On Strict (Outer-)Confluent Graphs

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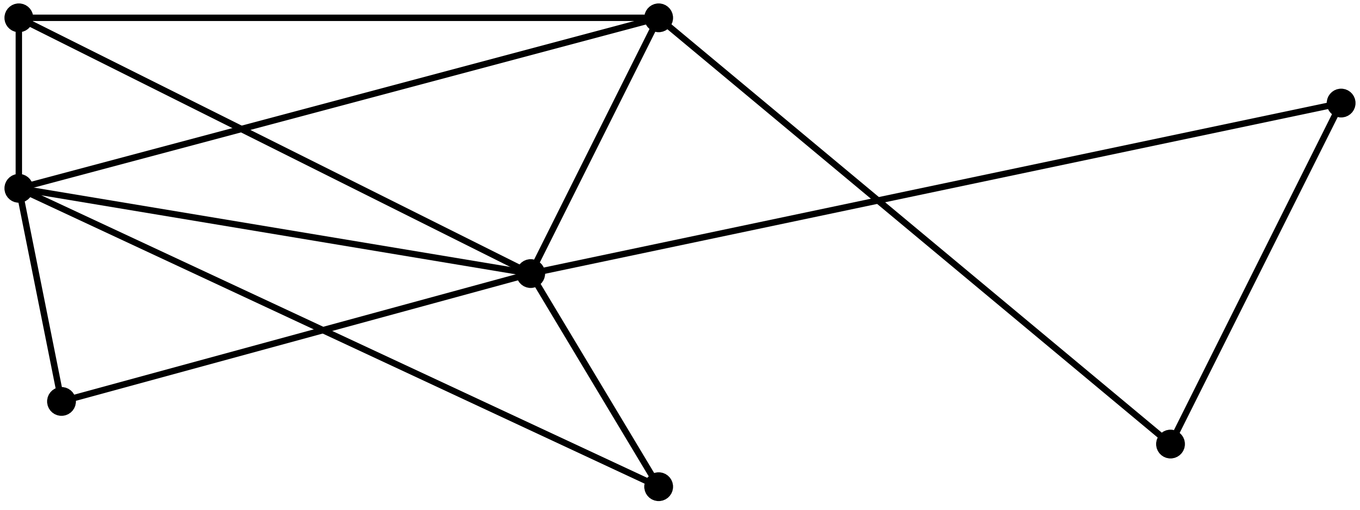
Graph Drawing 2019 · September 18, 2019

ALGORITHMS AND COMPLEXITY GROUP

EBERHARD KARLS UNIVERSITÄT TÜBINGEN

Confluence

Technique to bundle edges



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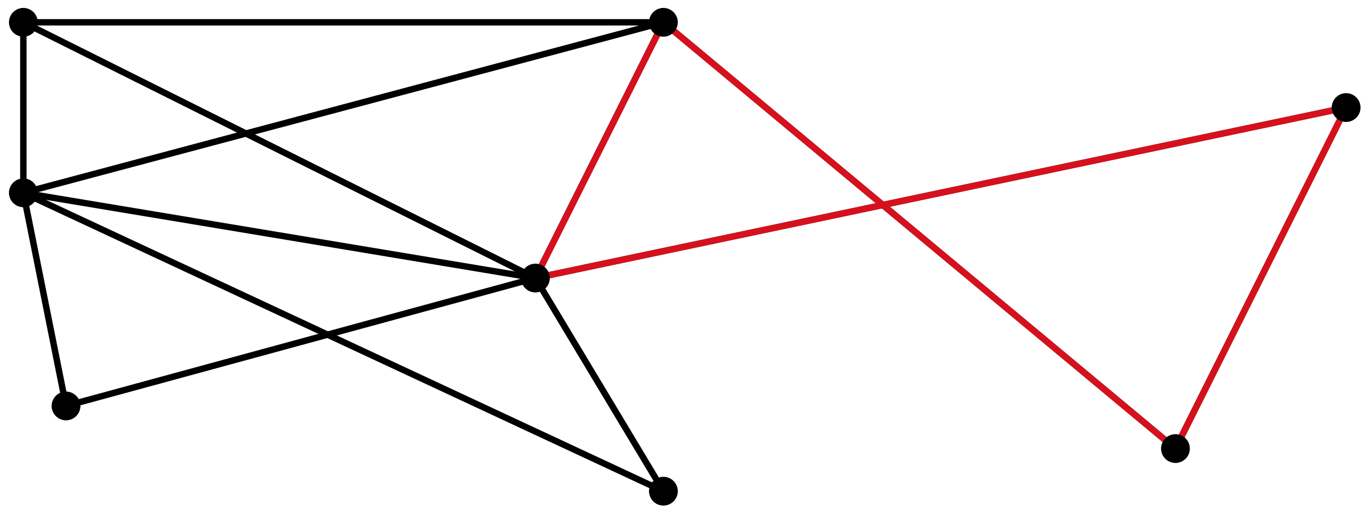

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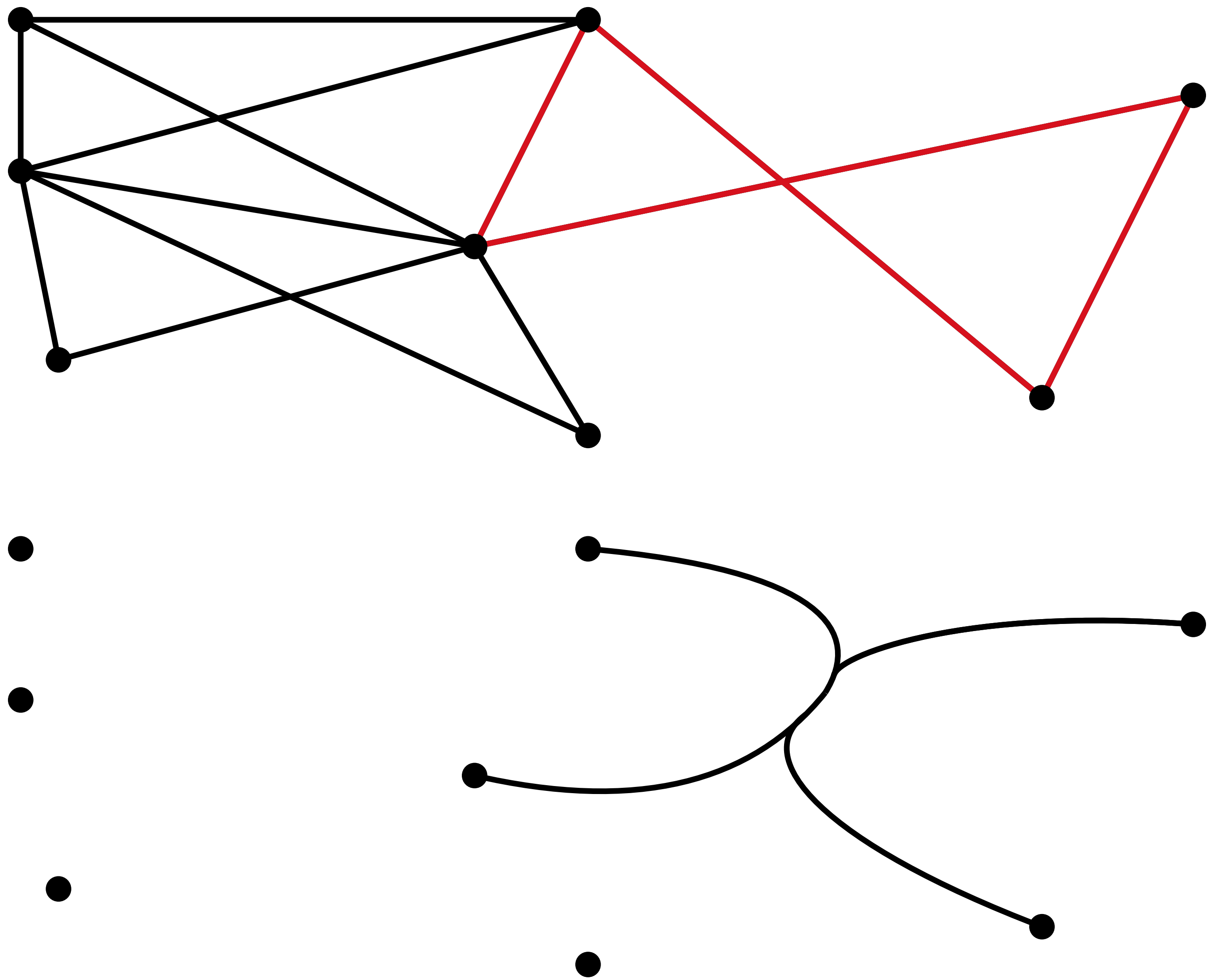

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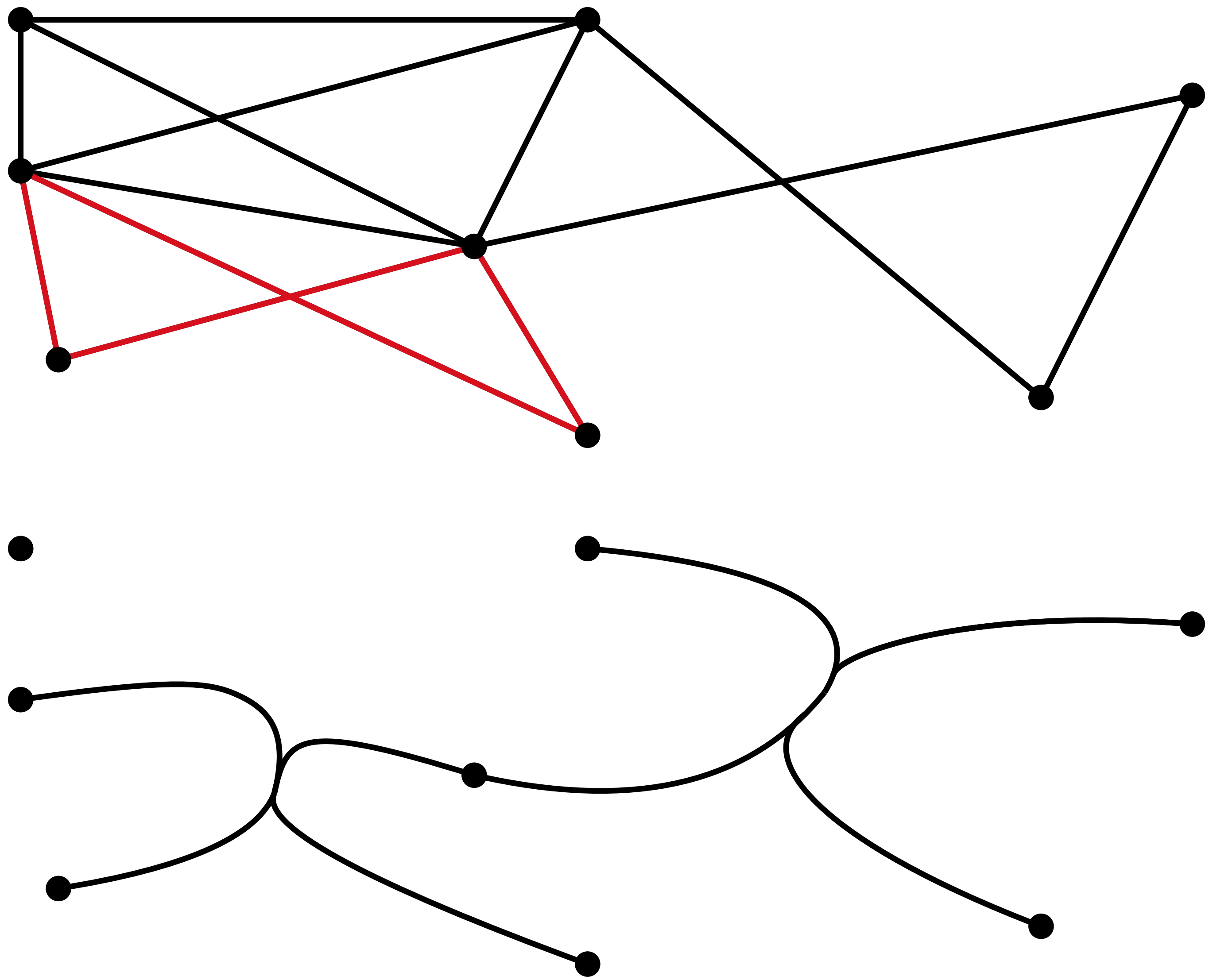

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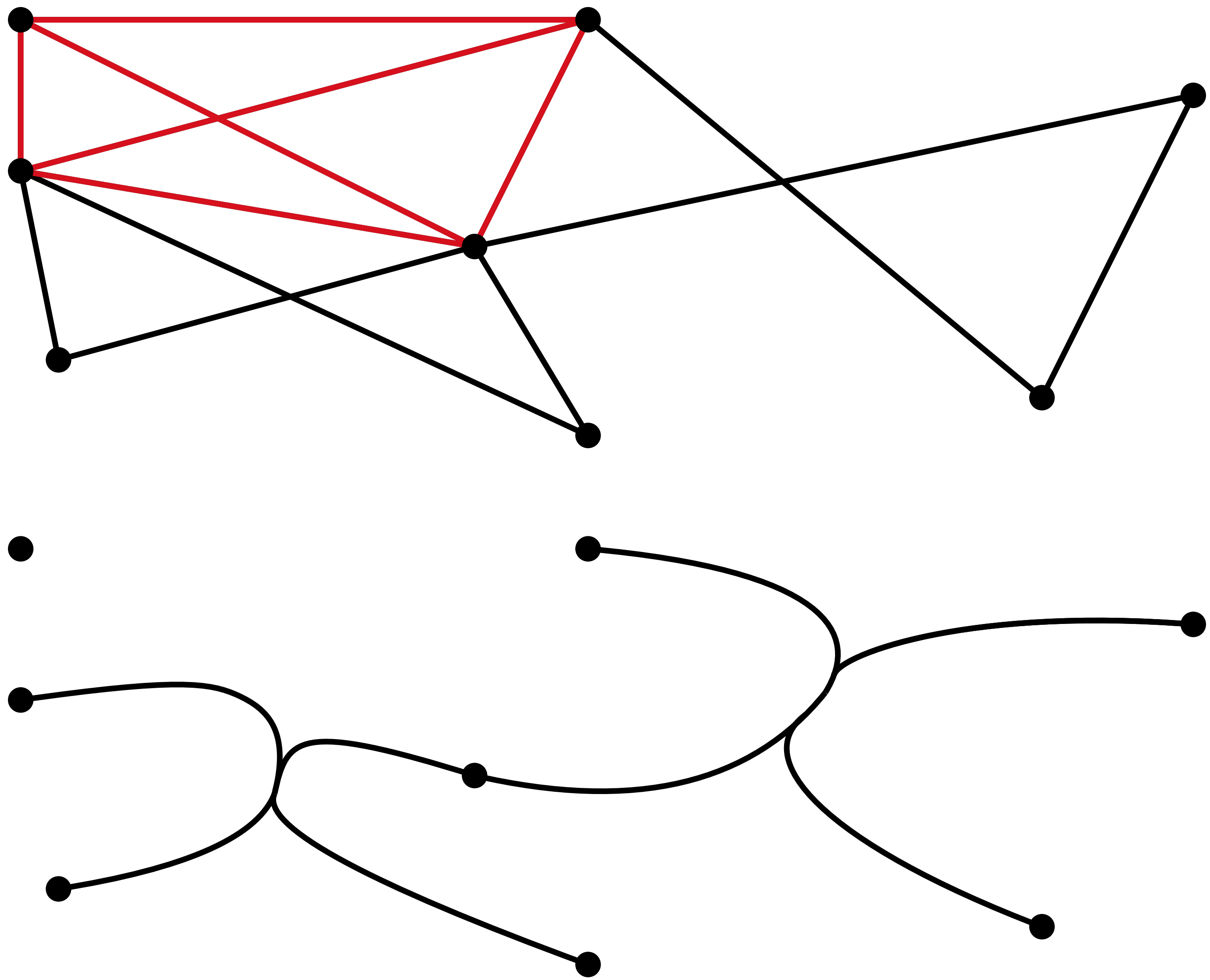

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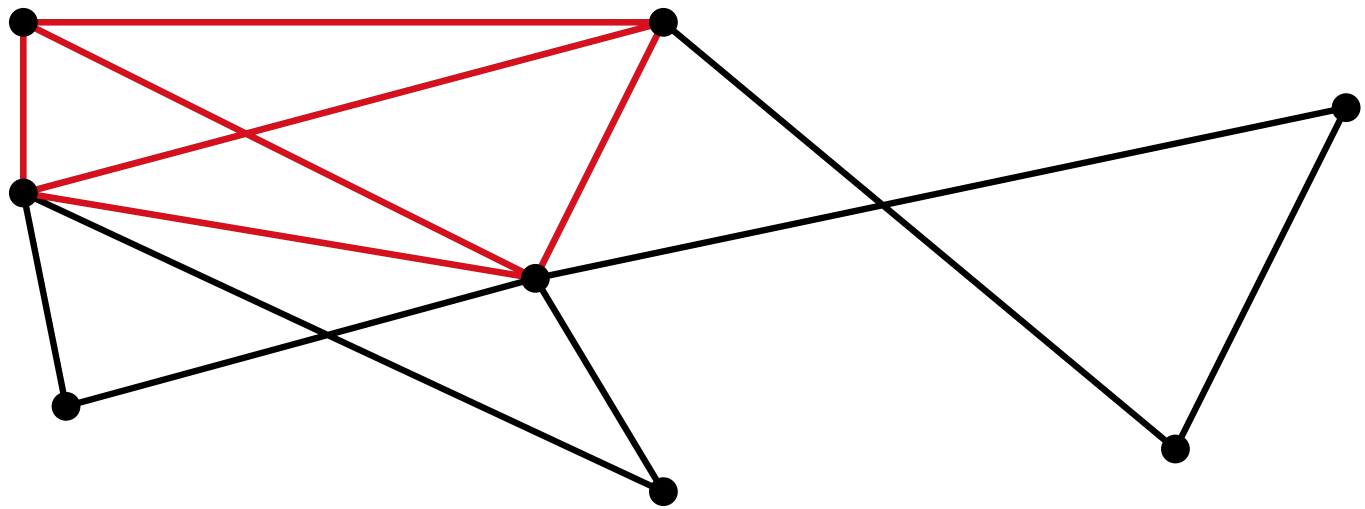

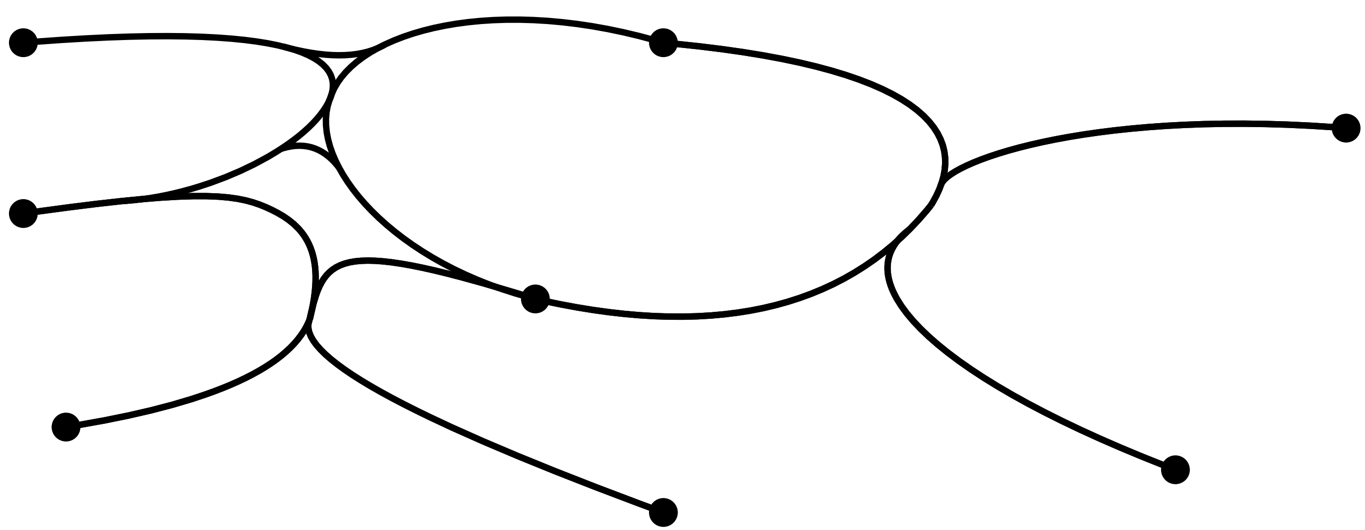

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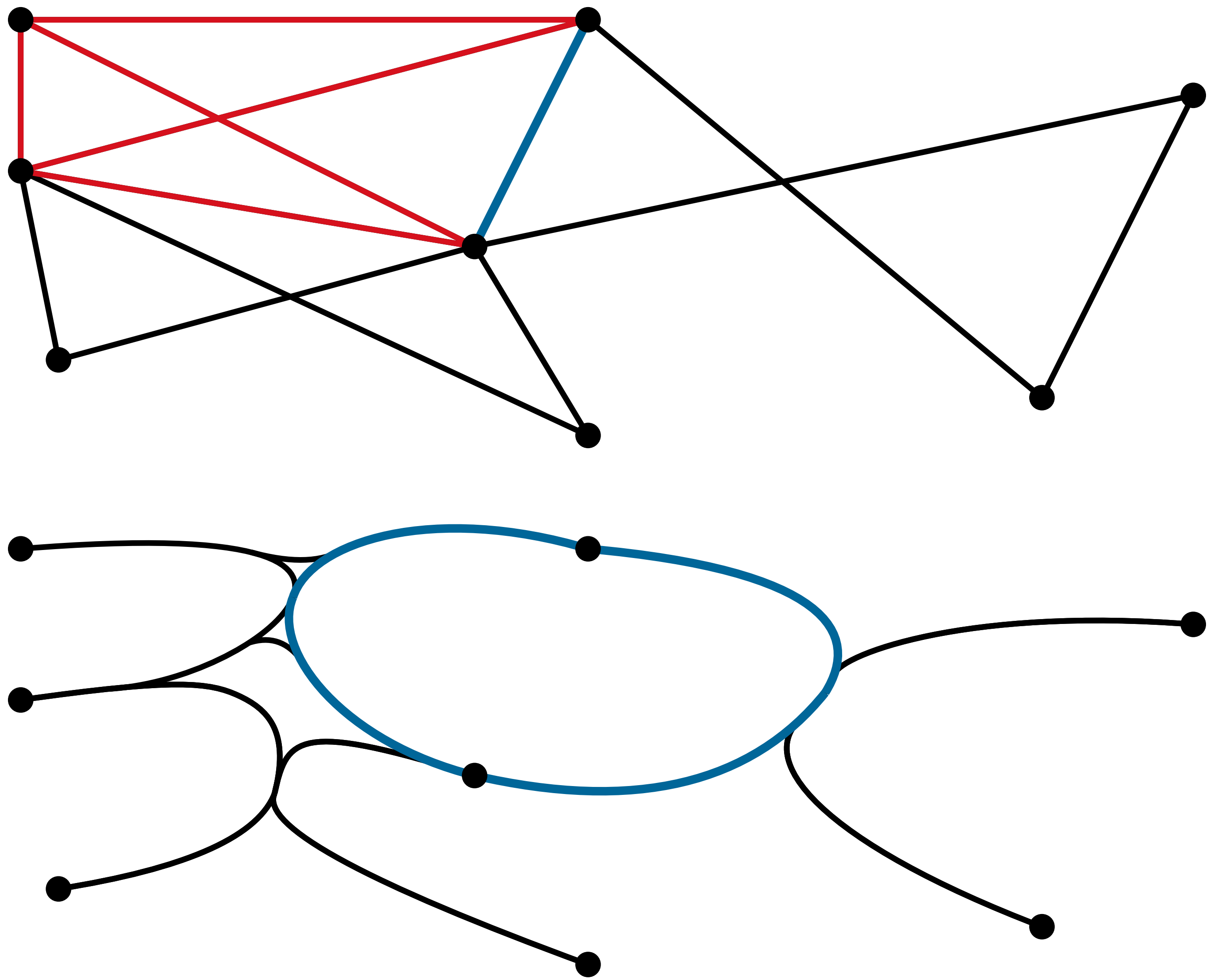

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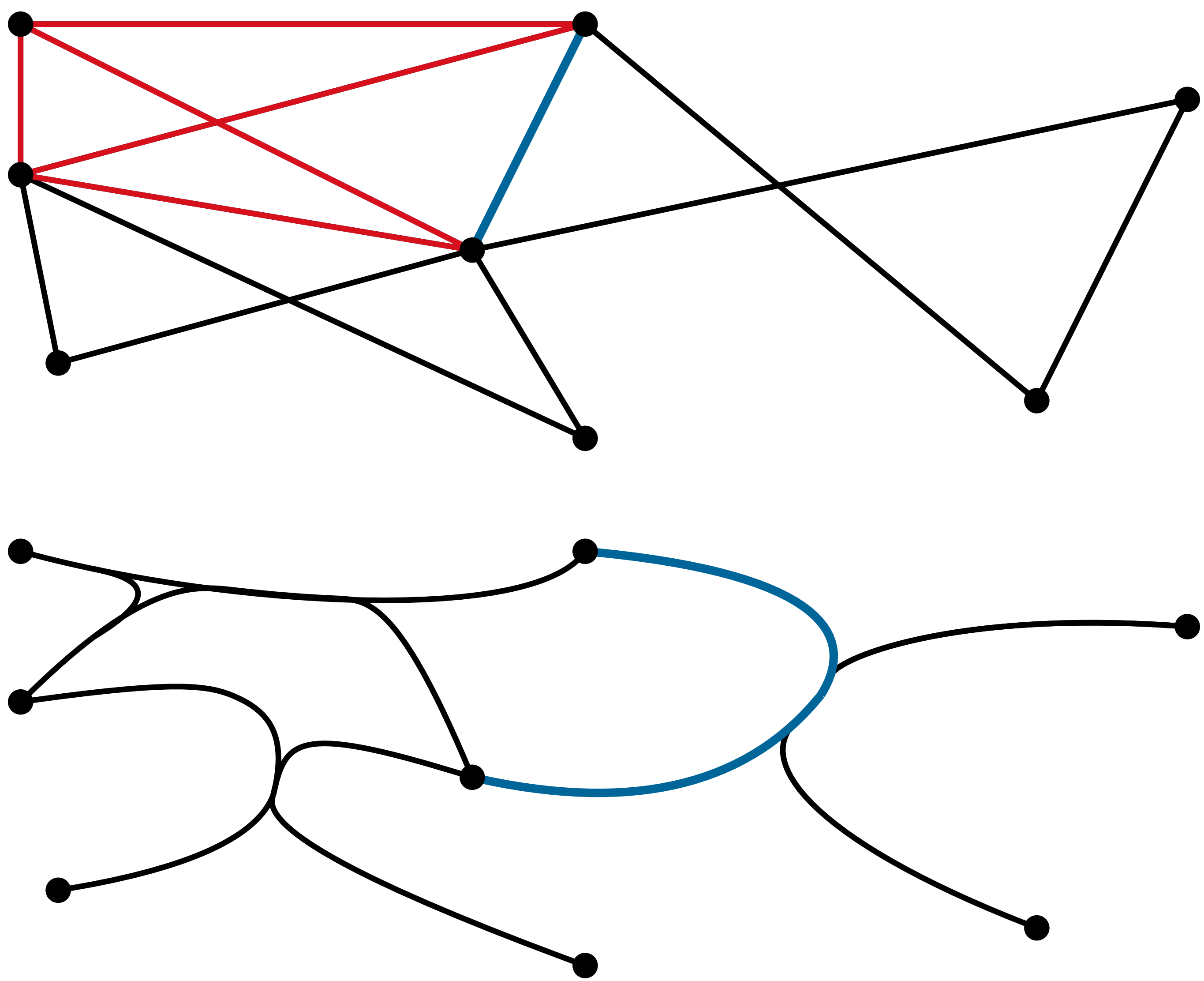

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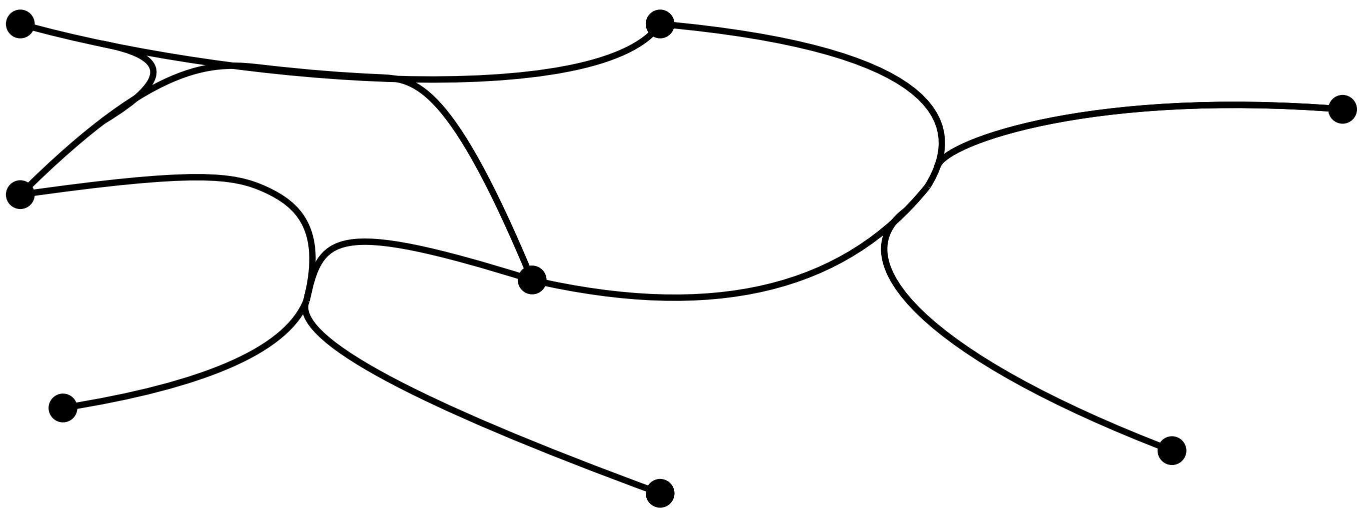

Plane drawing, i.e. no “real” intersections
No wrong adjacencies

No double paths $\Rightarrow$ strict confluency
All vertices on a circle $\Rightarrow$ outer confluency

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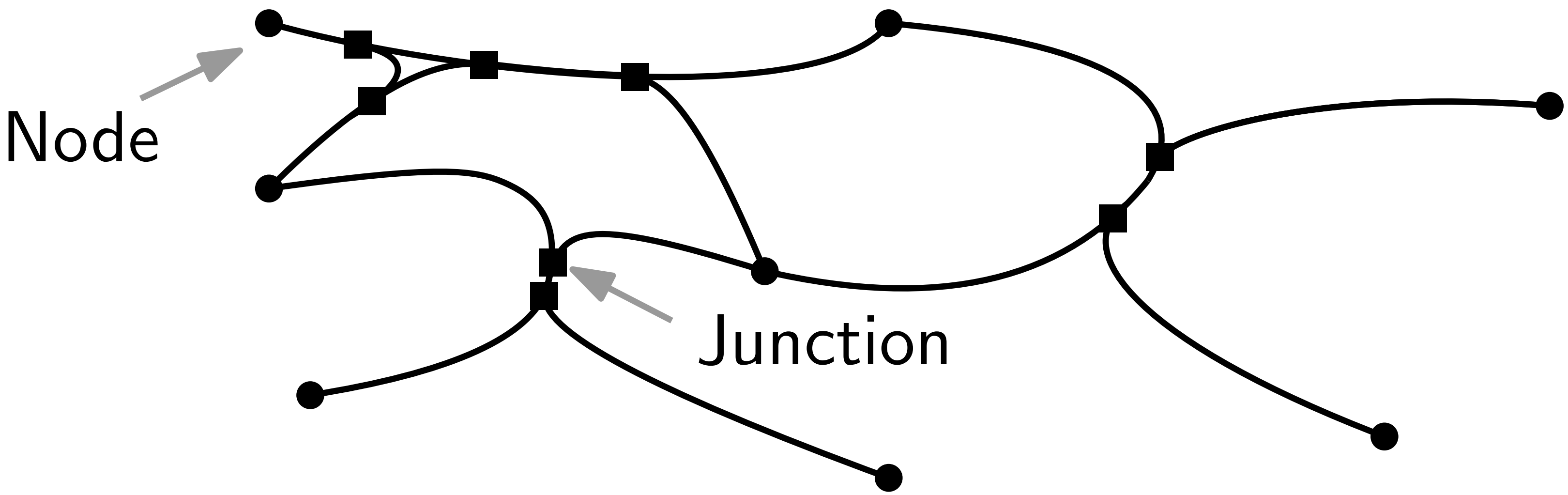

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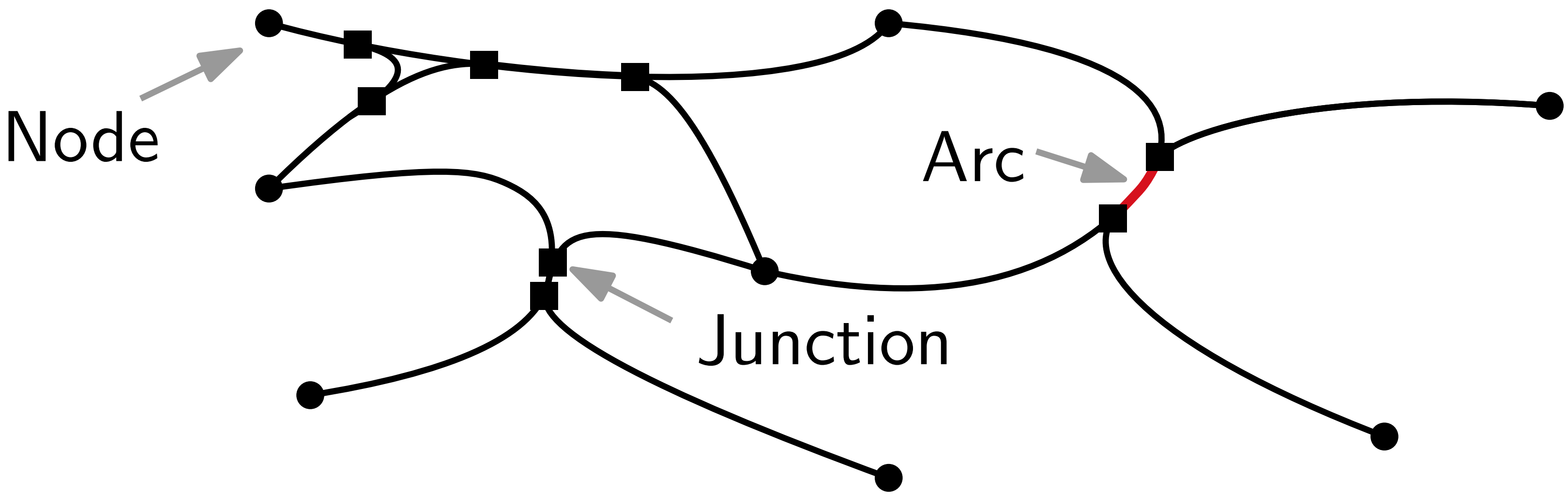

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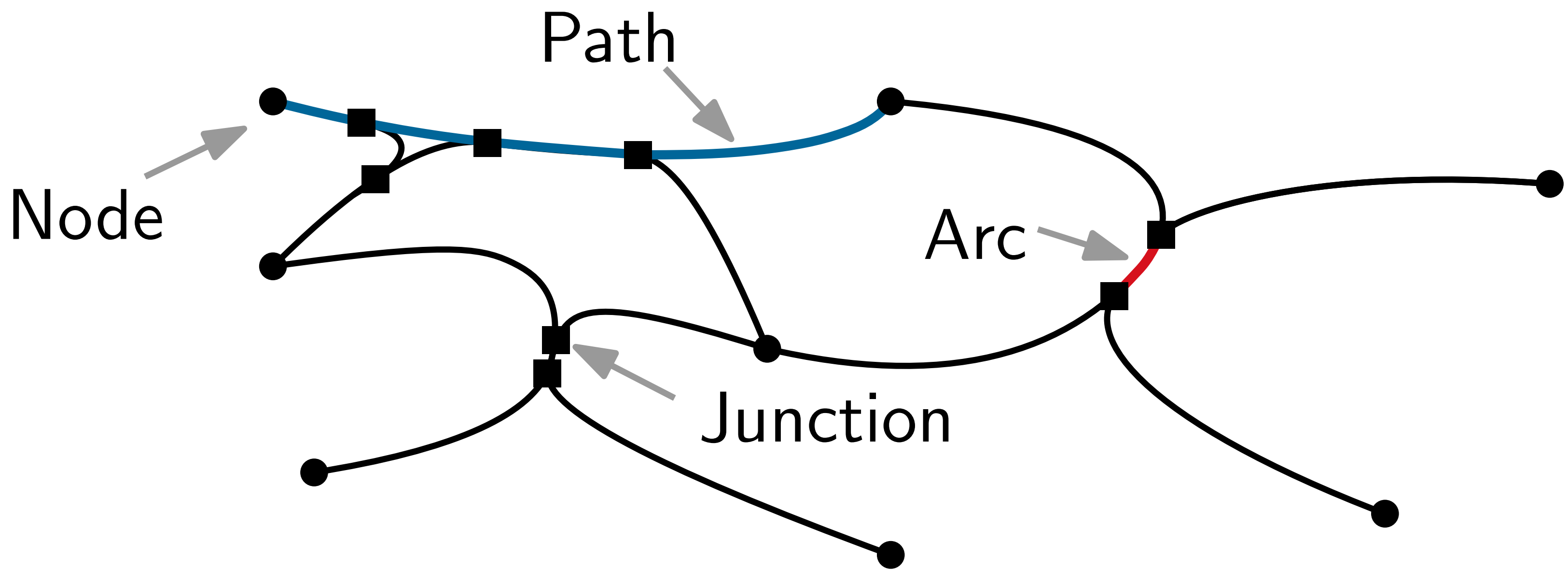

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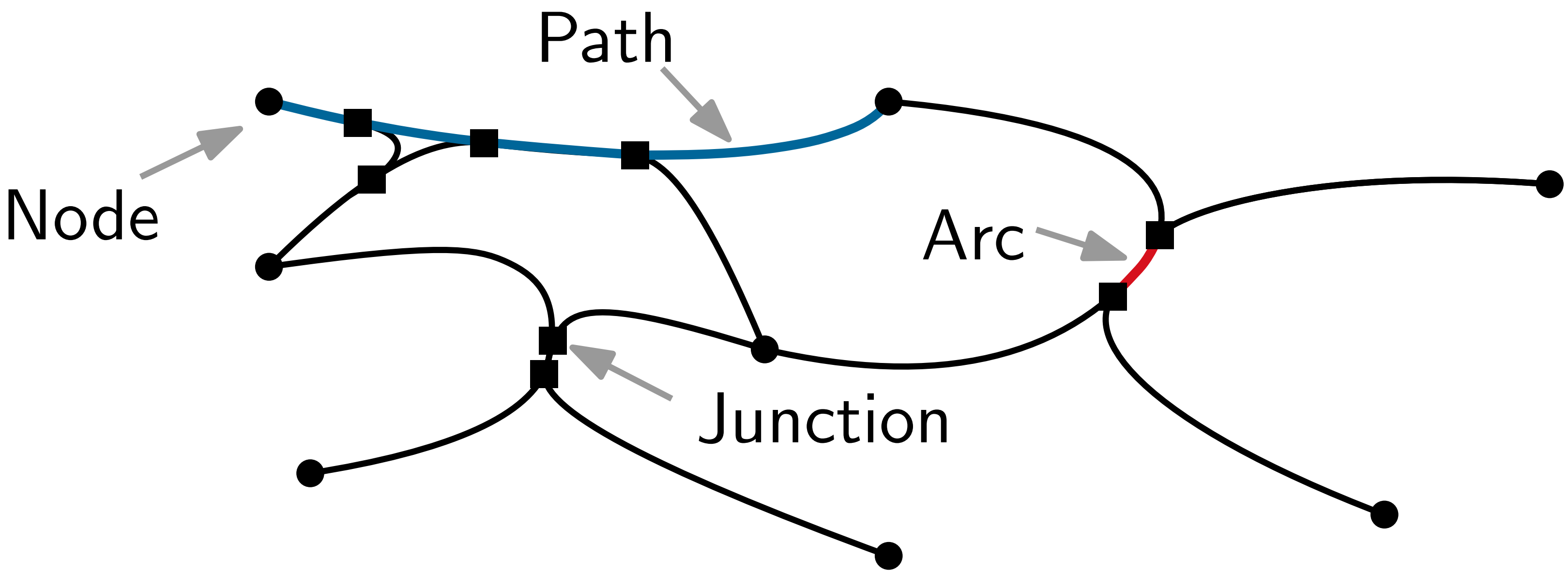

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Every plane drawing is (strict) confluent
⇒ Questions mainly interesting for non-planar graphs

Known Results

Does a graph G admit a confluent drawing?

- Only known for a few classes of graphs
  E.g. Interval graphs, bipartite permutation graphs
  [Dickerson et al 2005, Hui et al. 2007]
- Negative case also only known for a few classes of graphs
  E.g. Petersen graph, Chordal graphs [Dickerson et al. 2005]
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  [Eppstein et al. 2016]

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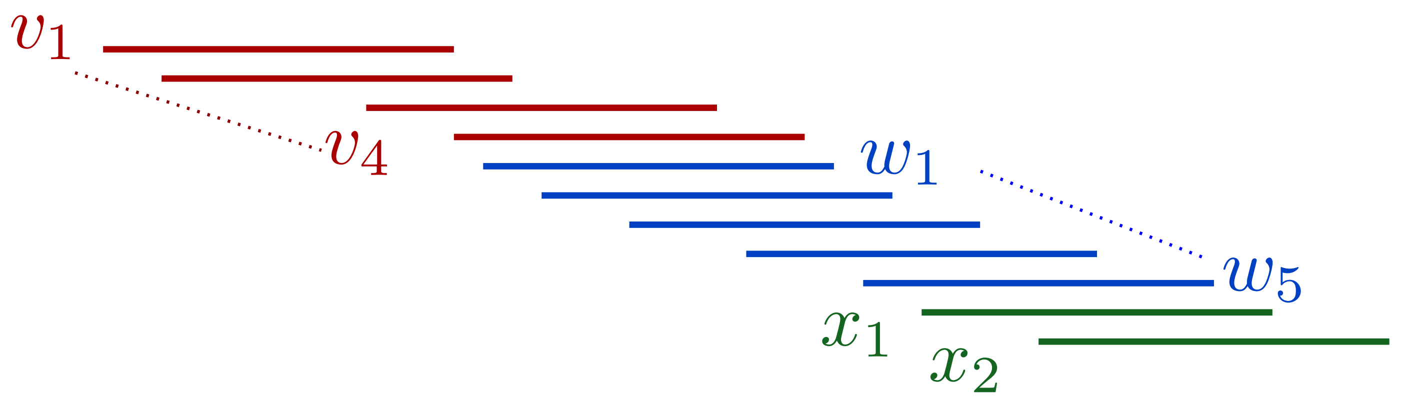
One main open question

Which graphs have a strict outer-confluent drawing?

