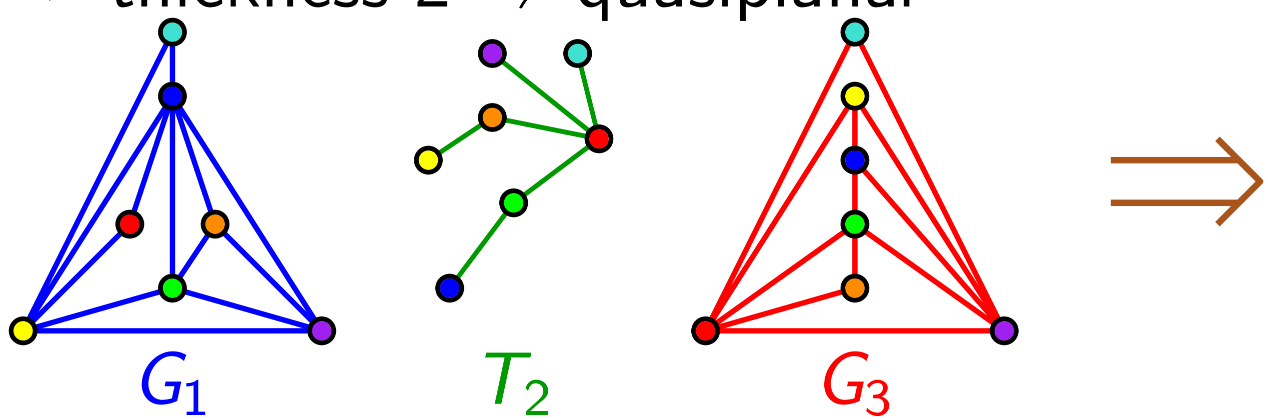
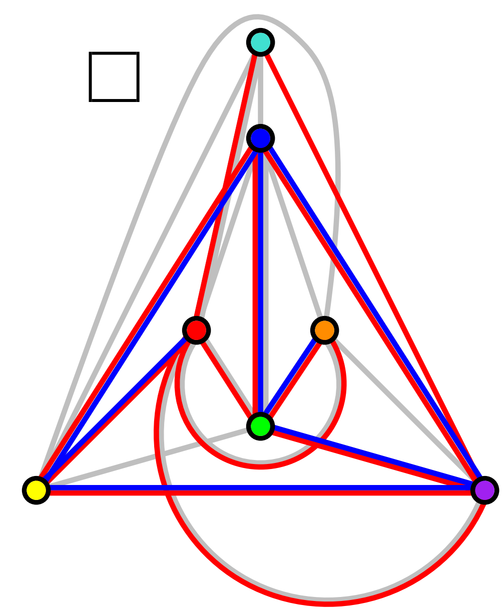
  - $G_3 \setminus G_1$ planar
  - thickness 2 $\Rightarrow$ quasiplanar

$\square$





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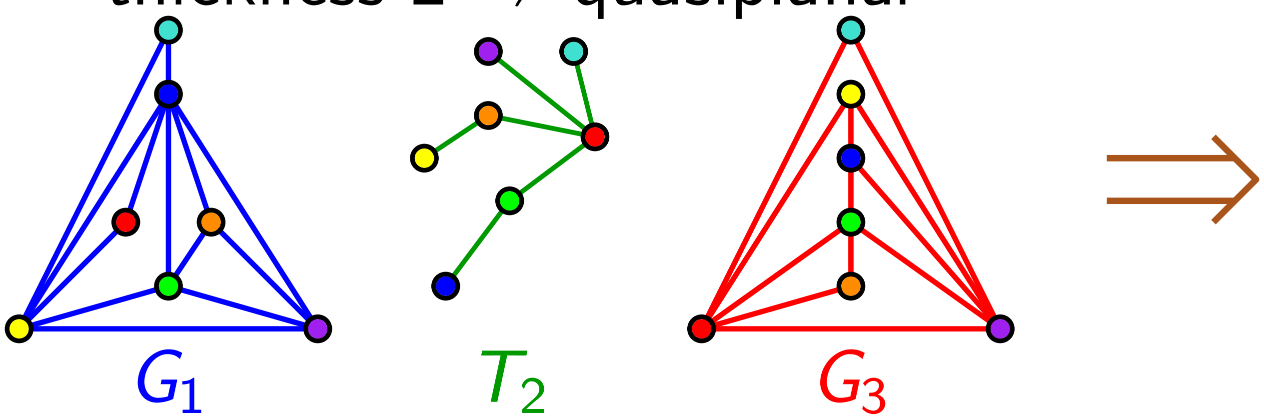

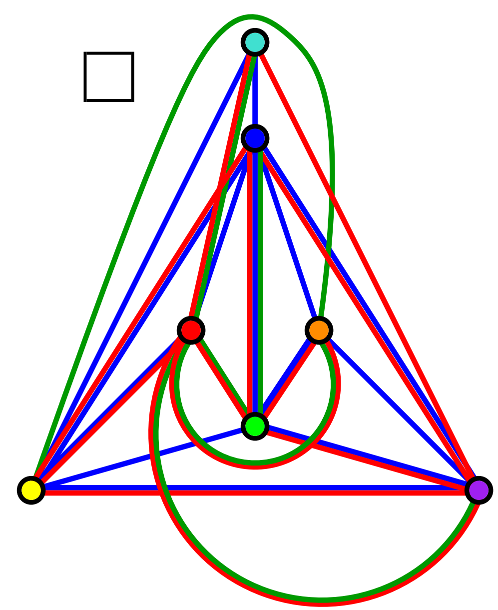

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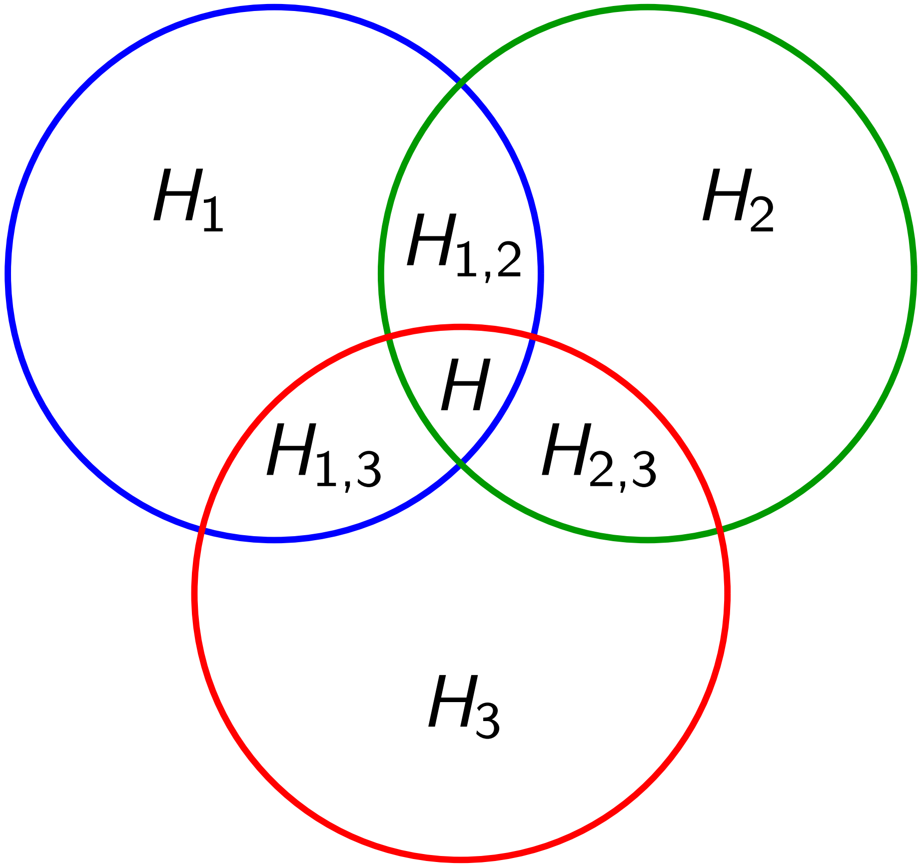