SC Drawings of Unit Interval Graphs

Graphs that can be represented as intersection of unit intervals



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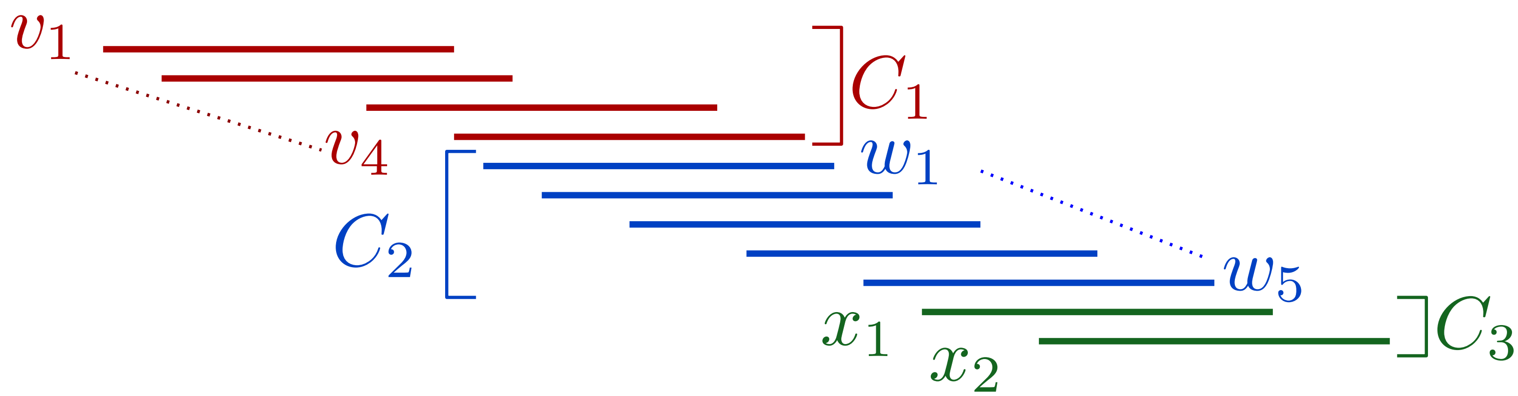

Decompose into cliques

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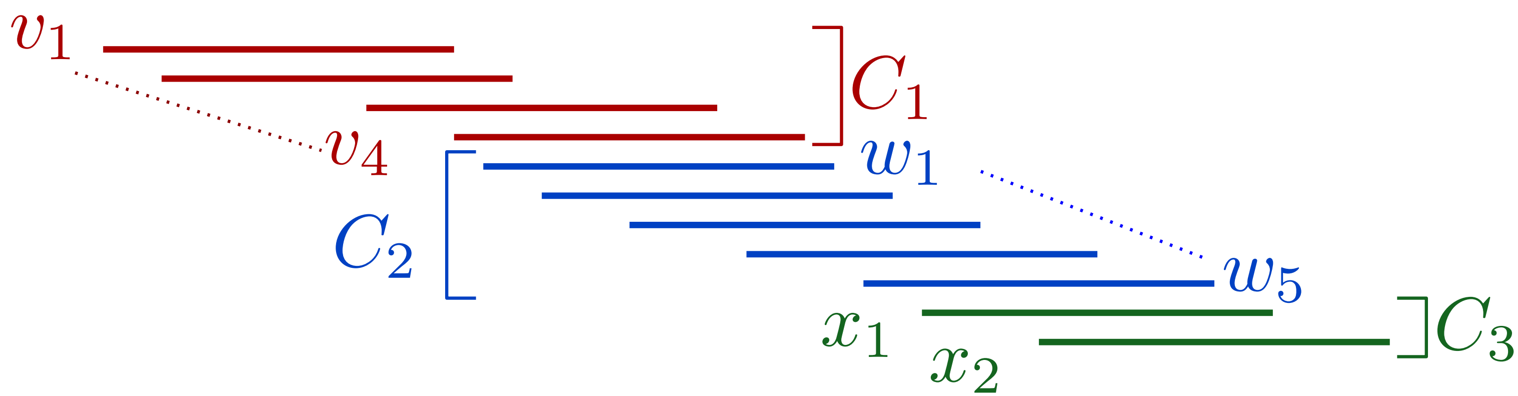

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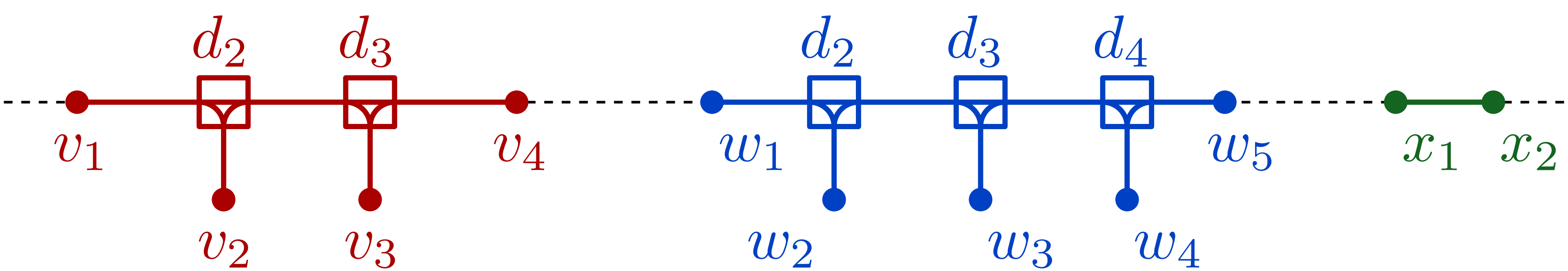

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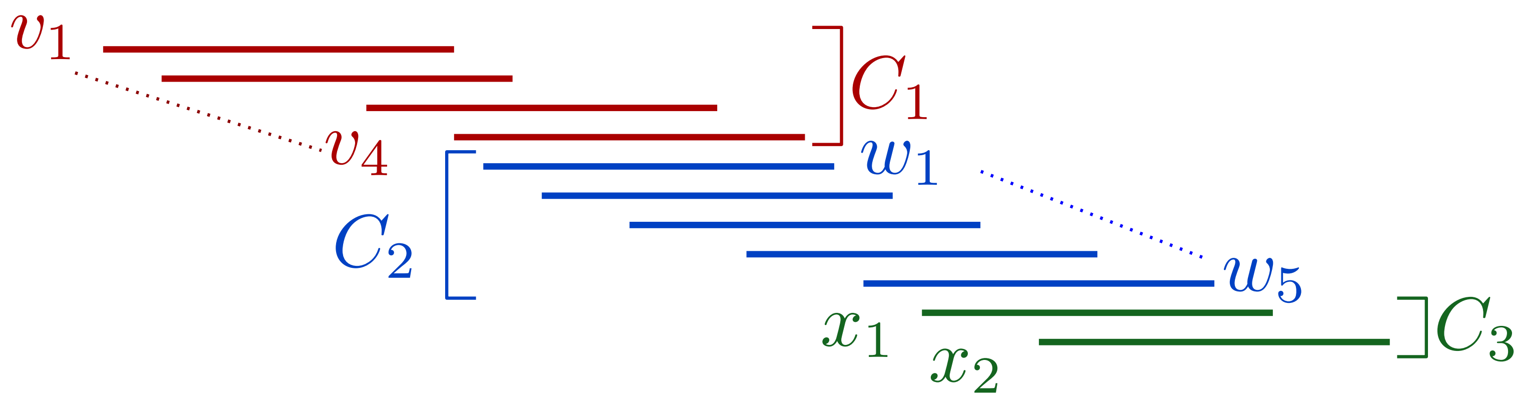

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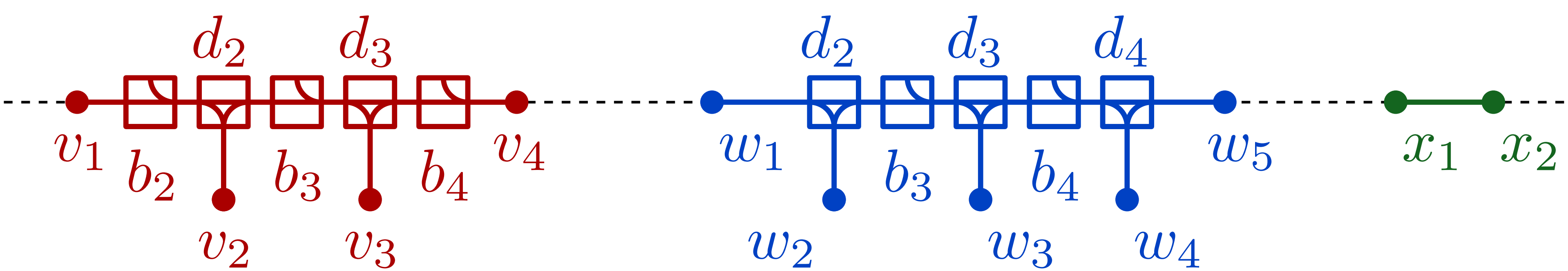

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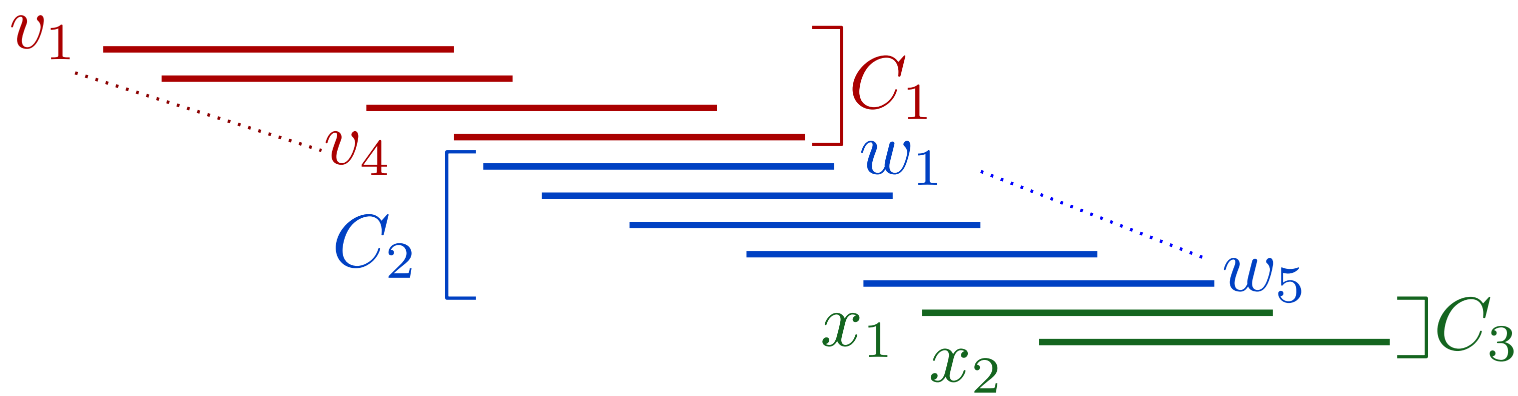

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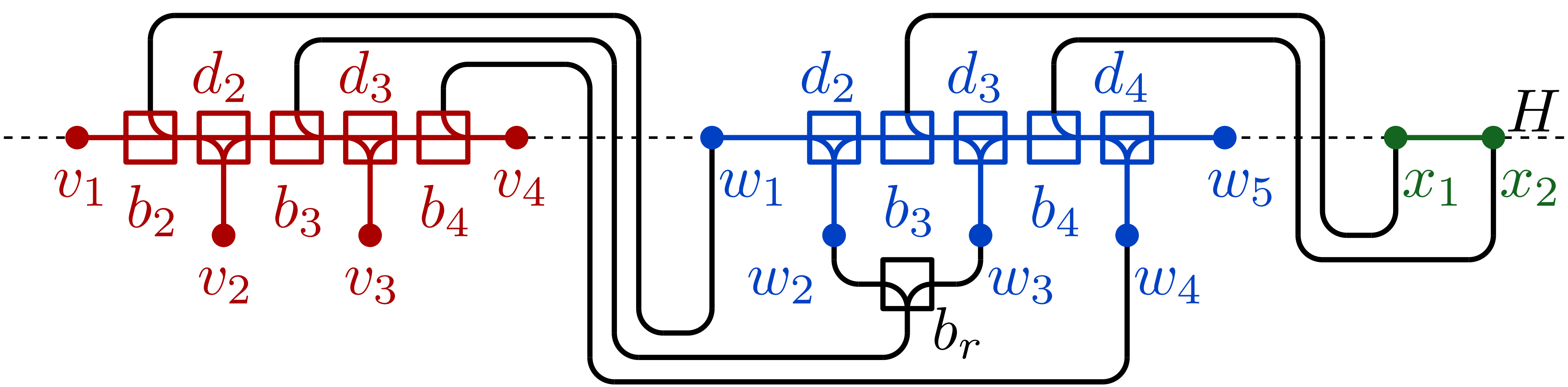
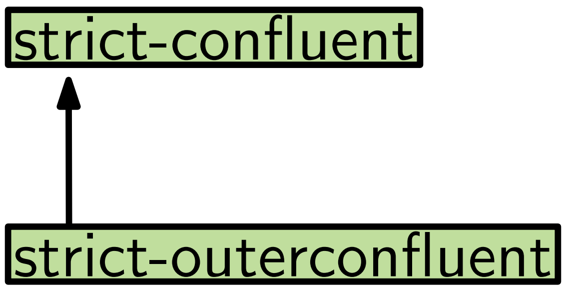
Connect the cliques with confluent paths



Our Results



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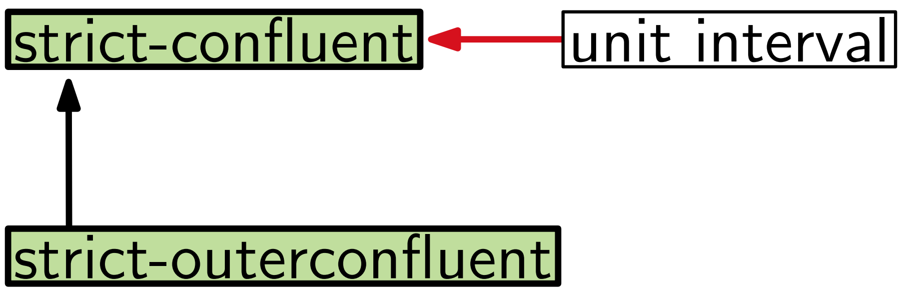


Our Results

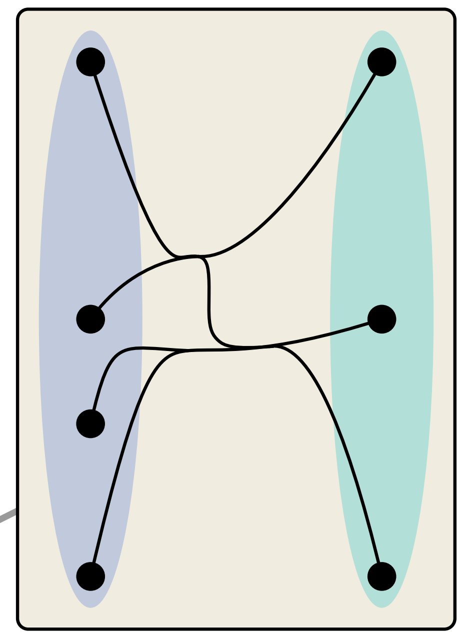
strict-confluent ← unit interval

↑

strict-outerconfluent

↑

strict bipartite outerconfluent



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bipartite permutation ∩ domino-free

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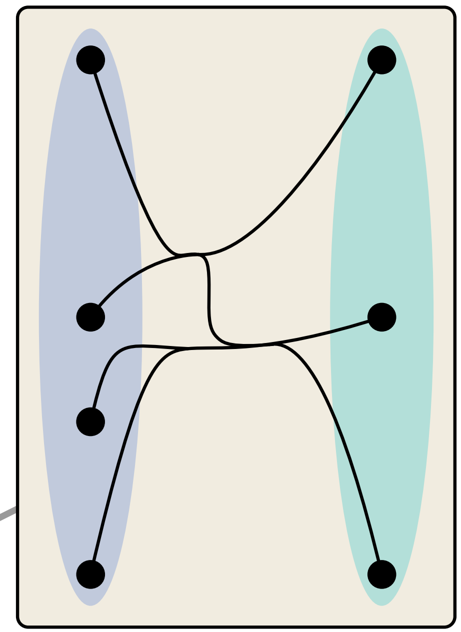

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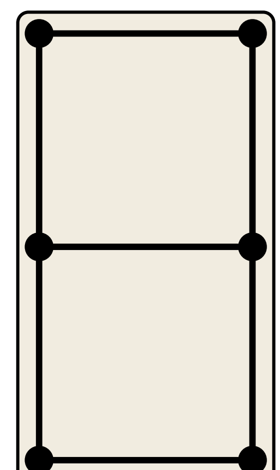

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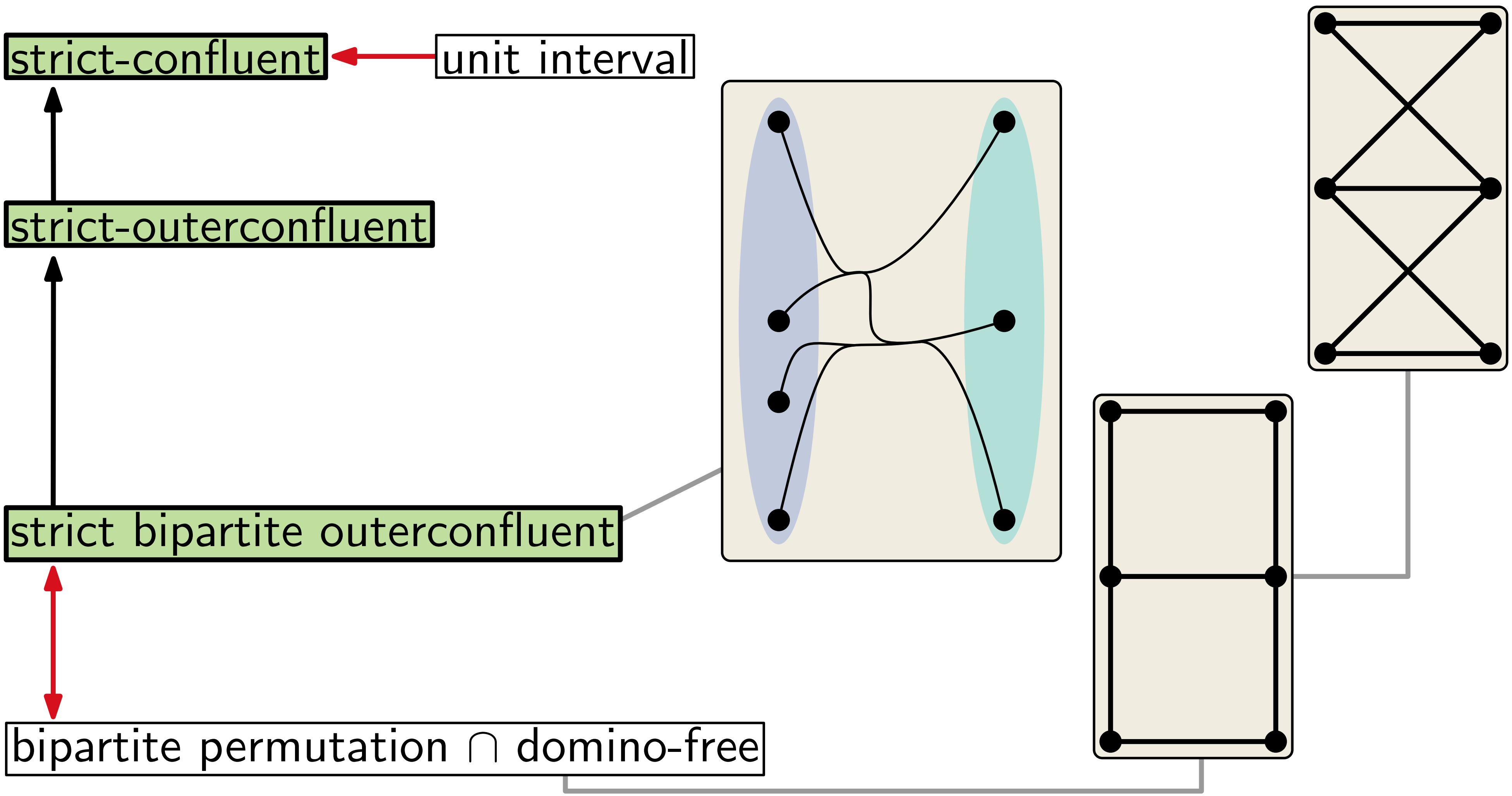
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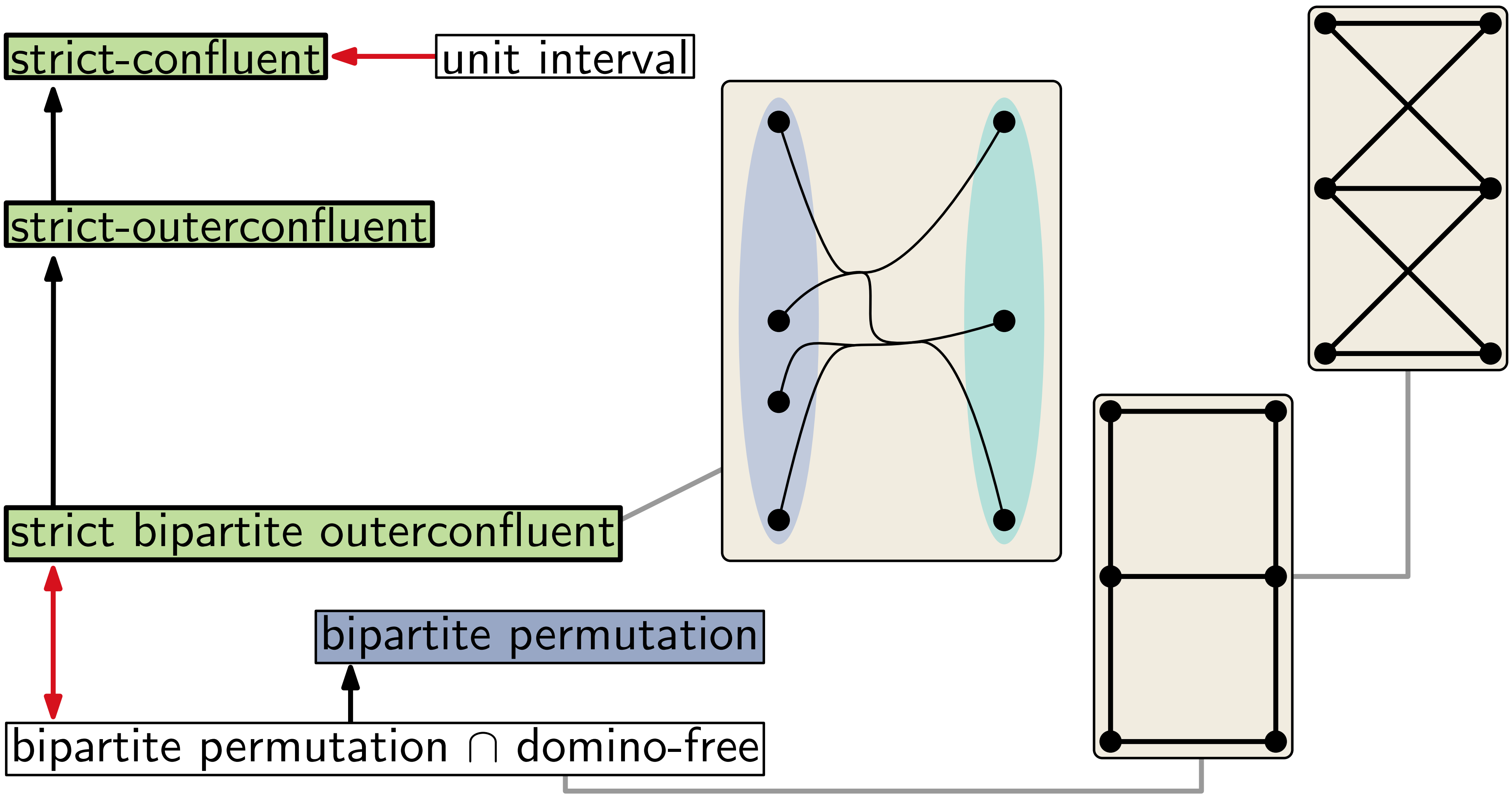
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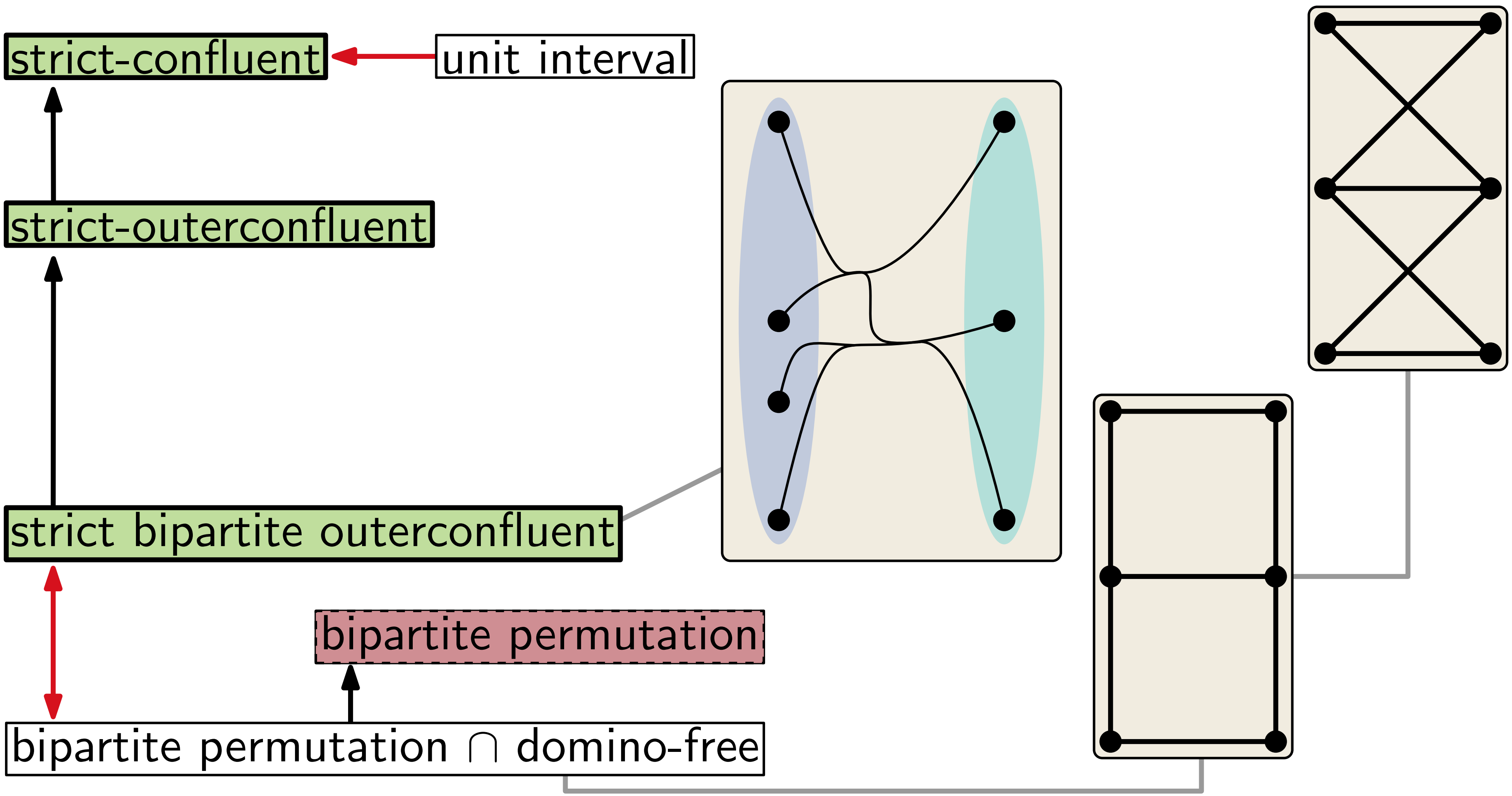

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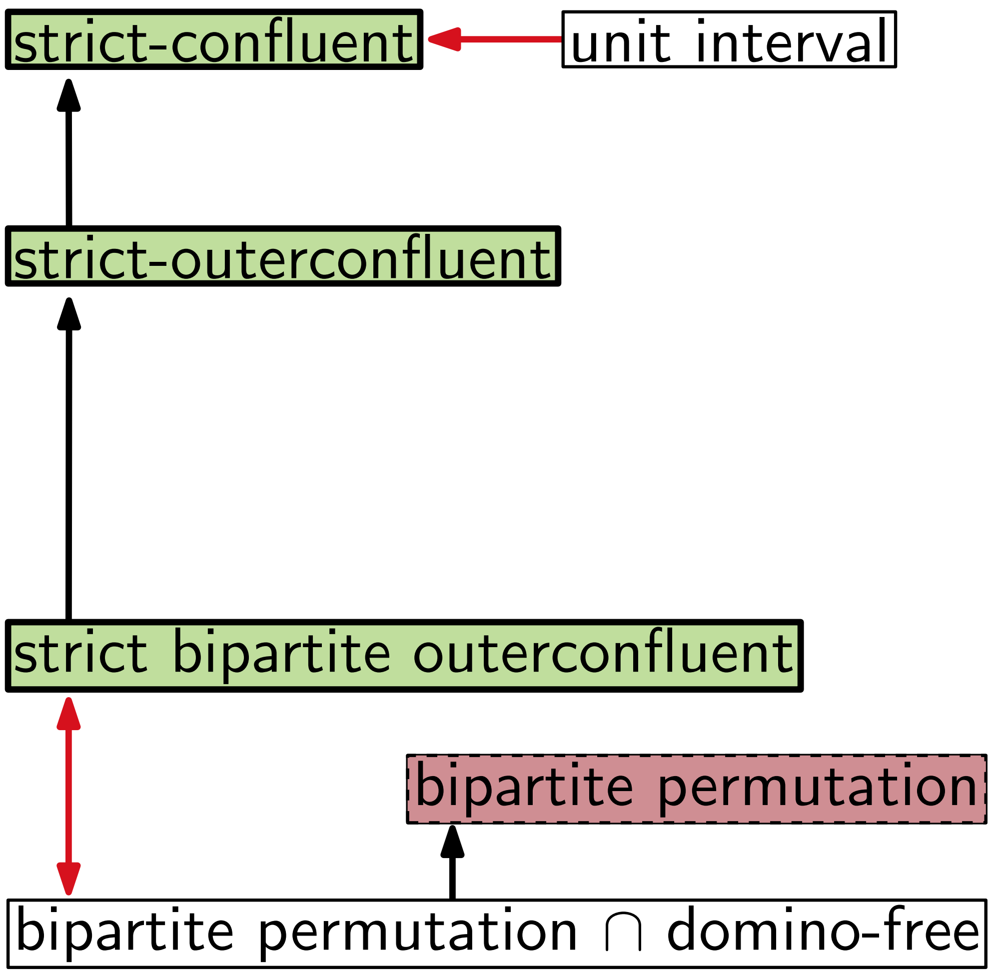

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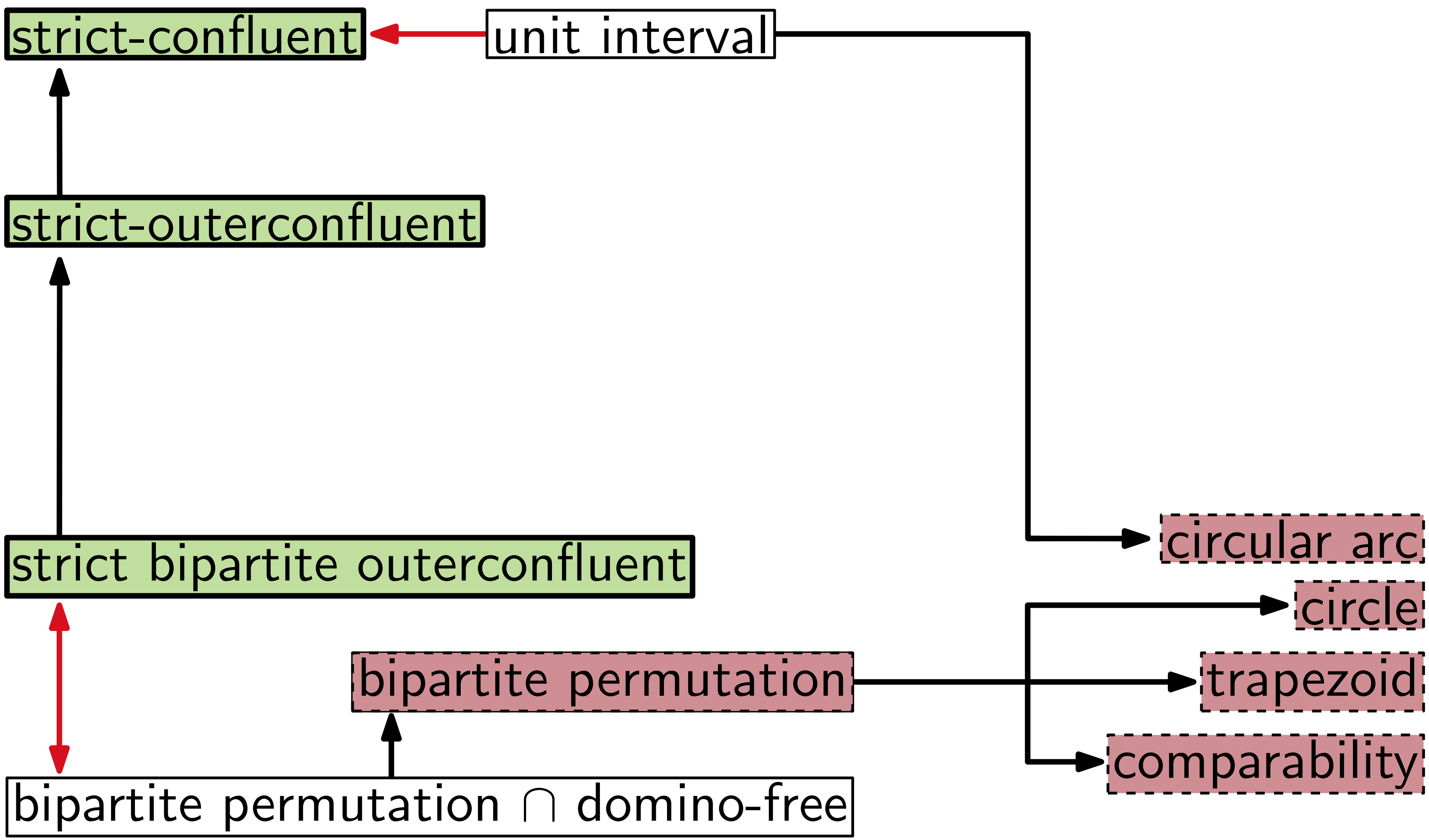
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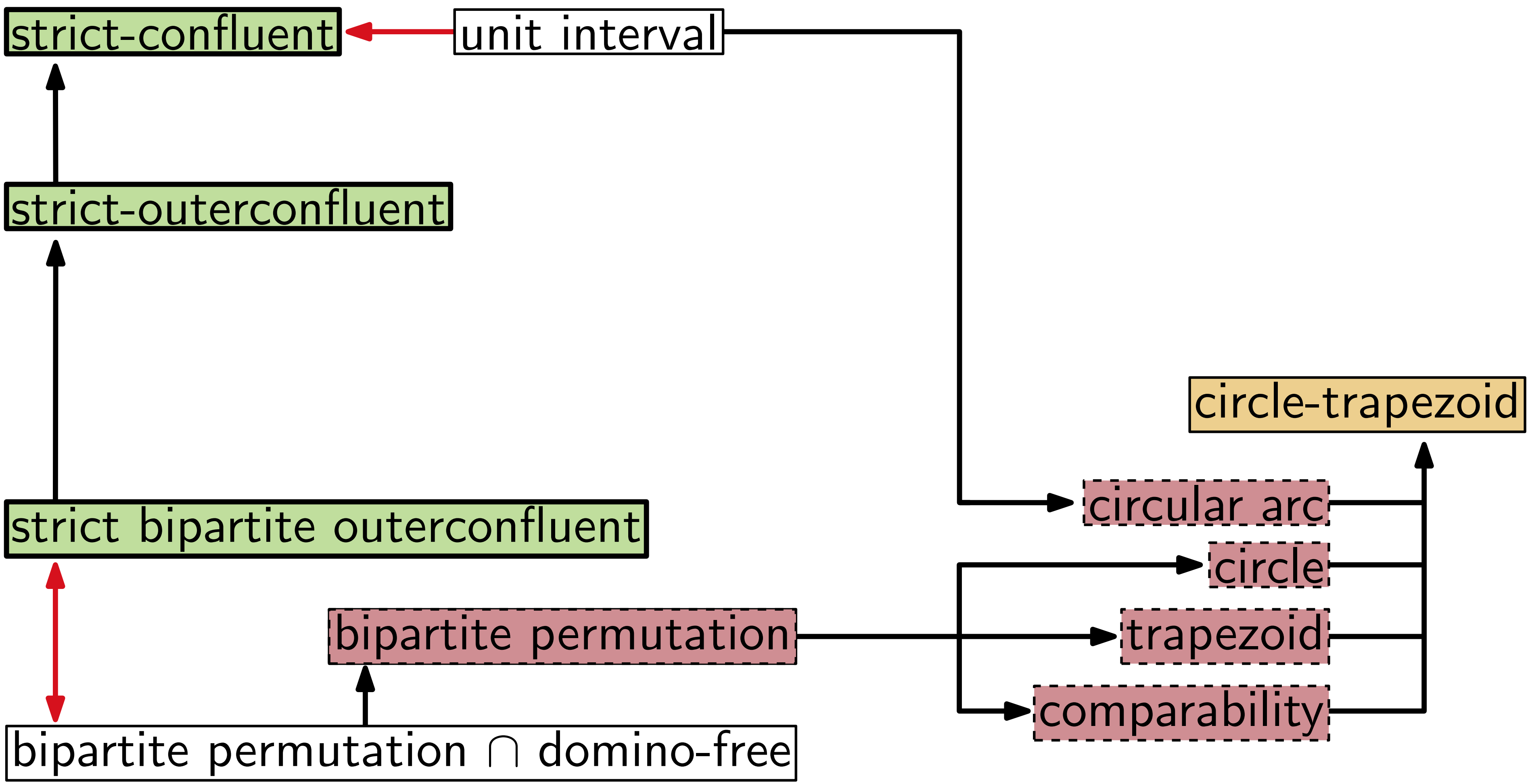
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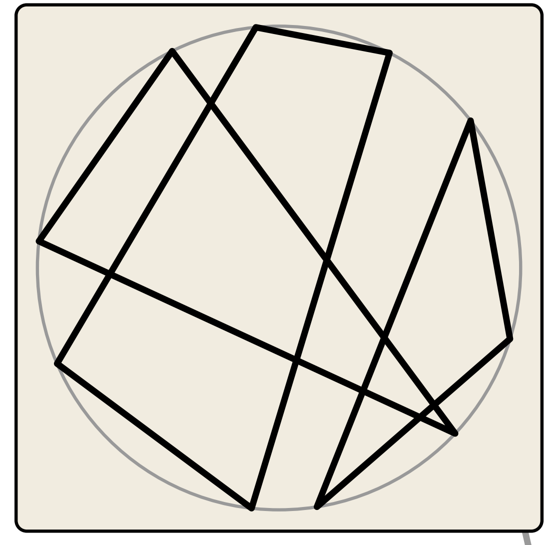

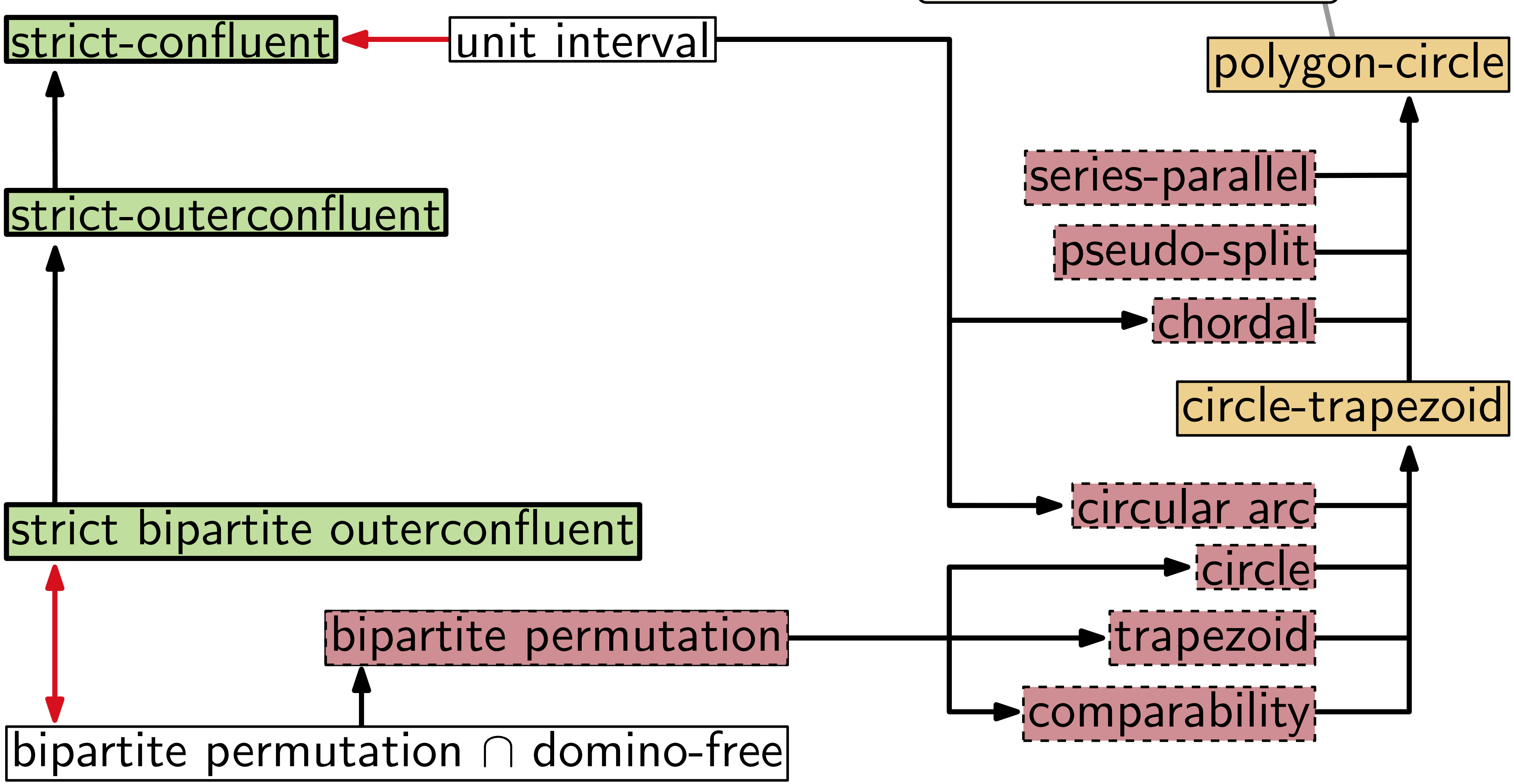
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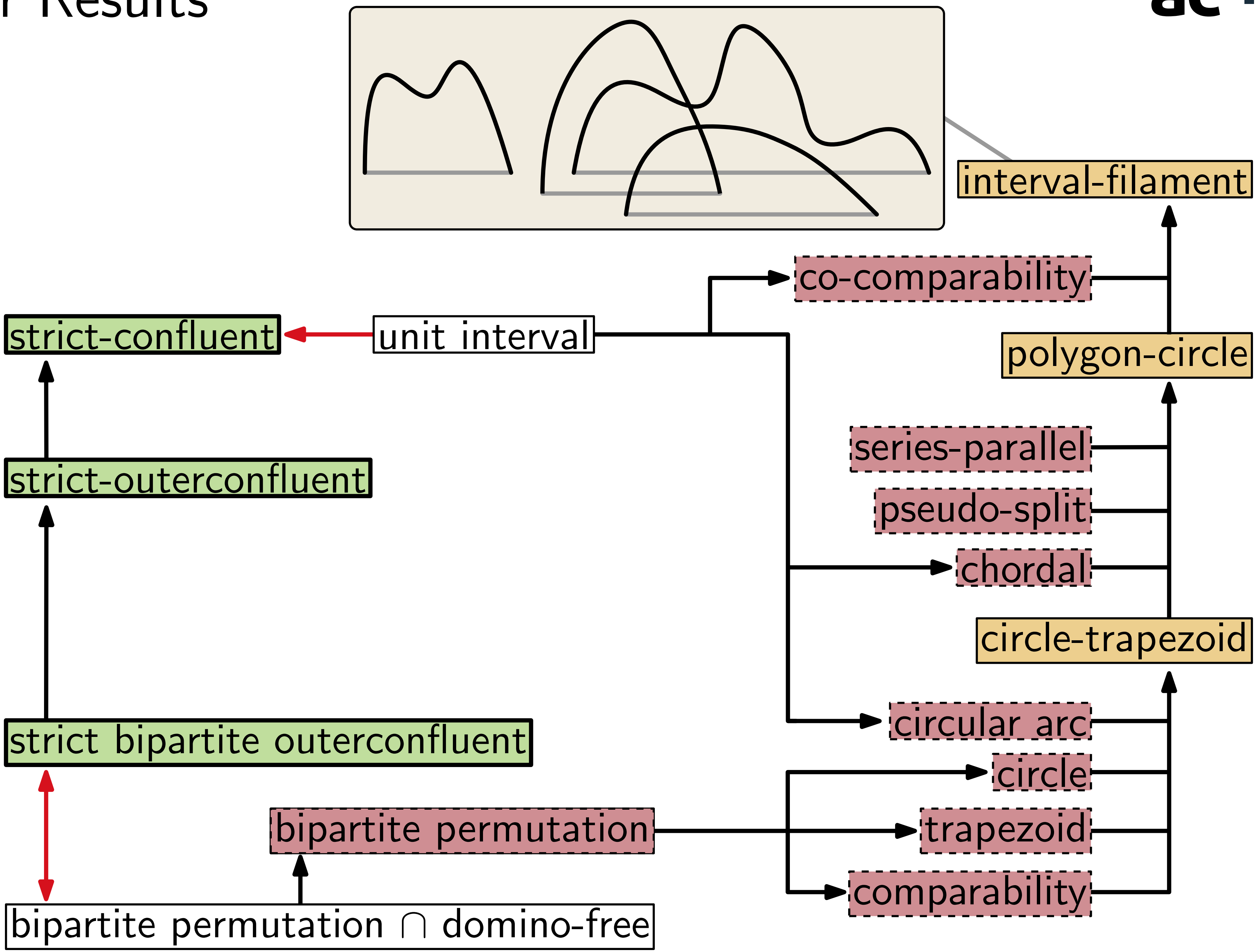
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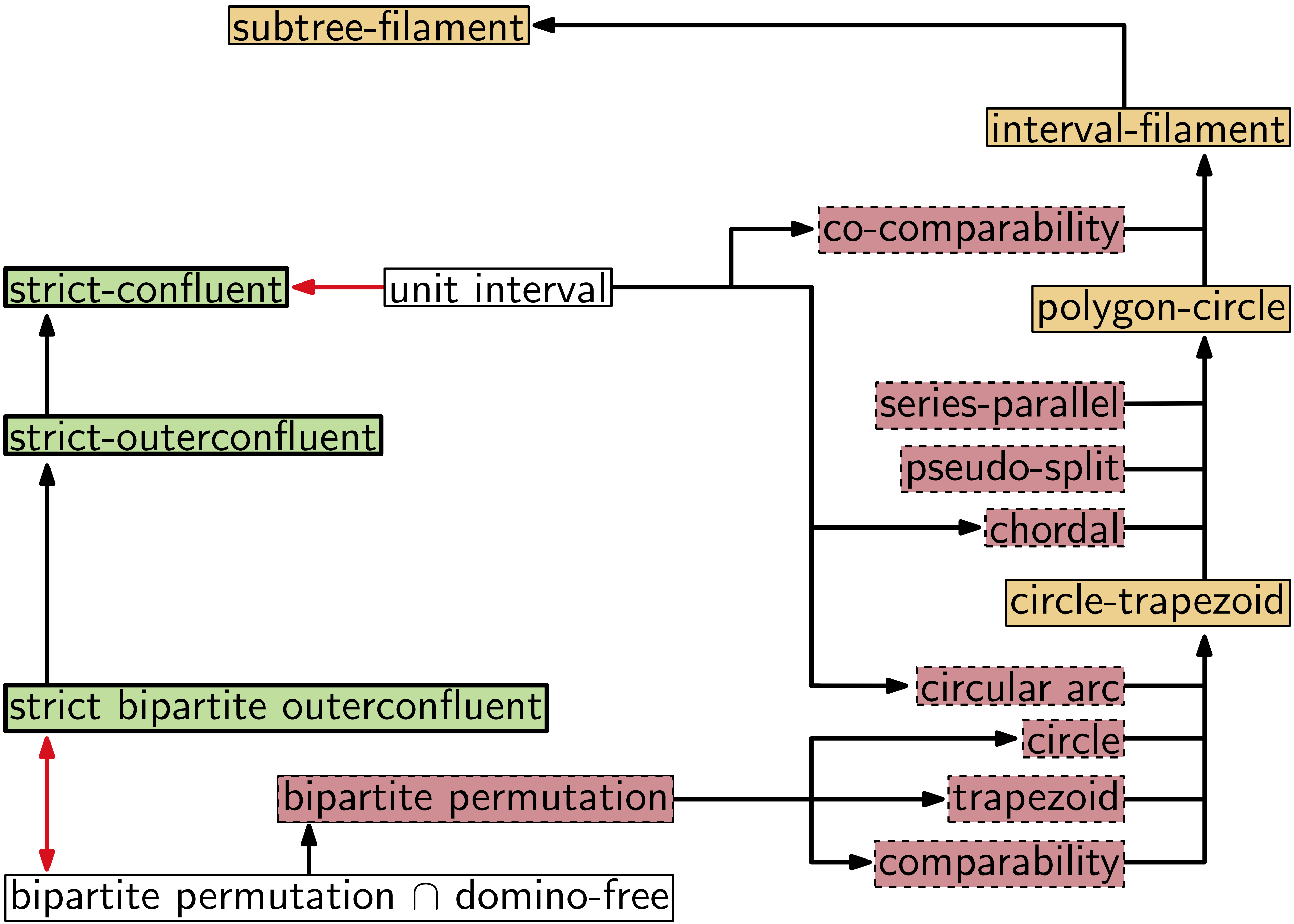
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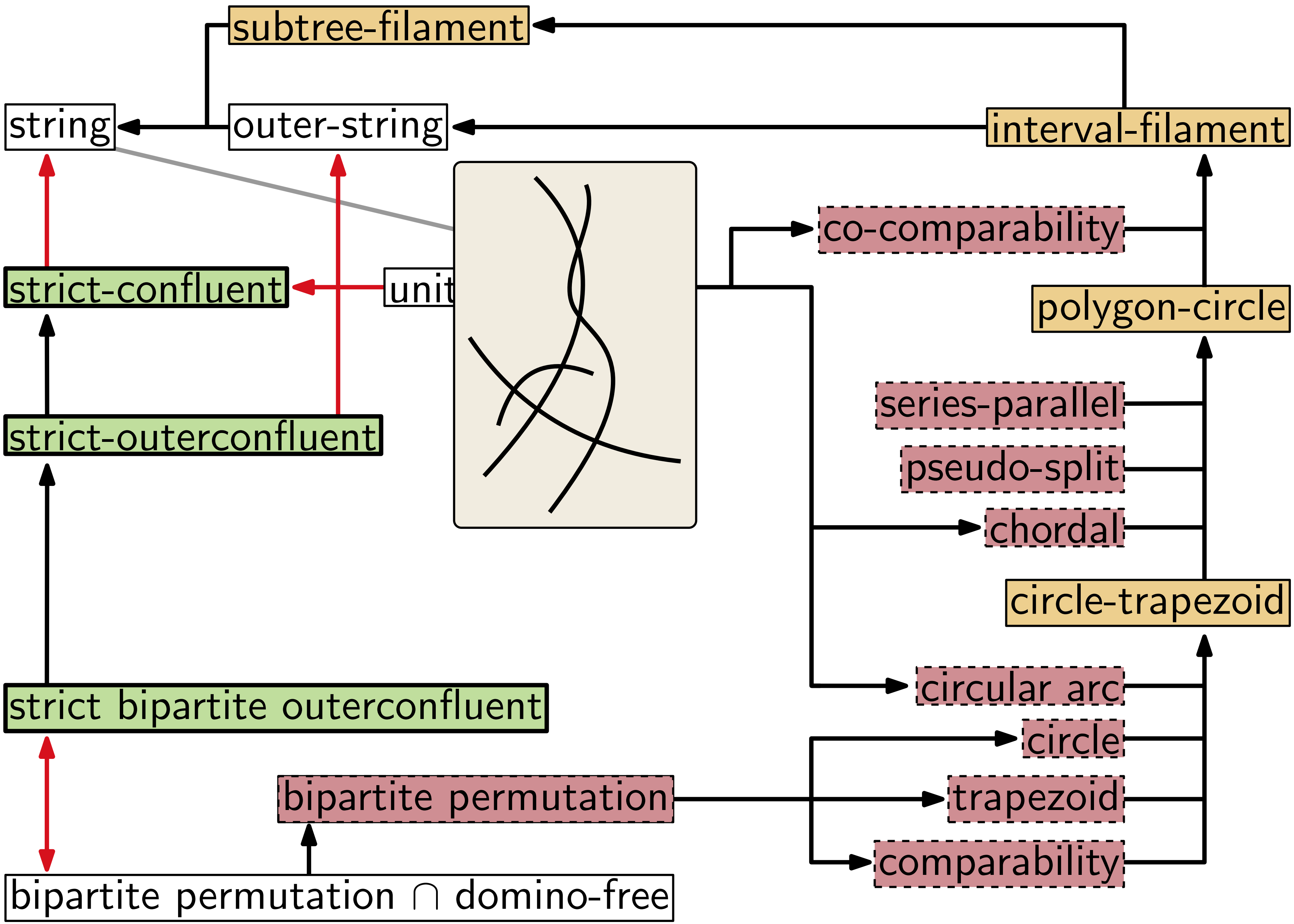
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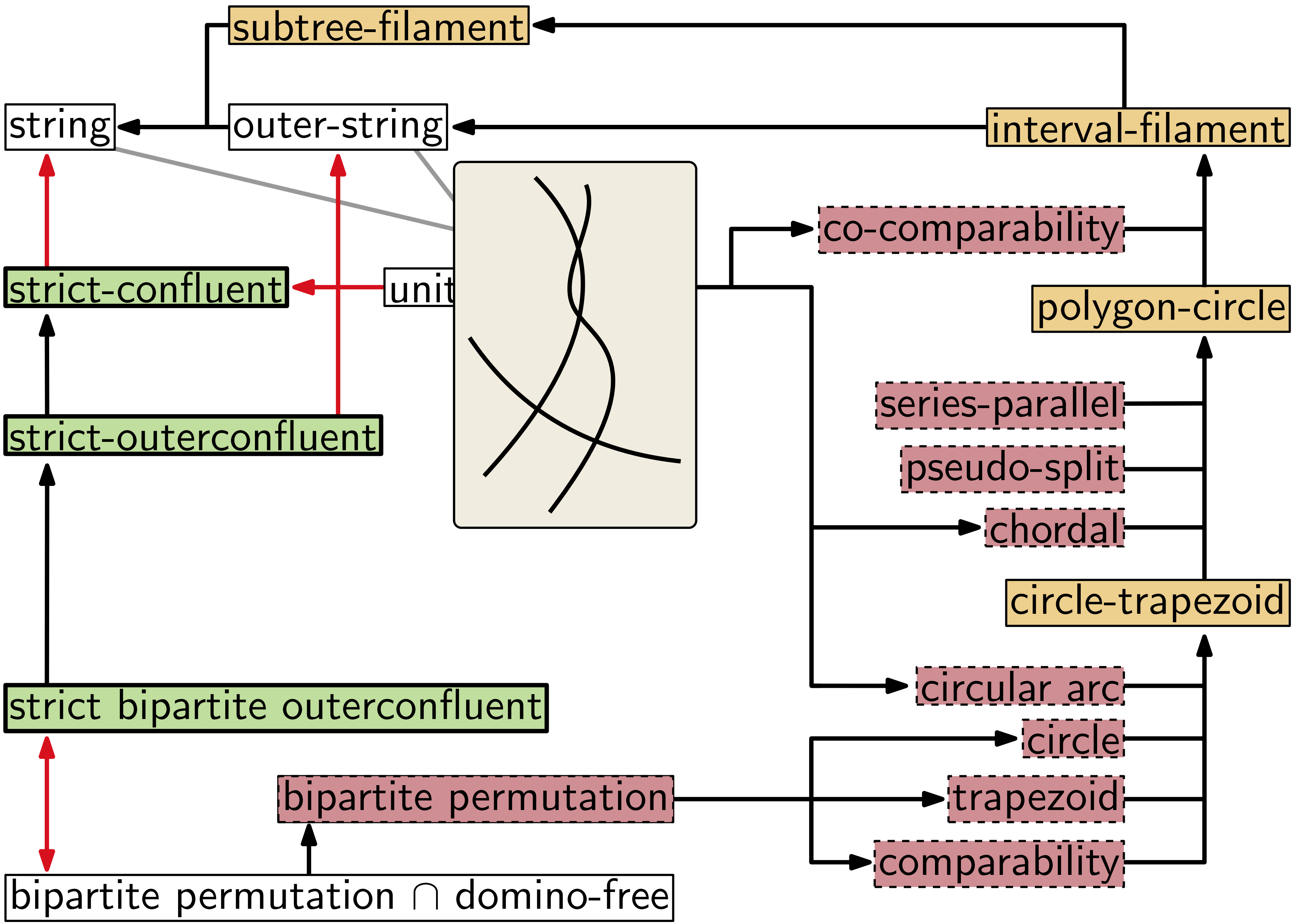