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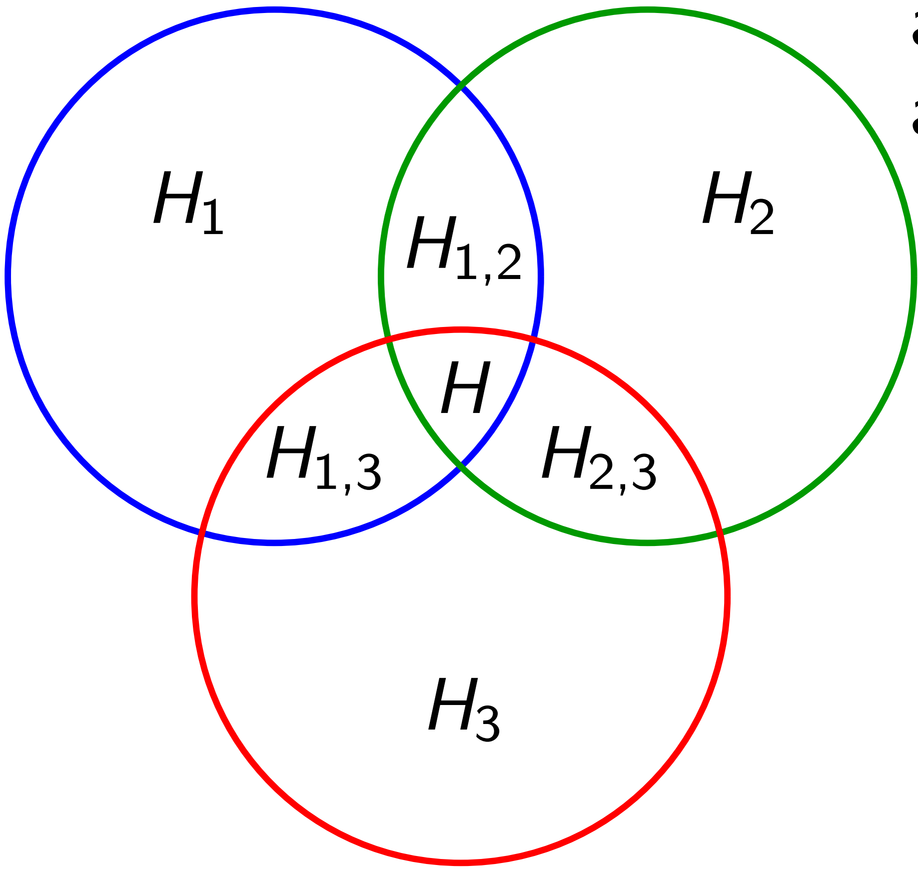

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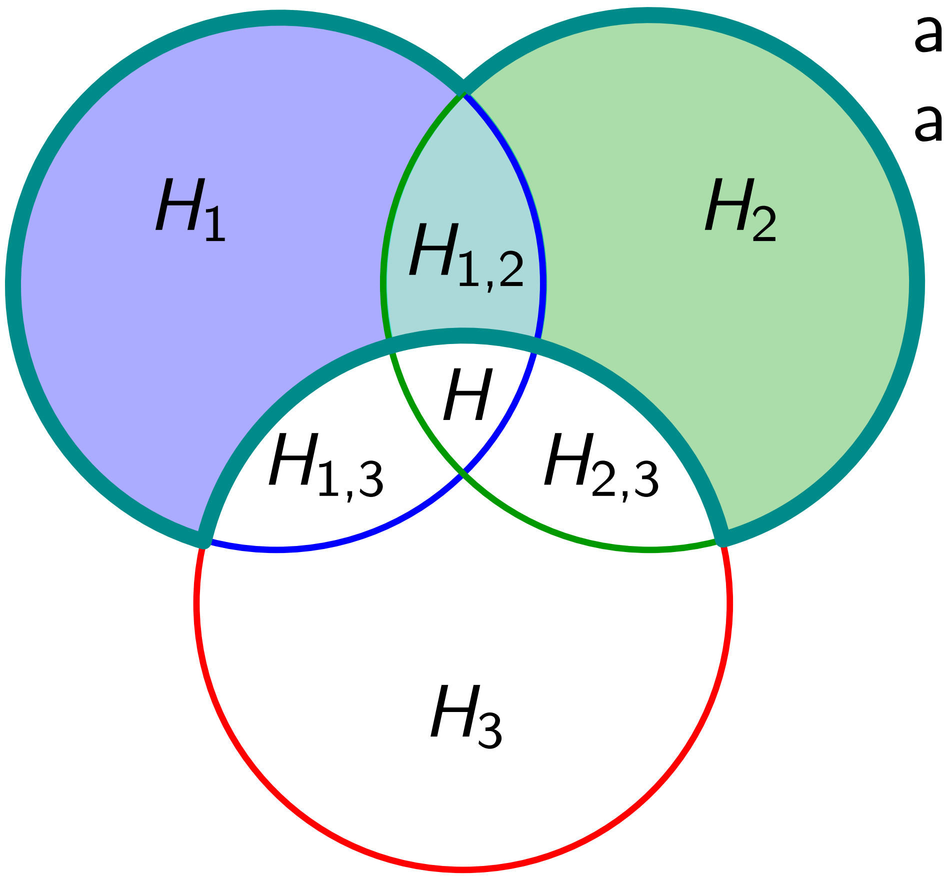

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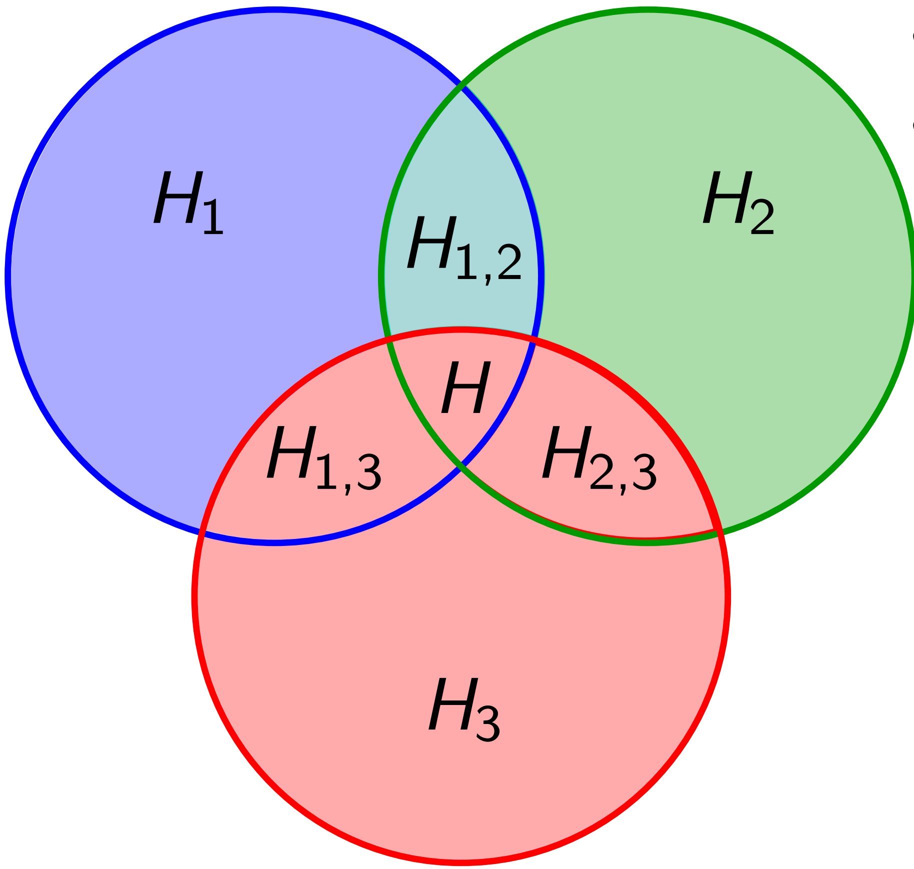