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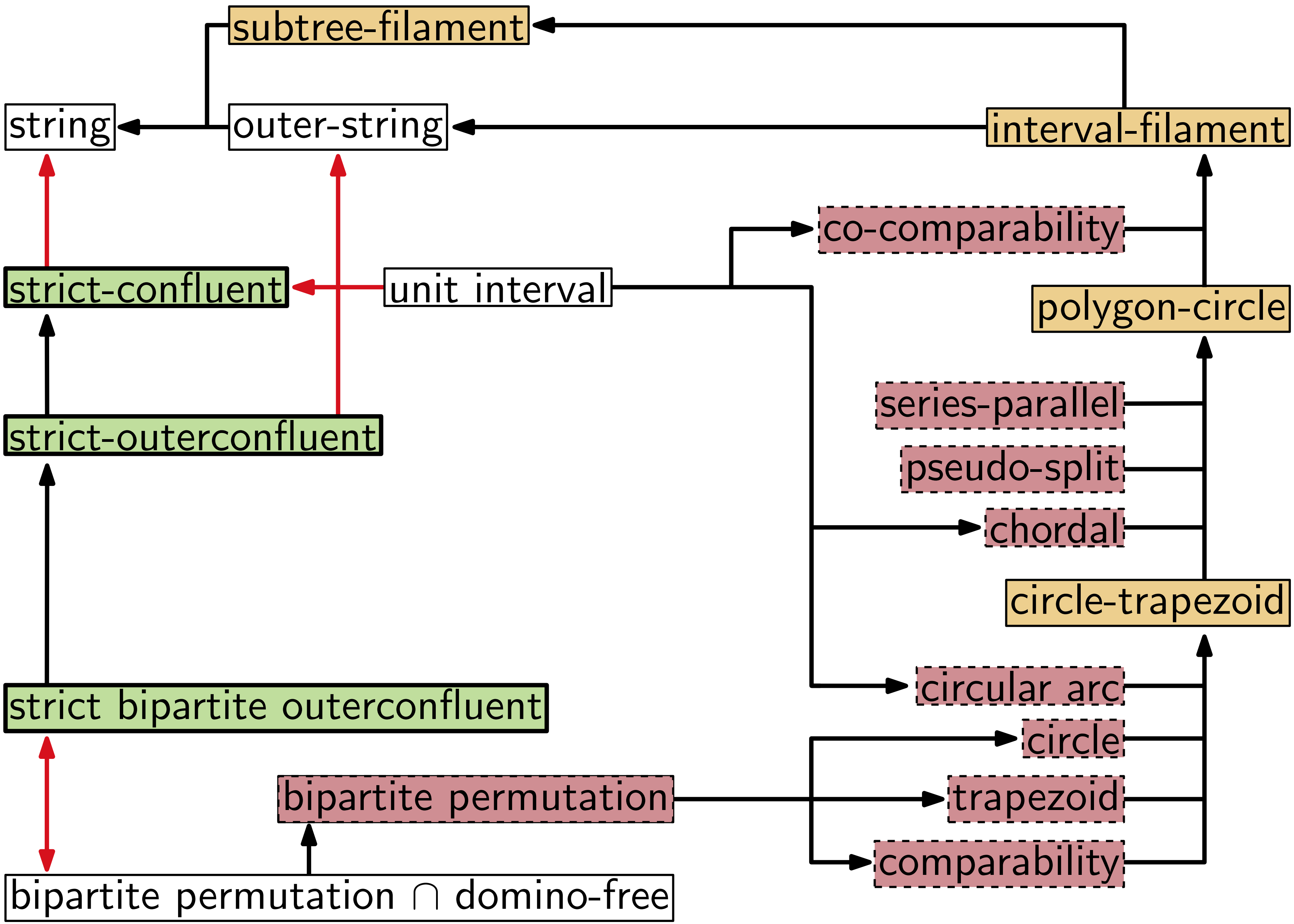
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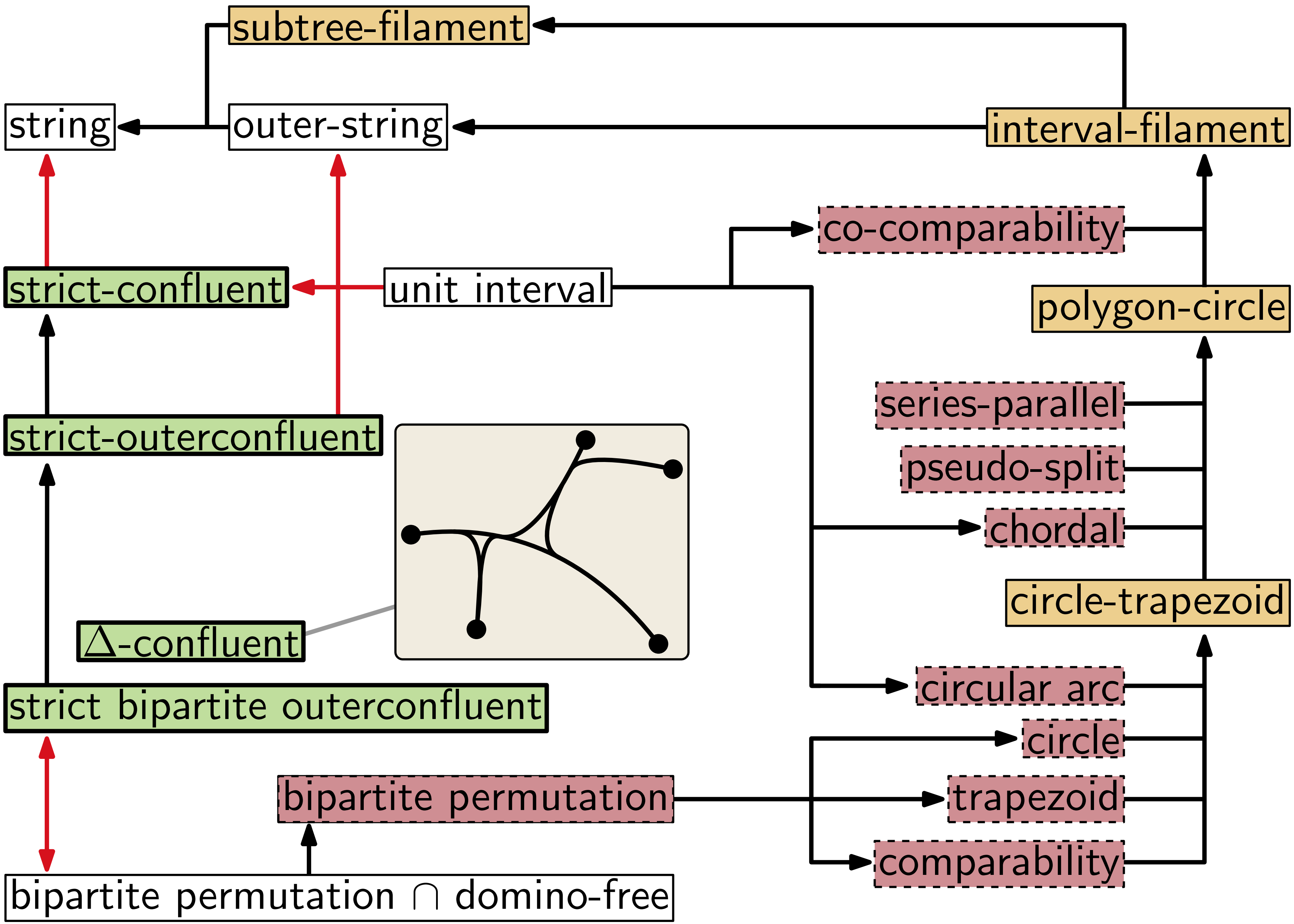

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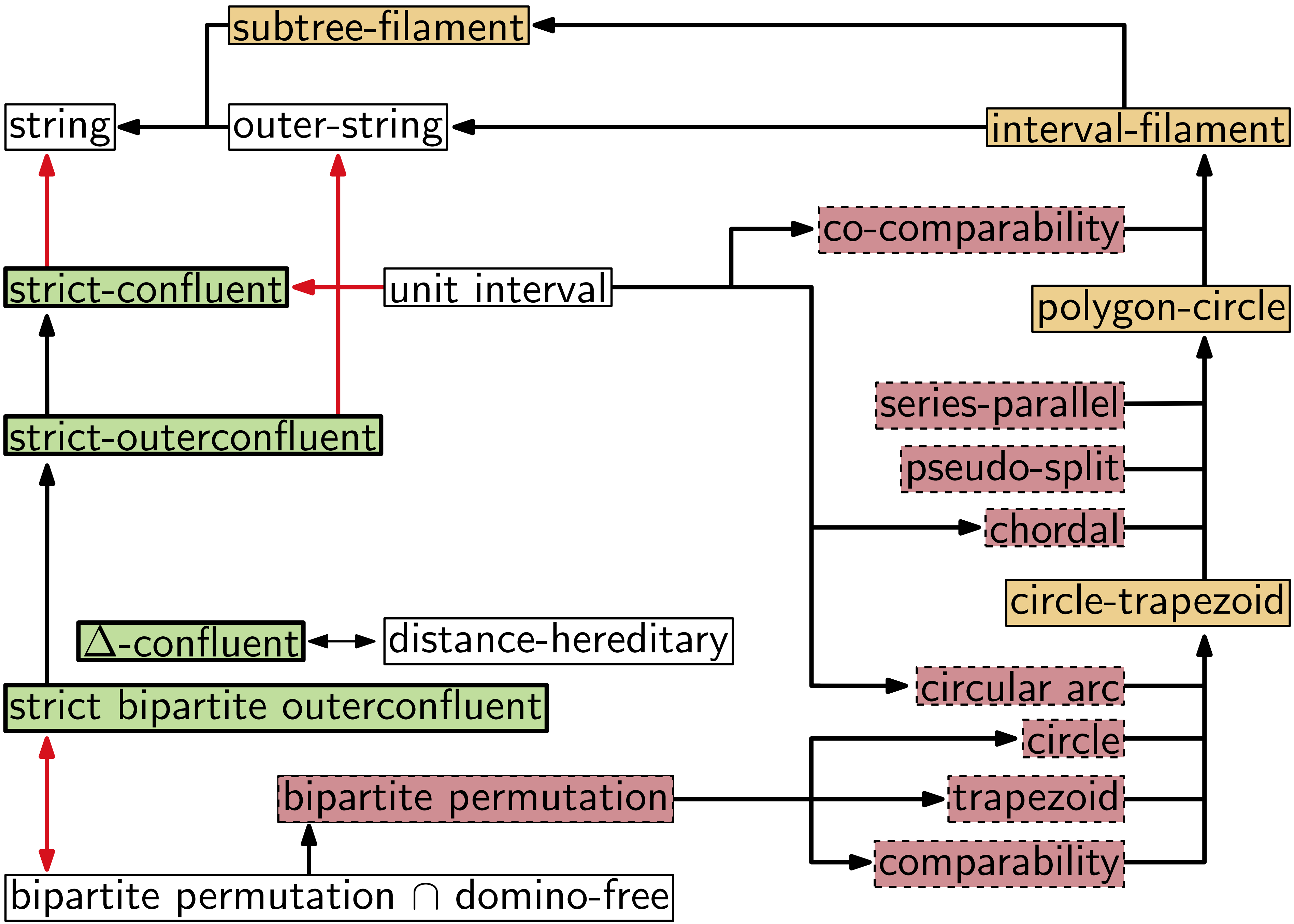
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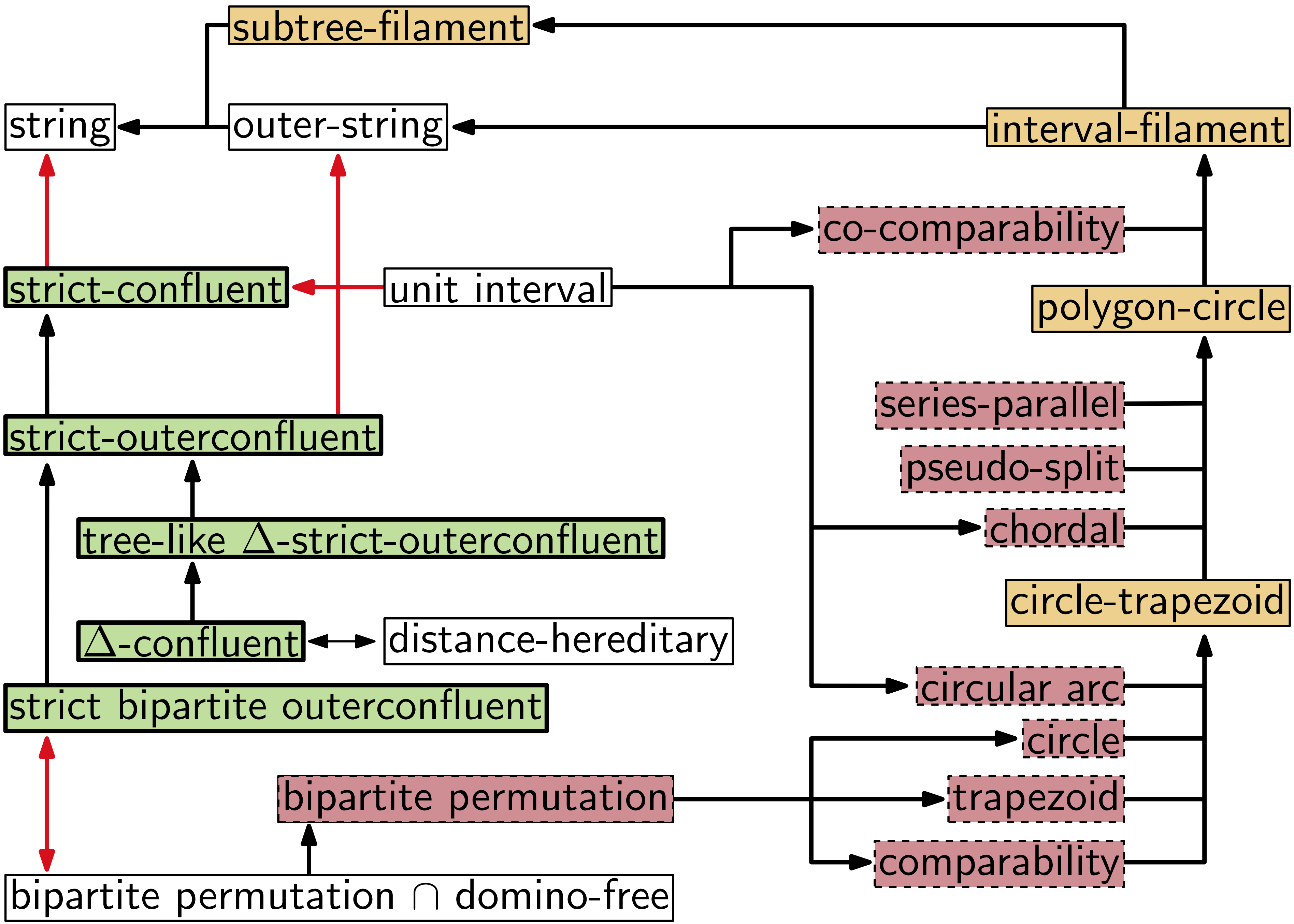
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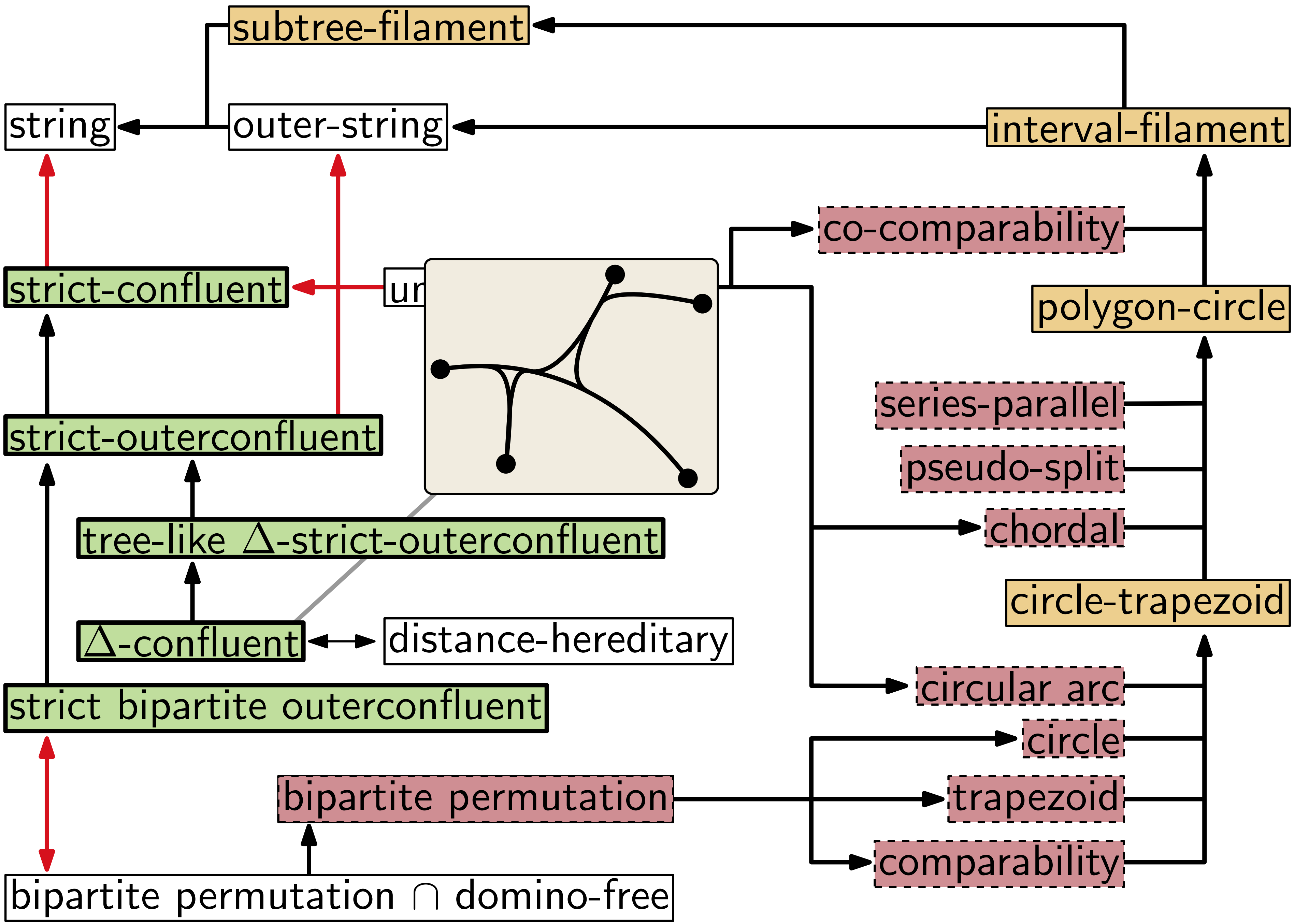
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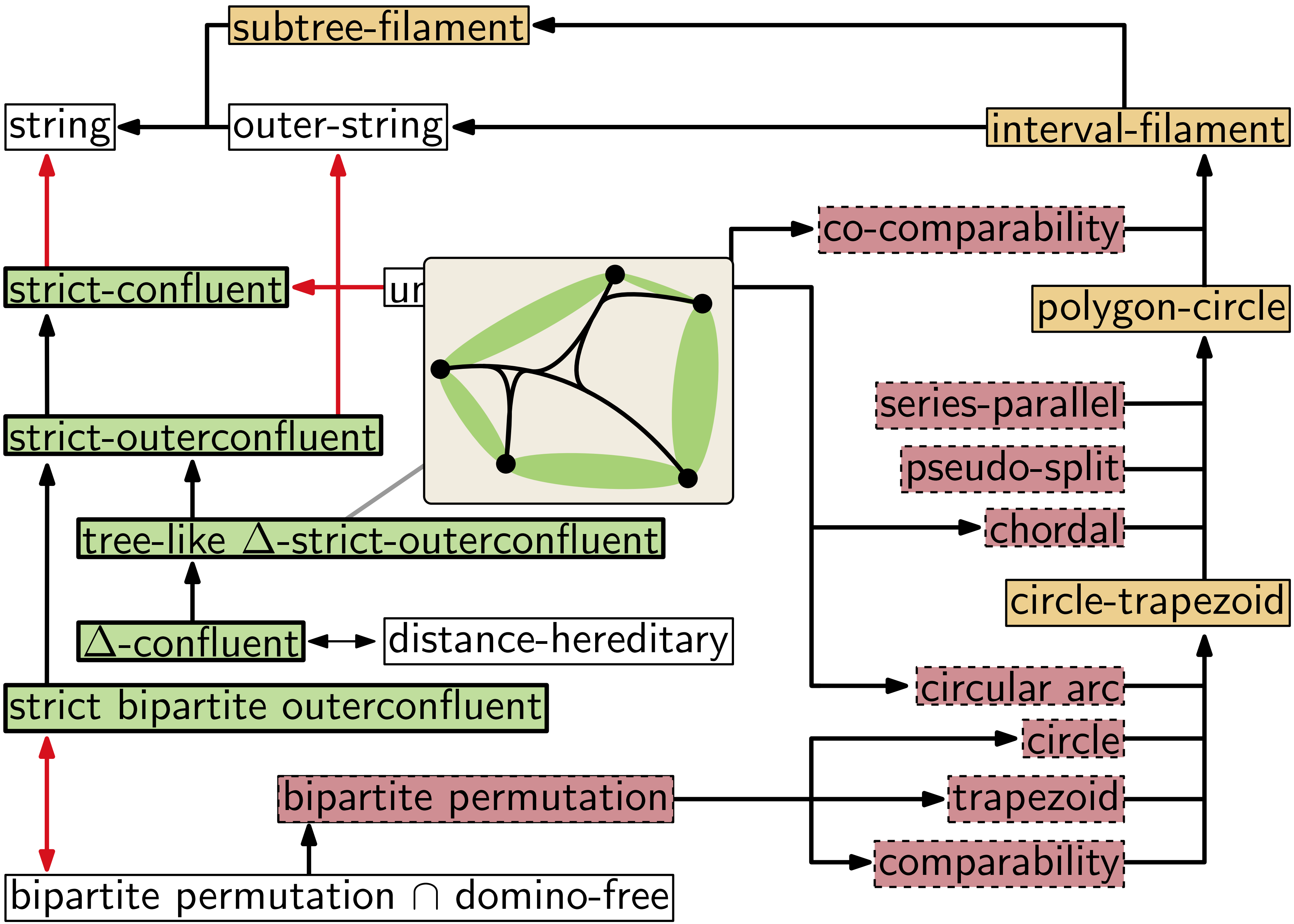

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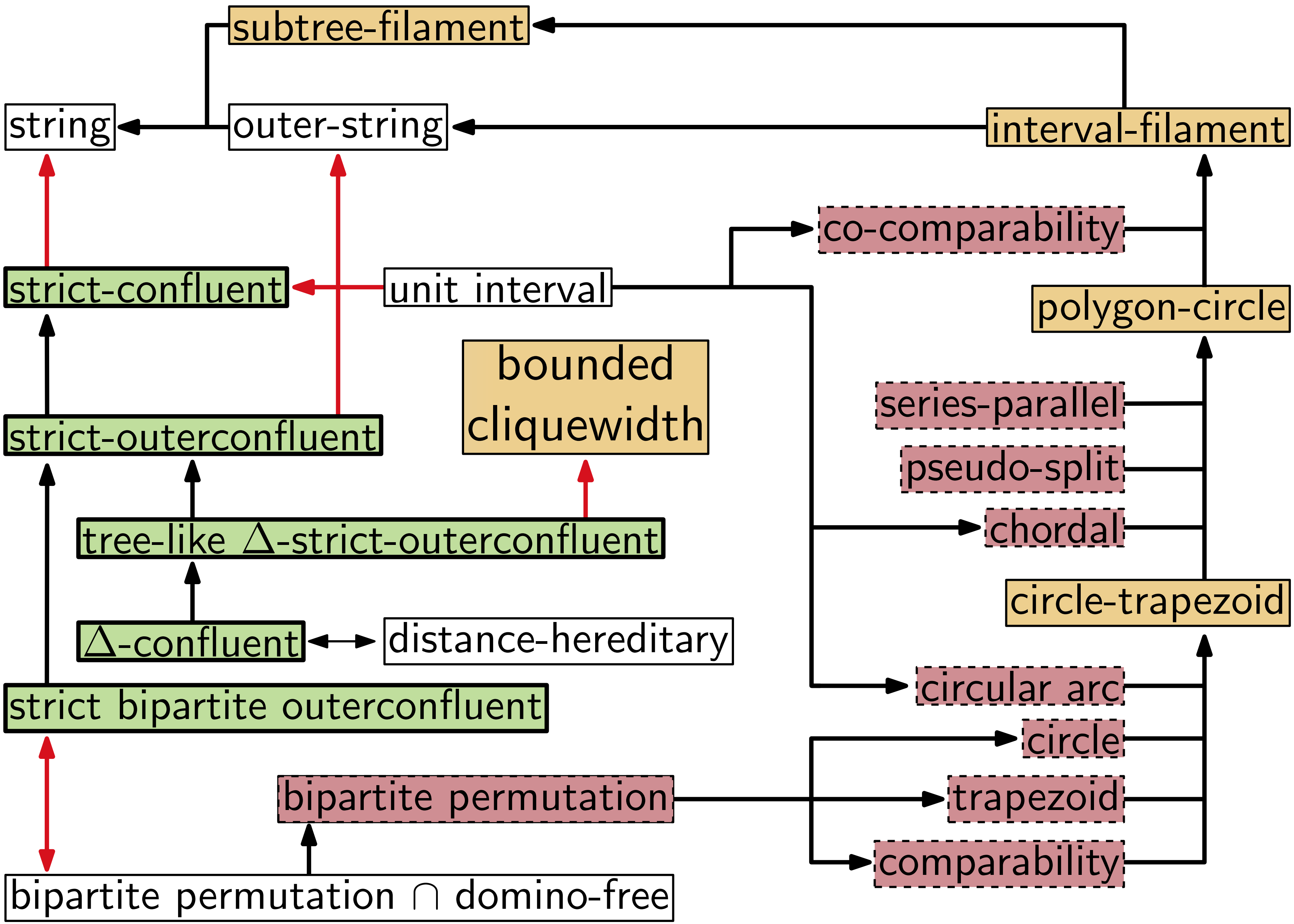
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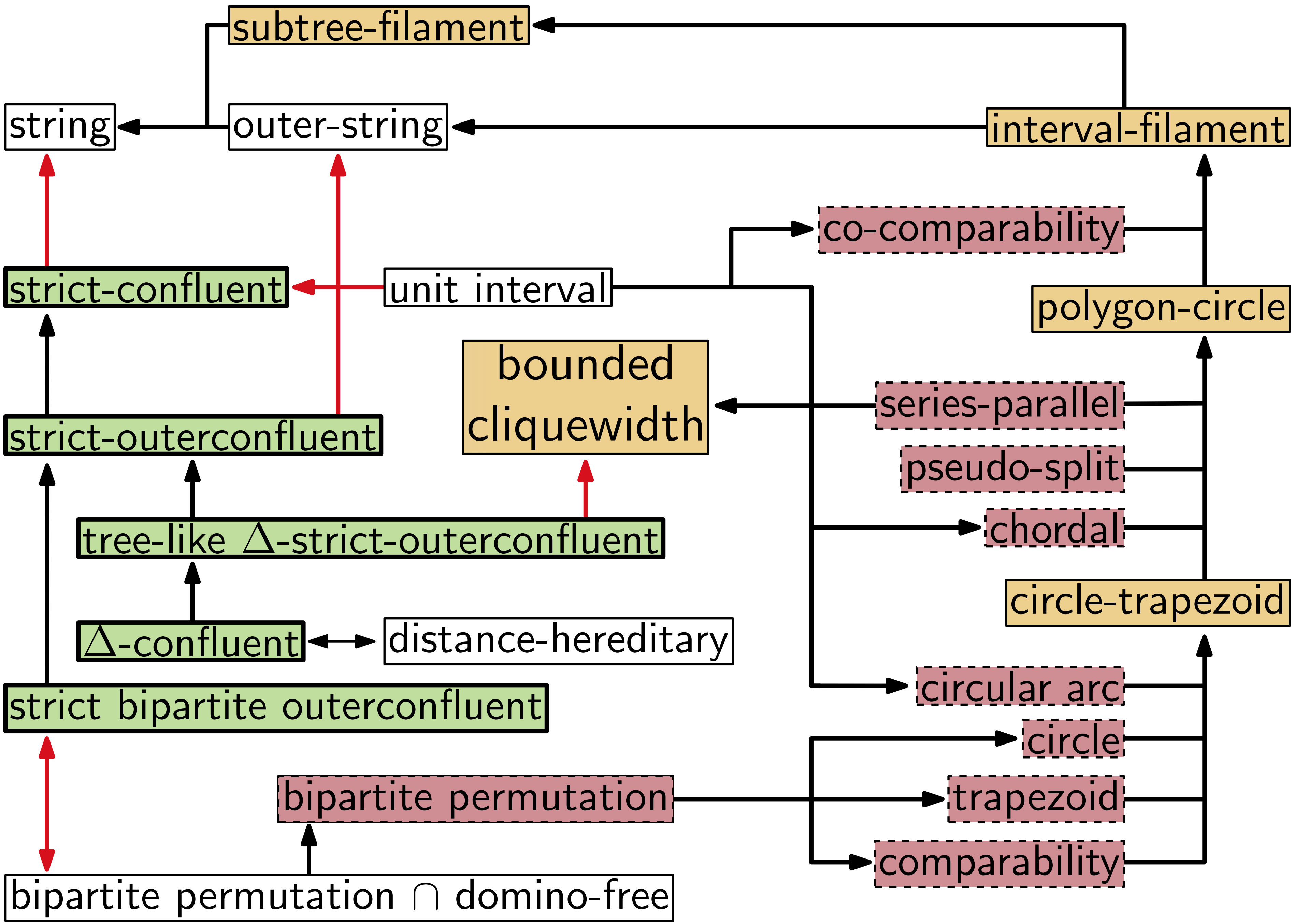
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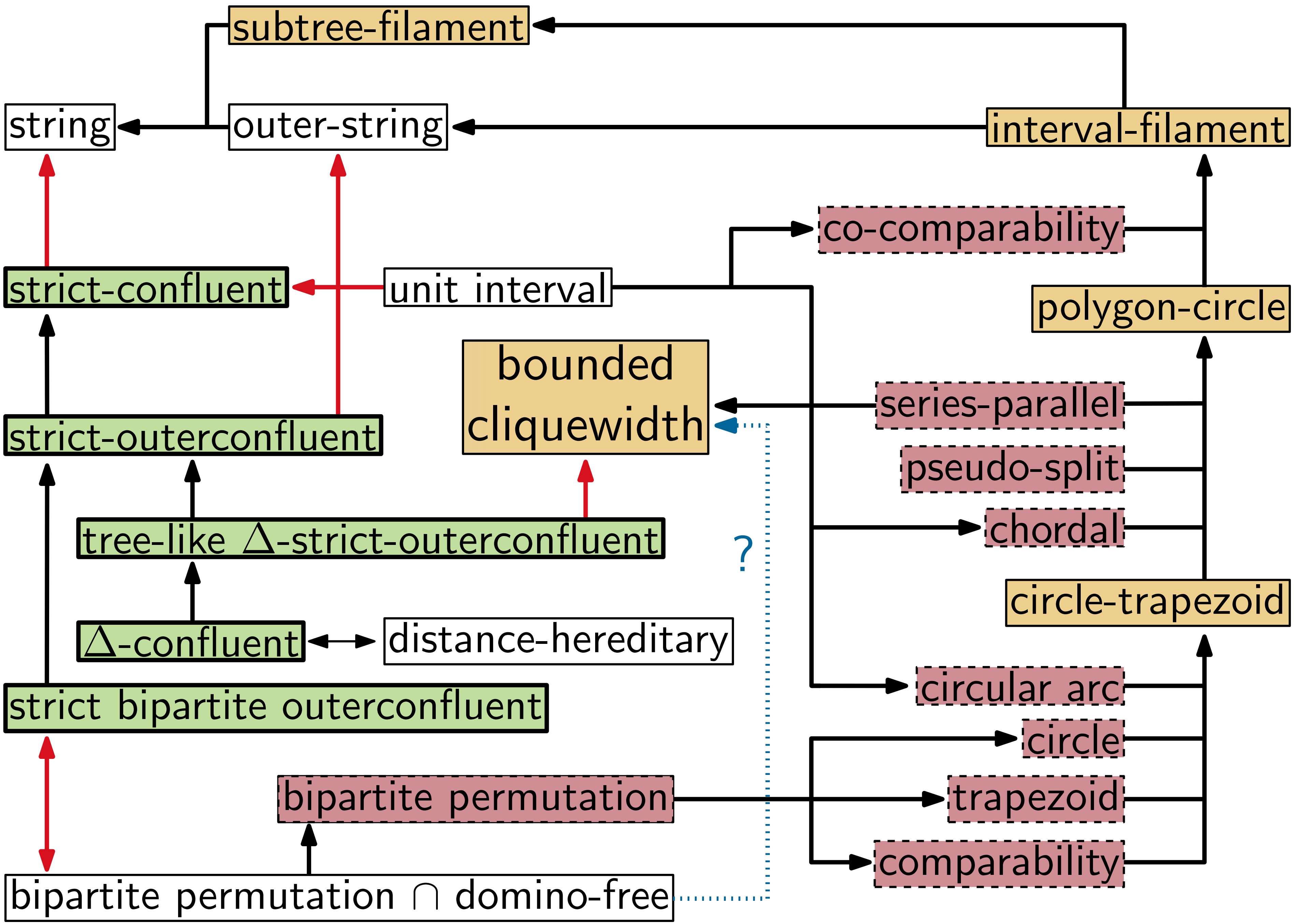

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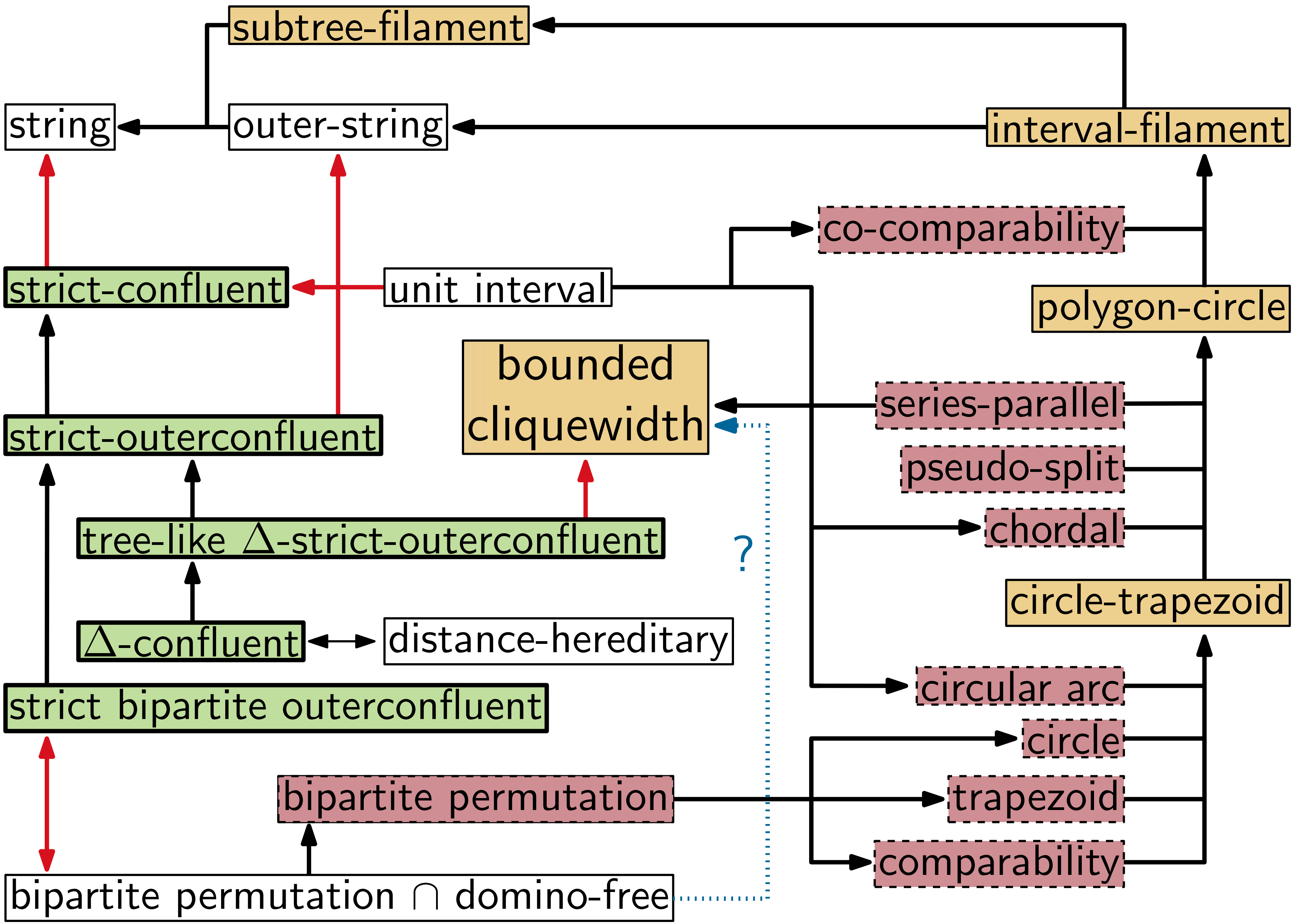
distance-hereditary
strict bipartite outerconfluent
circular arc
circle
bipartite permutation
trapezoid
comparability
bipartite permutation ∩ domino-free

Our Results

Cop number: c [Gavenčiak et al. 18]



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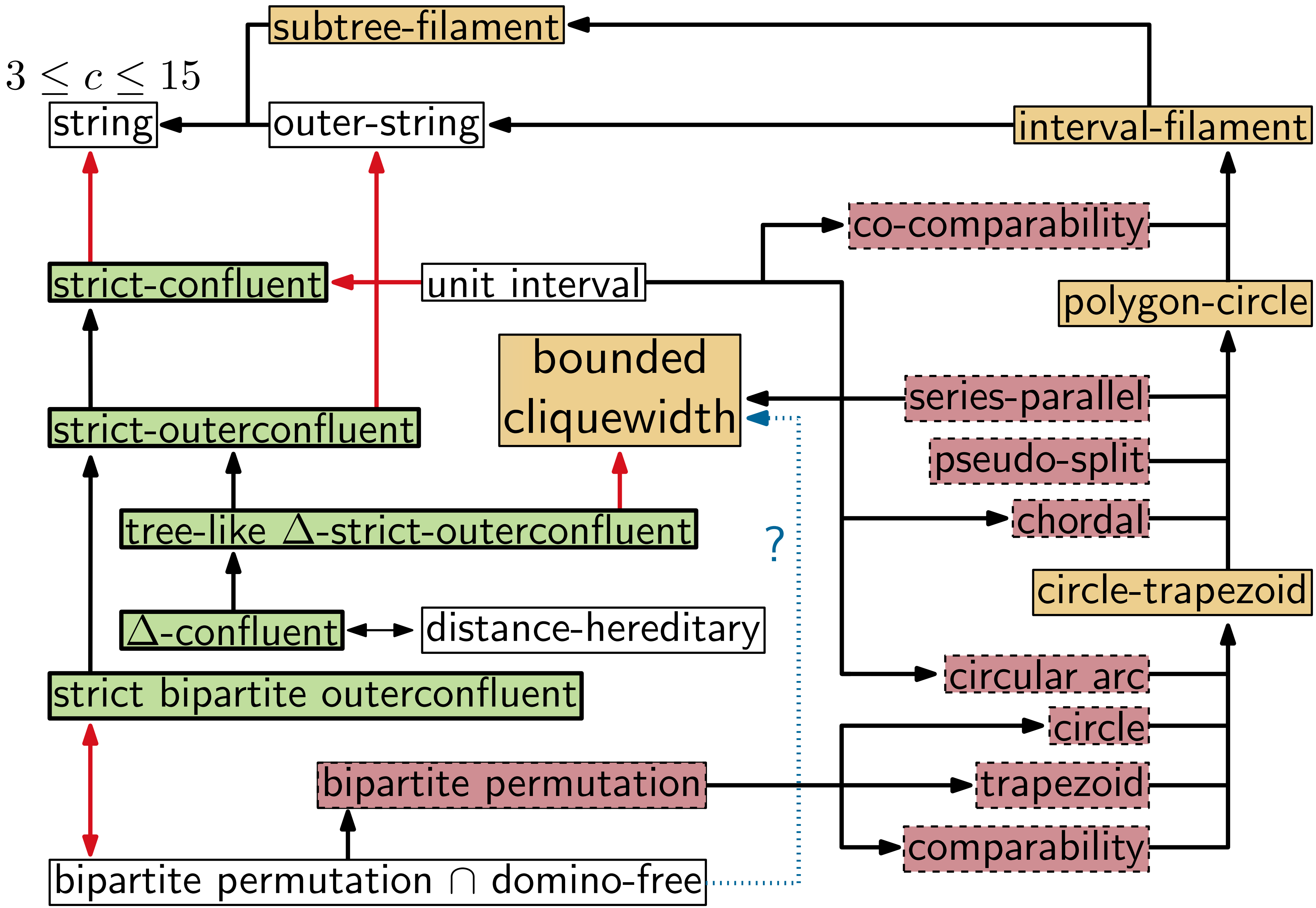