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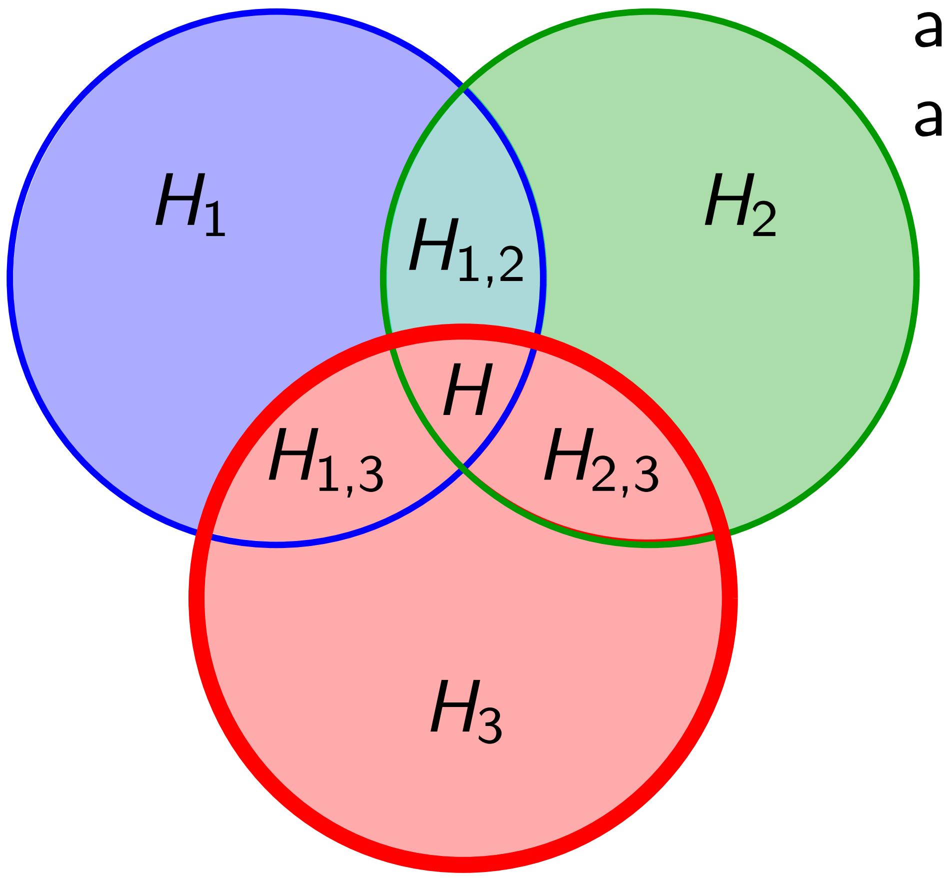

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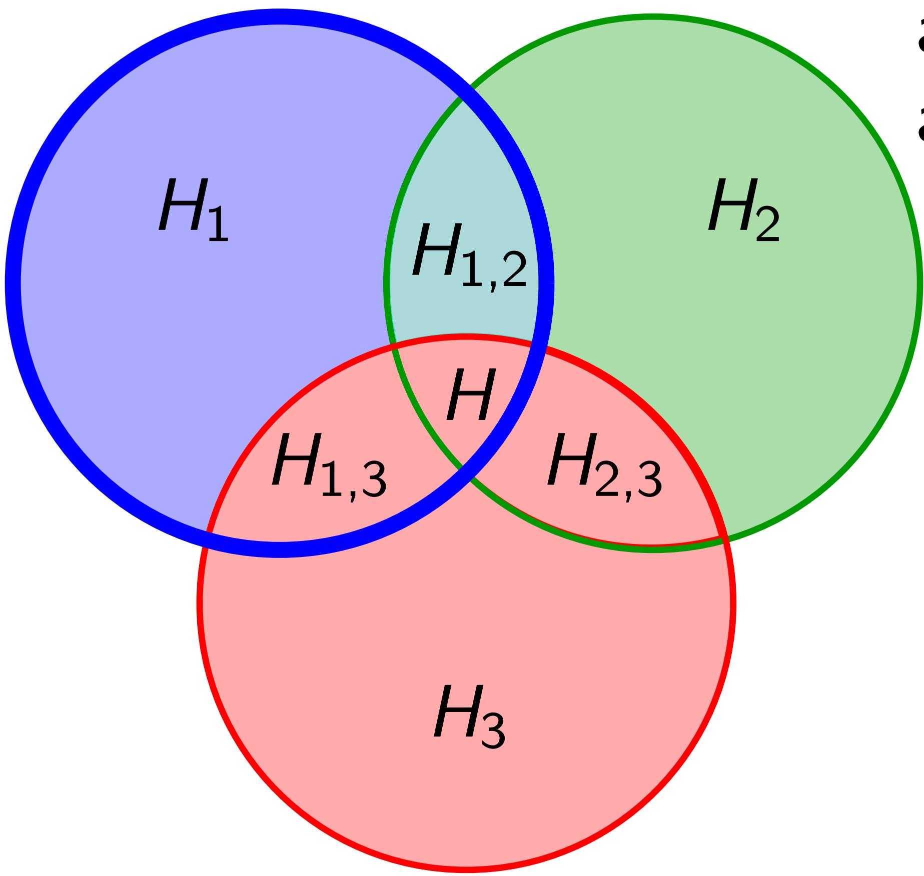

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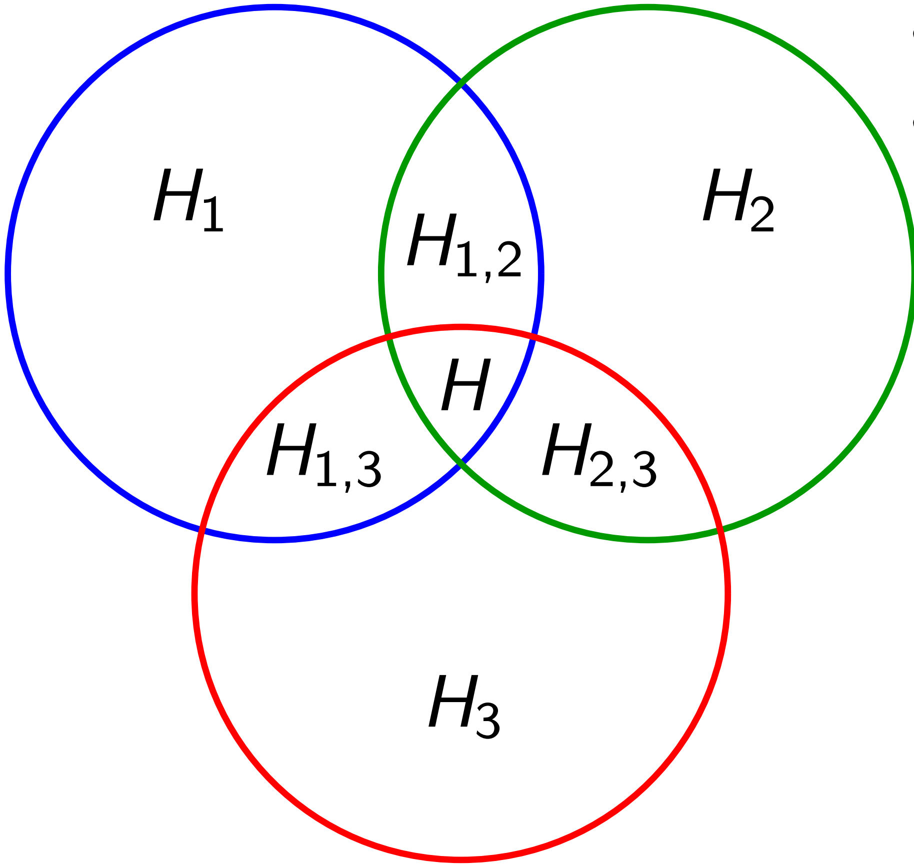