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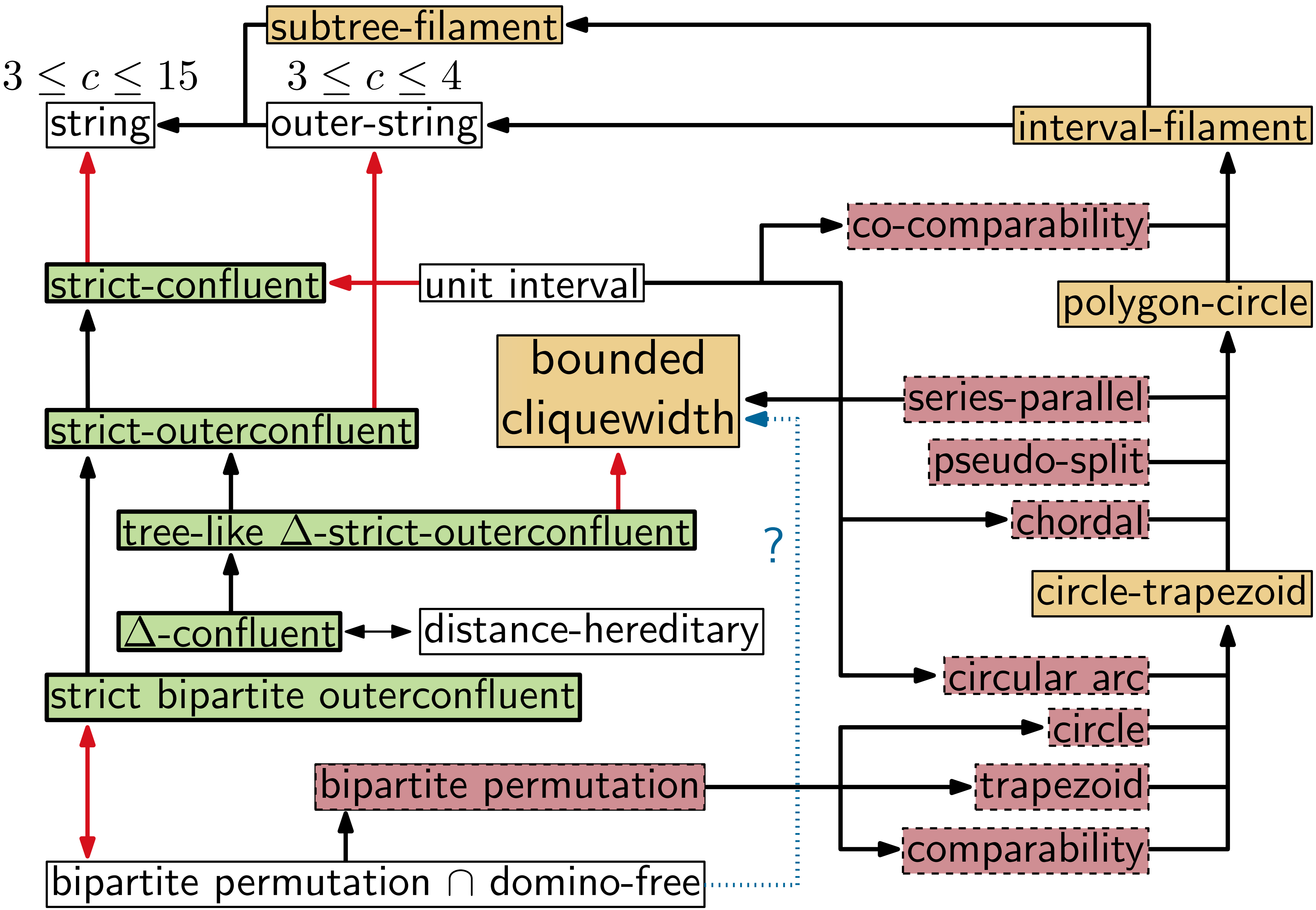

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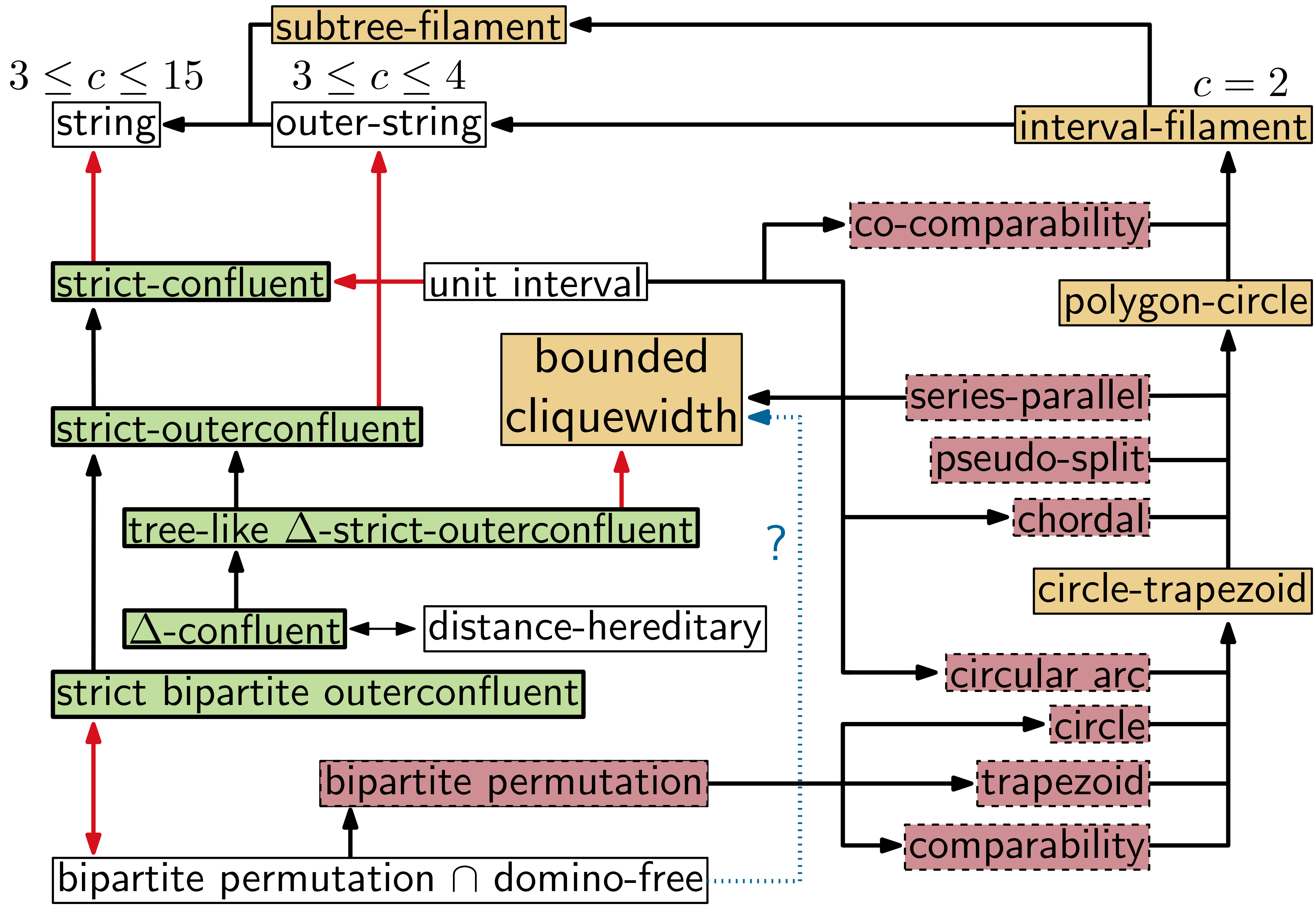

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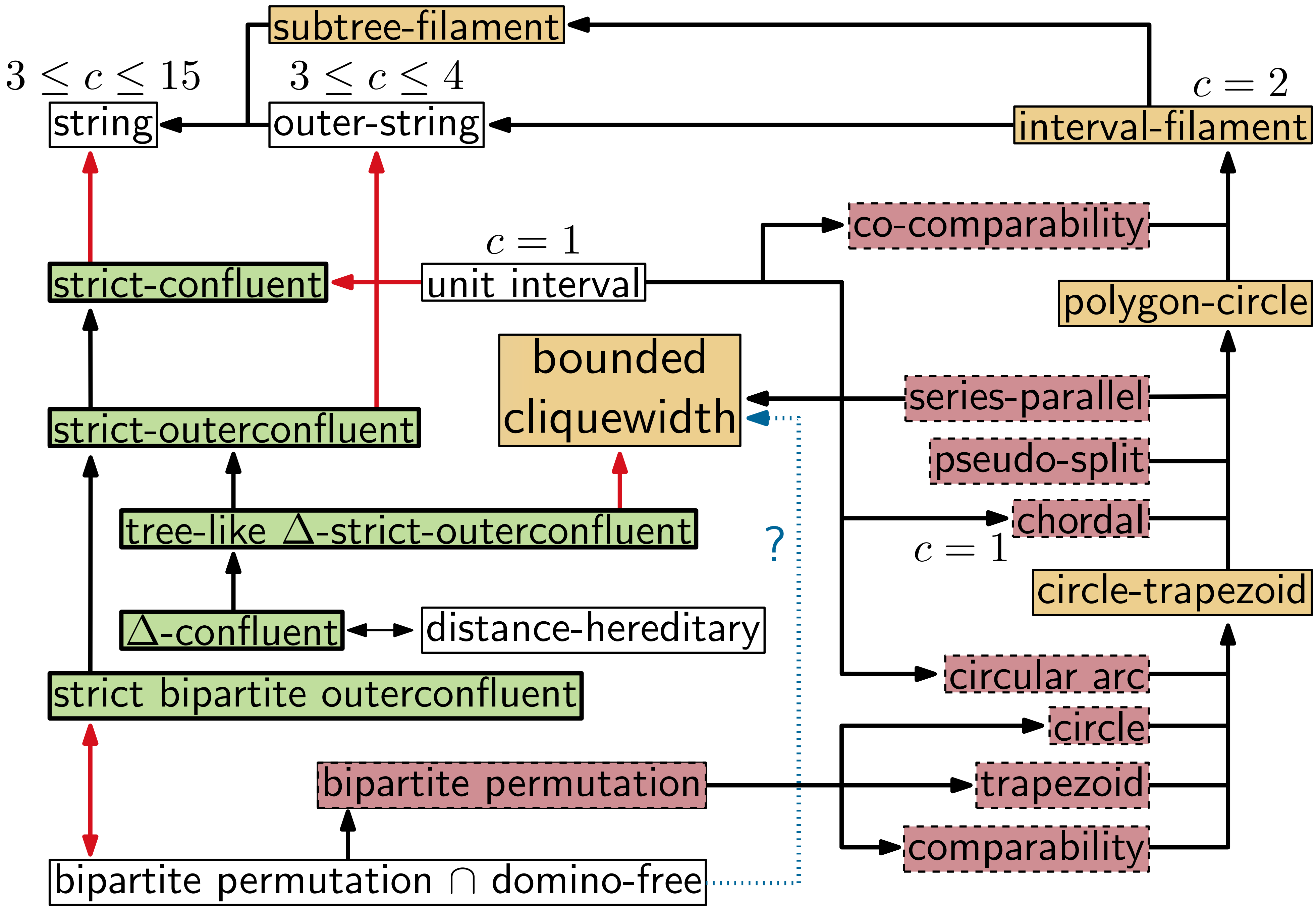

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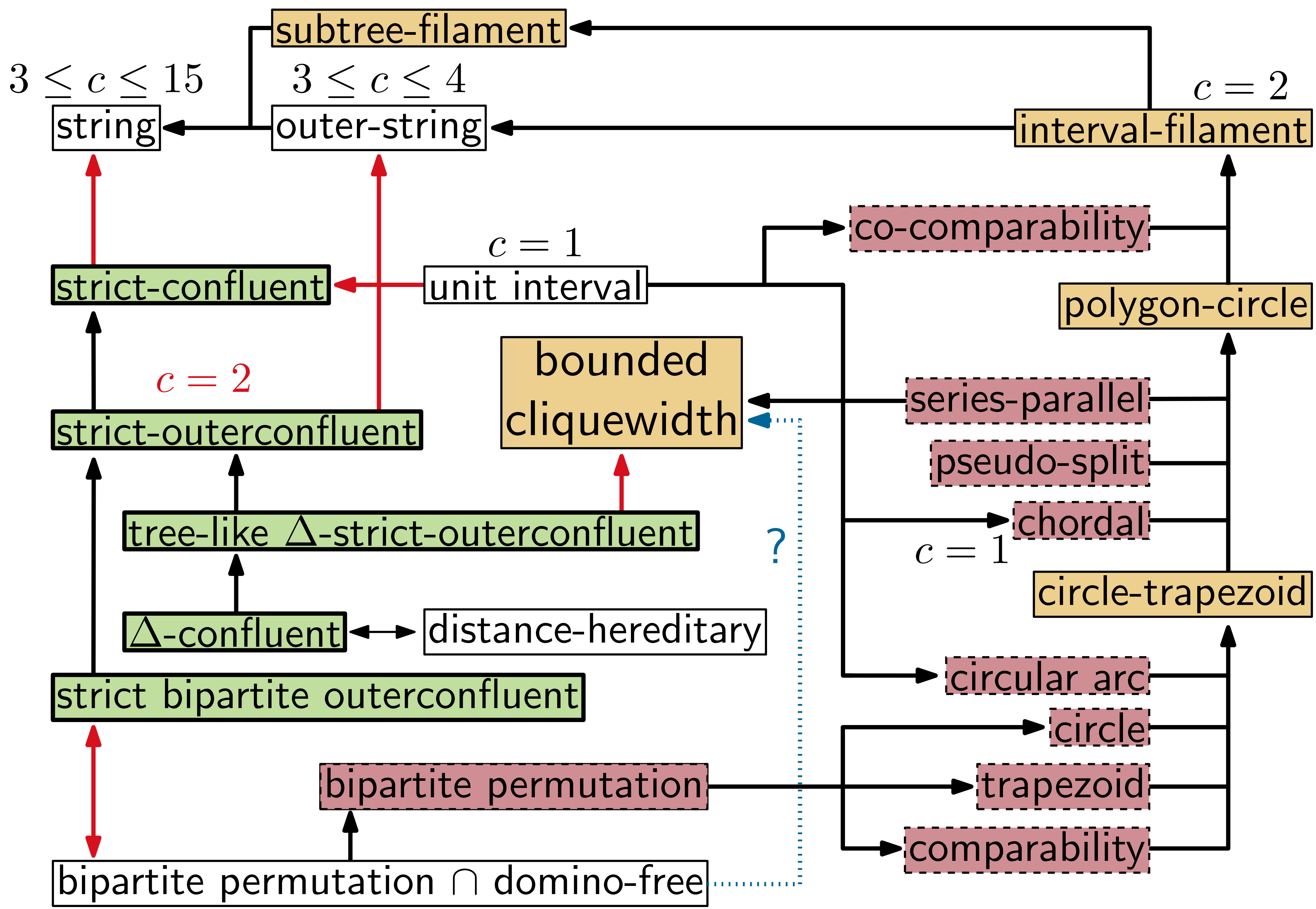

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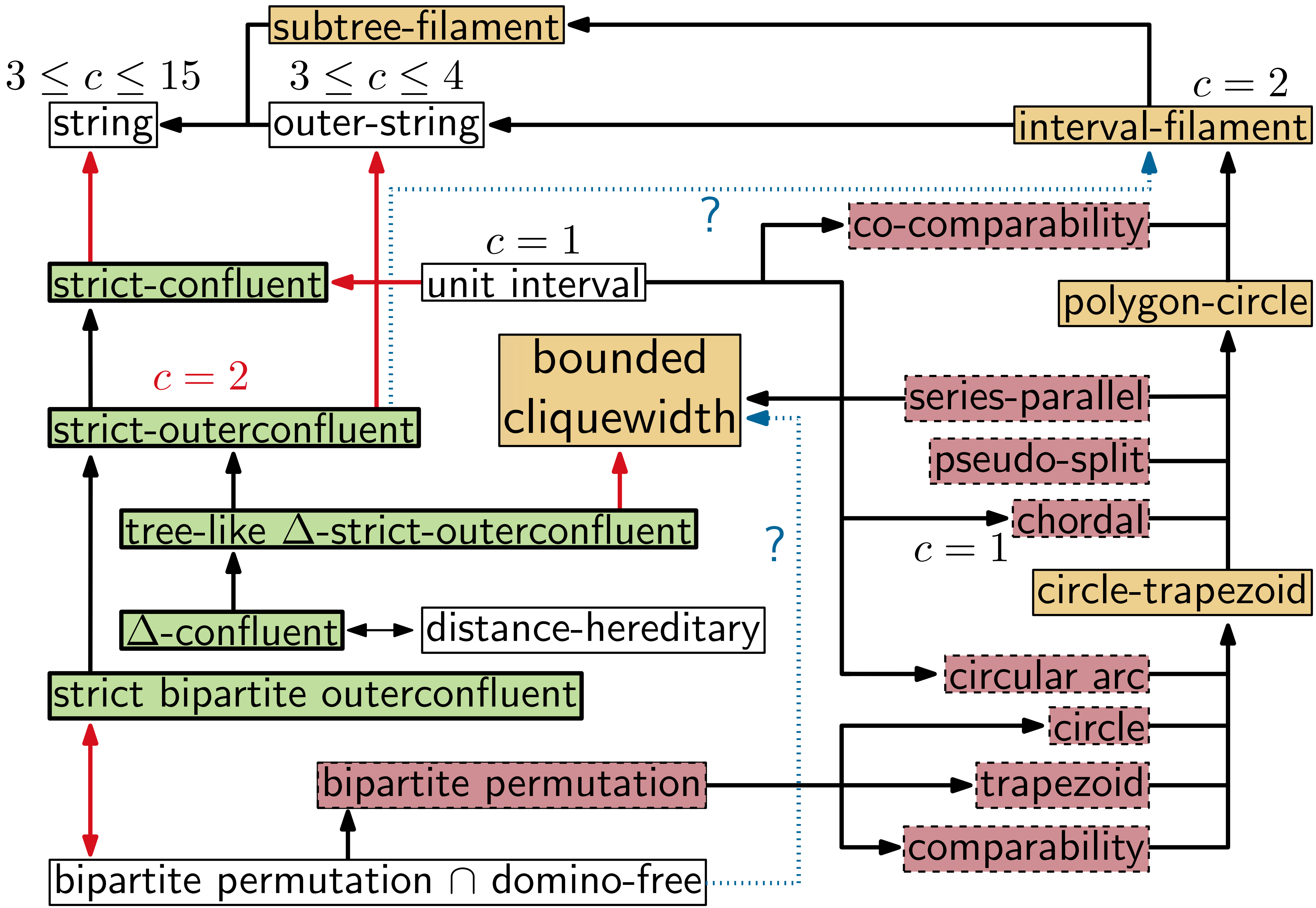

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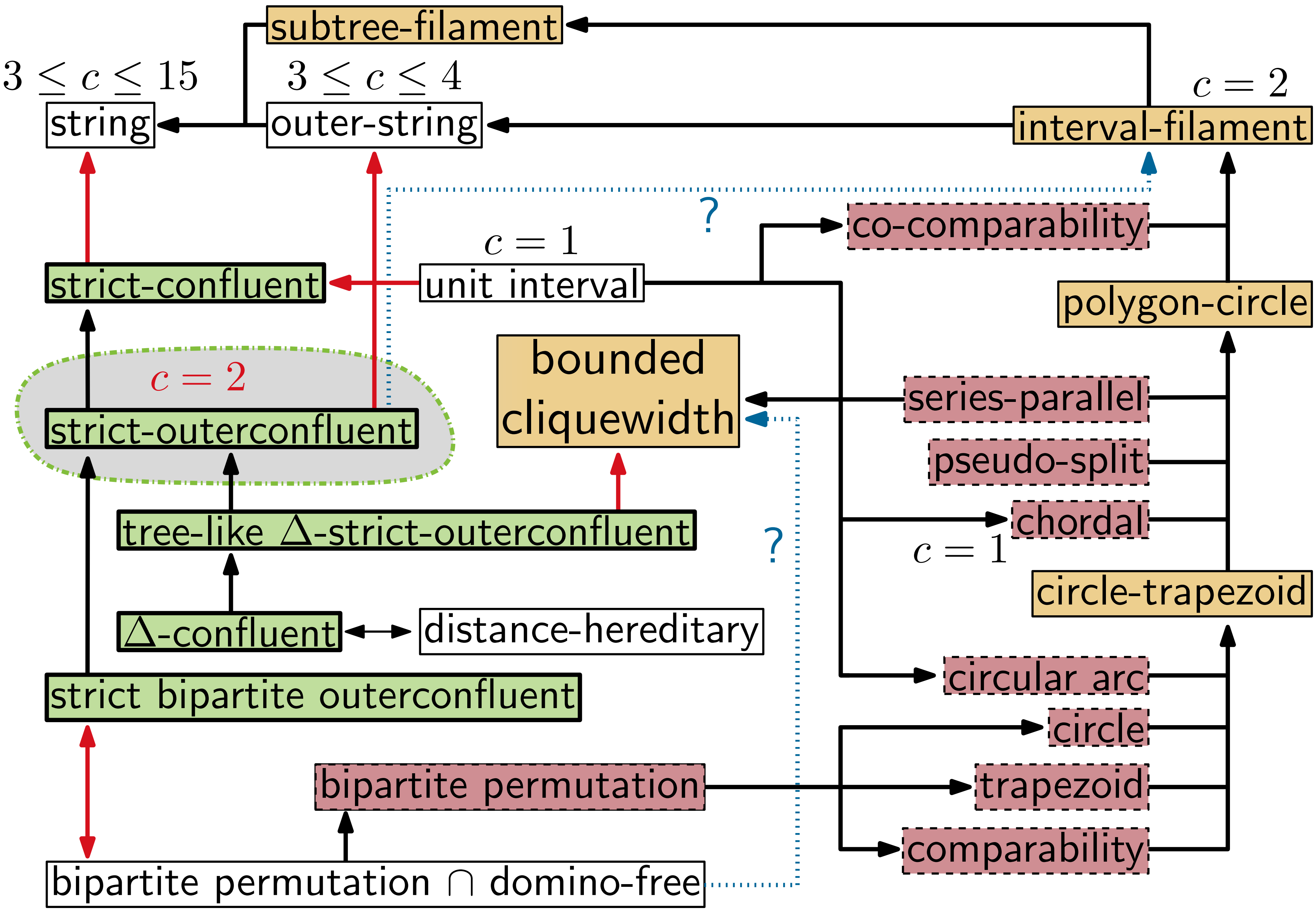

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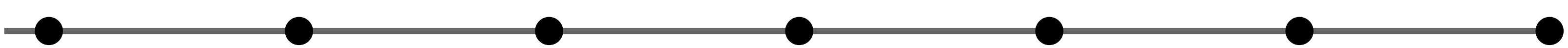

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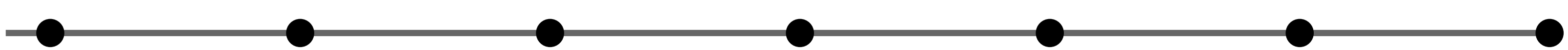

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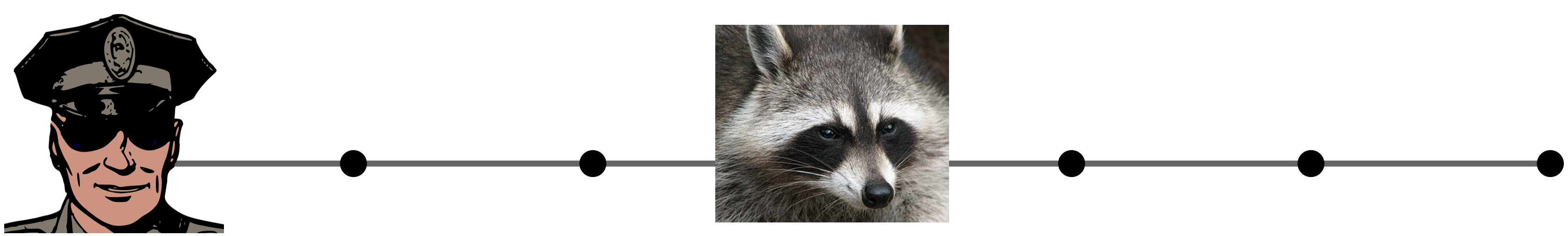

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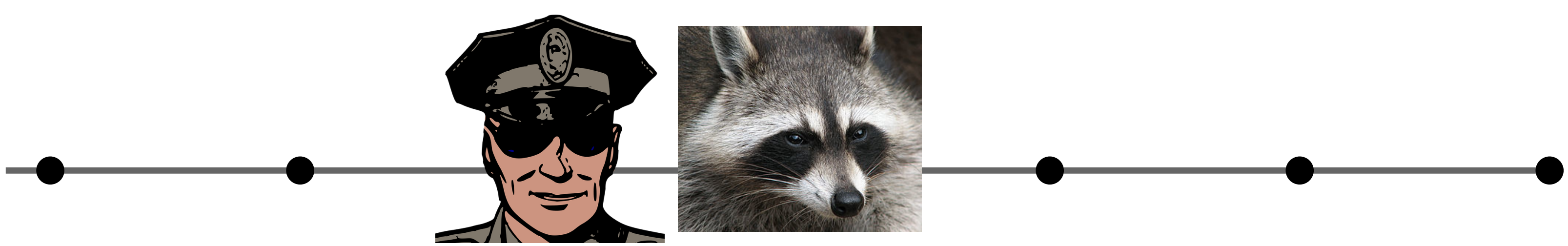

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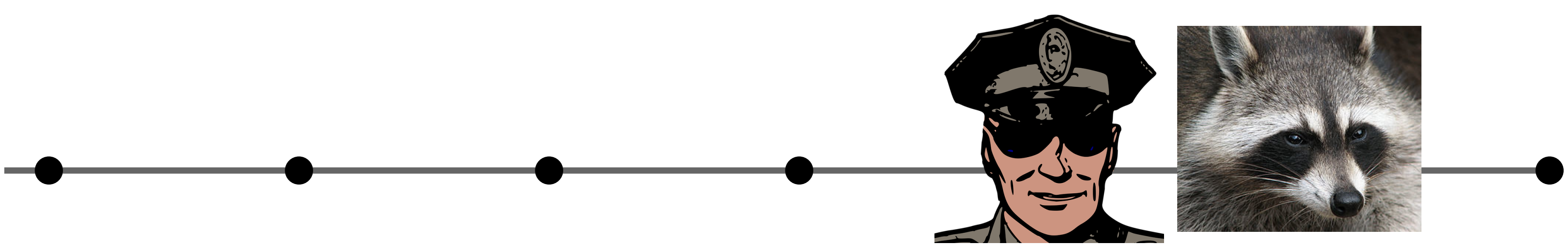

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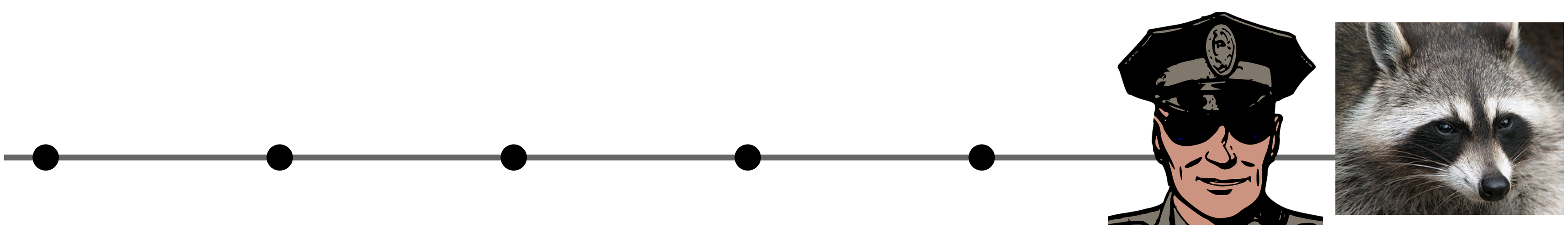

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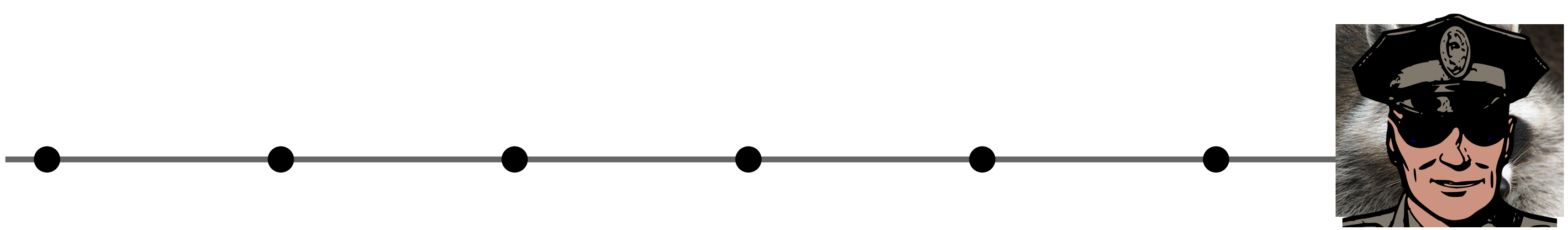

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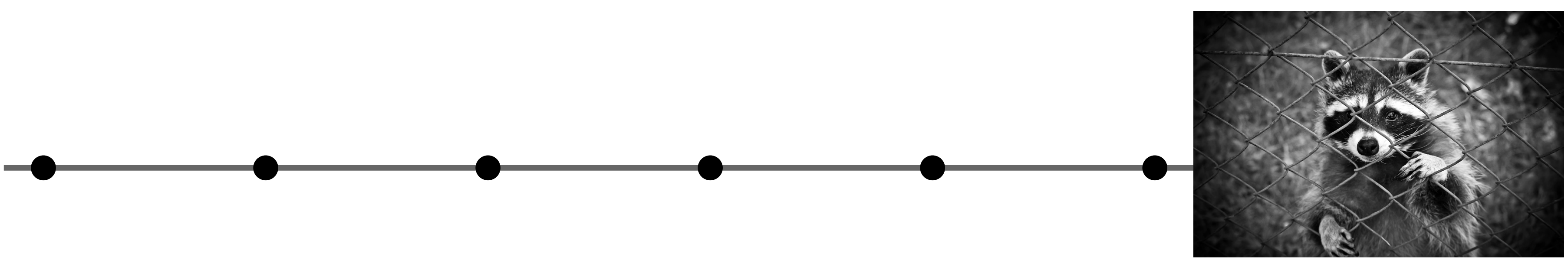

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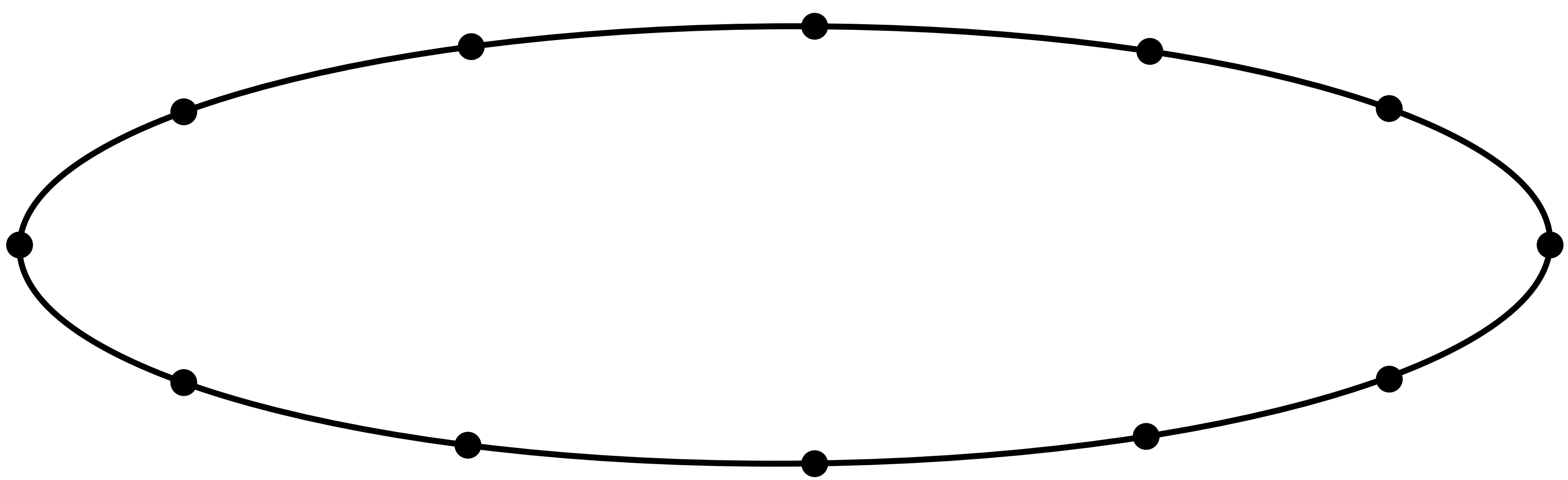

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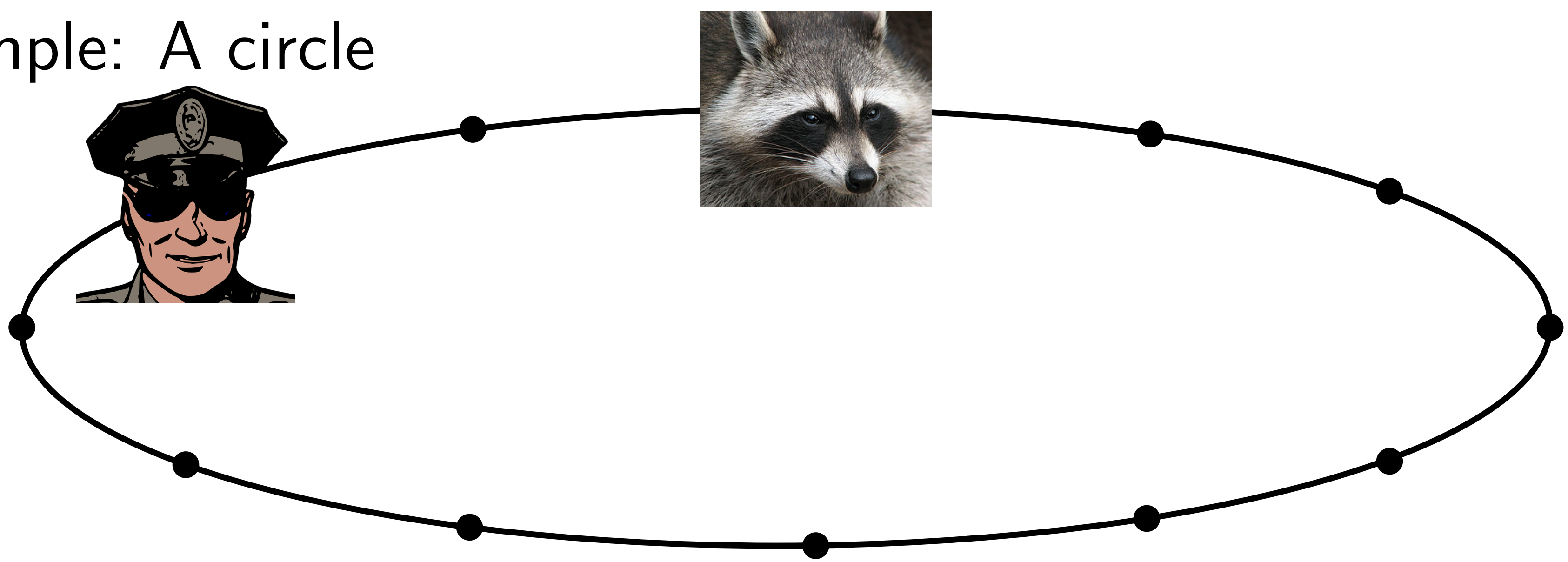

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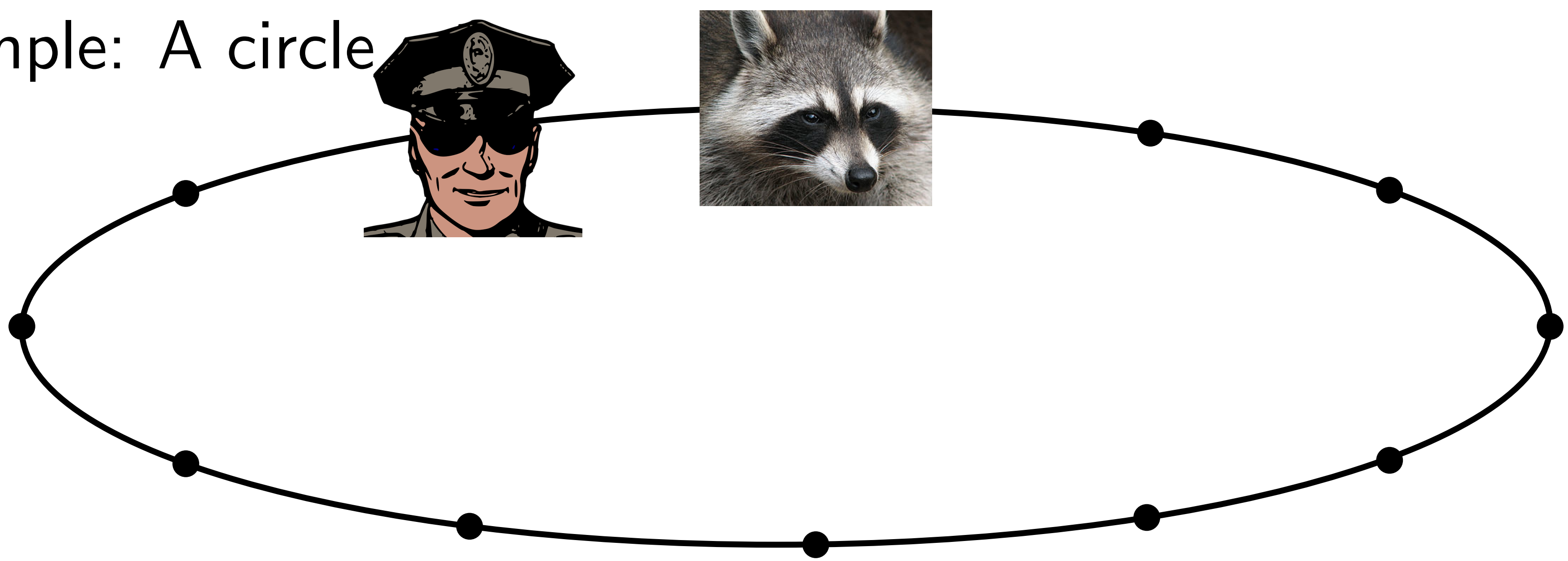

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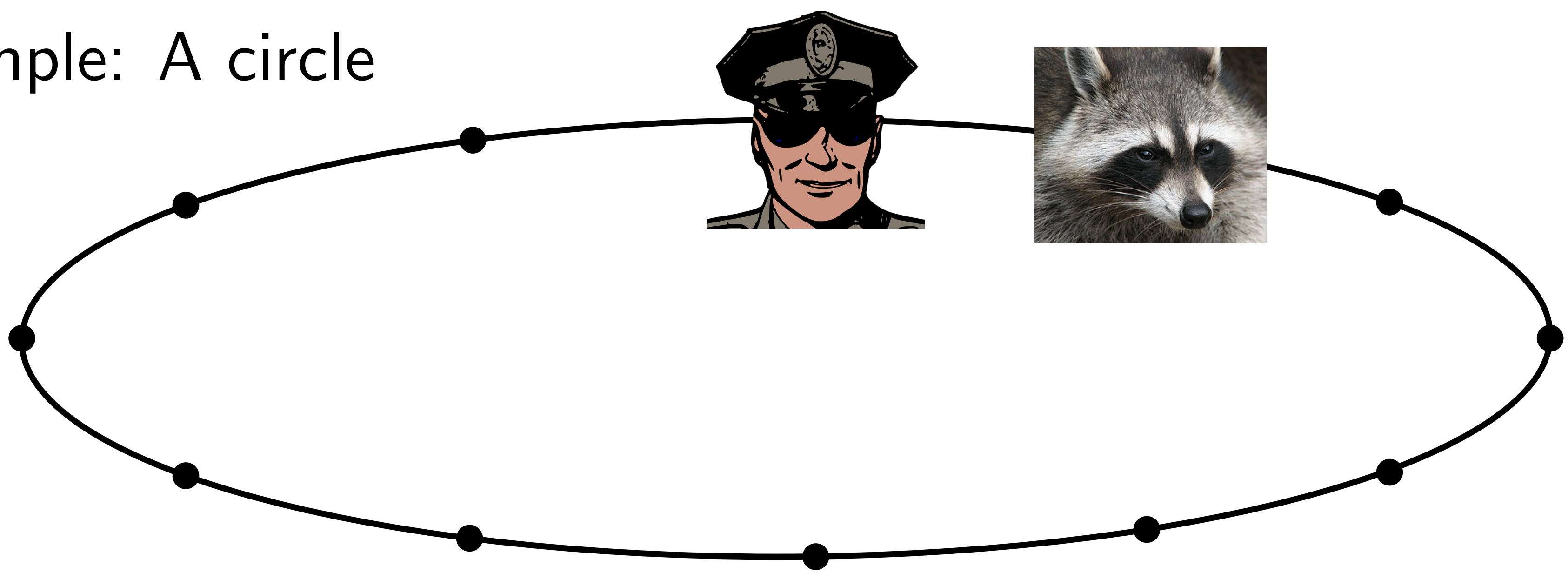

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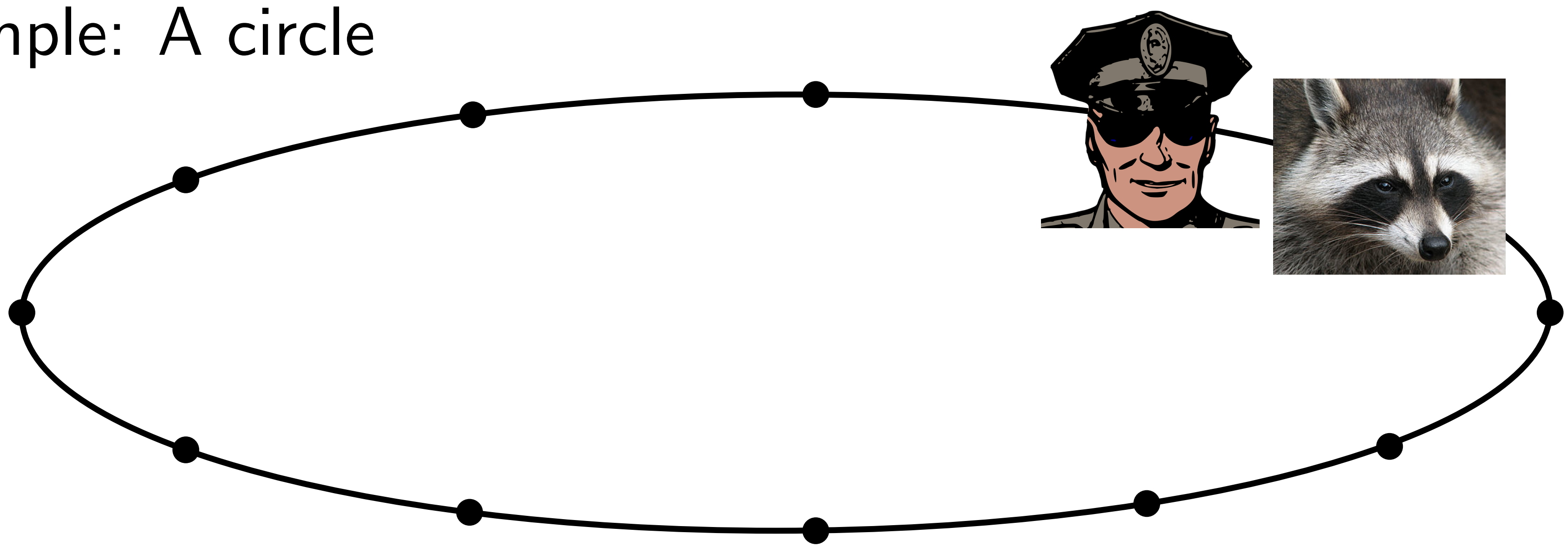

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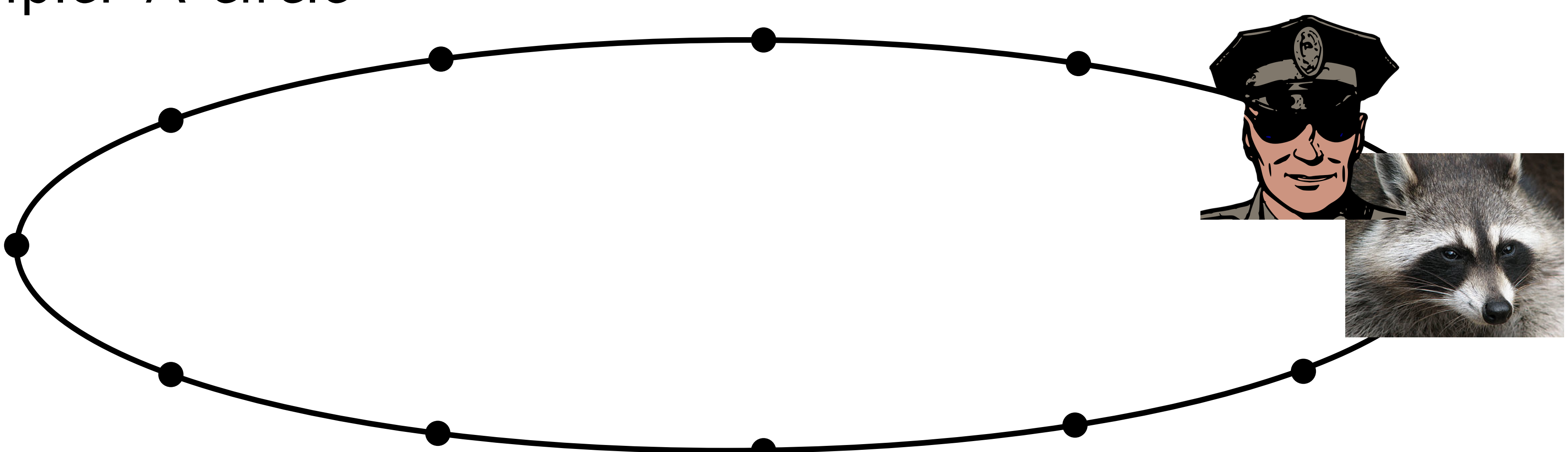