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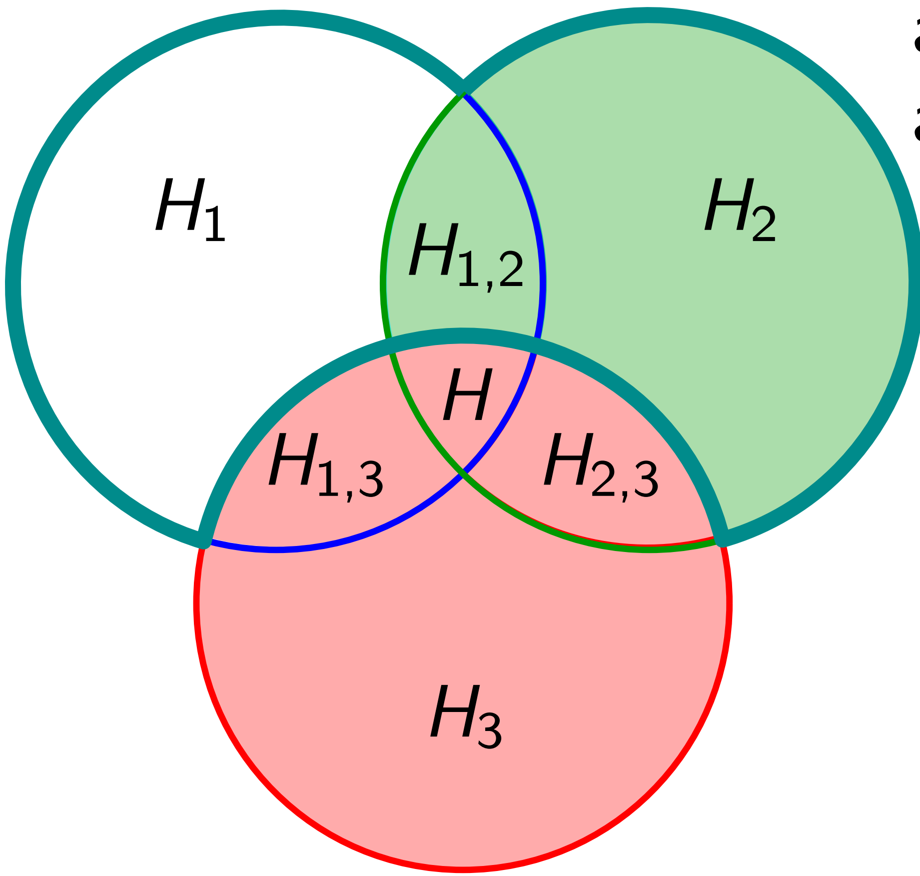

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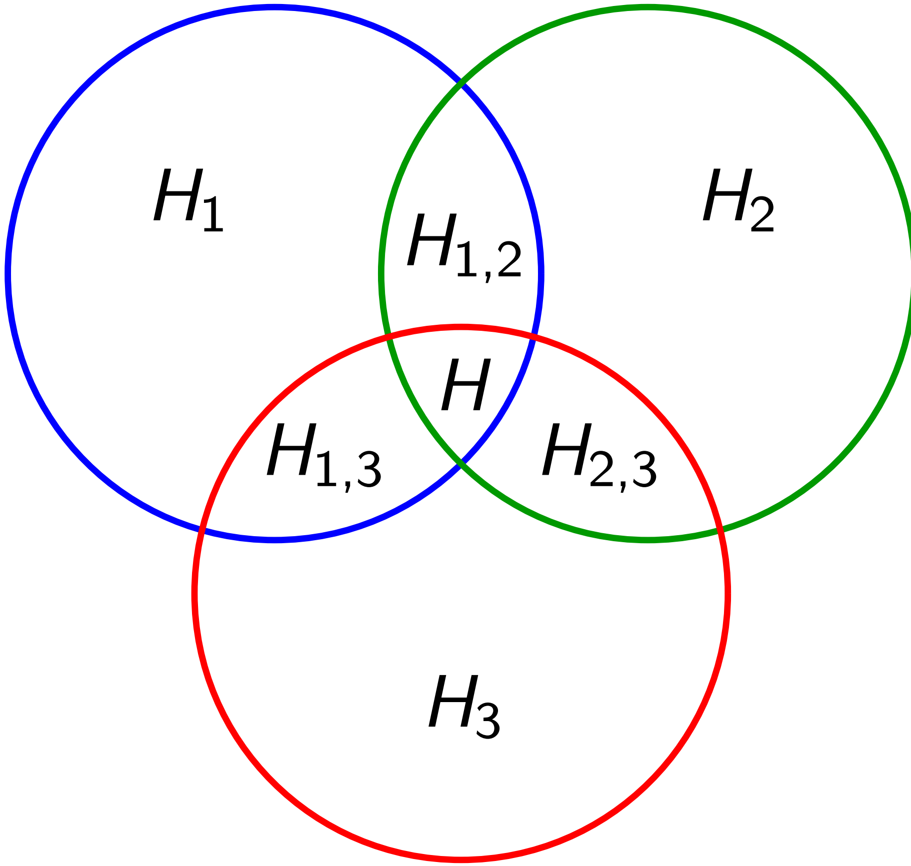

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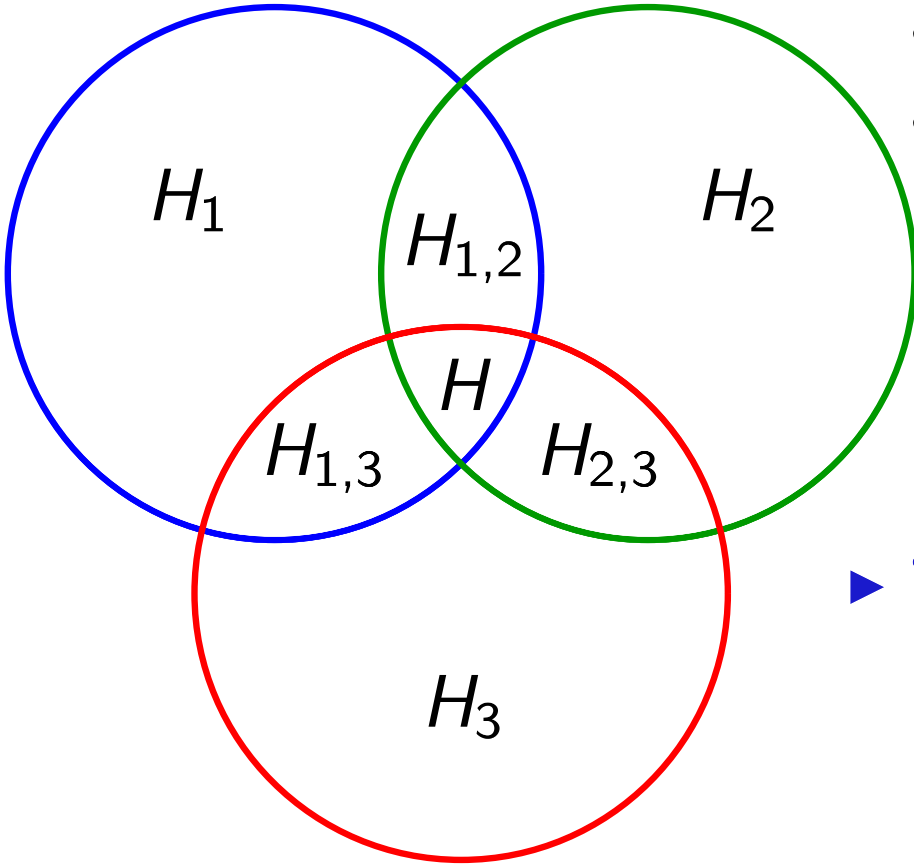

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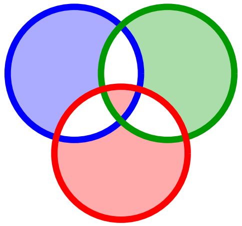

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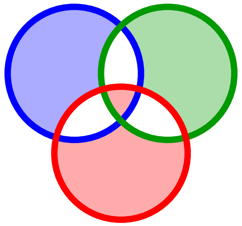

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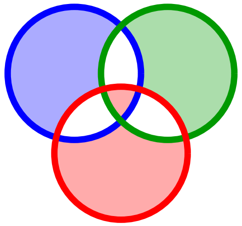

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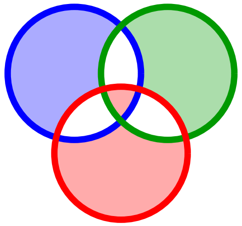

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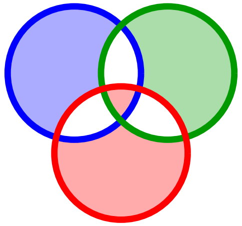