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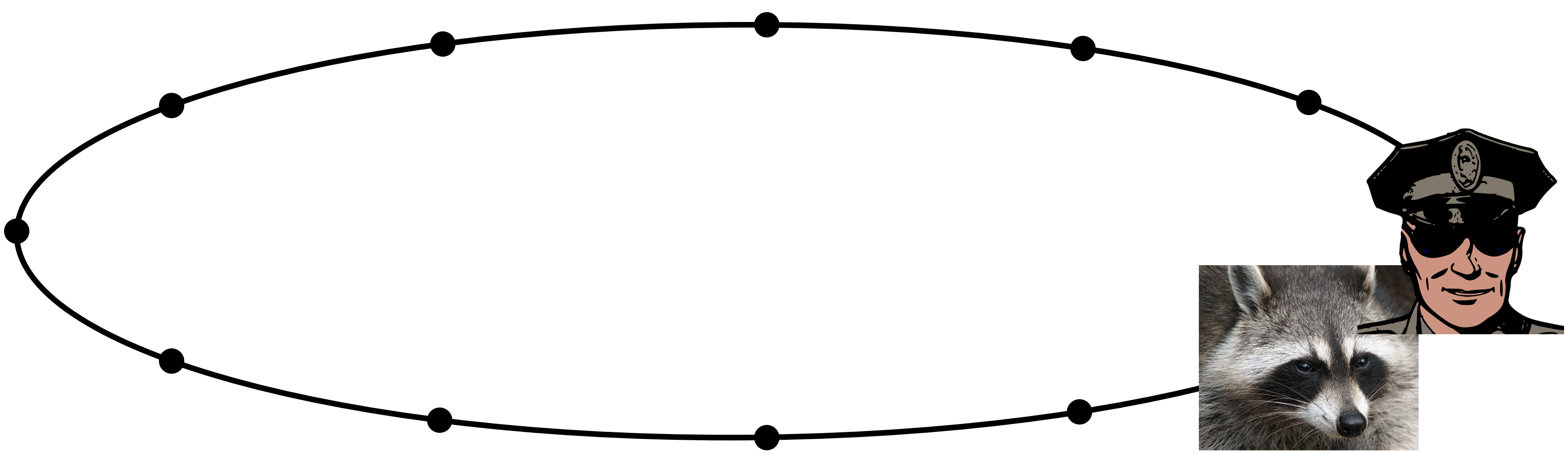

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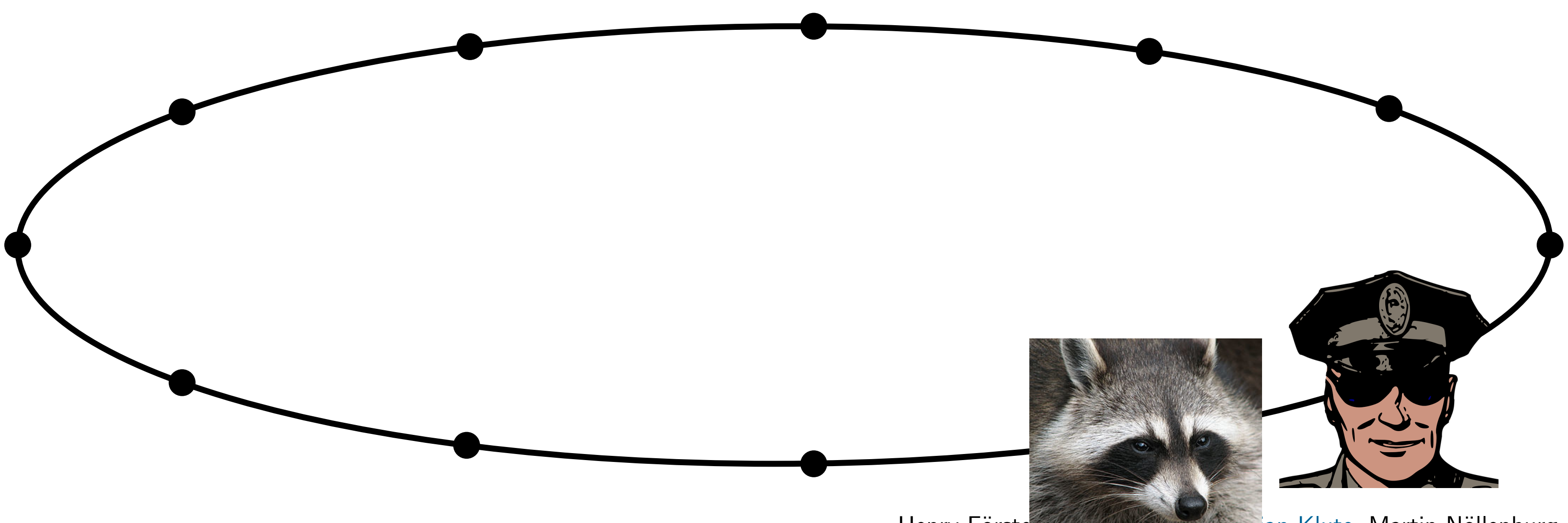

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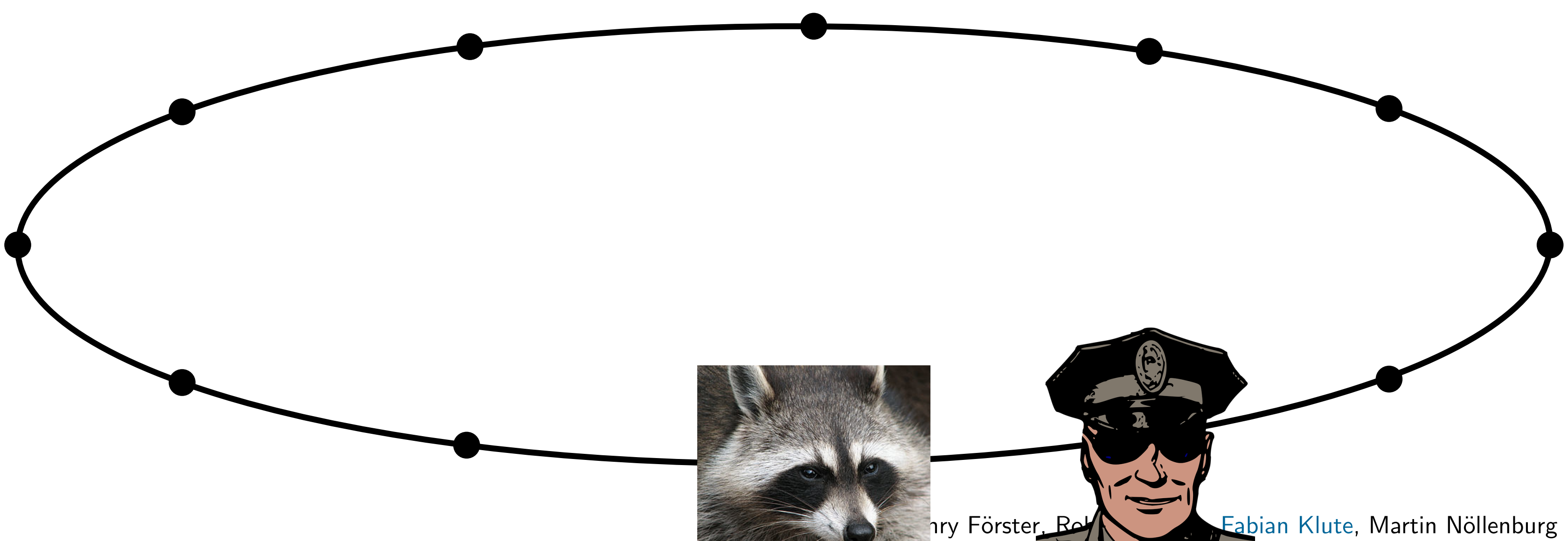

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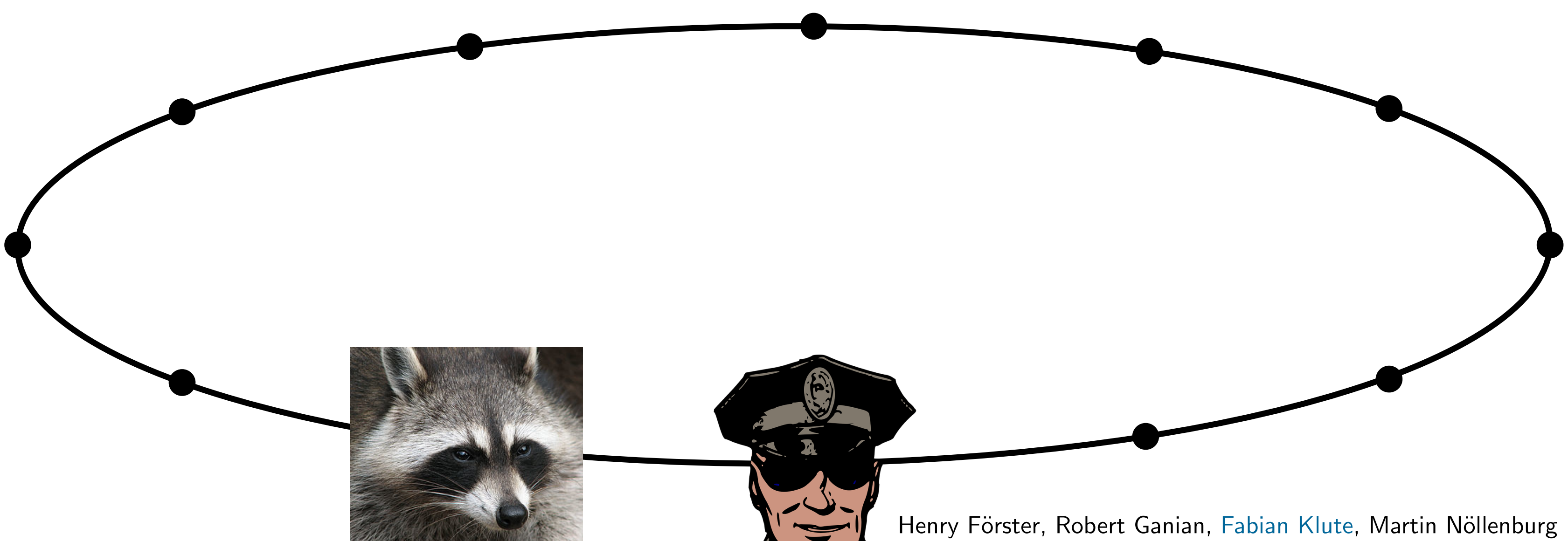

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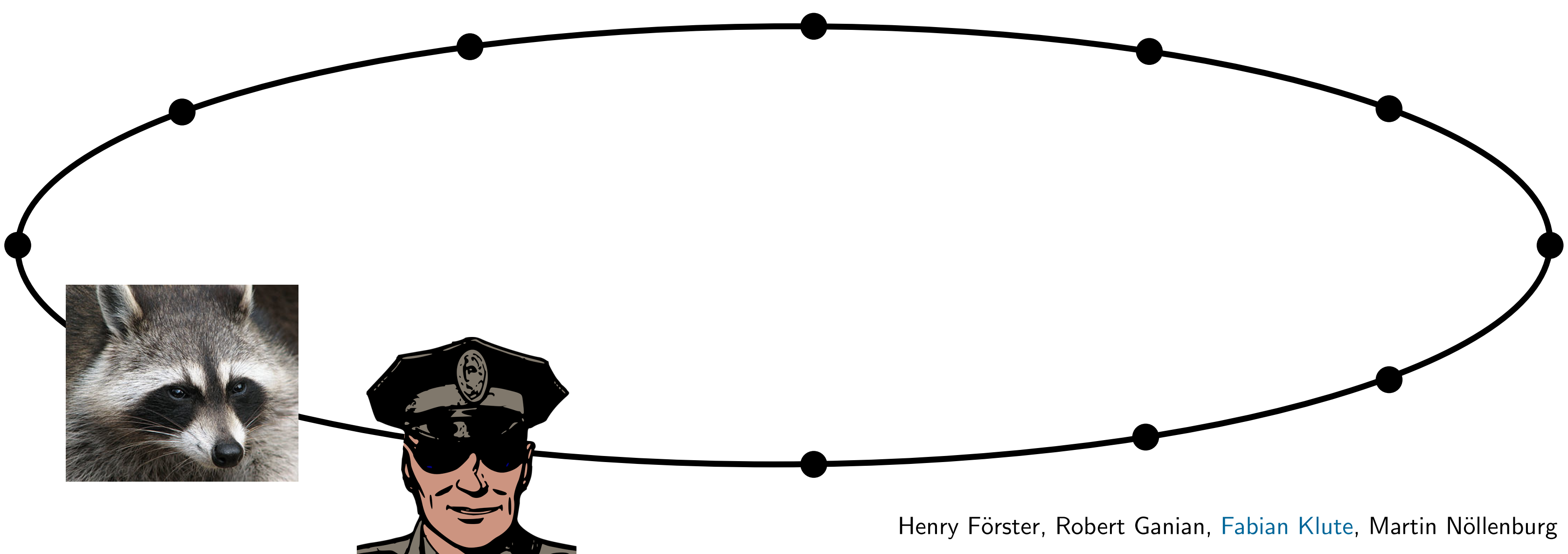

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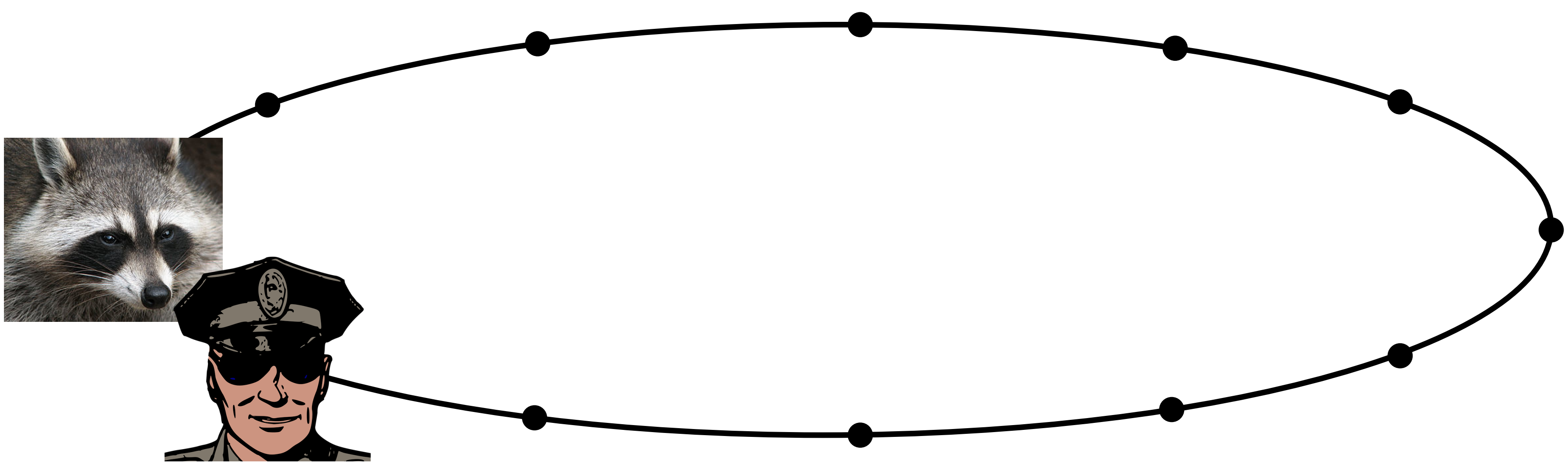

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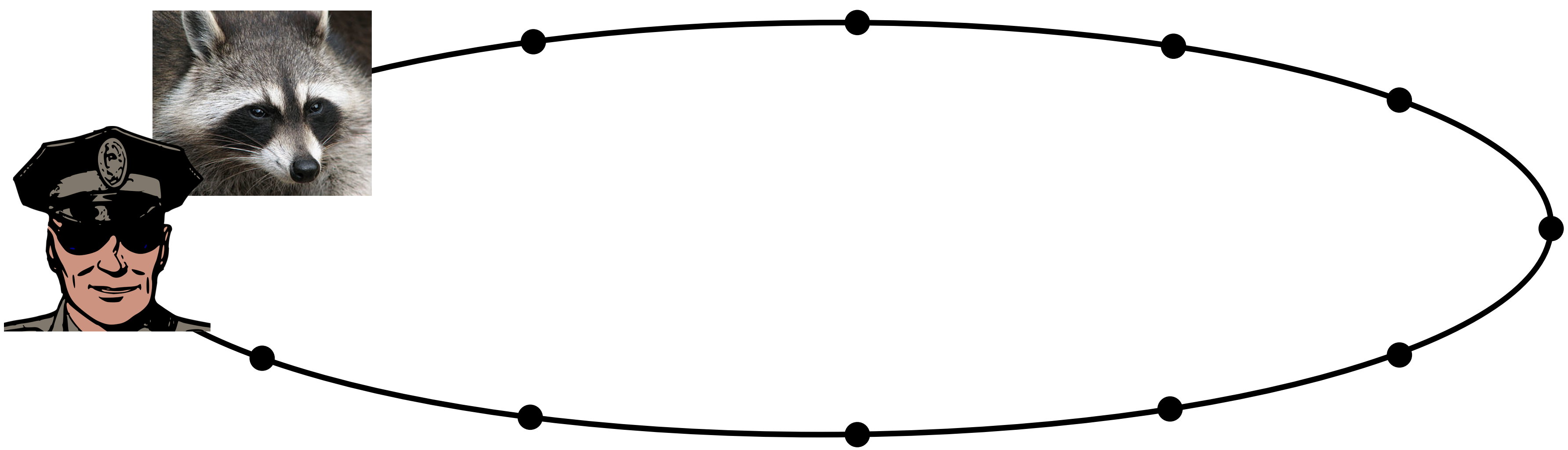

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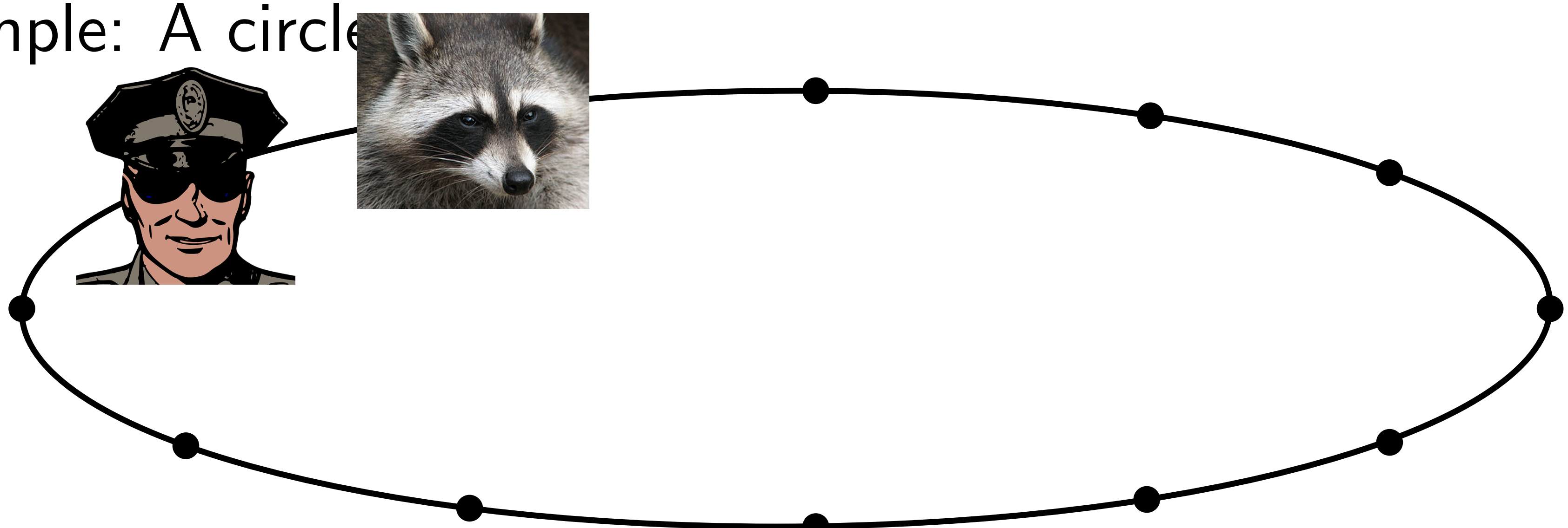

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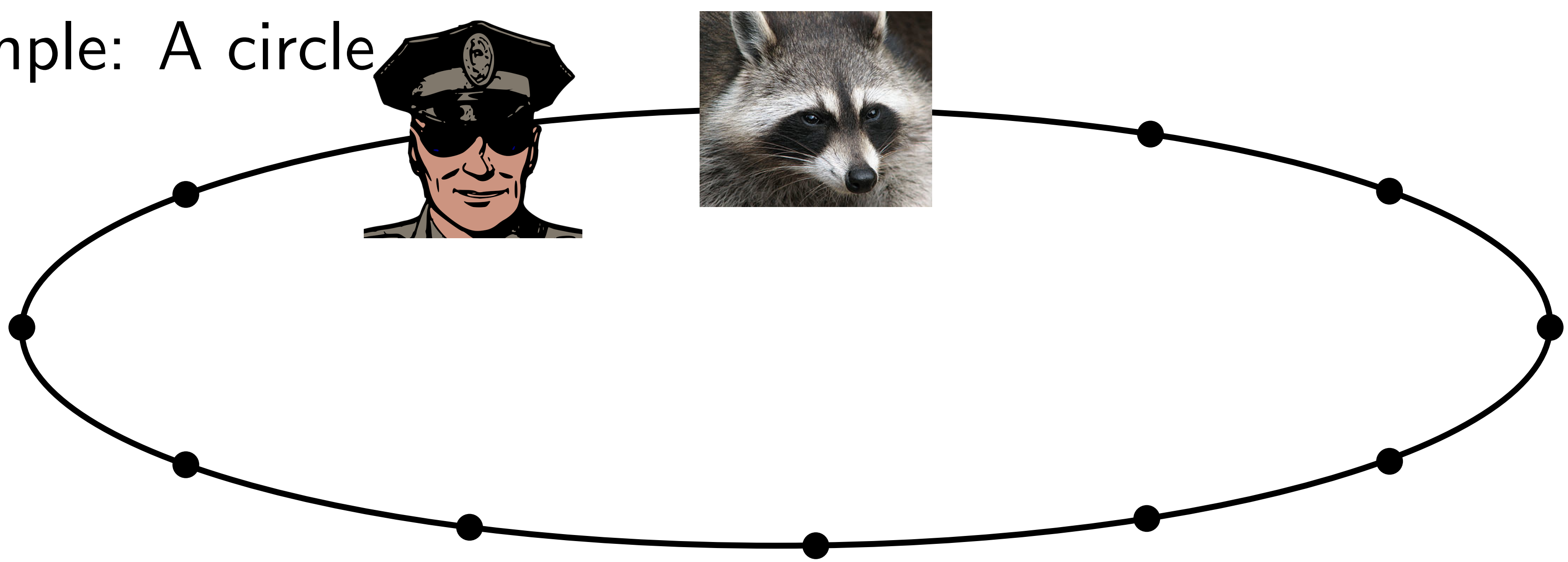

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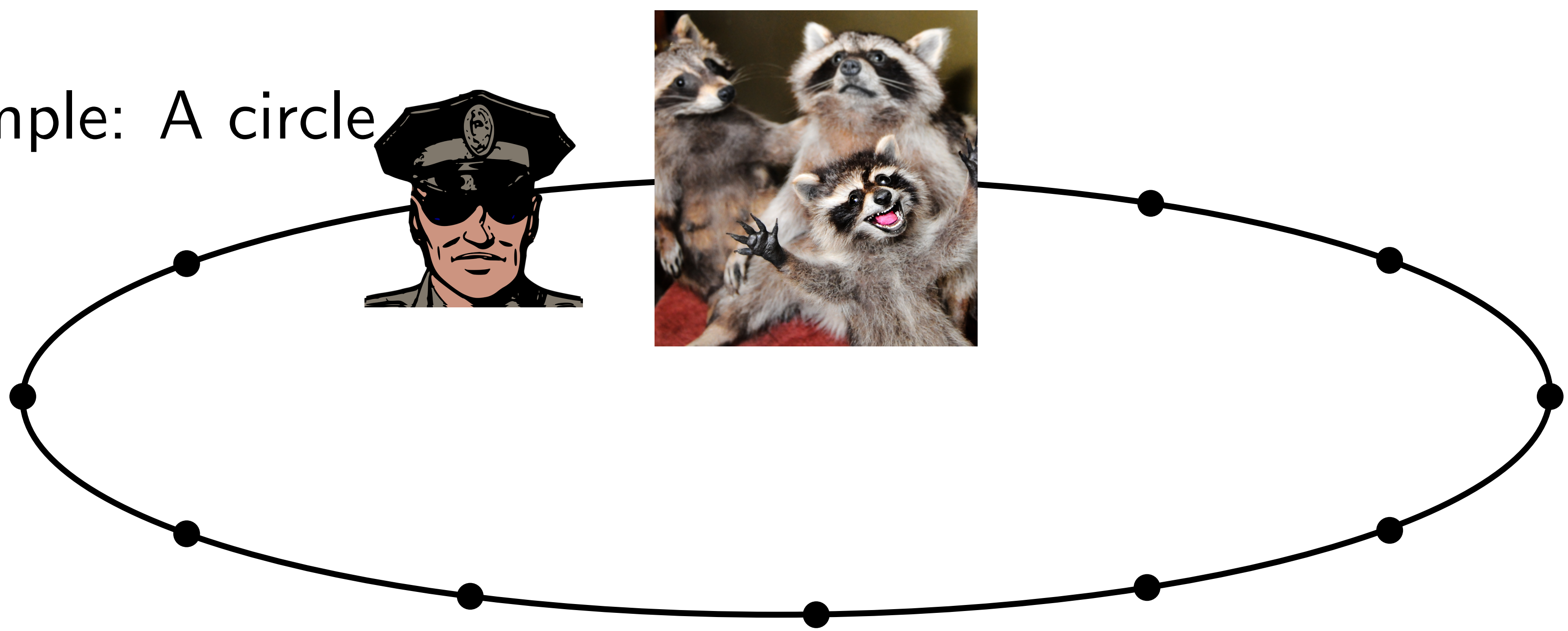

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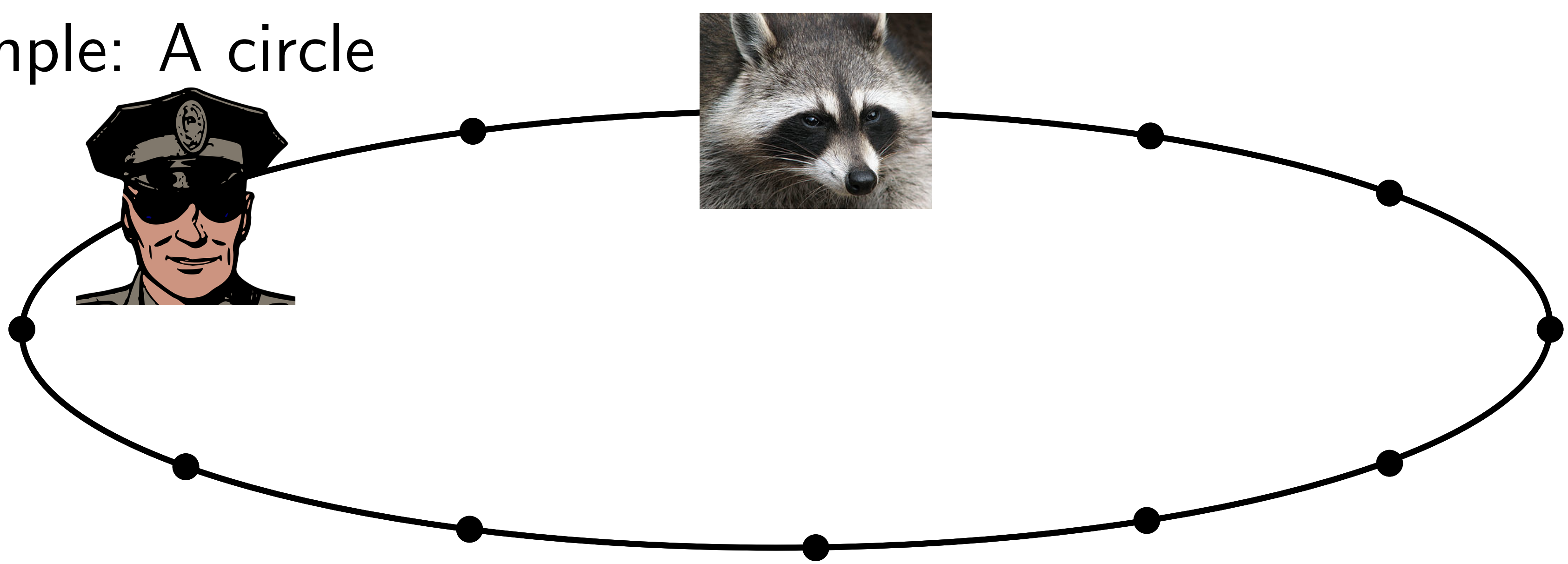

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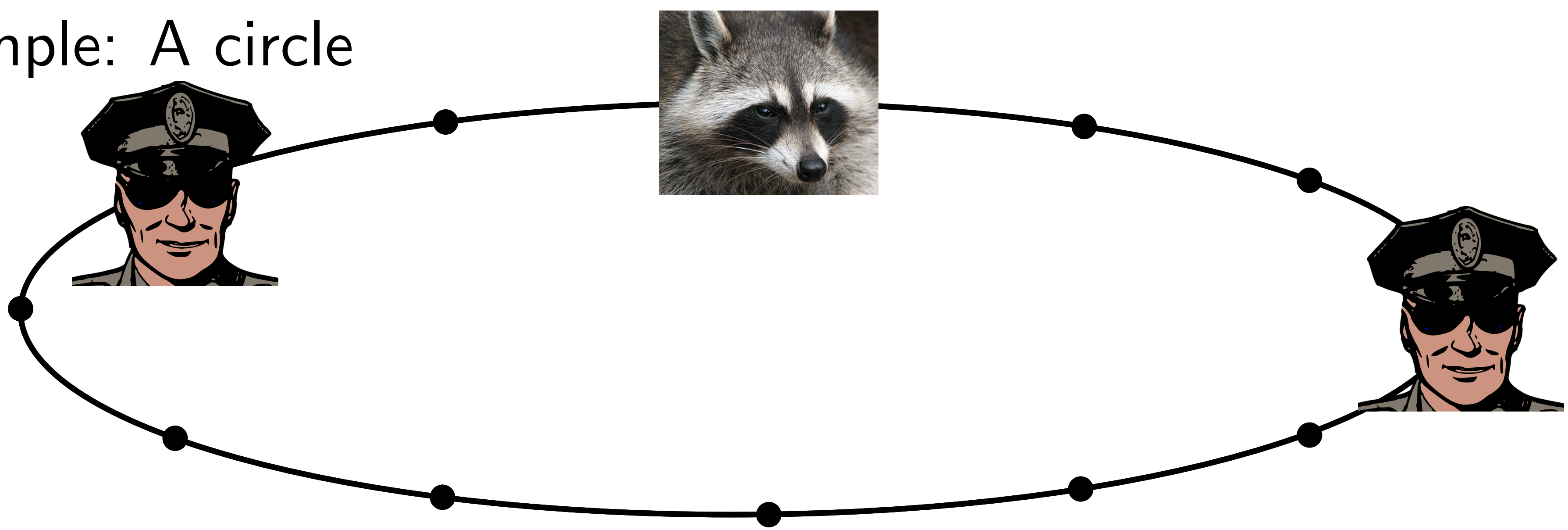

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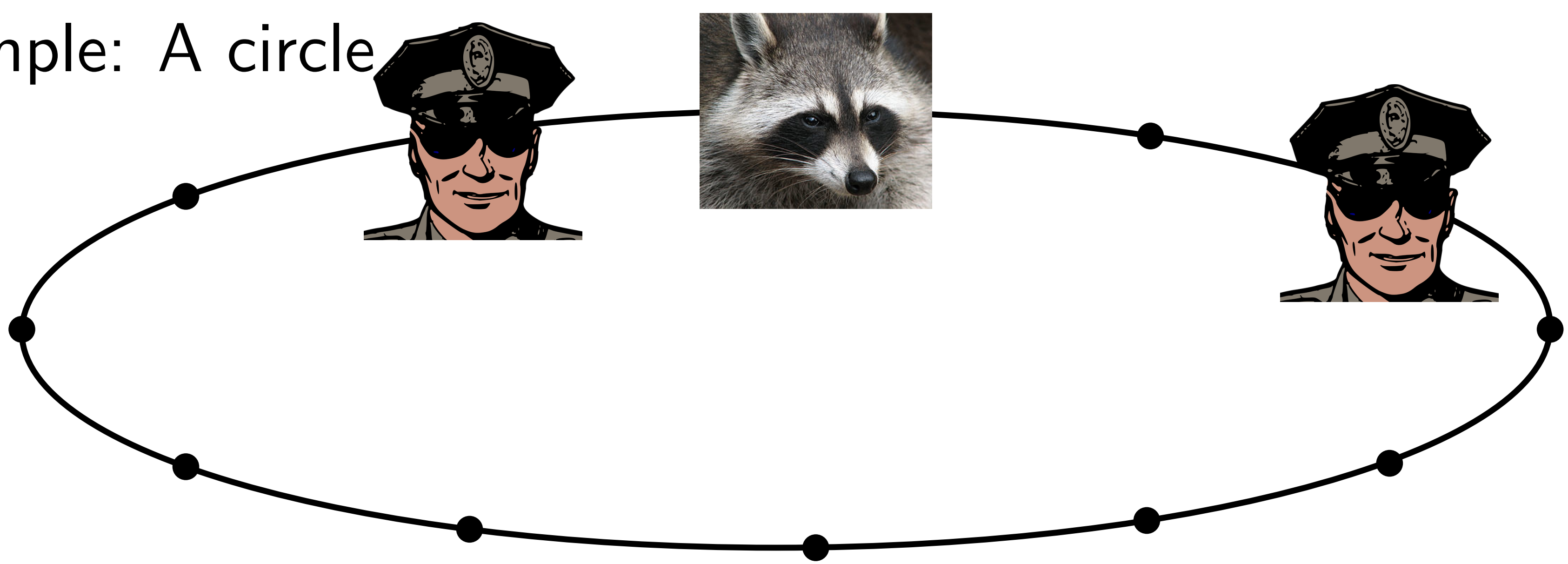

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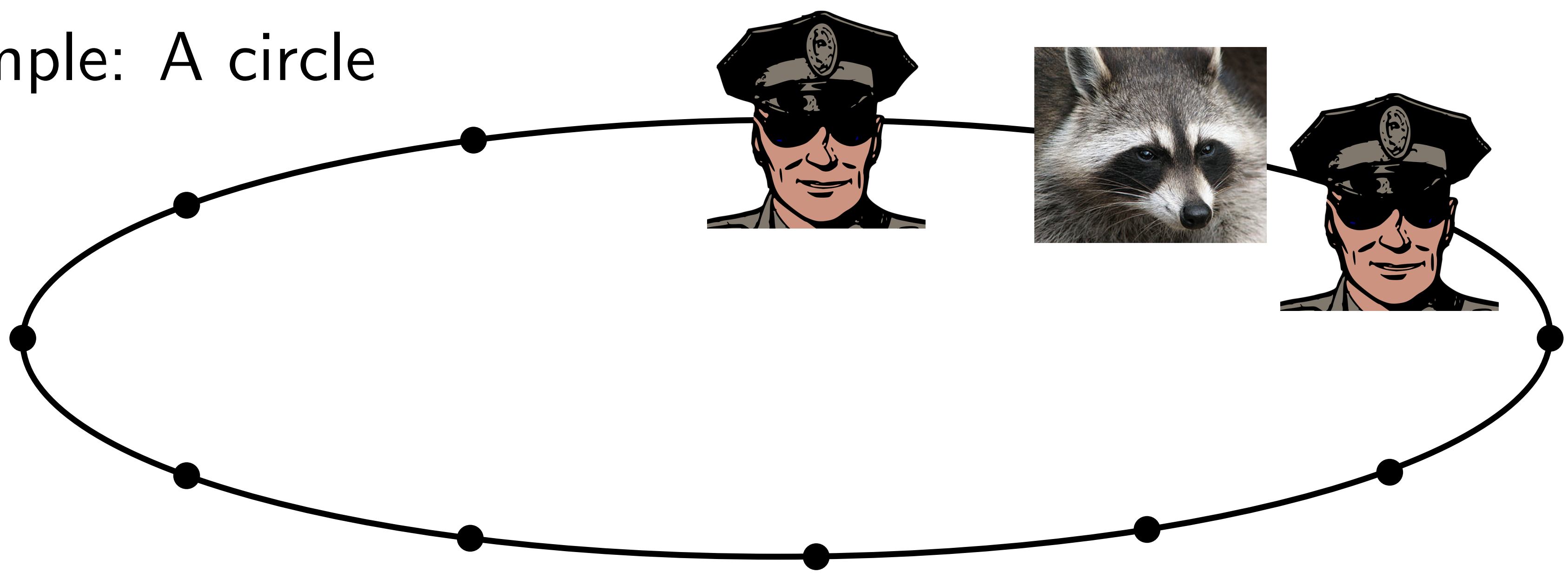

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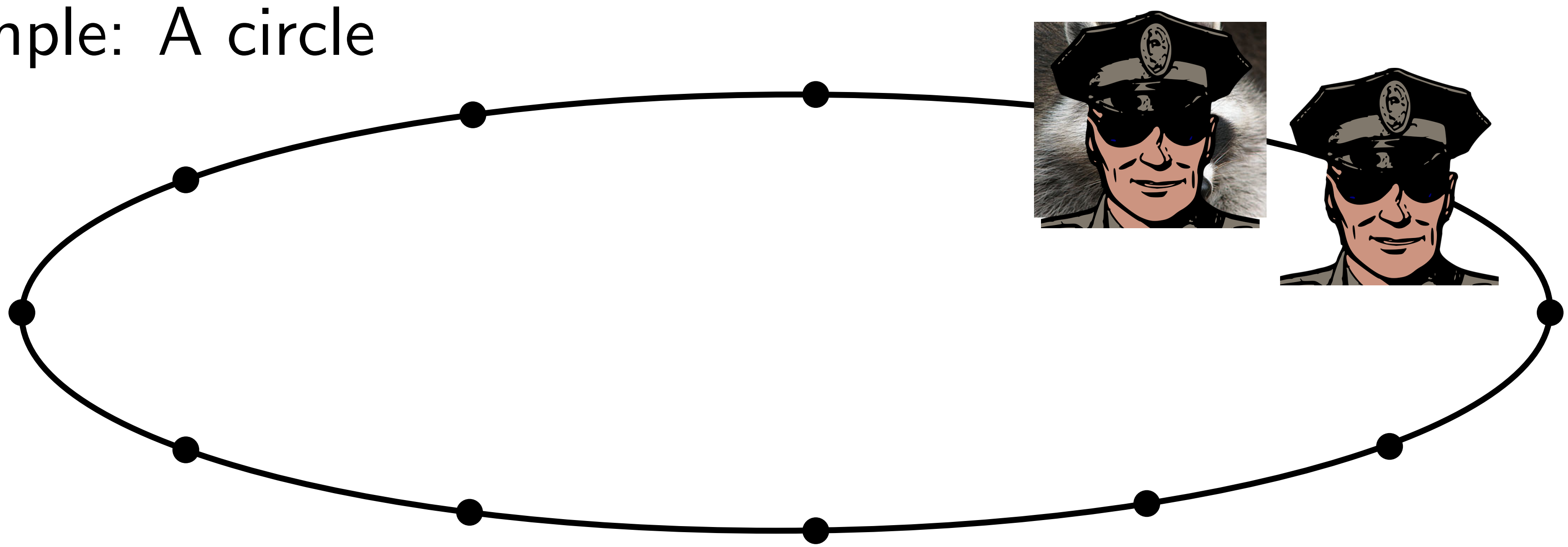

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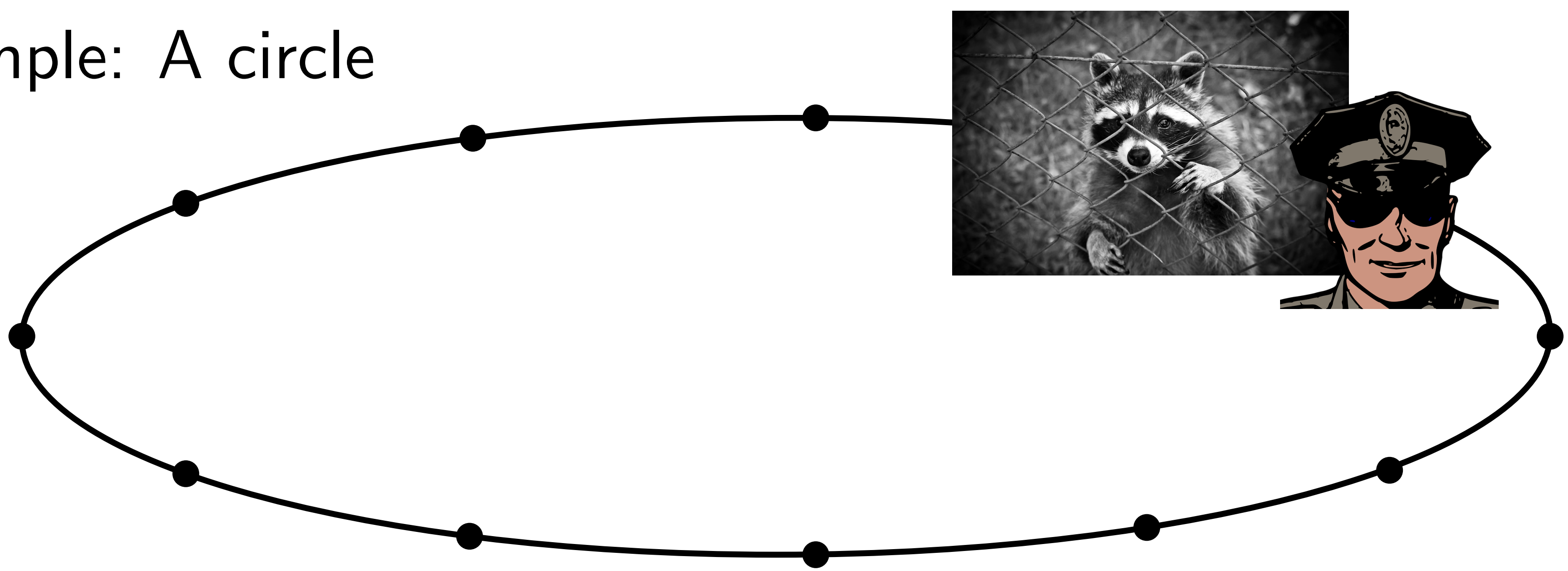

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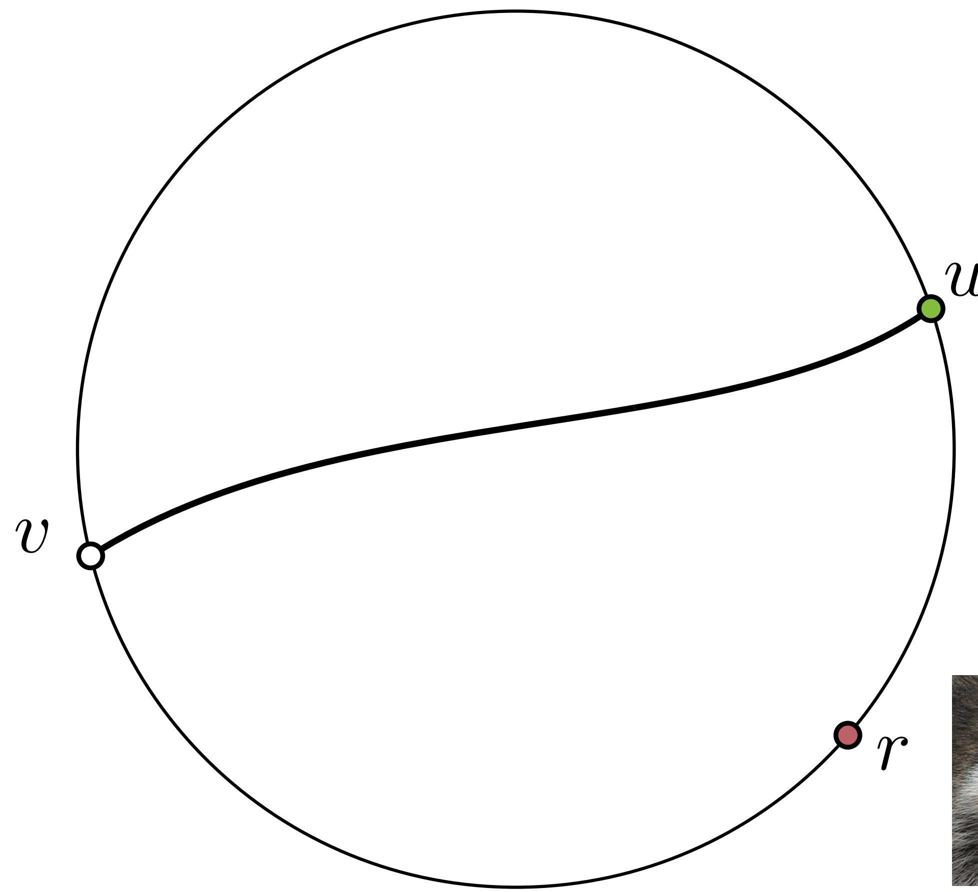

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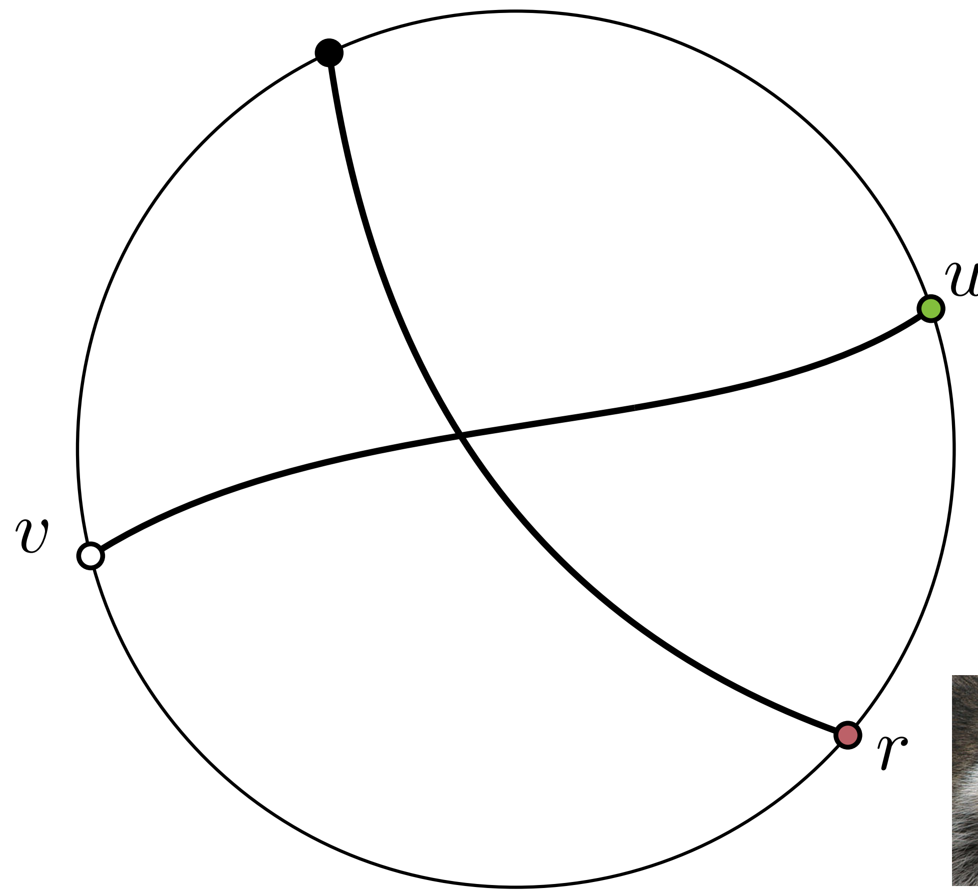

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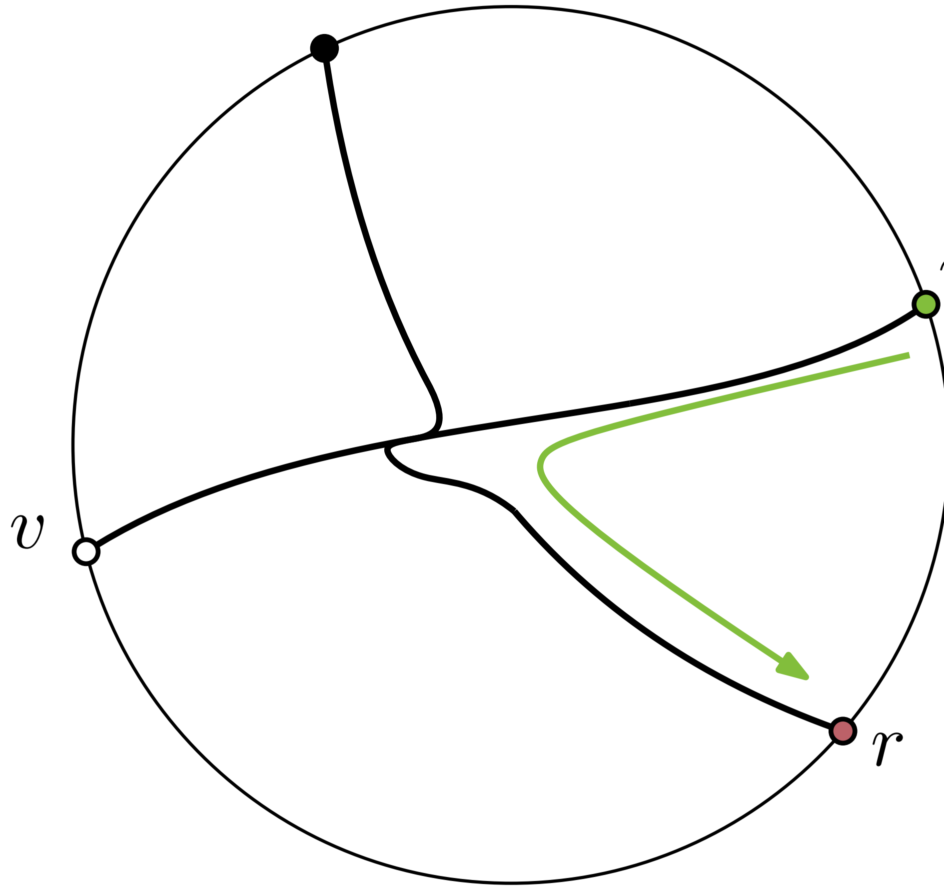

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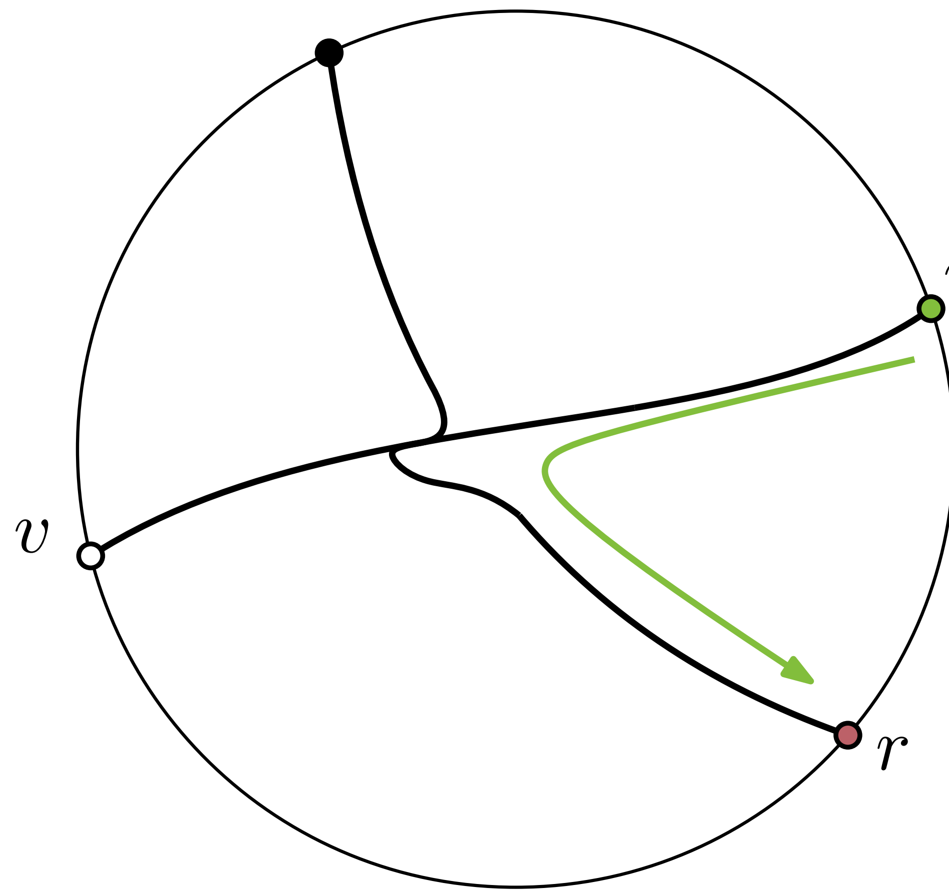

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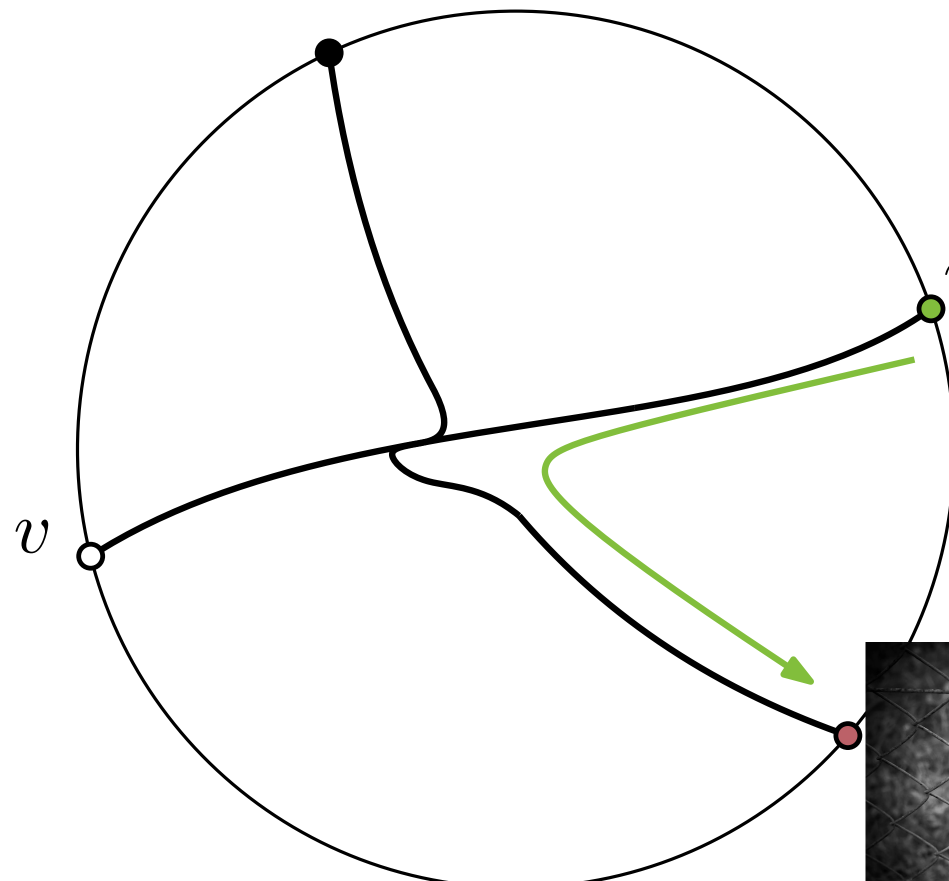

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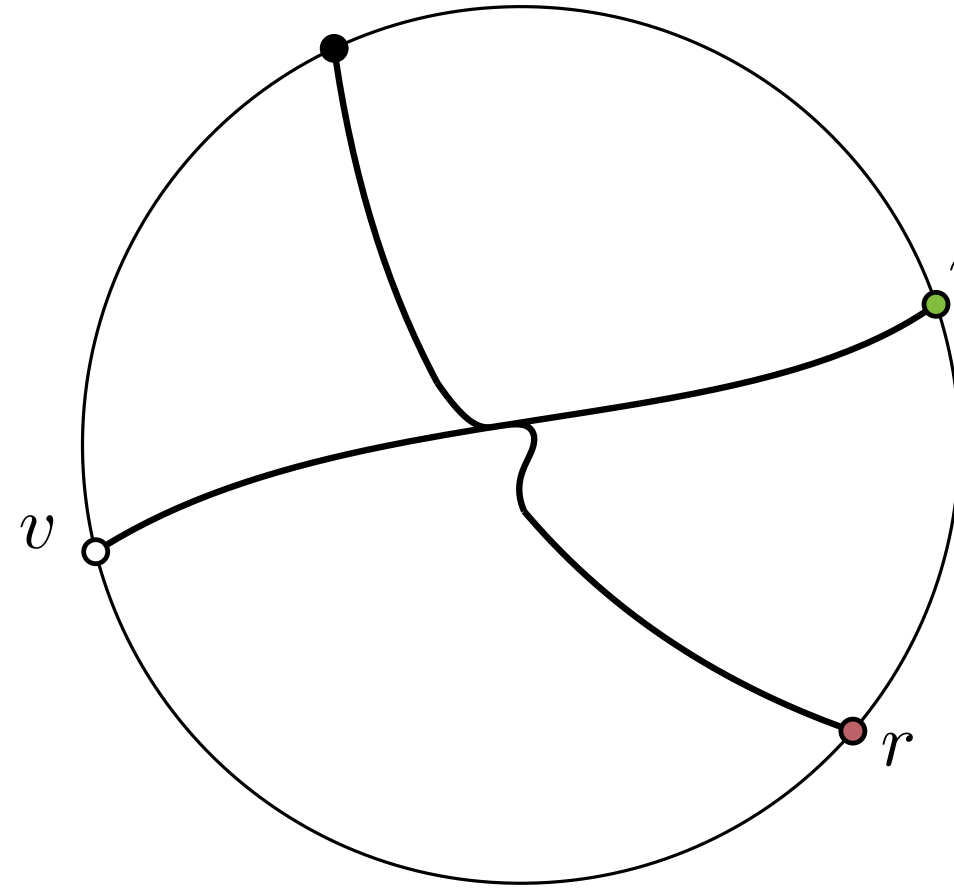

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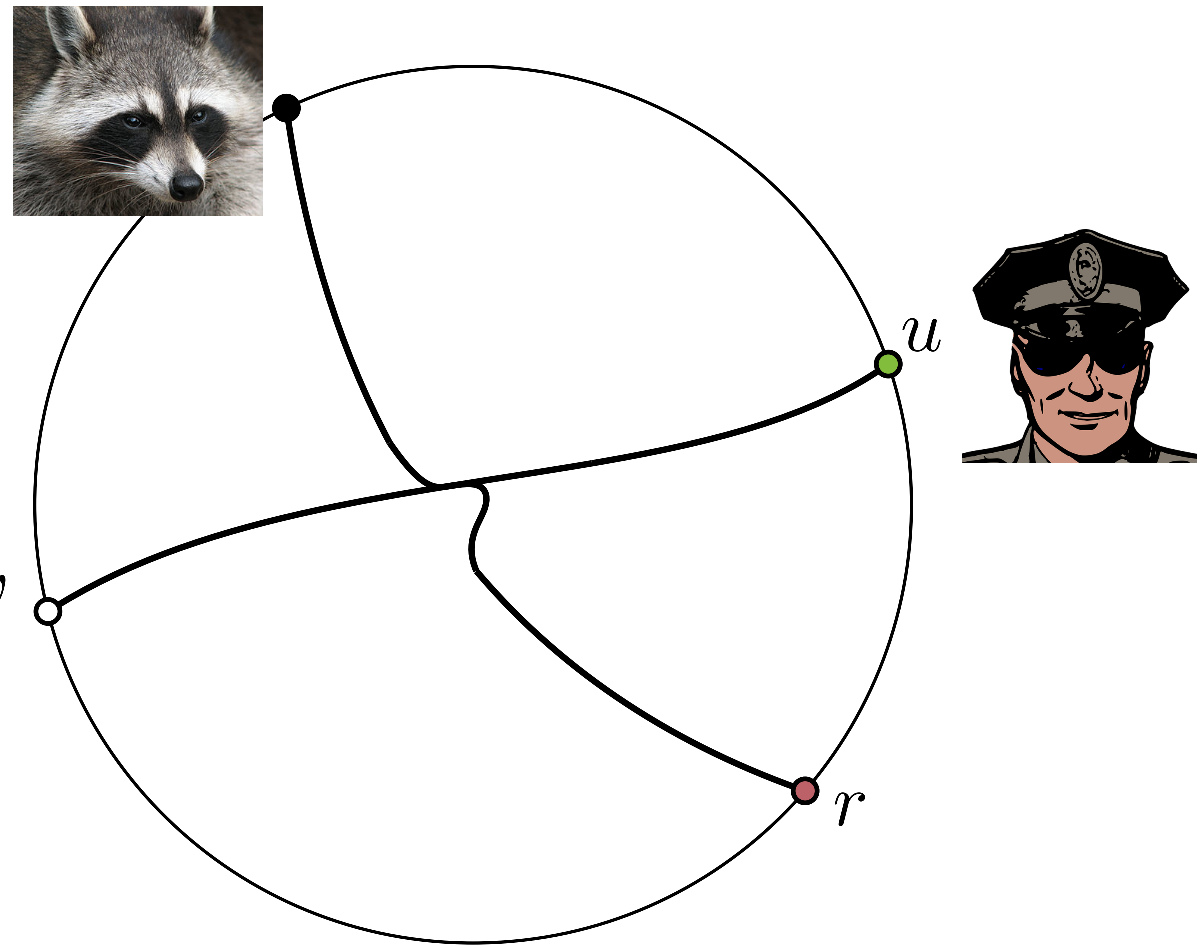

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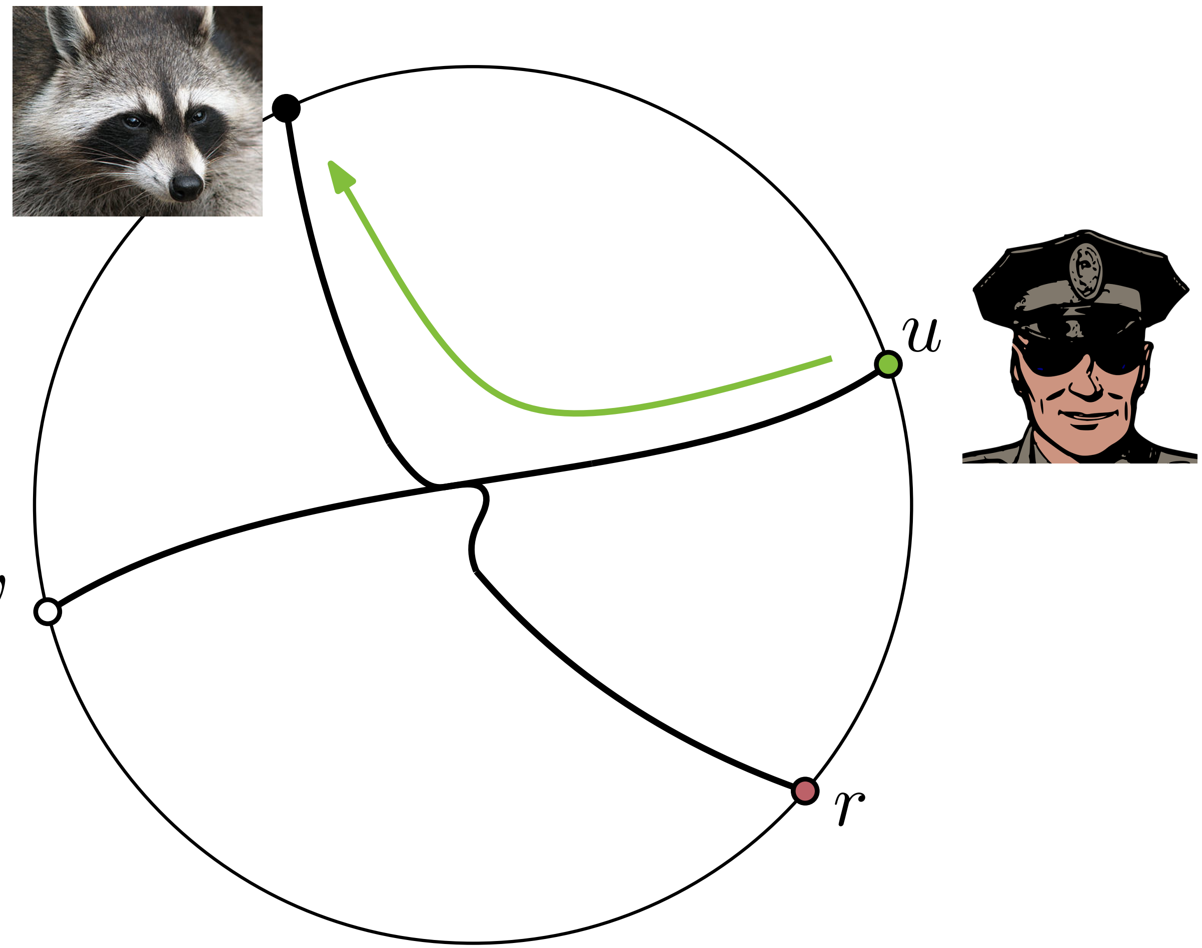

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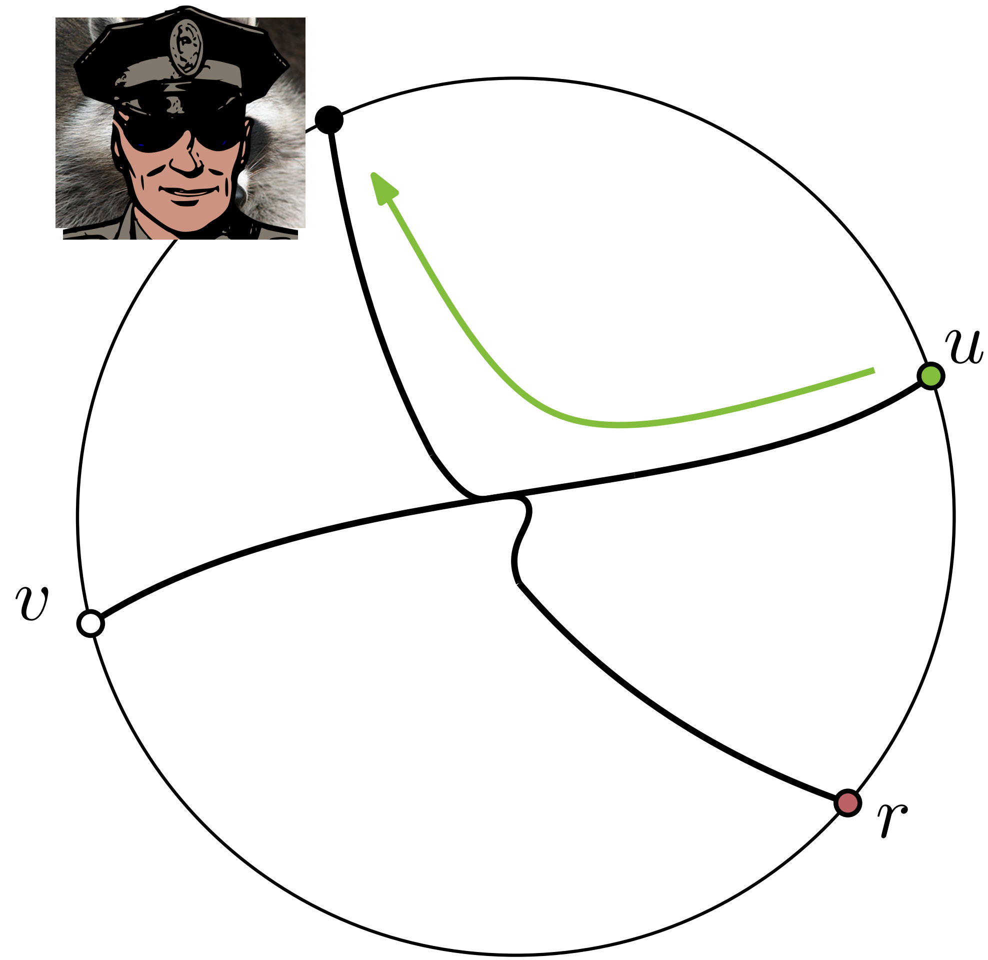

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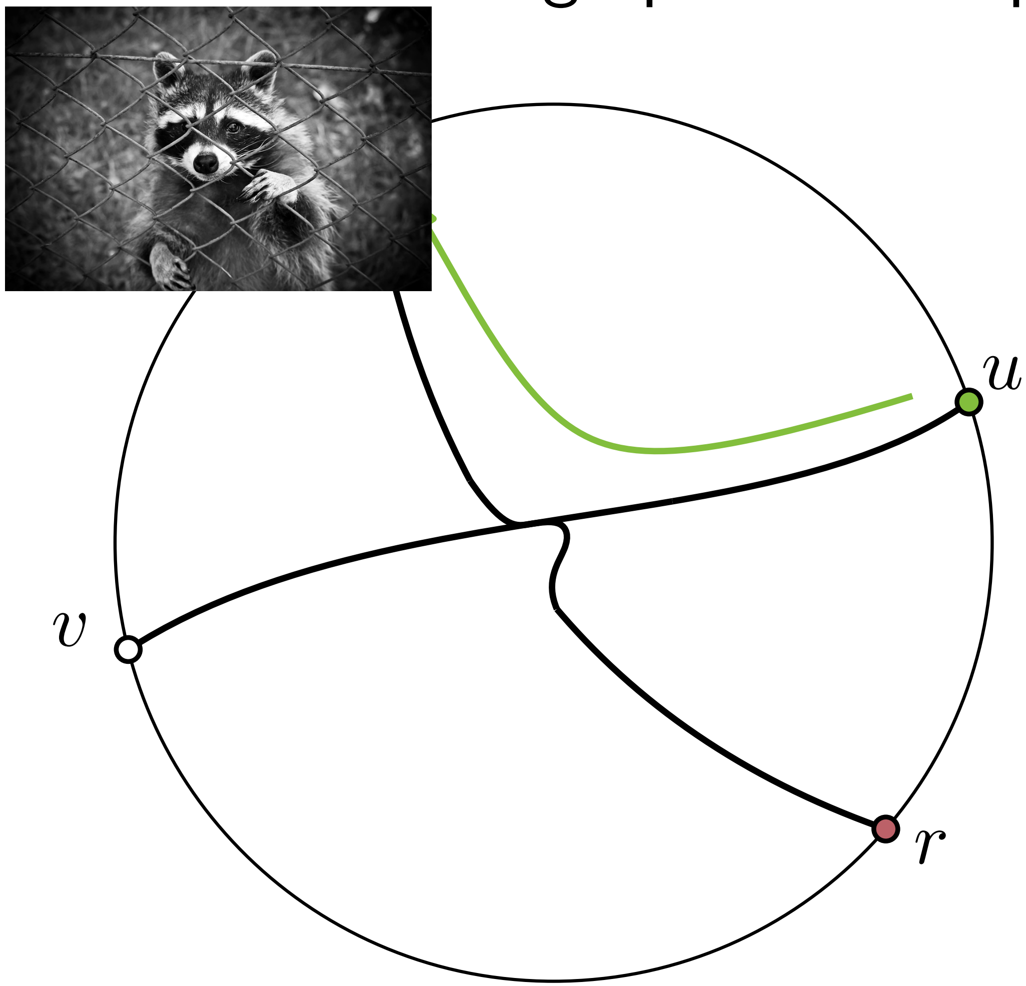

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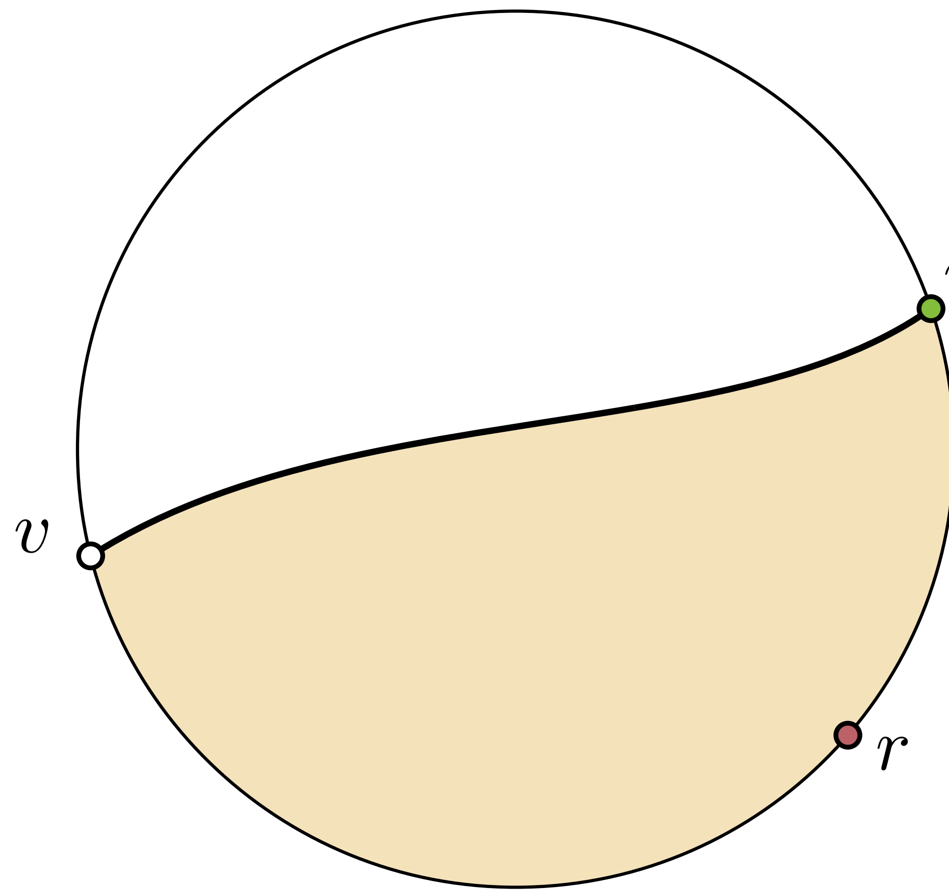

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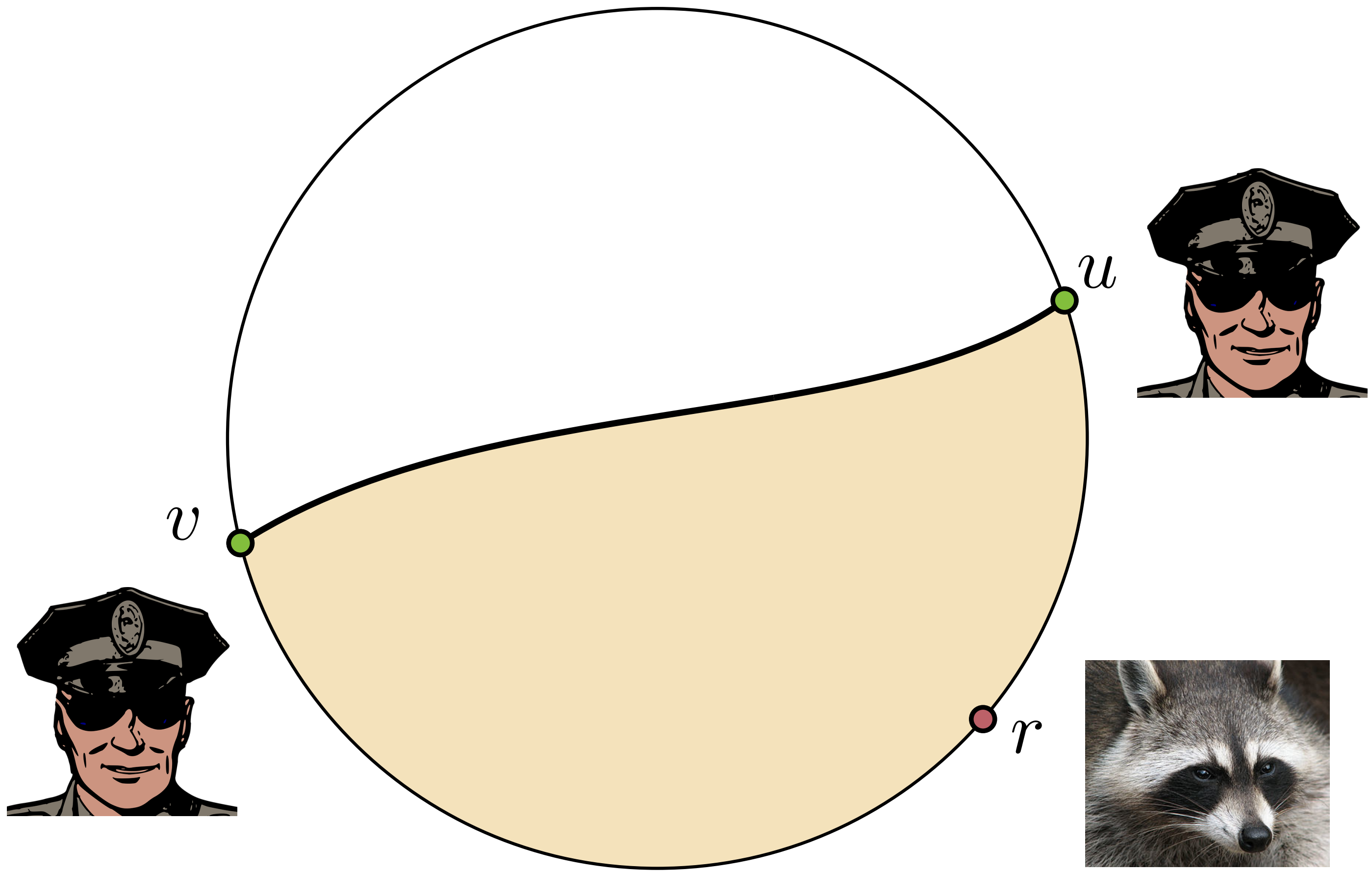

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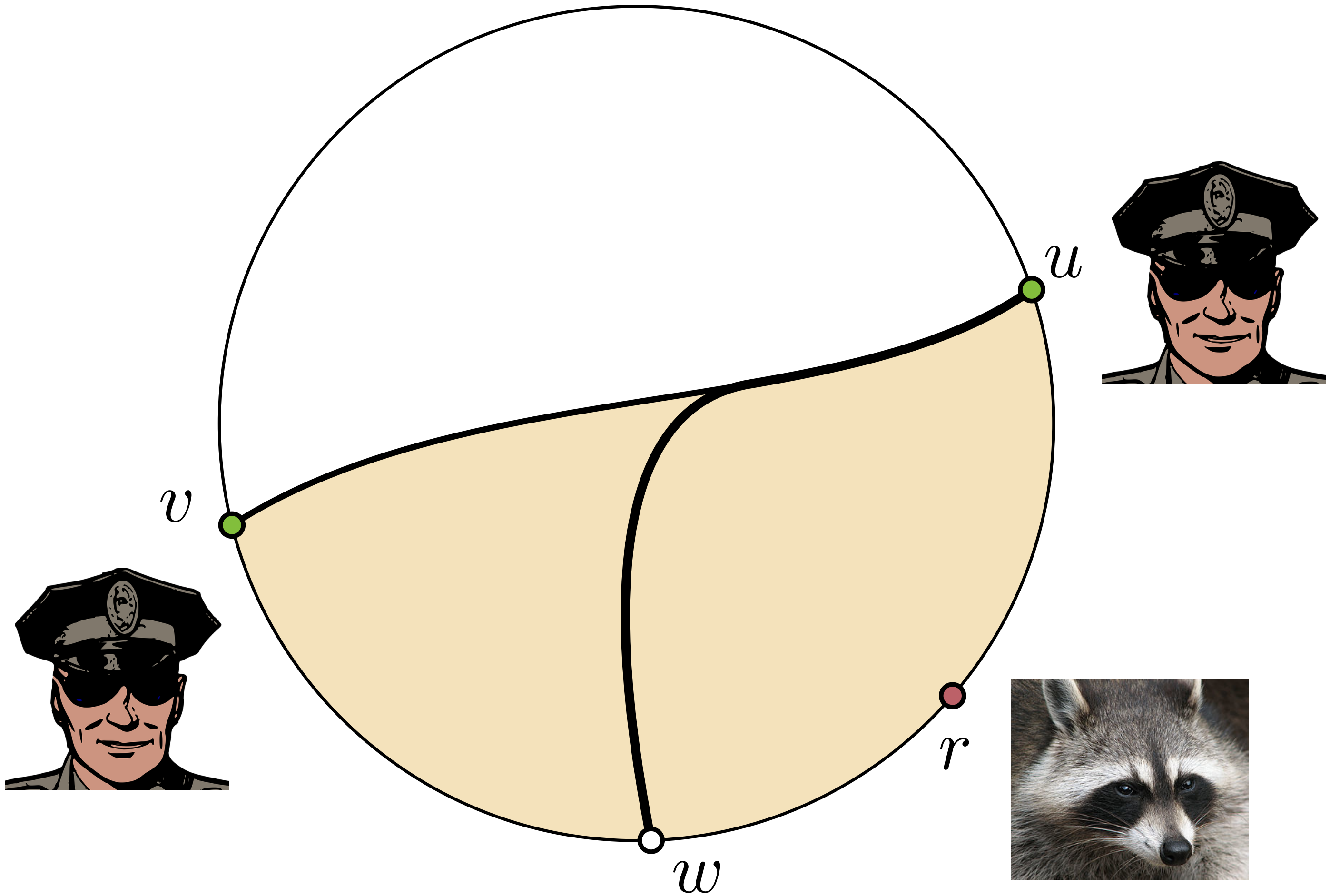
How to lock robber to smaller set of vertices?

Moving cops – Case 1

Result: Strict outer-confluent graphs have cop-number 2



$\exists uw$-path under which we can lock the robber

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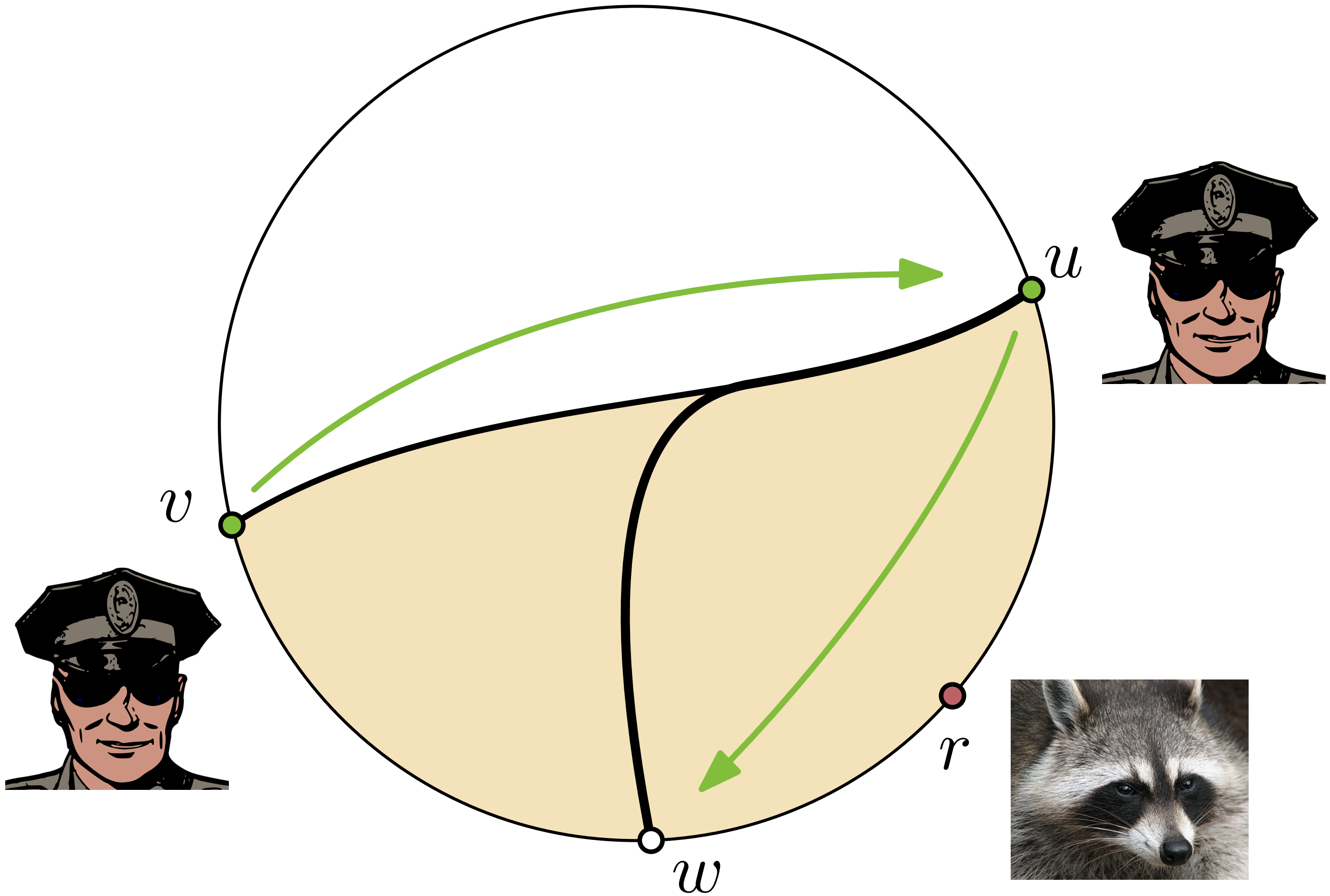

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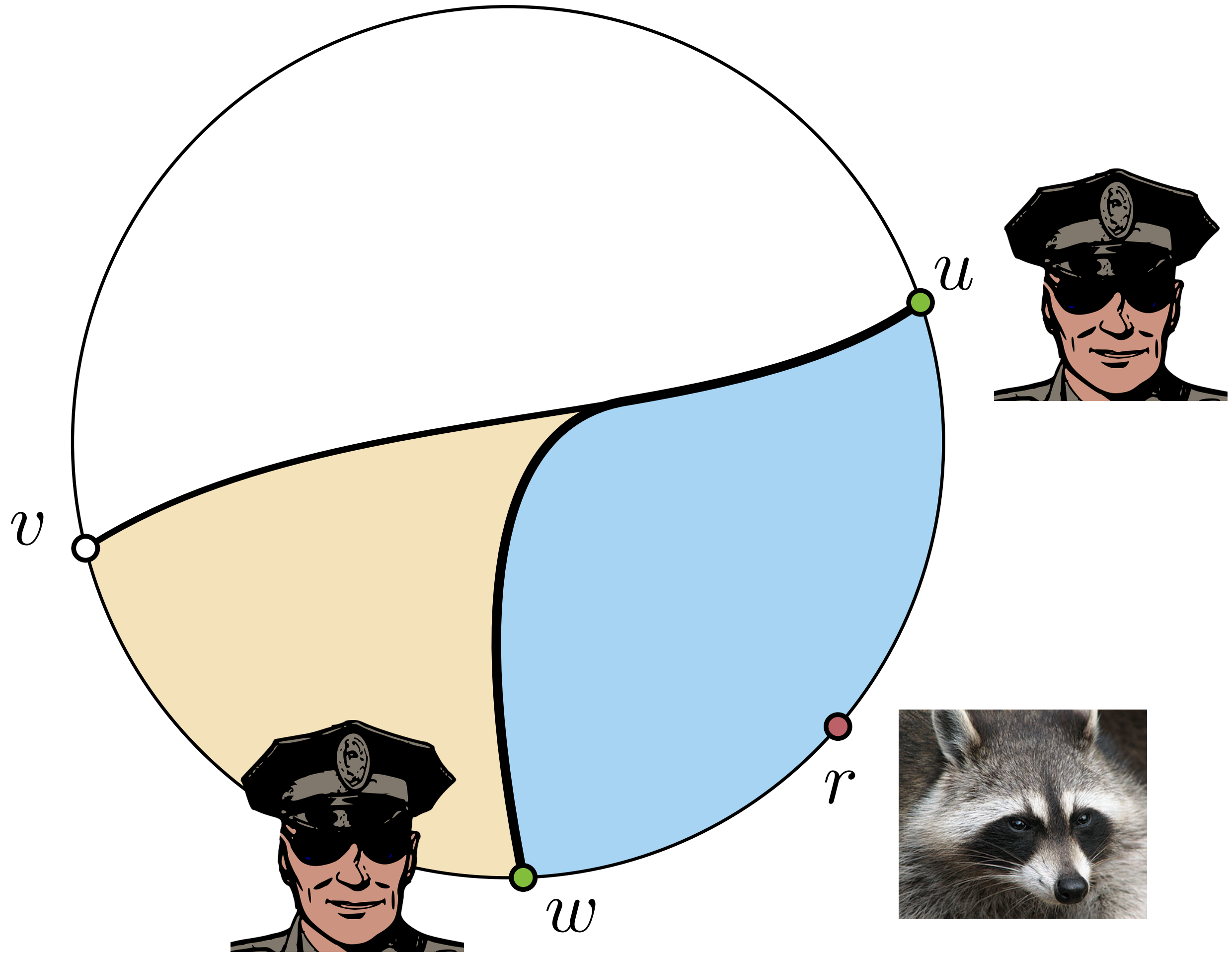

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Moving Cops – Case 2

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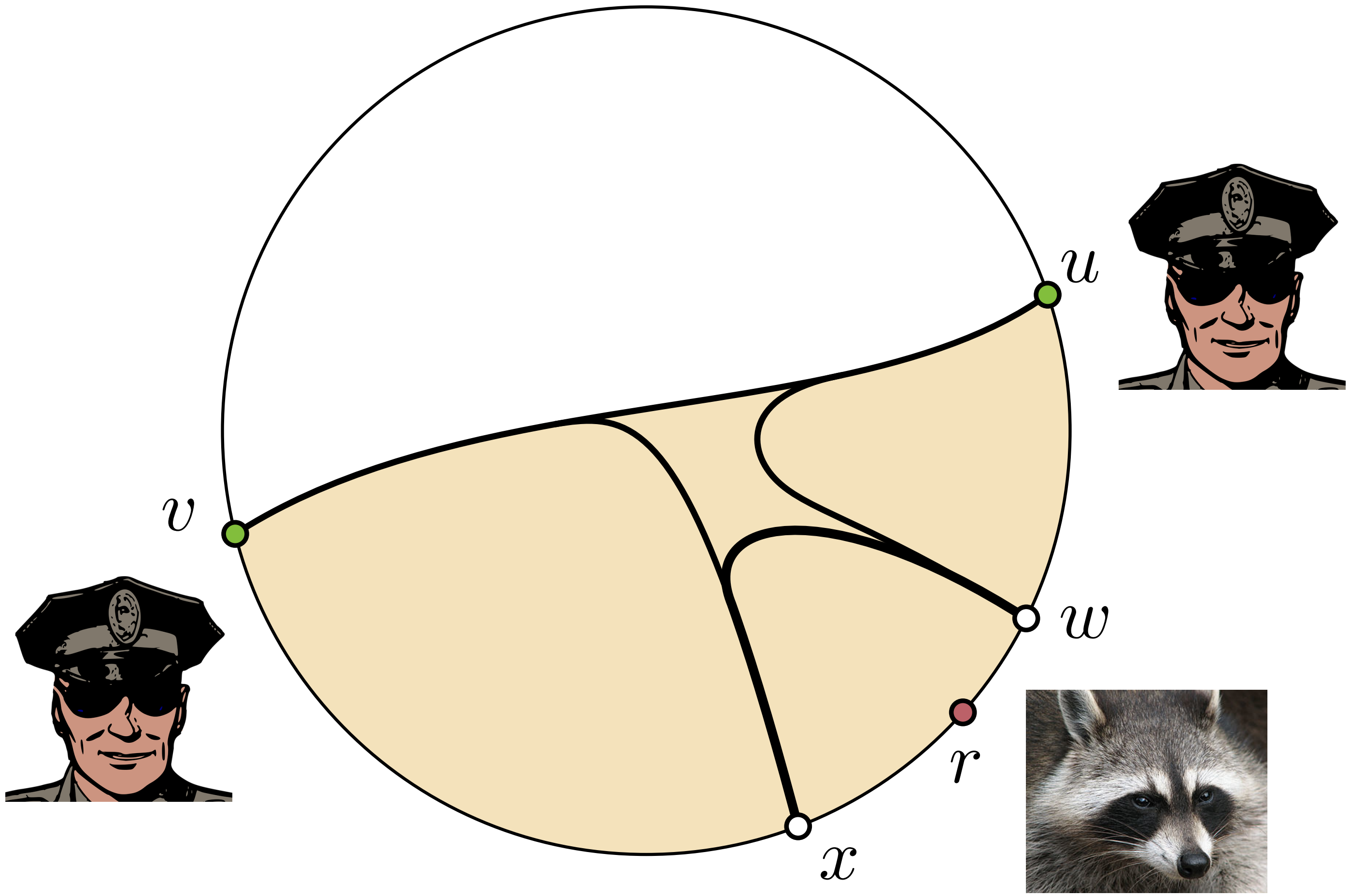