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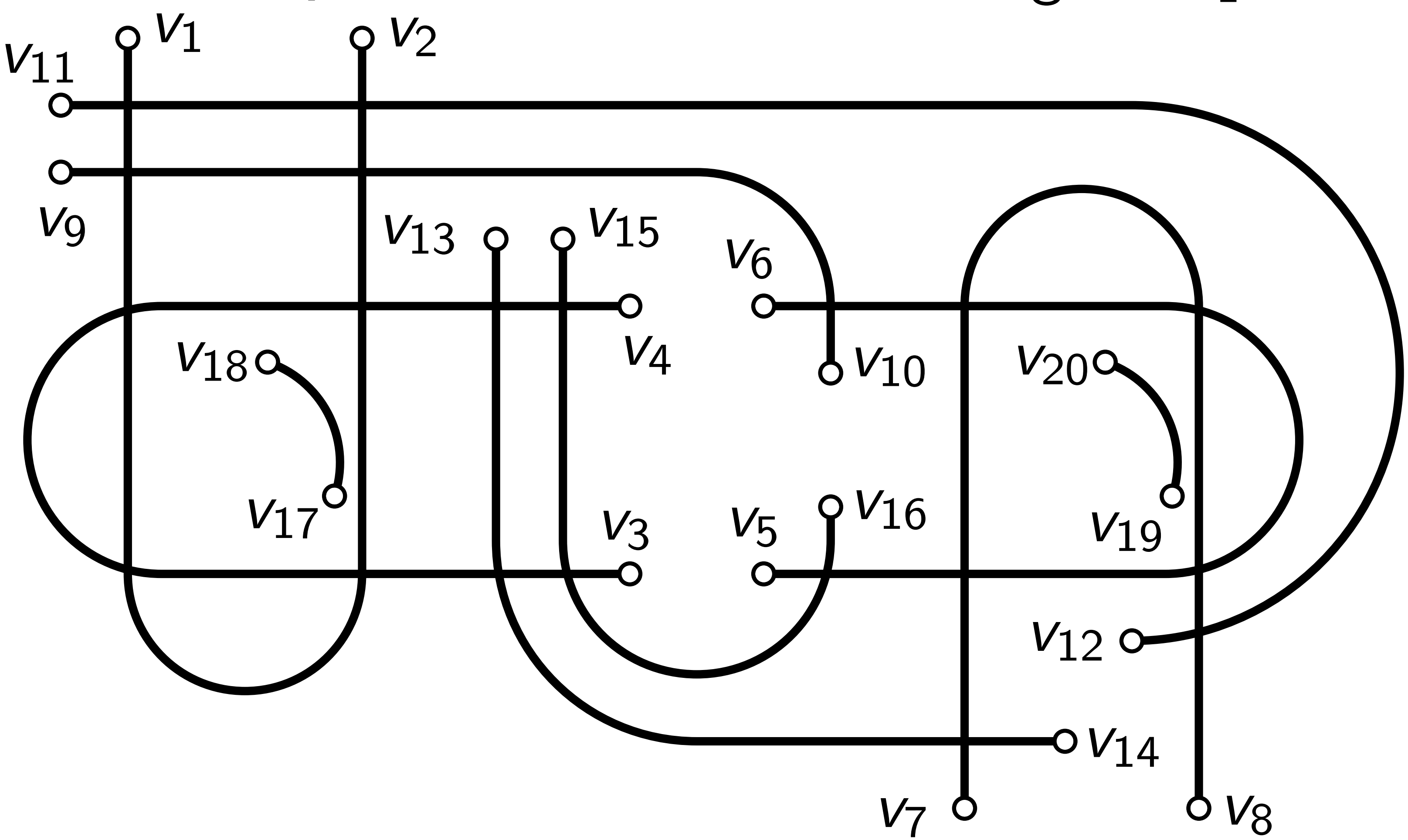

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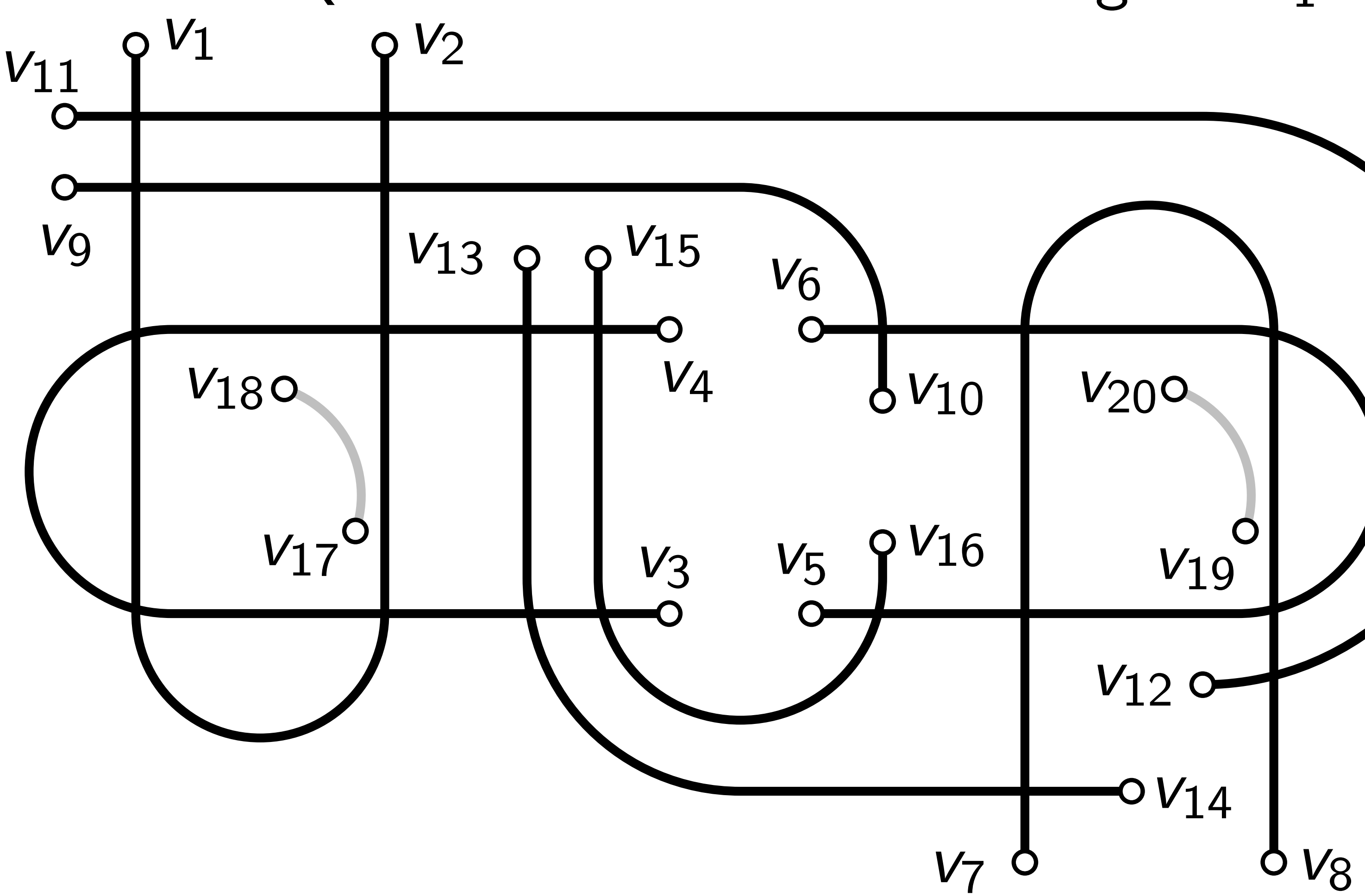

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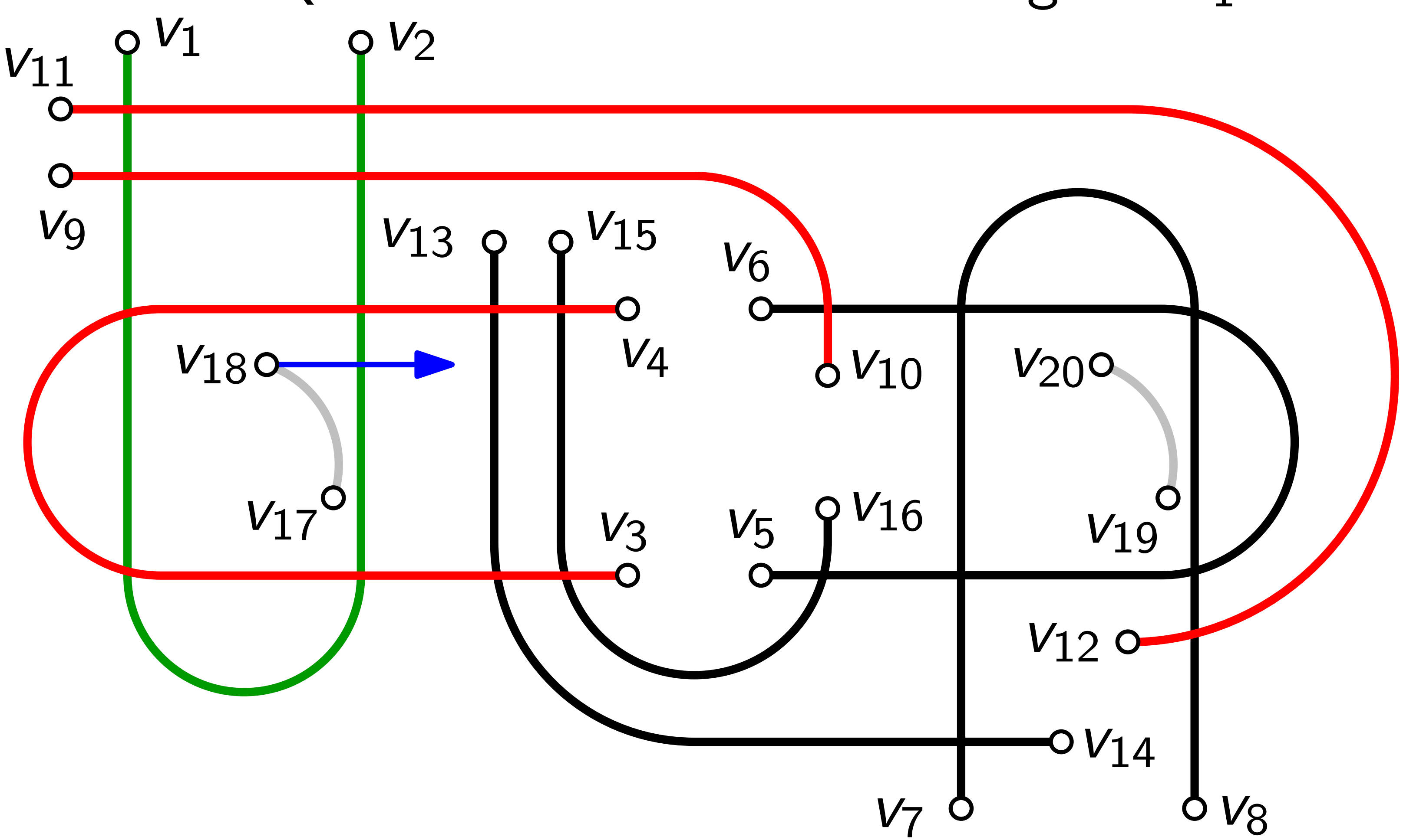

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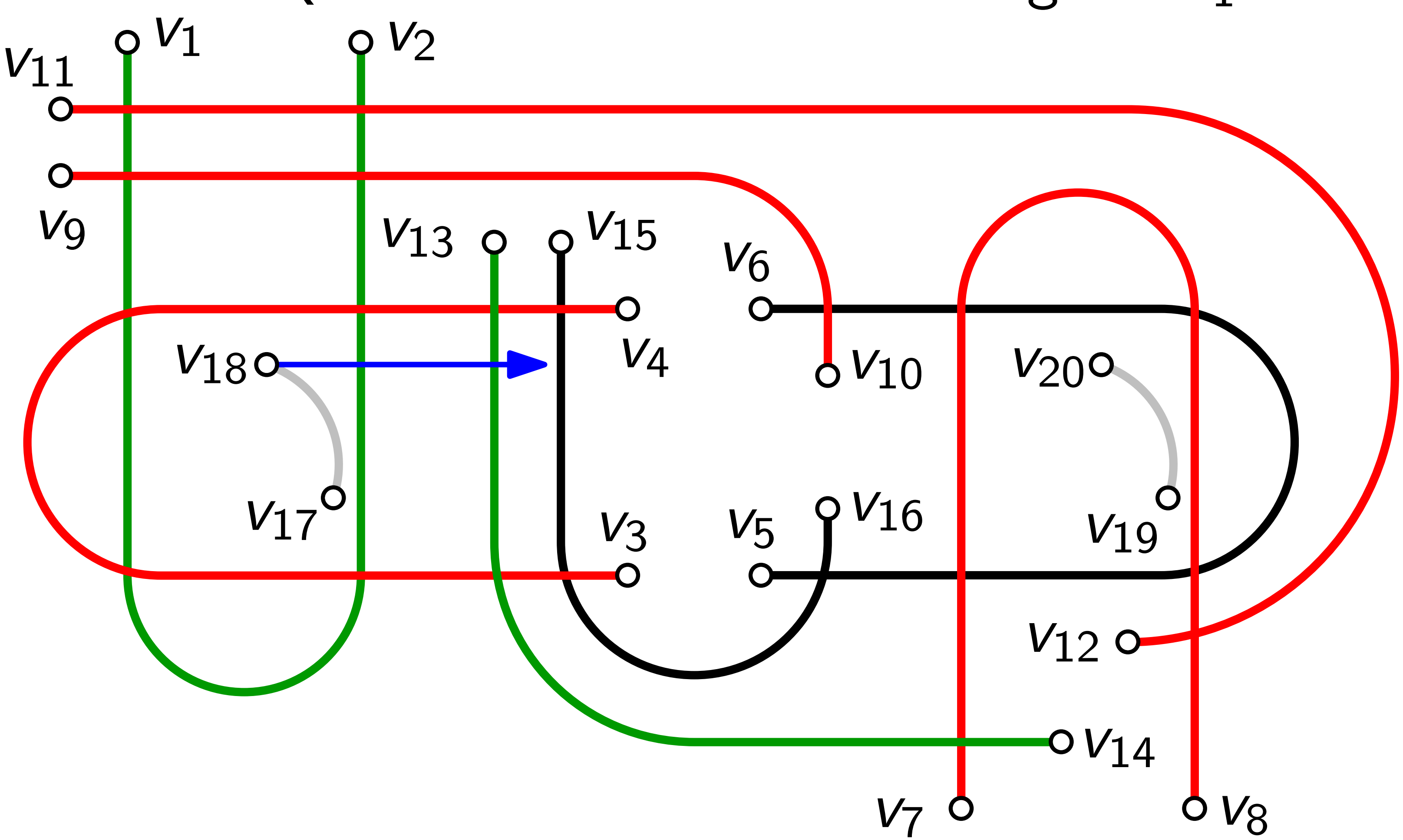

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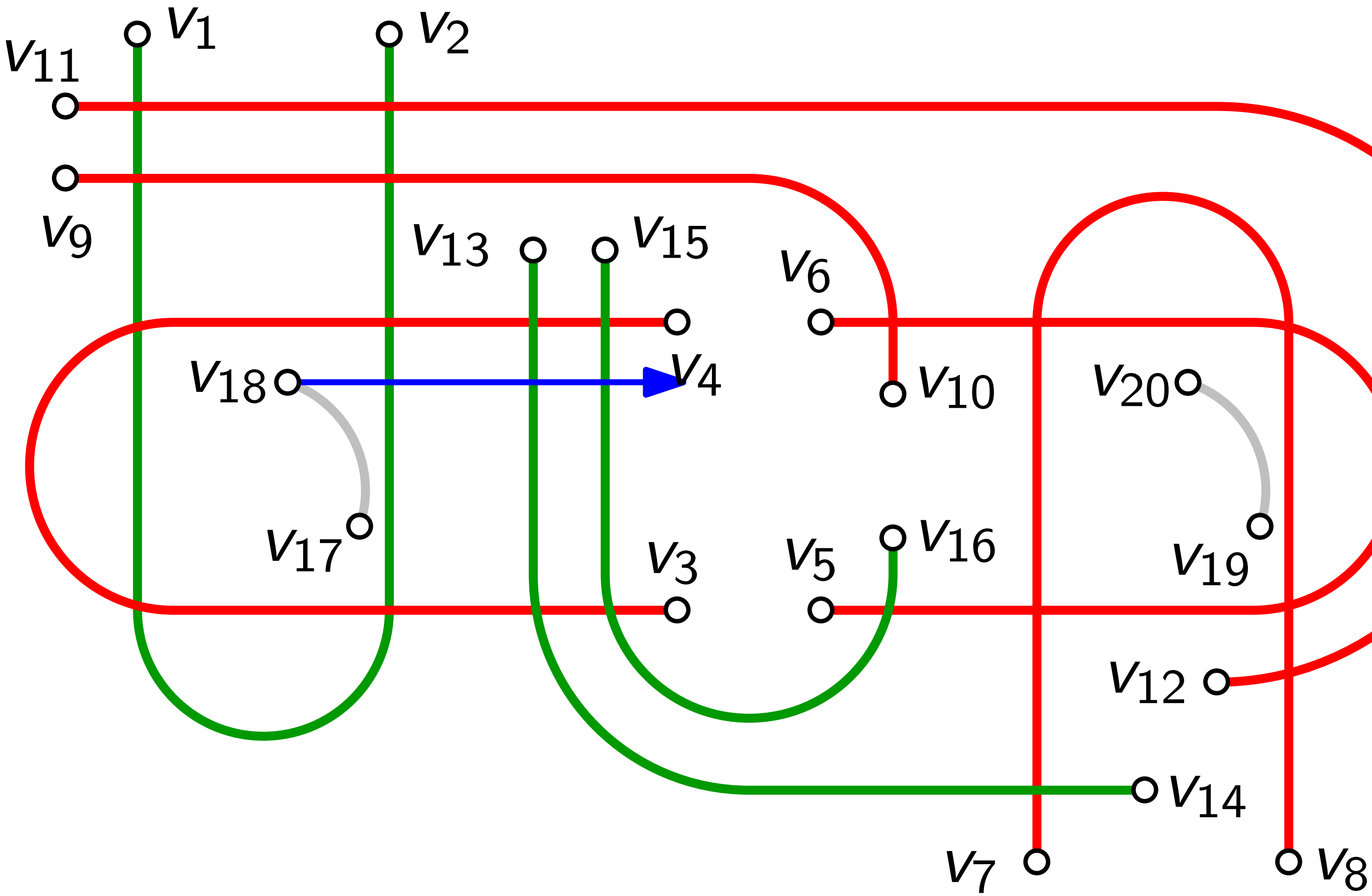