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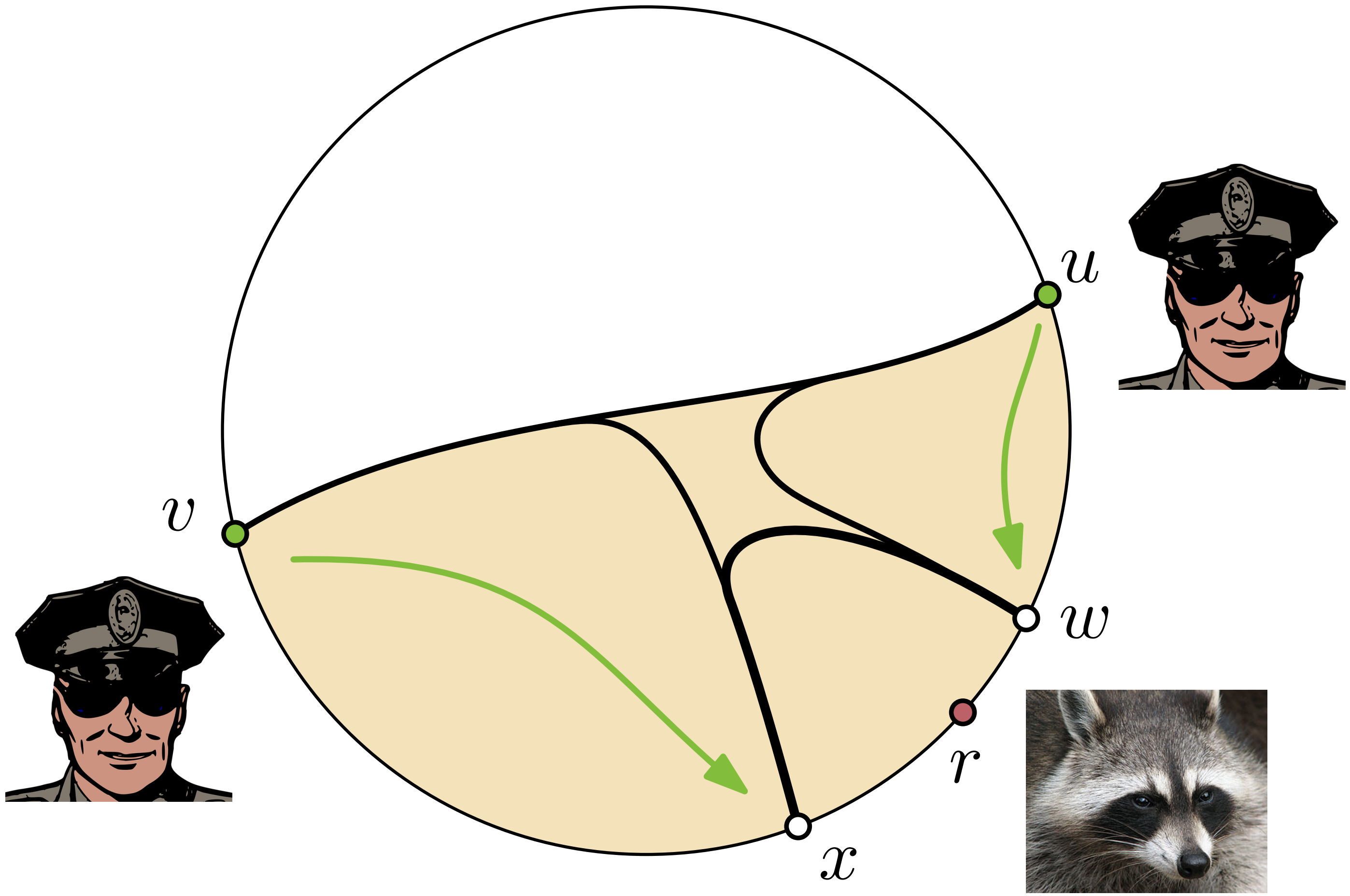

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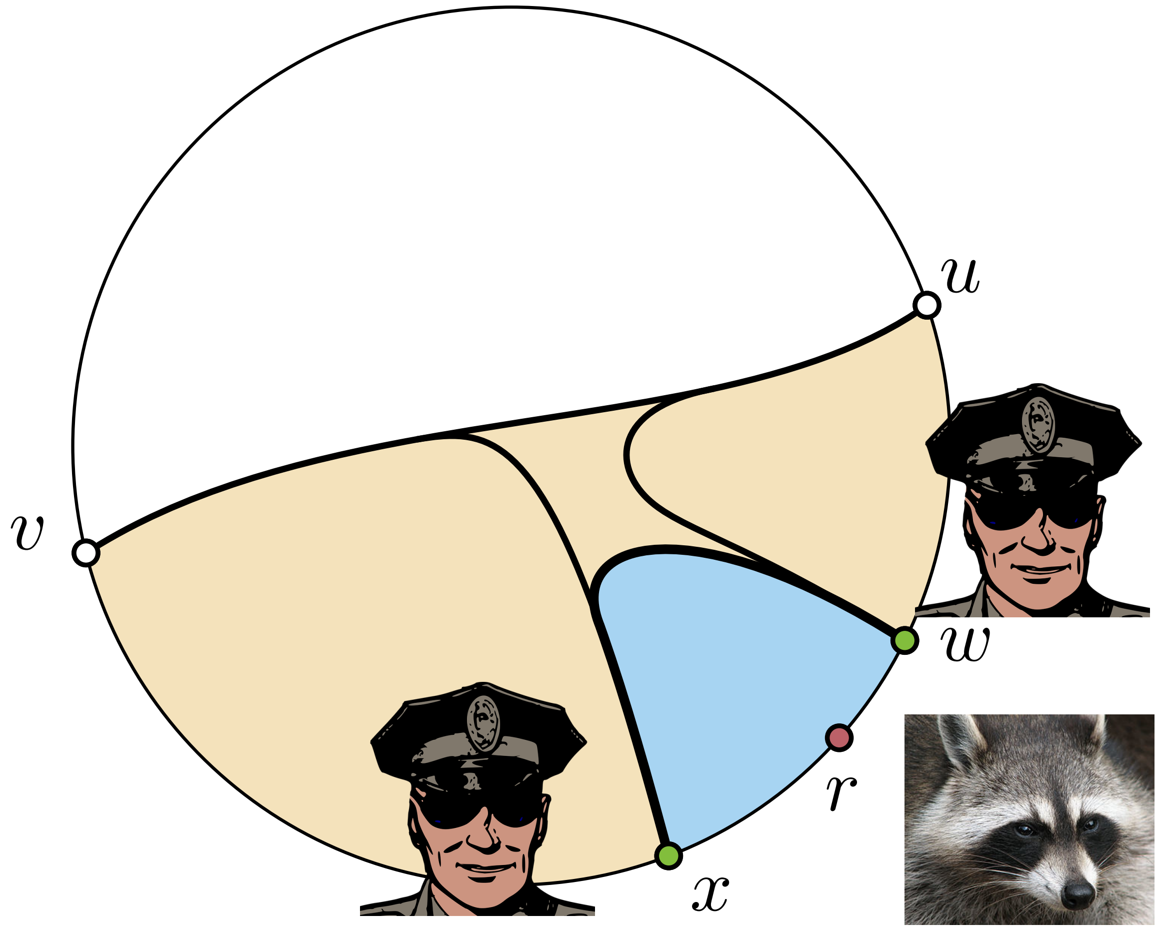

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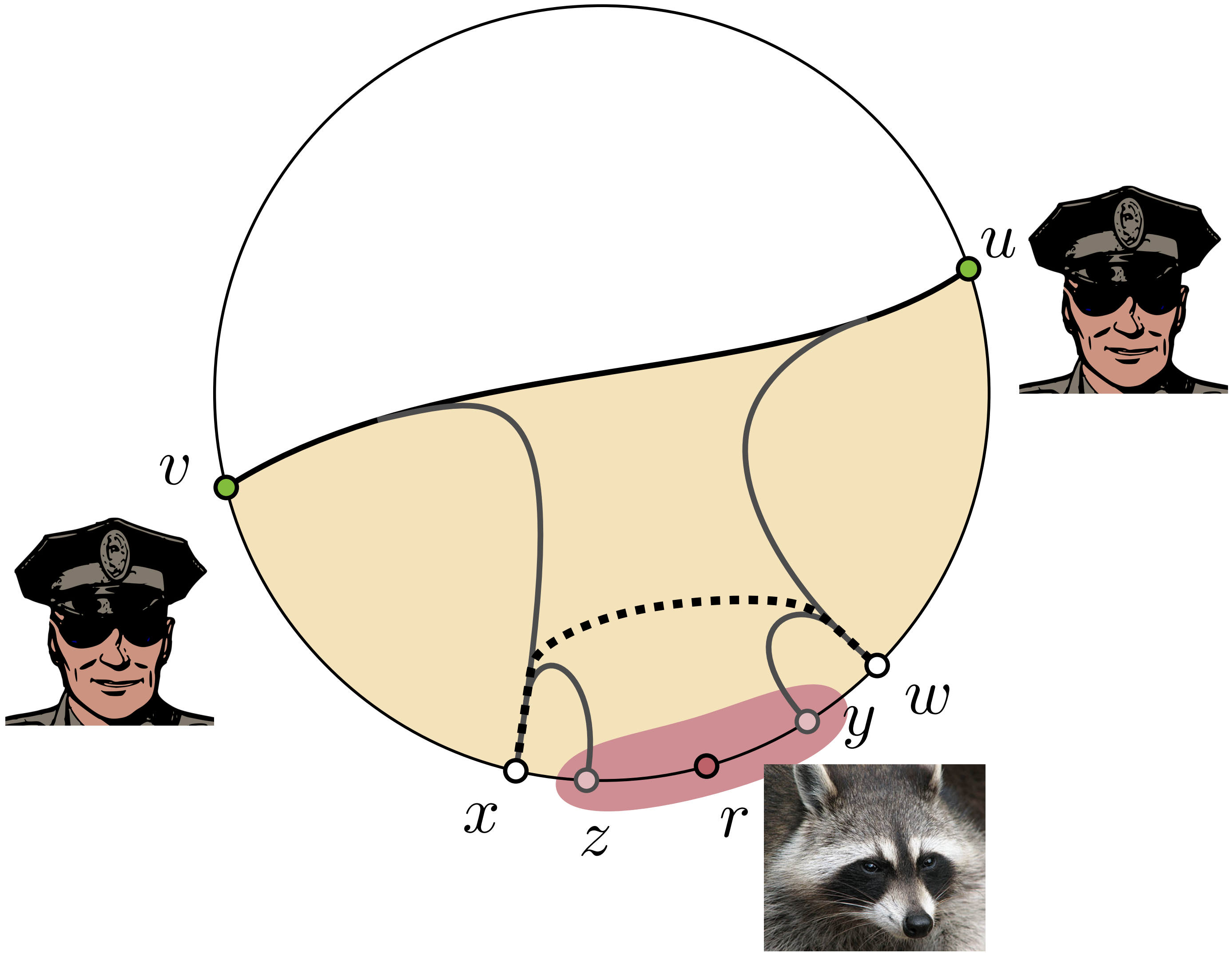
Moving Cops – Case 3

Result: Strict outer-confluent graphs have cop-number 2



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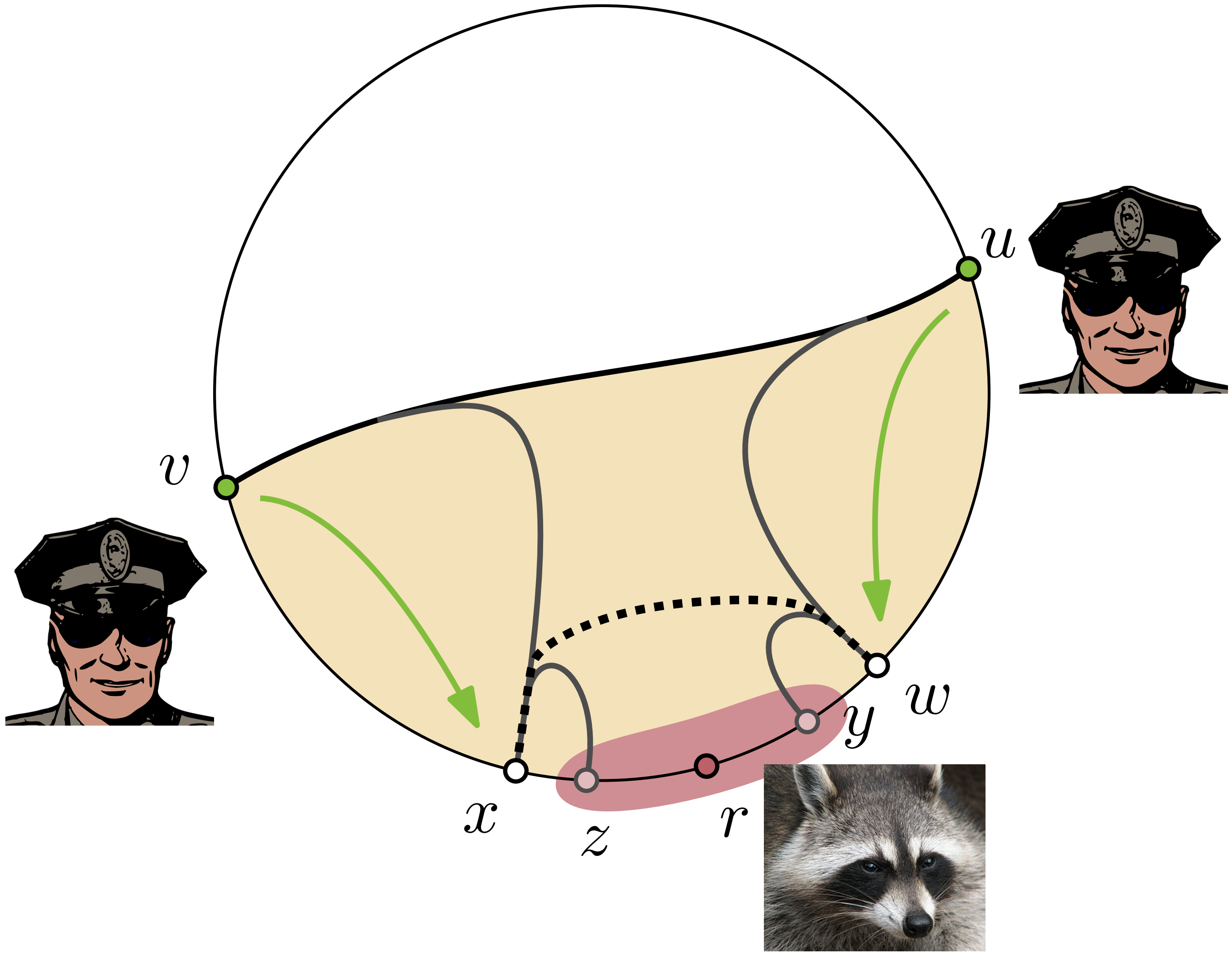

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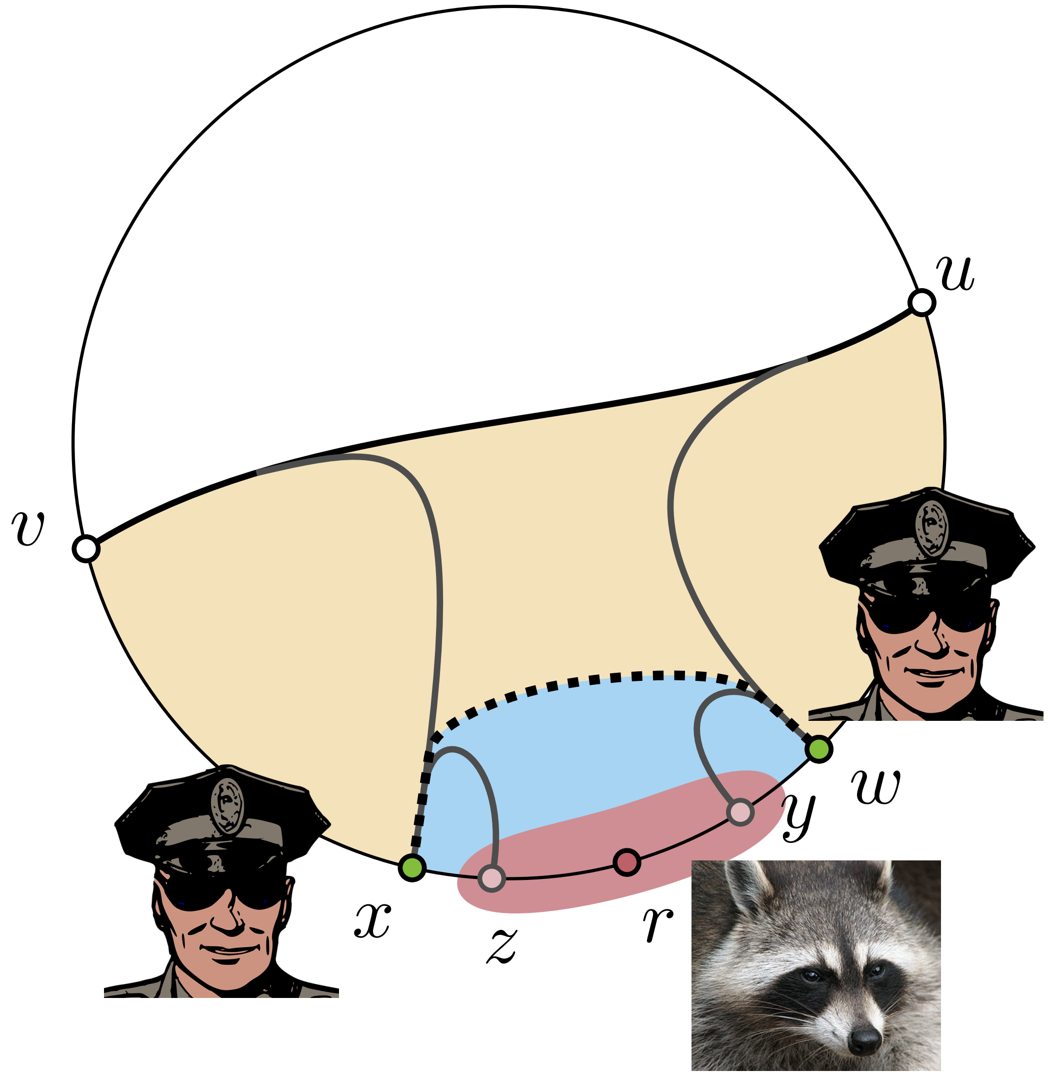
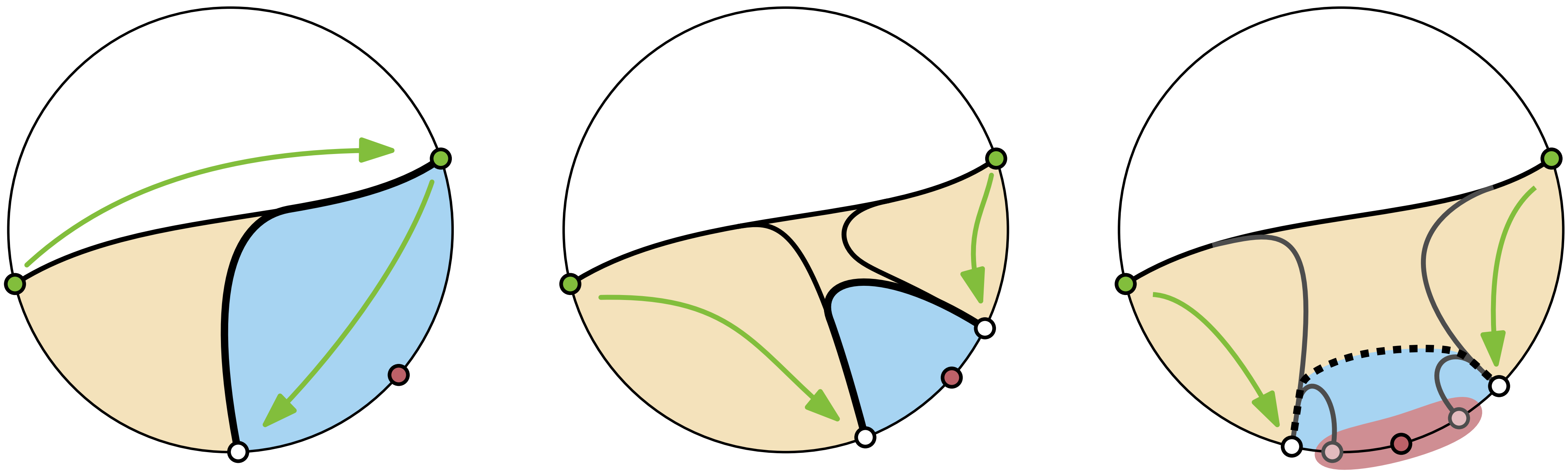


If no wx-path exists, we find y, z such that we can lock a robber that is inside the red region

Catching the Robber

Result: Strict outer-confluent graphs have cop-number 2



Each case makes the set of nodes the robber can use smaller

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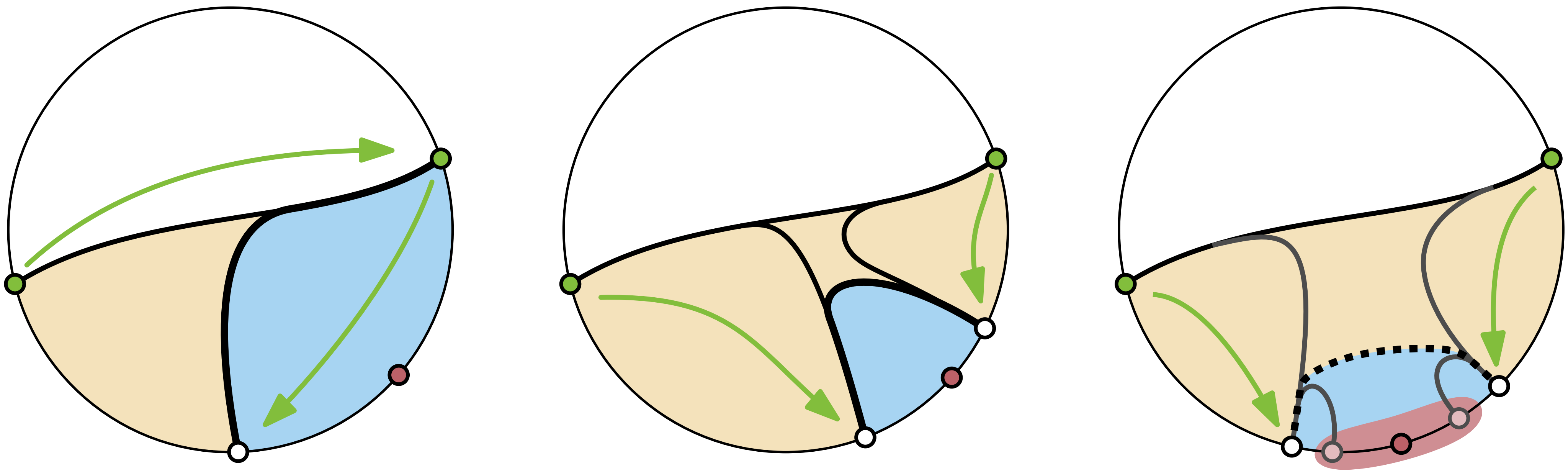

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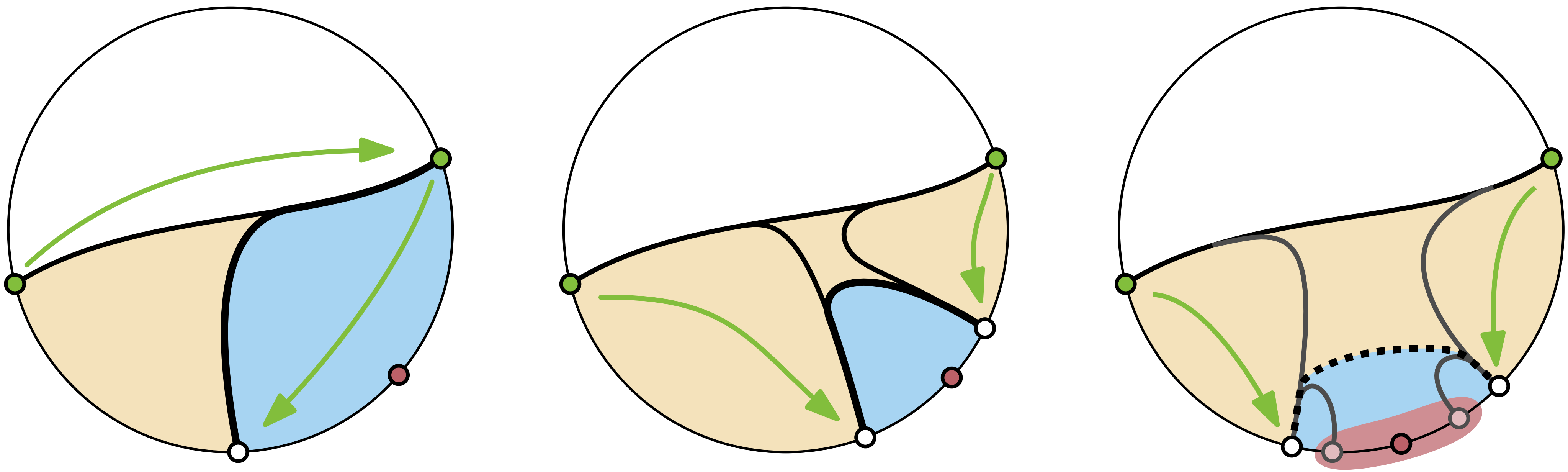

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Many other questions

Do strict outer-confluent graphs have bounded cliquewidth?

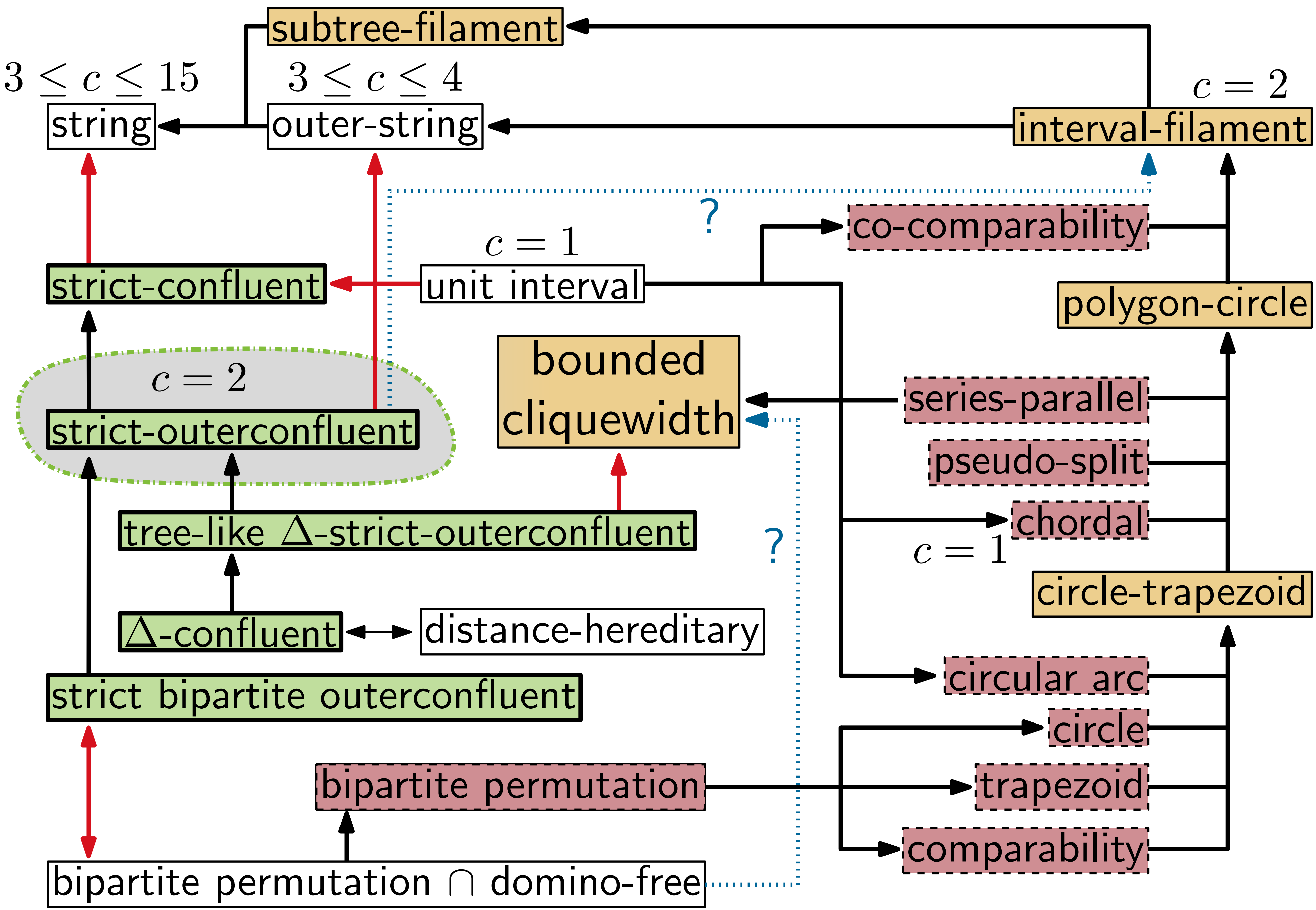
Complexity of other types of confluence?

What other graph classes admit (strict) confluent drawings?

Confluence, but with some allowed crossings? [Bach et al. 16]

Our Results

Cop number: c



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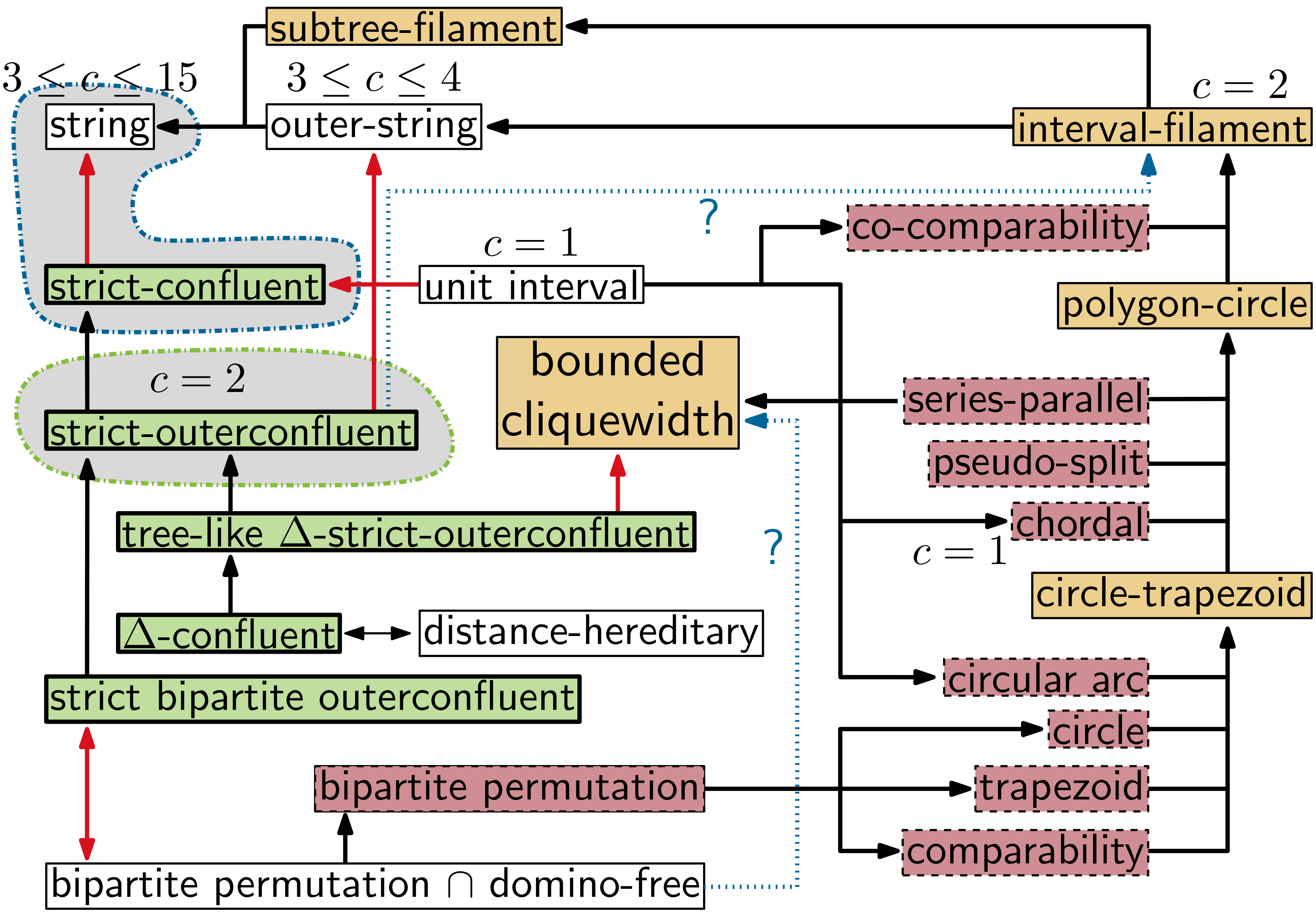

Construction of Traces

Given strict confluent drawing D

For each u define trace $t(u)$ (these will be the strings)

Each trace starts at u in D

Viewed from u, $t(u)$ stays on the left side of the paths

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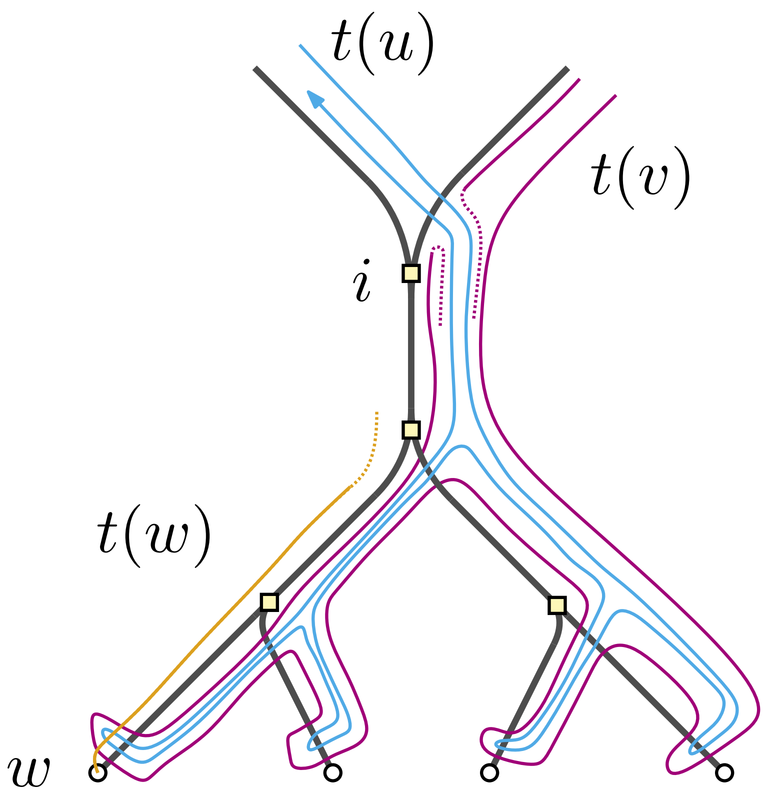
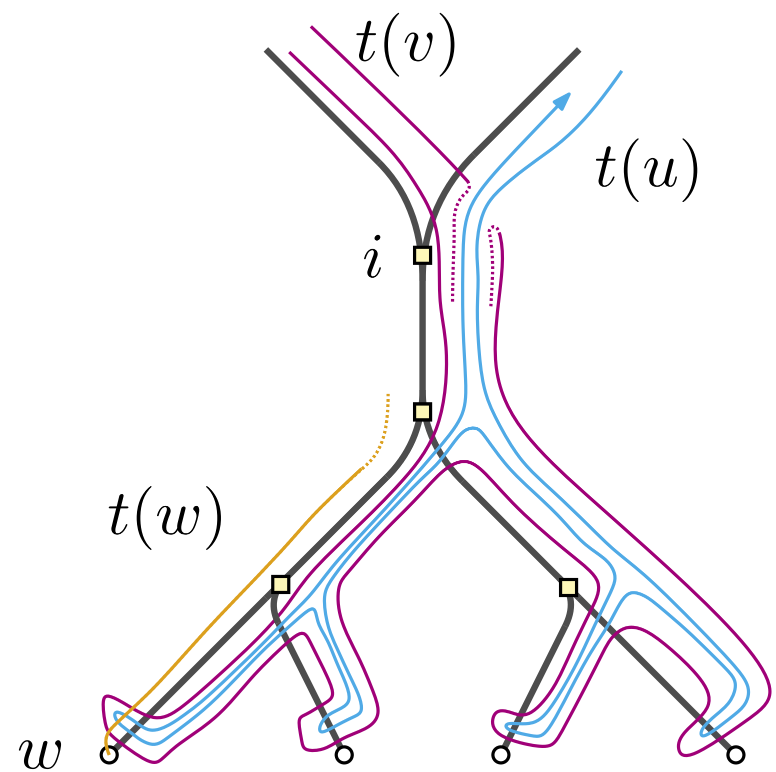
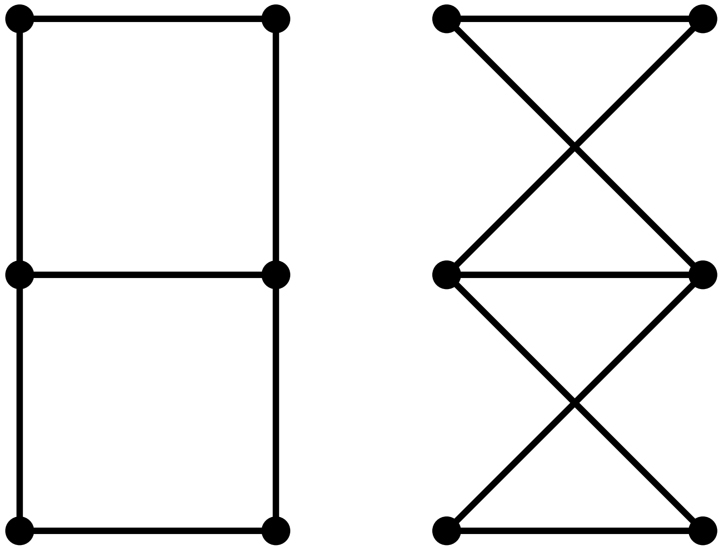




Two Problems for Strictness

Domino



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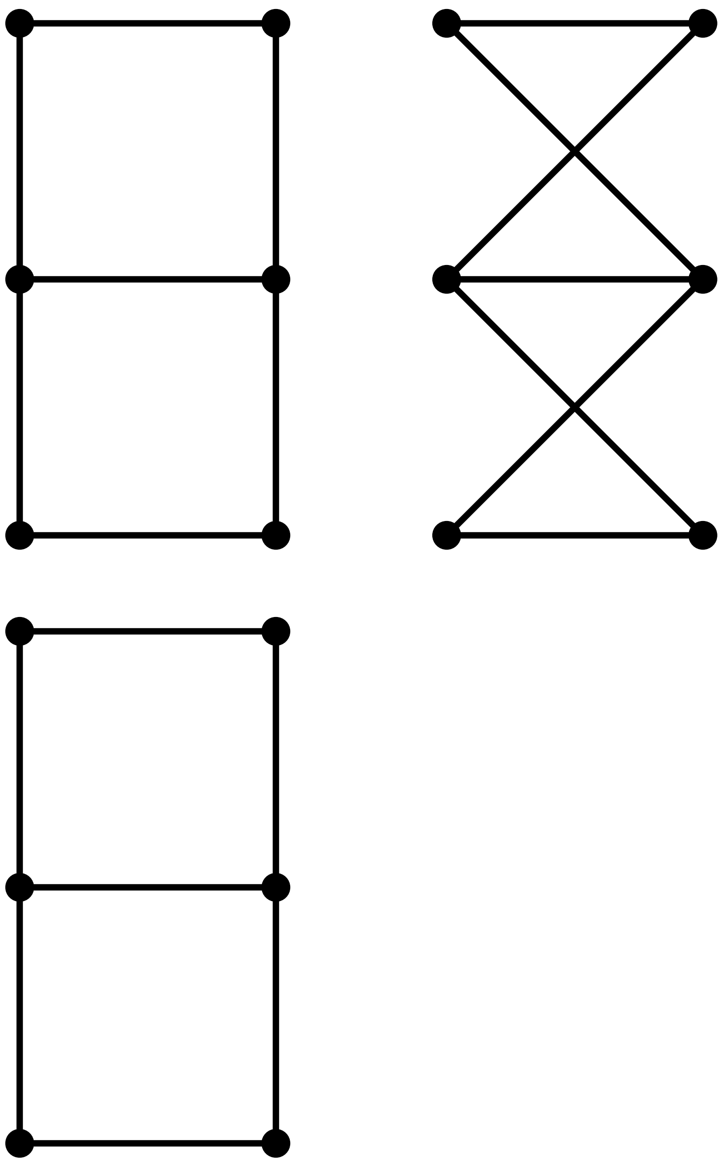

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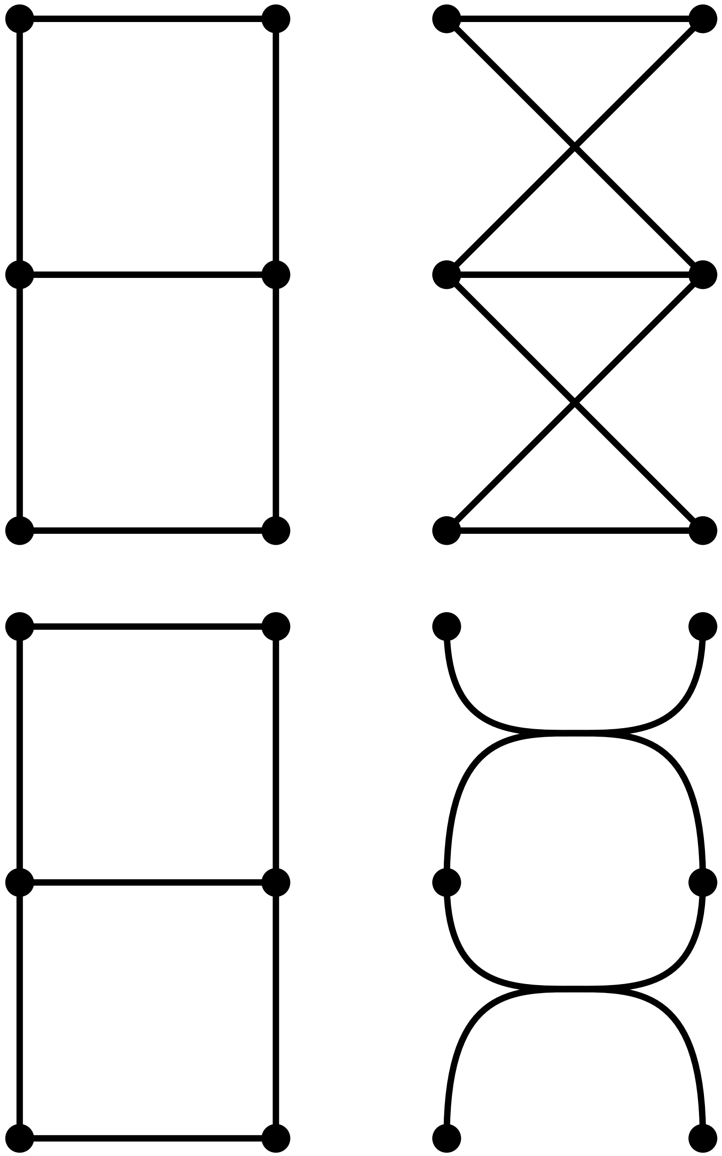

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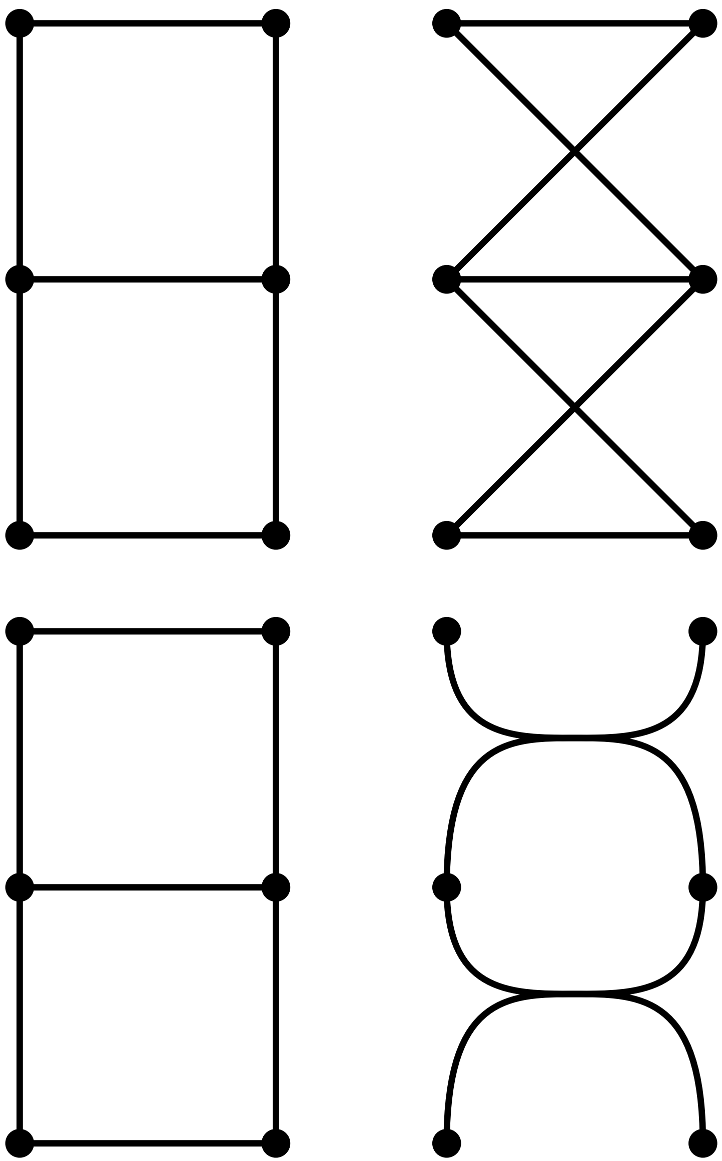
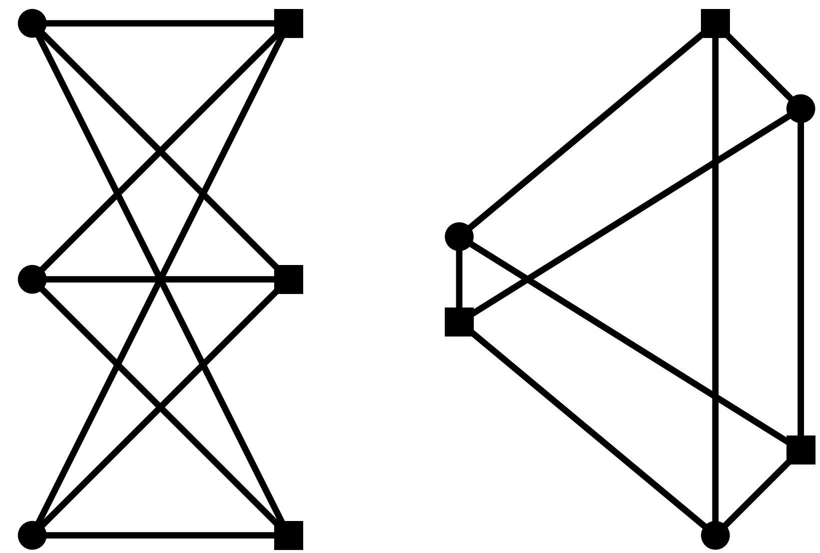