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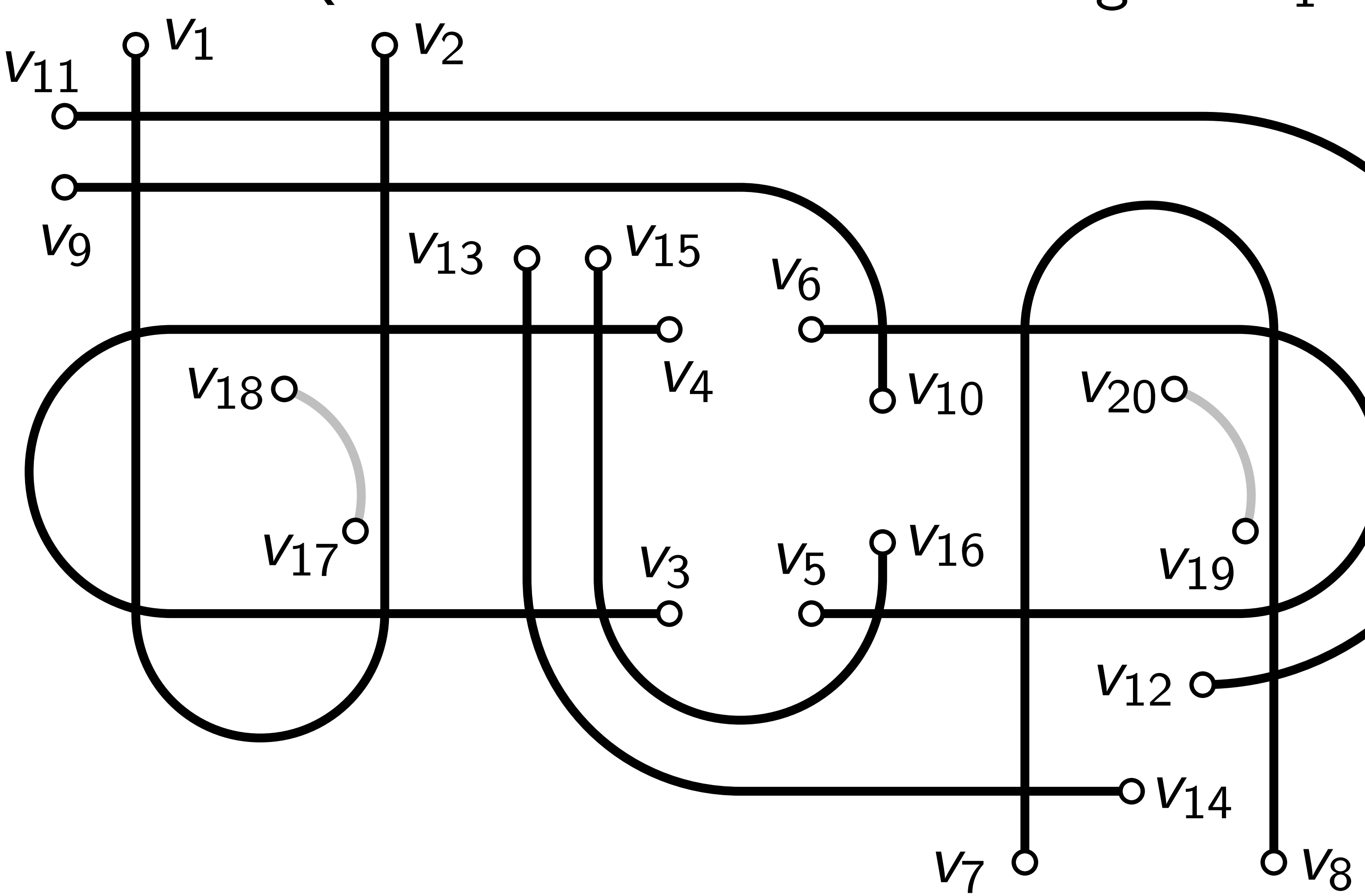

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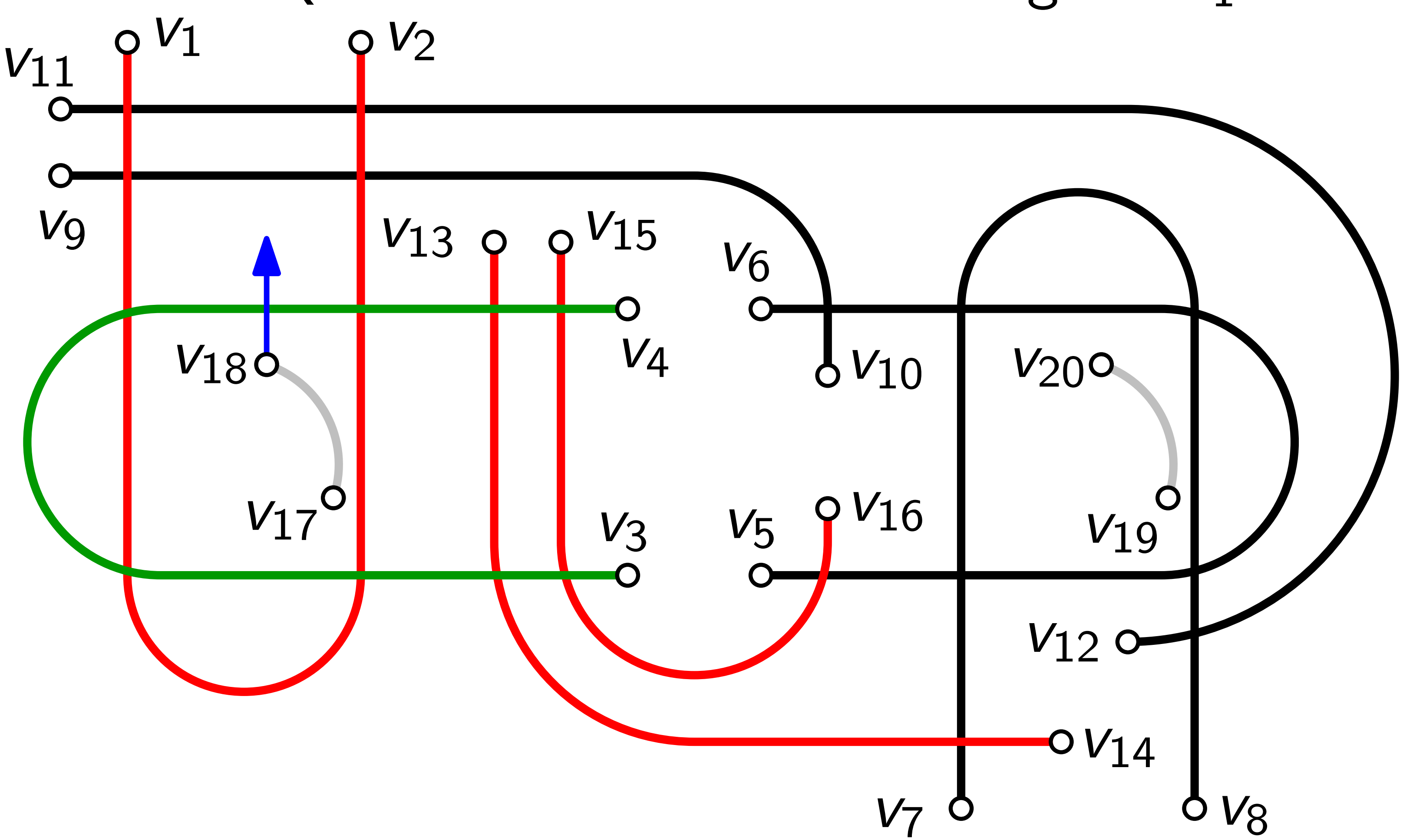

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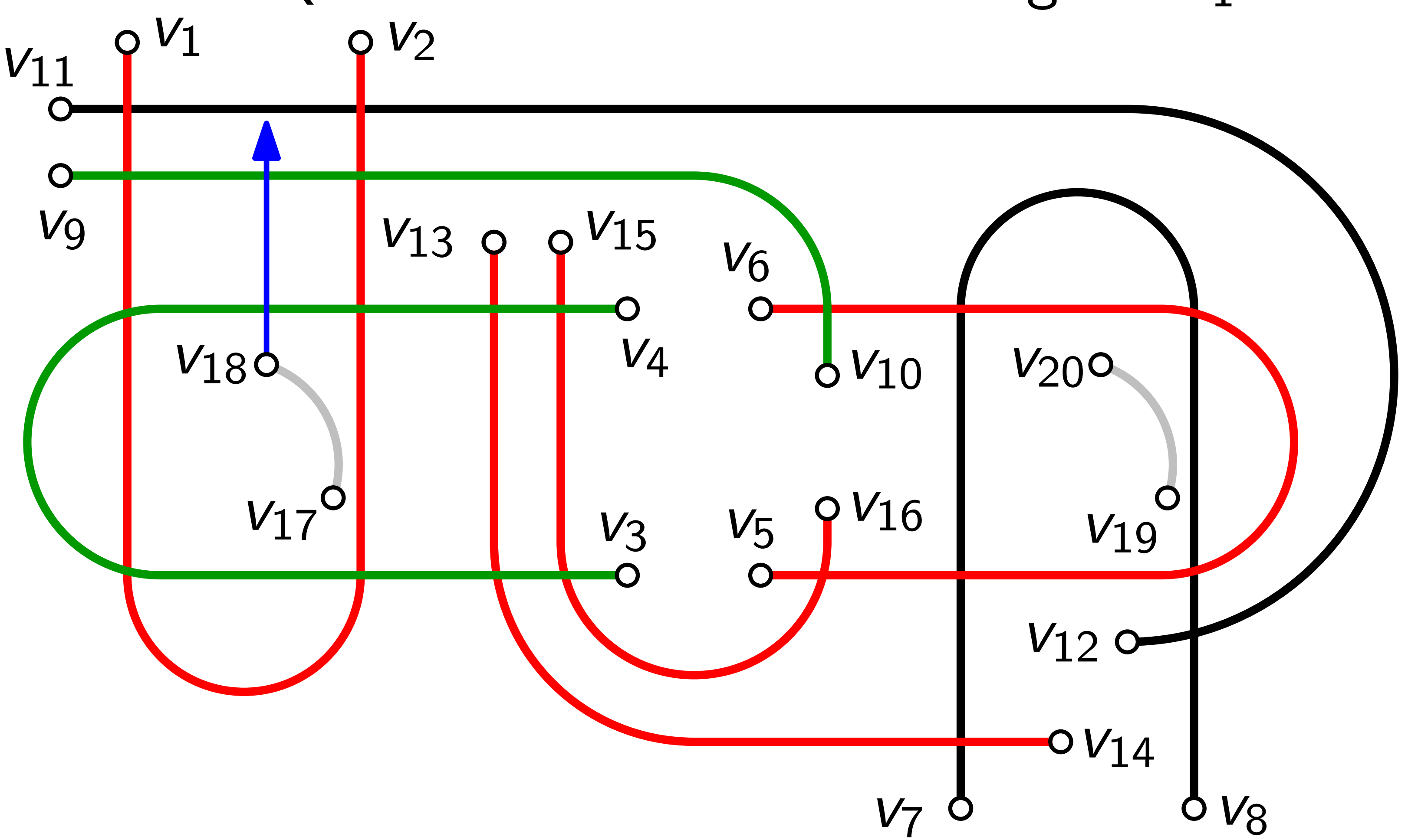

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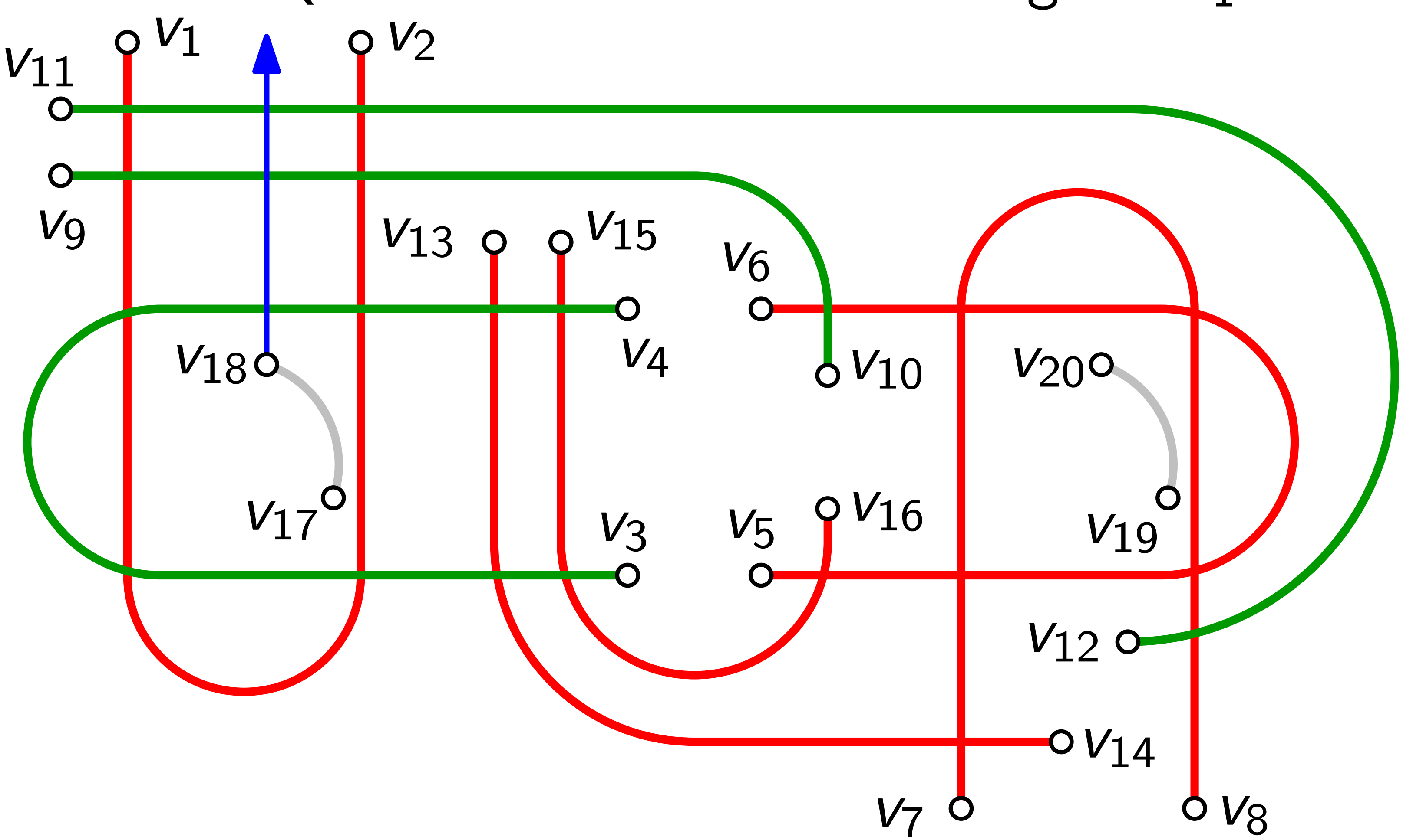

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