$K_{3,3}$



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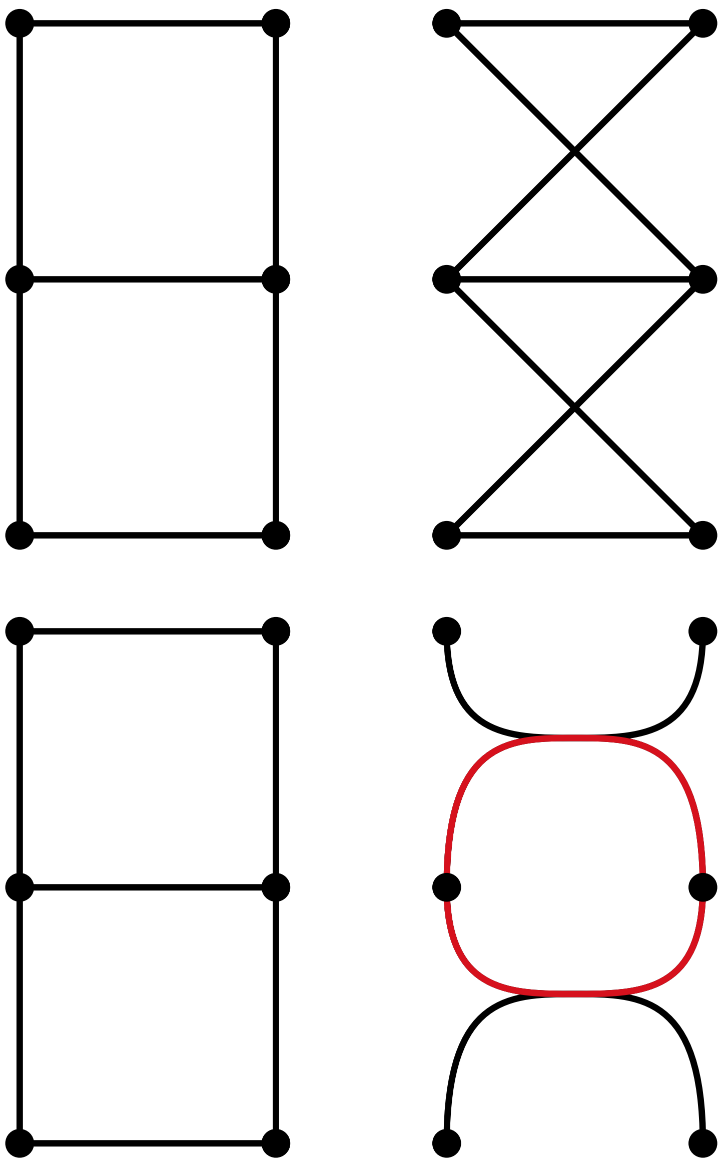

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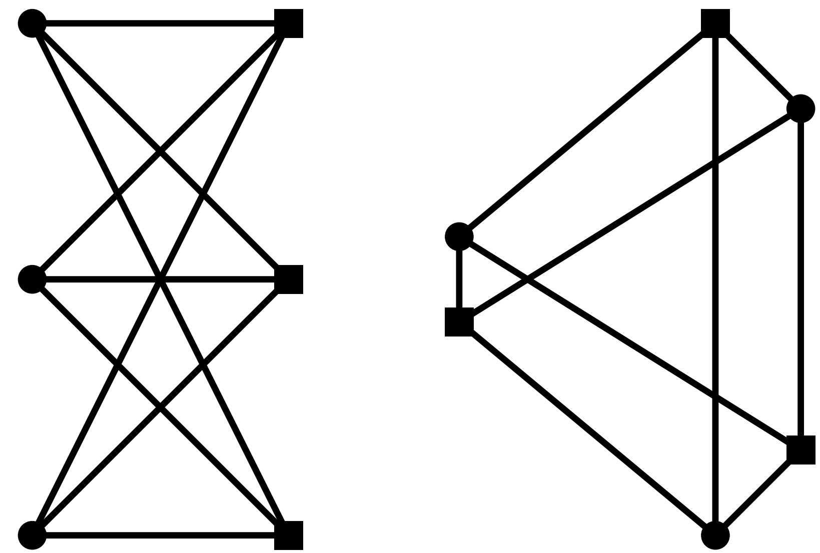

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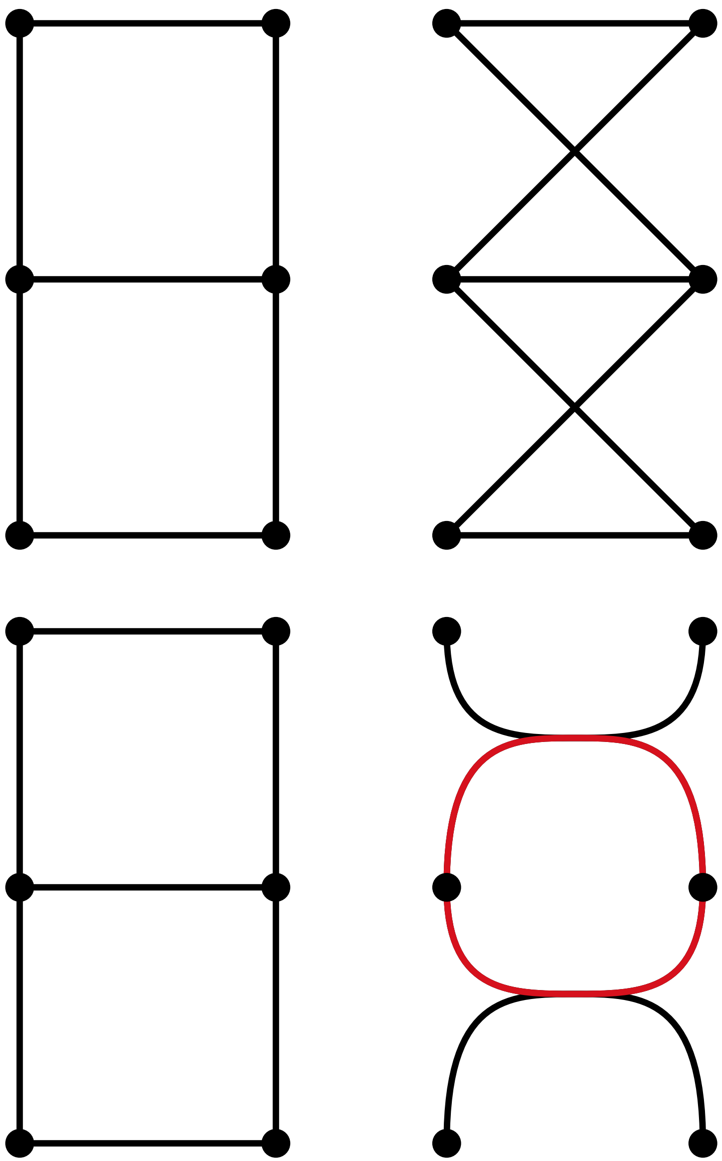

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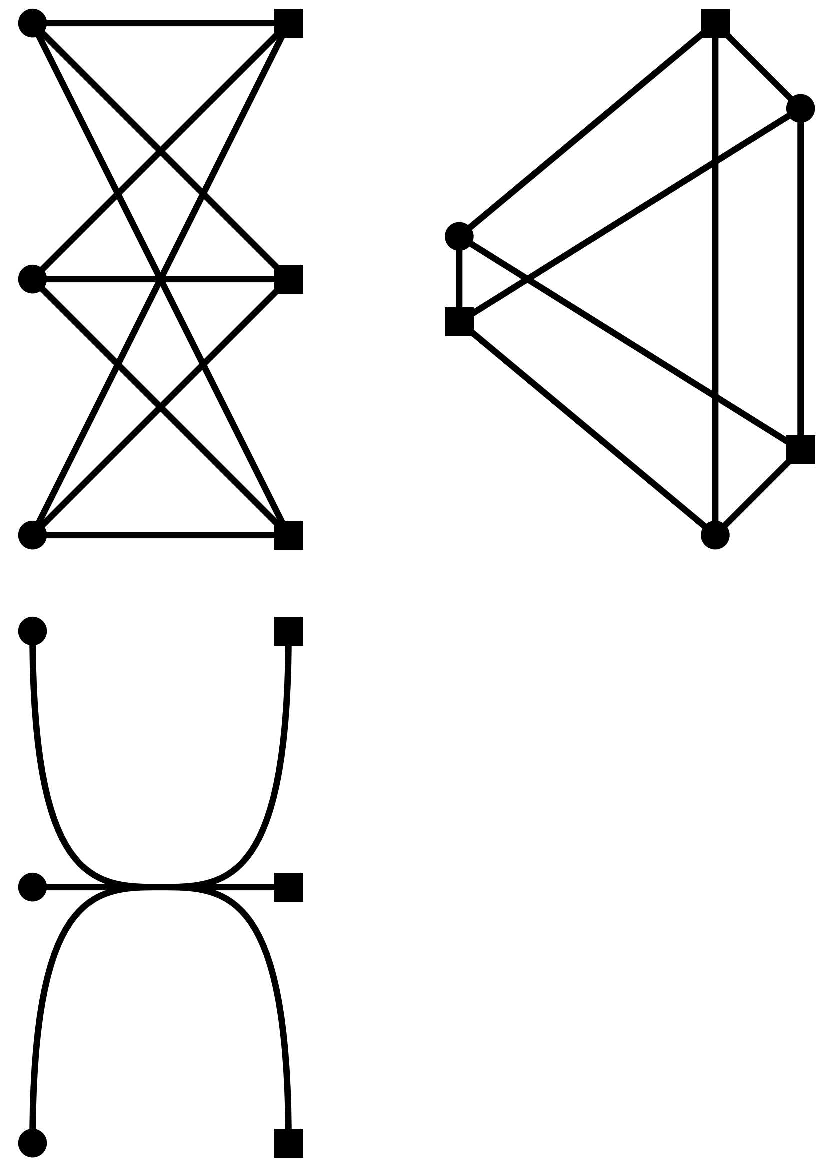

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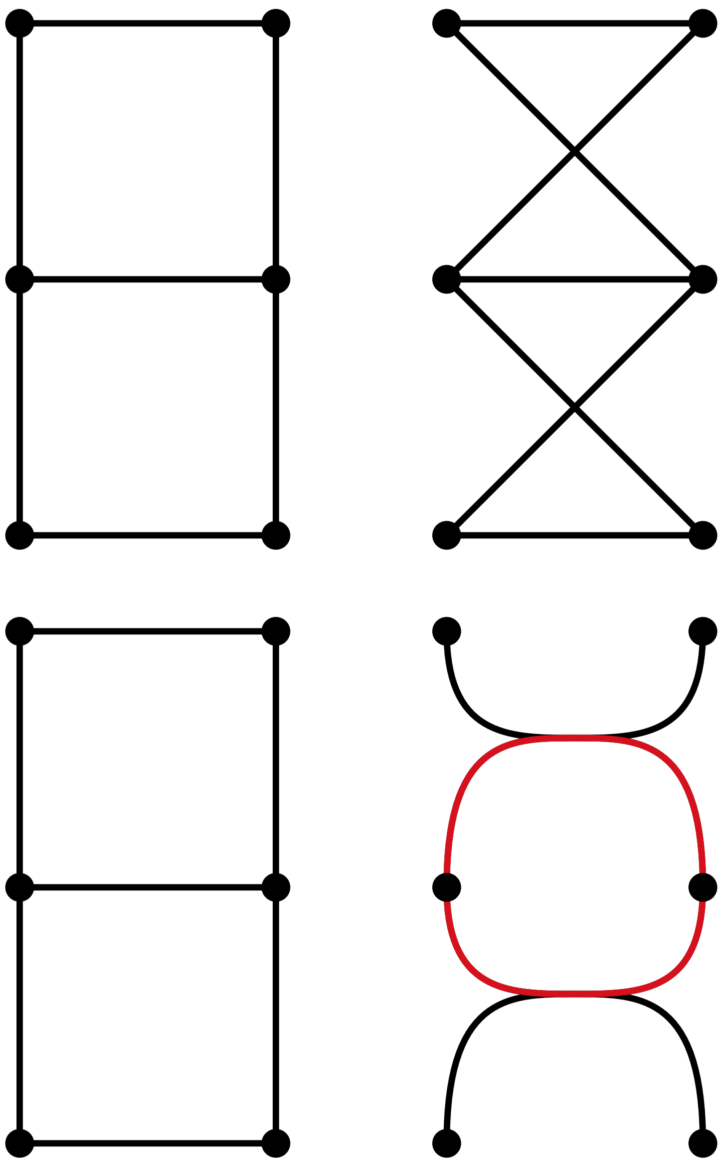

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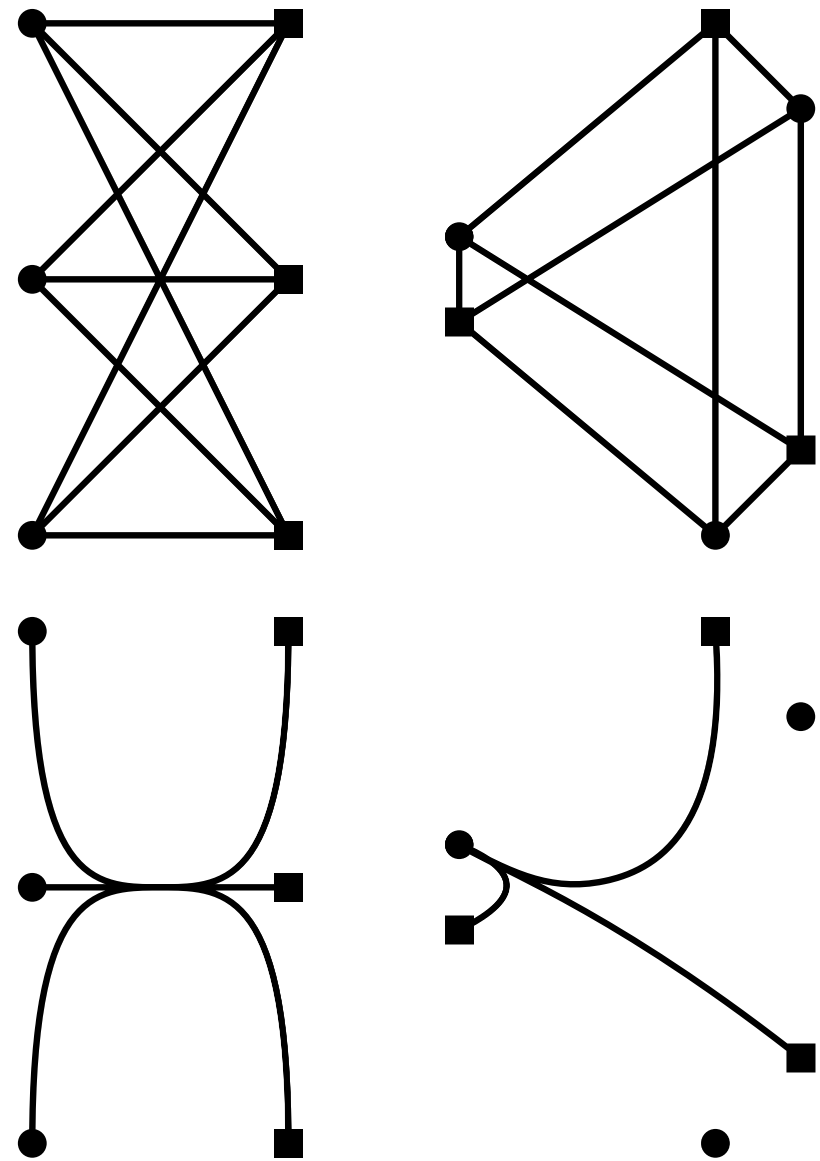

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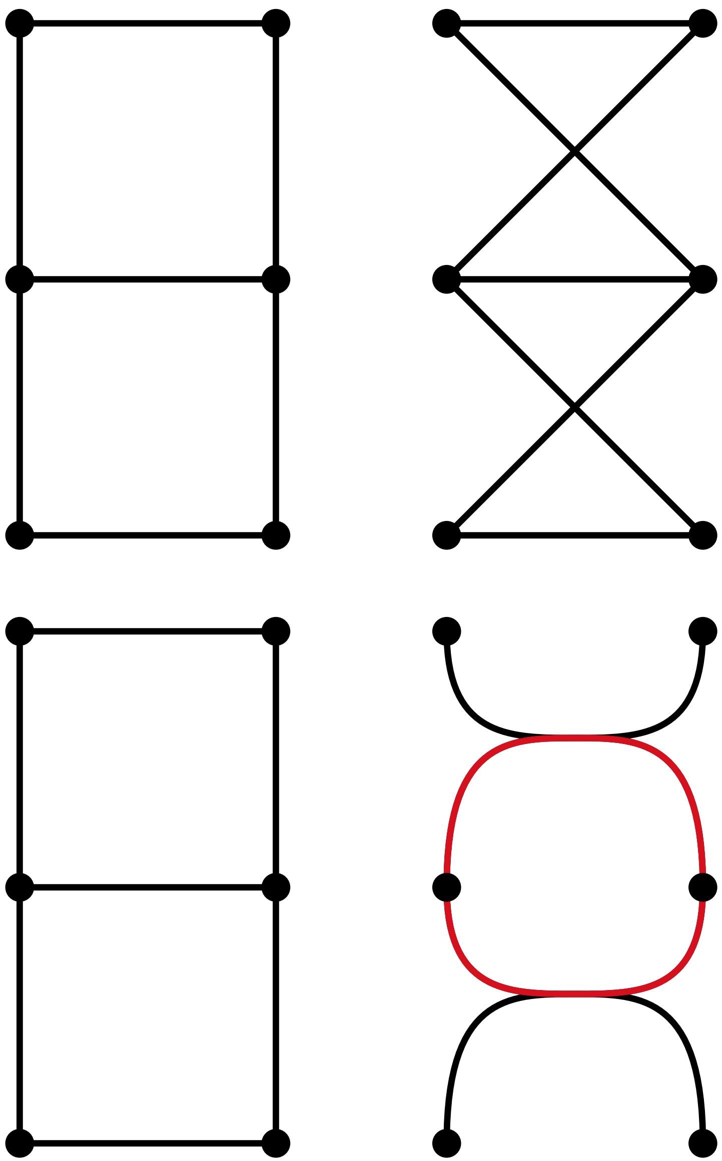

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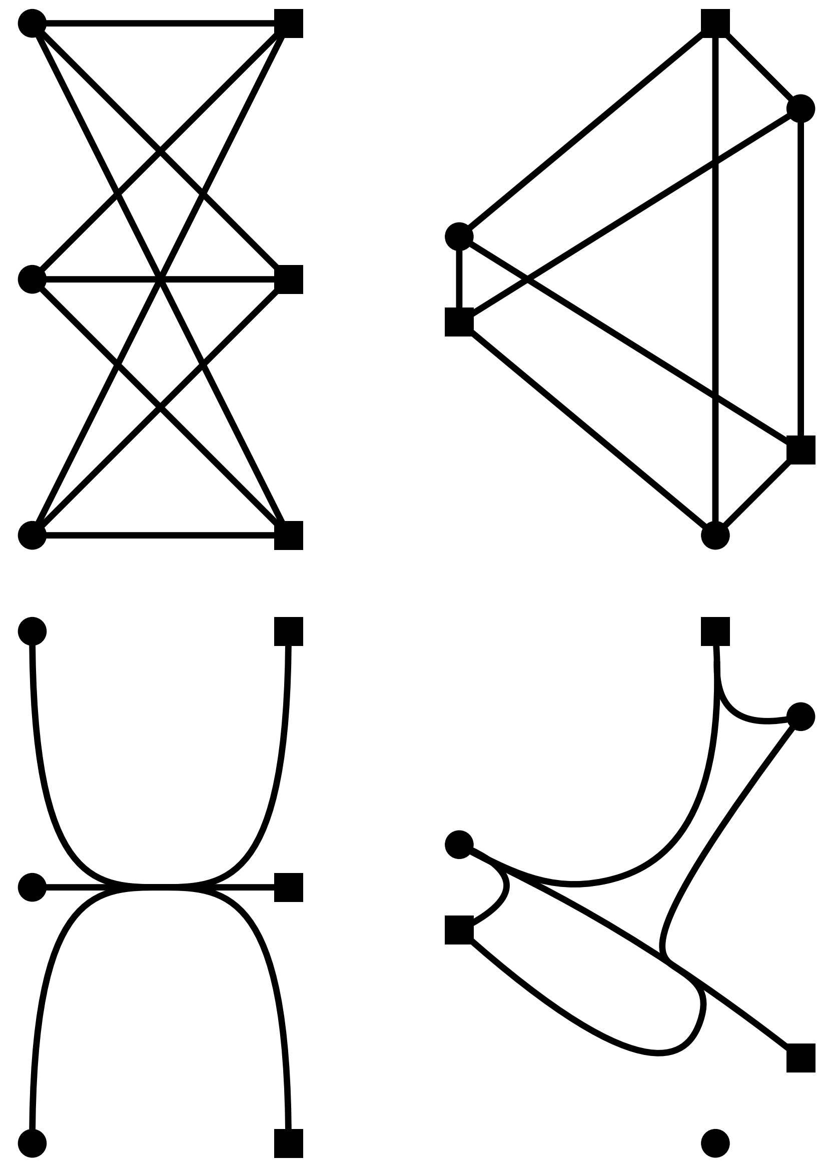

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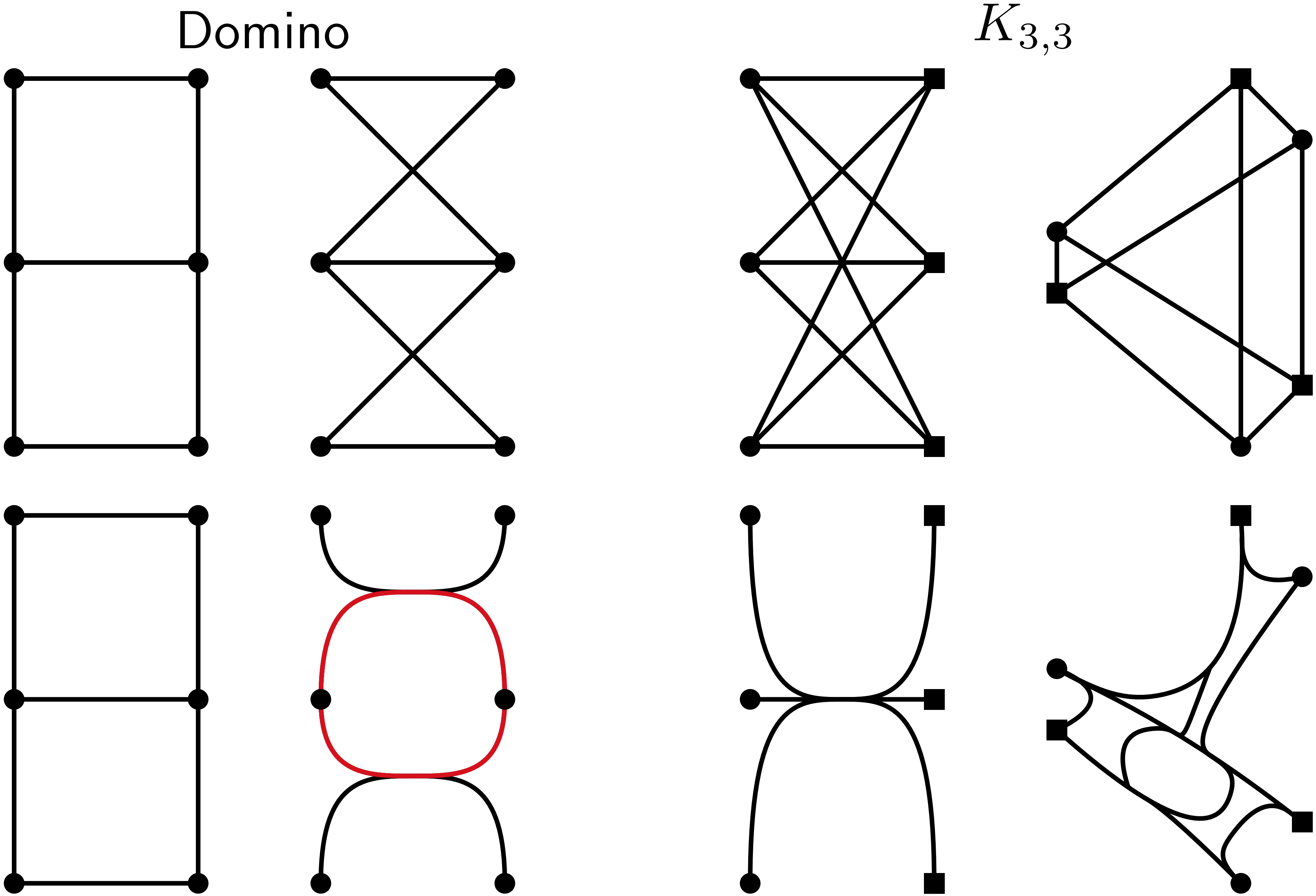

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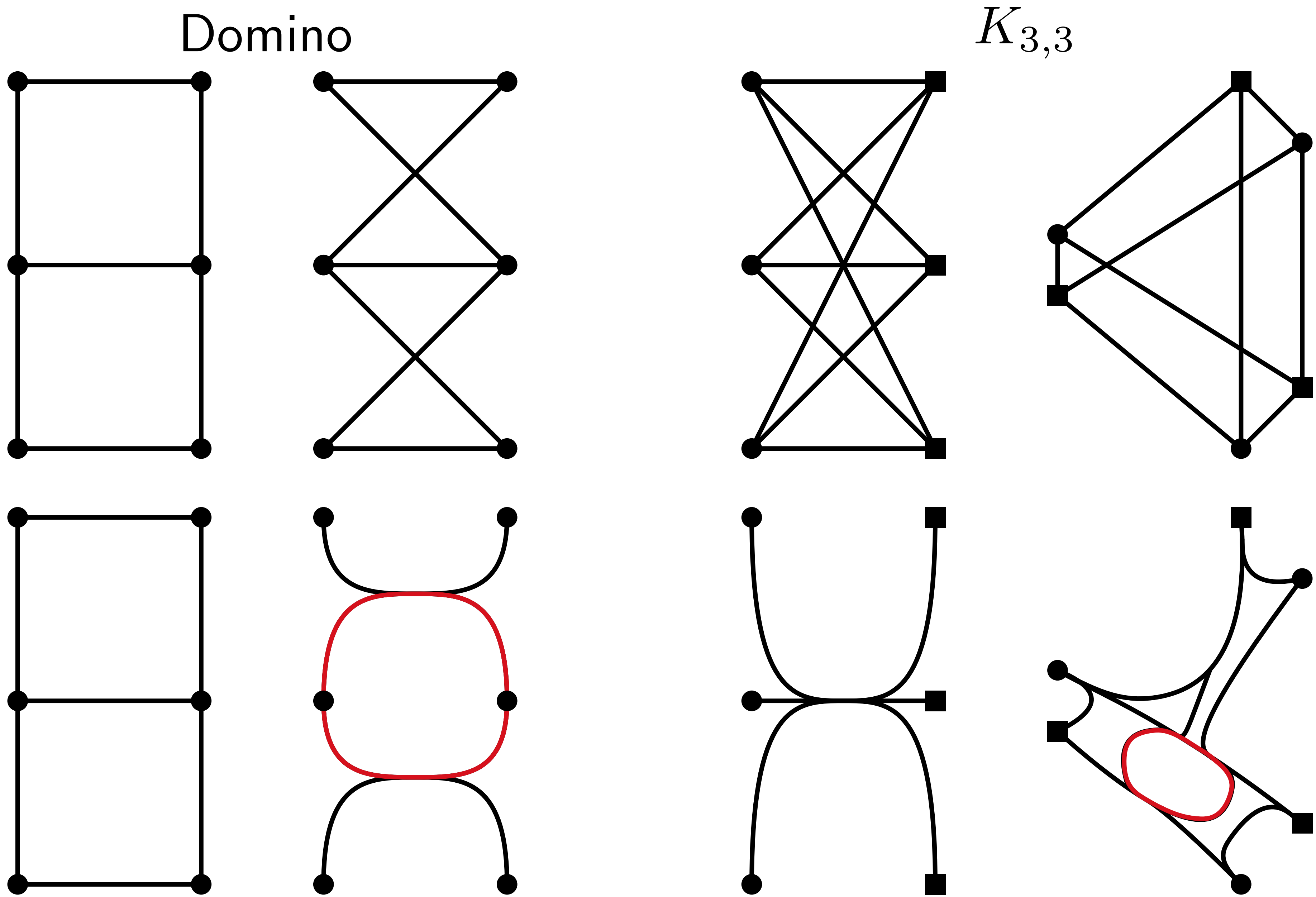

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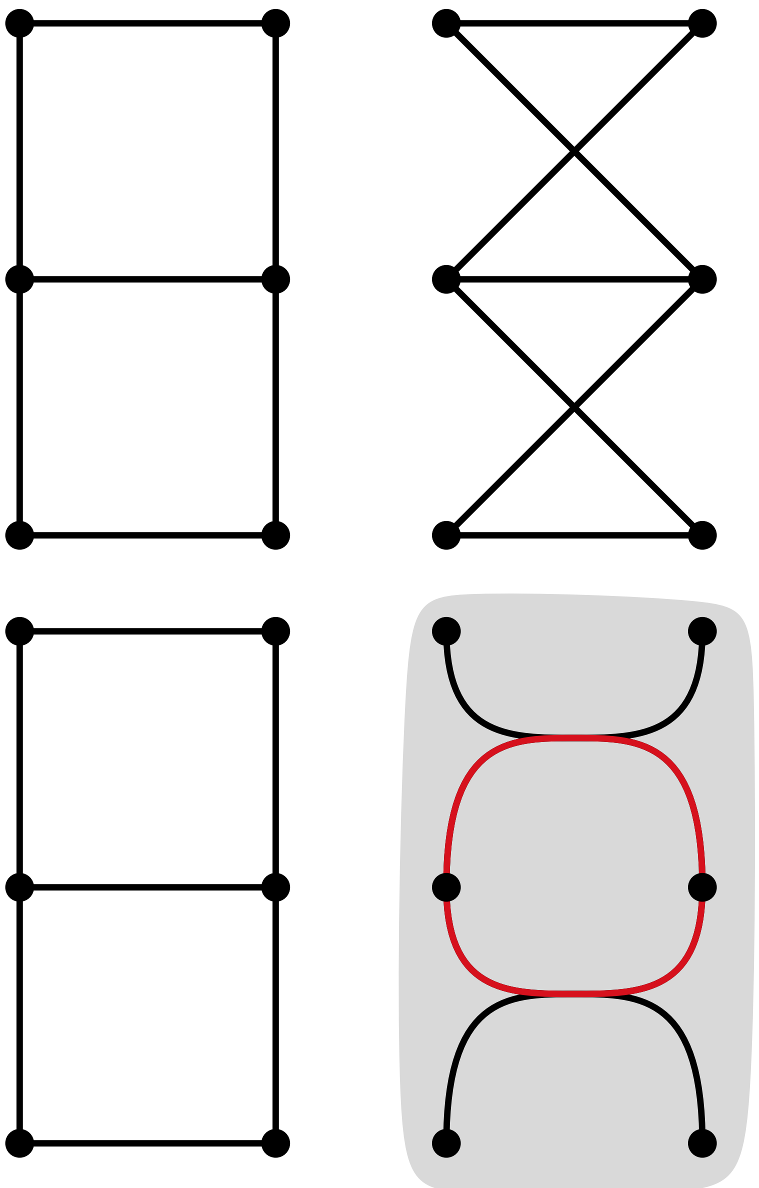
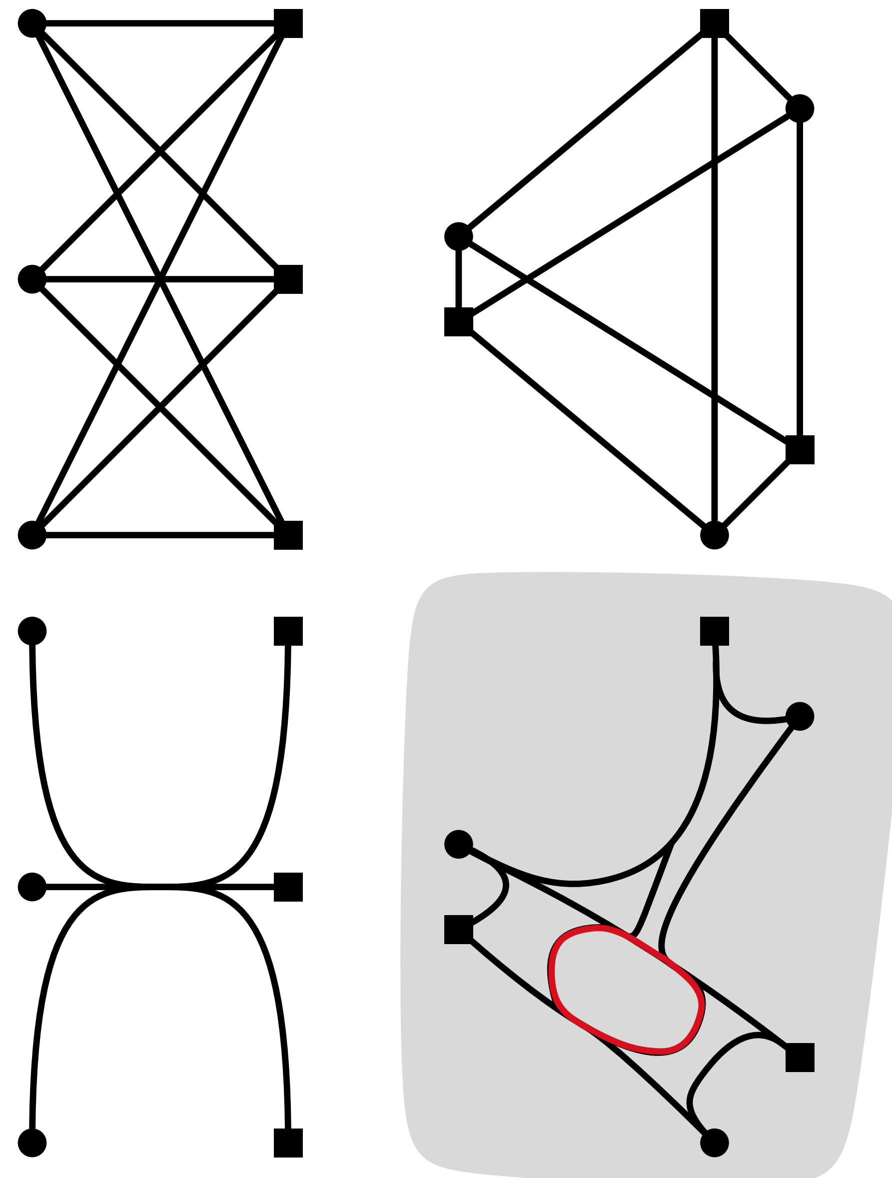


Twisted domino

$K_{3,3}$



Alternating $K_{3,3}$

SOC Drawings of Bipartite Permutation Graphs

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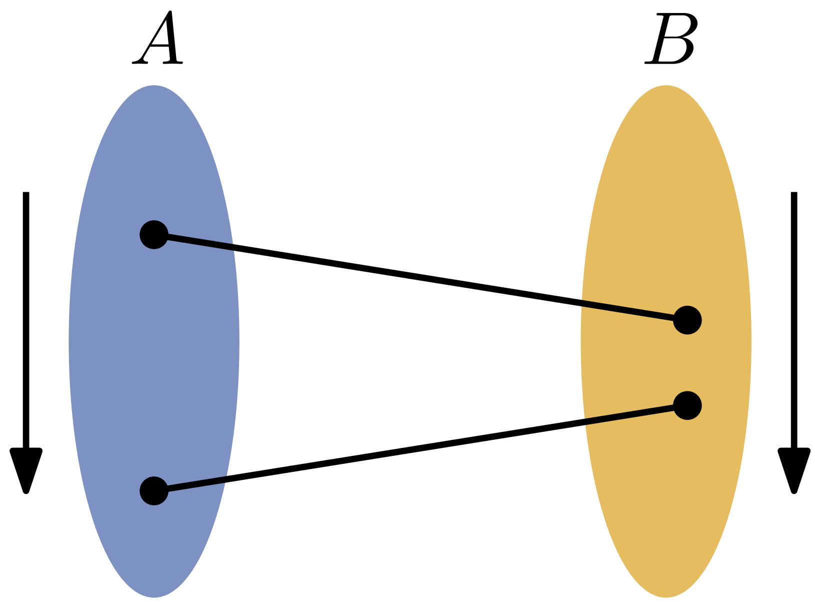

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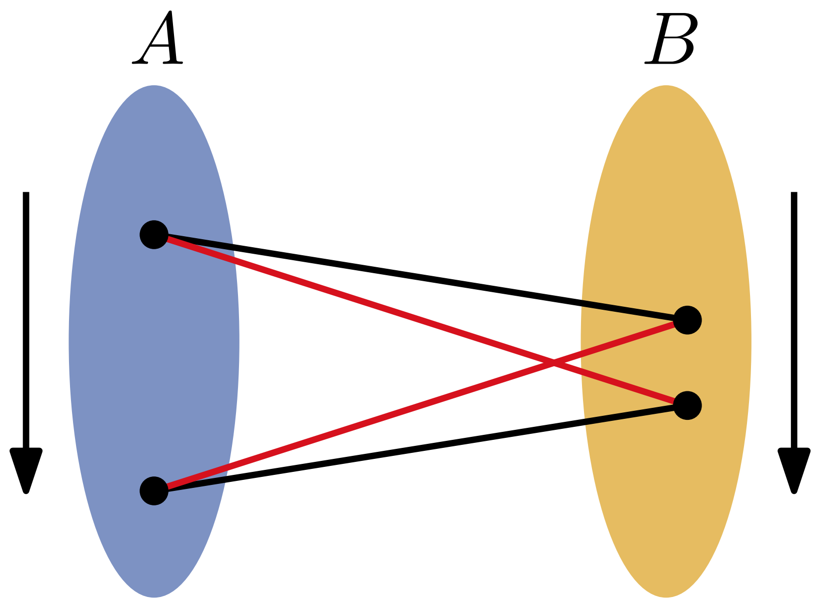

$\Rightarrow$ Confluent drawing easily possible (Formal proof [Hui et al. 09])

Domino graph is a Bipartite Permutation graph $\Rightarrow$ strictness?

BP Without Dominos

BP without dominos have soc drawings

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Draw given graph with algorithm by Hui et al.

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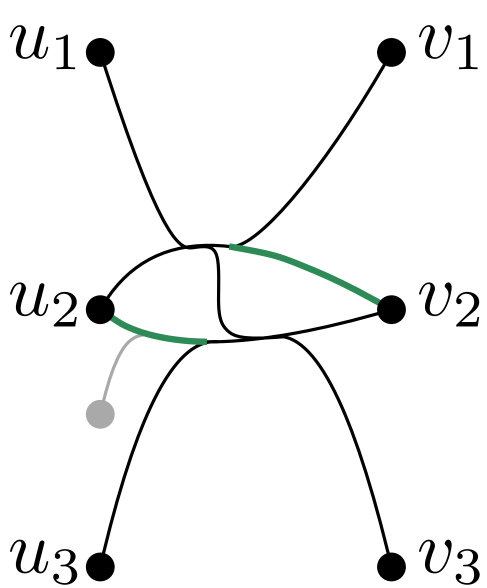

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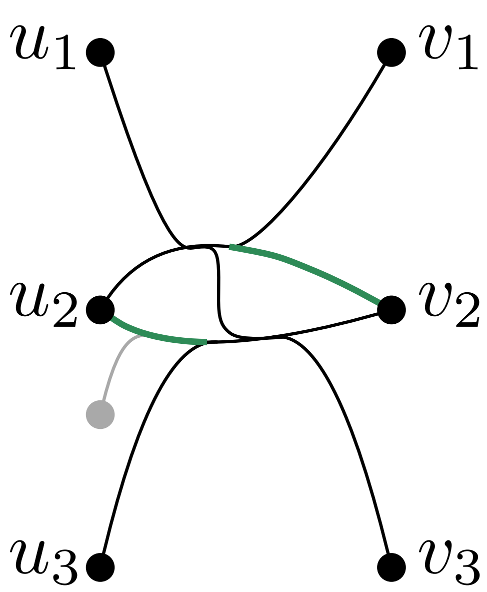
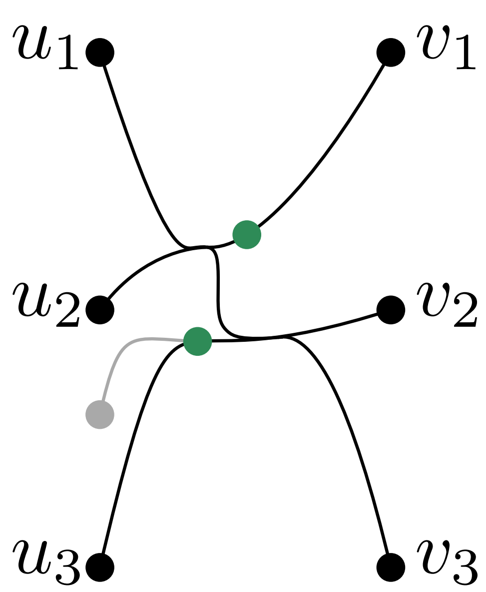
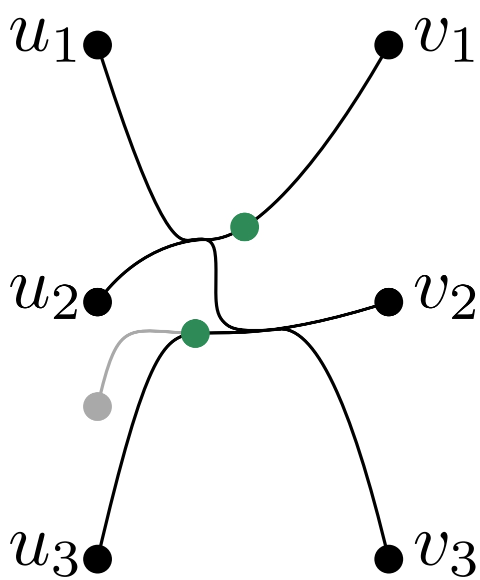




Bipartite Strict Confluent Drawings

Drawings such that



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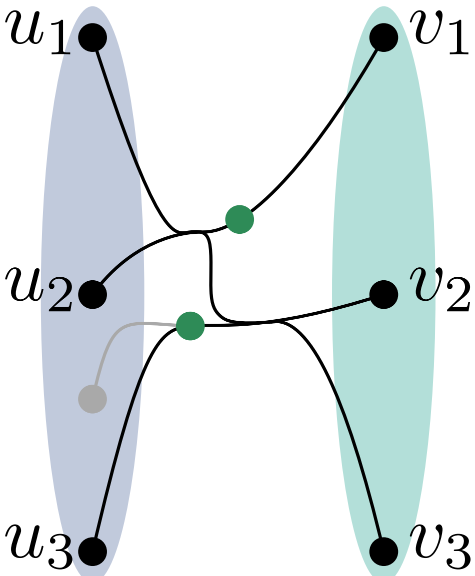

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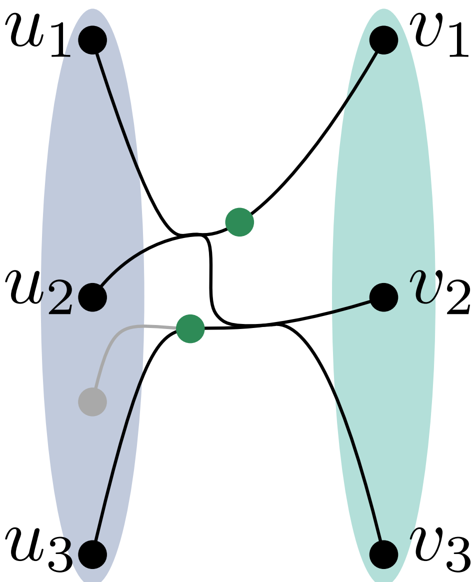

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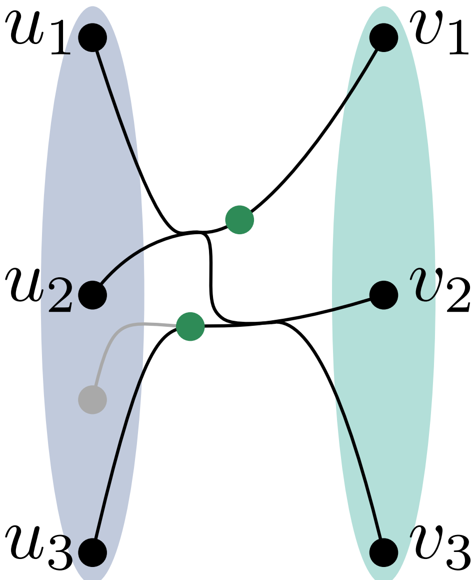

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Slide before

Bipartite permutation graphs without domino have such drawings

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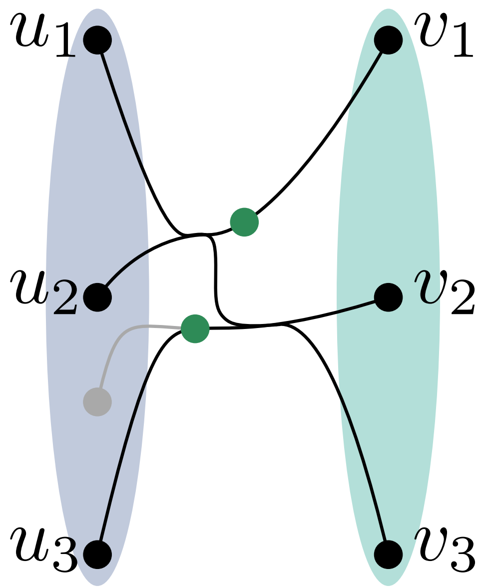

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Slide before

Bipartite permutation graphs without domino have such drawings

Such drawings are bipartite permutation graphs without dominos:

- Twisted domino is only order admitting bipartite confluent drawing
- But twisted domino can not be drawn strict outer-confluent

Counterexample for BP $\subset$ SOC