Scheduling Problems and Algorithms in Traffic and Transport

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Basic Rolling Stock Rostering Problem = Multicommodity Flow Problem

- Can be solved efficiently for networks with $10^9$ arcs

Constraints complicating rolling stock rostering

- Discretization: Space/Time ("Multiscale Problems")
- Robustness: Delay Propagation
- Path Constraints: Maintenance, Parking
- Configuration Constraints: Track Usage, Train Composition, Uniformity
Integrated Routing and Scheduling
Integrated Routing and Scheduling

Routing

Scheduling

Routing Problems in Traffic and Transport
Train Routes are Flexible in Space and Time
Scheduling Problems in Traffic and Transport
Track Allocation Graph

Scheduling Problems in Traffic and Transport
Track Allocation/Train Timetabling Problem

Combinatorial Optimization Problem

Path Packing Problem
Literature

- Charnes and Miller (1956), Szpiegel (1973), Jovanovic and Harker (1991),
- Semet and Schoenauer (2005),
- Caprara, Monaci, Toth and Guida (2005)
- Kroon, Dekker and Vromans (2005),
- Caprara, Kroon, Monaci, Peeters, Toth (2006)
- Fischer, Helmberg, Janßen, Krostitz (2008)
- Caimi (2009), Klabes (2010)
- ...
Path Packing Model

\((\text{APP})\) \quad \max \sum_{i \in I} \sum_{a \in A} c_{ai} x_{ai}^i

(i) \quad \sum_{a \in \delta^+_i(v)} x_{ai}^i - \sum_{a \in \delta^-_i(v)} x_{ai}^i = \beta_i(v) \quad \forall v \in V, i \in I \quad \text{Flow}

(ii) \quad \sum_{(a, i) \in k} x_{ai}^i \leq 1 \quad \forall k \in K \quad \text{Conflicts}

(iii) \quad x_{ai}^i \in \{0,1\} \quad \forall a \in A, i \in I \quad \text{Integ.}
Configuration Model

Scheduling Problems in Traffic and Transport
Packing- and Configuration Model

### (APP)

\[
\text{(APP)} \quad \max \sum_{i \in I} \sum_{a \in A} c_{ai} x_{ai}
\]

\[
(i) \quad \sum_{a \in \delta^+(v)} x_{ai} - \sum_{a \in \delta^-(v)} x_{ai} = \beta_i(v) \quad \forall v \in V, i \in I \quad \text{Flow}
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(ii) \quad \sum_{(a,i) \in k} x_{ai} \leq 1 \quad \forall k \in K \quad \text{Conflicts}
\]

\[
(iii) \quad x_{ai} \in \{0,1\} \quad \forall a \in A, i \in I \quad \text{Integ.}
\]

### (PCP)

\[
\text{(PCP)} \quad \max \sum_{i \in I} \sum_{p \in P_i} \sum_{a \in p} c_{ai} x_{ai}
\]

\[
(i) \quad \sum_{p \in P_i} x_{pi} \leq 1 \quad \forall i \in I \quad \text{Trains}
\]

\[
(ii) \quad \sum_{q \in Q_j} y_{q} \leq 1 \quad \forall j \in J \quad \text{Configs}
\]

\[
(iii) \quad \sum_{a \in p \in P} x_{ai} - \sum_{a \in q \in Q} y_{q} \leq 0 \quad \forall a \in A \quad \text{Coupling}
\]

\[
(iv) \quad x_{pi} \in \{0,1\} \quad \forall p \in P \quad \text{Integ.}
\]

\[
(v) \quad y_{q} \in \{0,1\} \quad \forall q \in Q \quad \text{Integ.}
\]
Theorem (B., Schlechte [2007]):

\[ v_{LP}(PCP) = v_{LP}(ACP) \]
\[ = v_{LP}(APP) = v_{LP}(PPP) \]
\[ \leq v_{LP}(APP'). \]

All LP-relaxations can be solved in polynomial time.

\[ v_{IP}(PCP) = v_{IP}(ACP) \]
\[ = v_{IP}(APP) = v_{IP}(PPP) \]
\[ = v_{IP}(APP'). \]
Packing- and Configuration Model

\[ \text{(APP)} \quad \text{max } \sum_{i \in I} \sum_{a \in A} c^i_a x^i_a \]

(i) \[ \sum_{a \in \delta^+_i(v)} x^i_a - \sum_{a \in \delta^-_i(v)} x^i_a = \beta_i(v) \quad \forall v \in V, i \in I \quad \text{Flow} \]

(ii) \[ \sum_{(a,i) \in k} x^i_a \leq 1 \quad \forall k \in K \quad \text{Conflicts} \]

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\[ \text{(PCP)} \quad \text{max } \sum_{i \in I} \sum_{p \in P_i} \sum_{a \in p} c^i_a x^i_p \]

(i) \[ \sum_{p \in P_i} x^i_p \leq 1 \quad \forall i \in I \quad \text{Trains} \]

(ii) \[ \sum_{q \in Q_j} y^i_q \leq 1 \quad \forall j \in J \quad \text{Configs} \]

(iii) \[ \sum_{a \in p \in P} x^i_p - \sum_{a \in q \in Q} y^i_q \leq 0 \quad \forall a \in A \quad \text{Coupling} \]

(iv) \[ x^i_p \in \{0,1\} \quad \forall p \in P \quad \text{Integ.} \]

(v) \[ y^i_q \in \{0,1\} \quad \forall q \in Q \quad \text{Integ.} \]
Configuration Model

(DUA) \( \min \sum_{i \in I} \gamma_i + \sum_{j \in J} \pi_j \)

(i) \( \gamma_i + \sum_{a \in p} \lambda_a \geq \sum_{a \in p} c_a \quad \forall p \in P_i, i \in I \) \text{ Paths}

(ii) \( \pi_j - \sum_{a \in q} \lambda_a \geq 0 \quad \forall q \in Q_j, j \in J \) \text{ Configs}

(iii) \( \gamma, \pi, \lambda \geq 0 \)

(PLP) \( \max \sum \sum \sum c^i_a x_p \)

(i) \( \sum_{p \in P_i} x_p \leq 1 \quad \forall i \in I \) \text{ Trains}

(ii) \( \sum_{q \in Q_j} y_q \leq 1 \quad \forall j \in J \) \text{ Configs}

(iii) \( \sum_{a \in p \in P} x_p - \sum_{a \in q \in Q} y_q \leq 0 \quad \forall a \in A \) \text{ Coupling}

(iv) \( x_p \geq 0 \quad \forall p \in P \) \text{ Integ.}

(v) \( y_q \geq 0 \quad \forall q \in Q \) \text{ Integ.}
(DUA) \( \min \sum_{i \in I} \gamma_i + \sum_{j \in J} \pi_j \)

(i) \( \gamma_i + \sum_{a \in p} \lambda_a \geq \sum_{a \in p} c^i_a \quad \forall p \in P_i, i \in I \) Paths

(ii) \( \pi_j - \sum_{a \in q} \lambda_a \geq 0 \quad \forall q \in Q_j, j \in J \) Configs

(iii) \( \gamma, \pi \geq 0 \)

**Proposition:**
Route pricing = acyclic shortest path problem with arc weights

\[ \bar{c}_a = -c_a + \lambda_a. \]
### Configuration Model

#### Proposition:

Config pricing = acyclic shortest path problem with arc weights

\[ \bar{c}_a = -\lambda_a. \]
(PLP) \[ \text{max} \sum_{i \in I} \sum_{p \in P_i} \sum_{a \in p} c^i_a x_p \]

(i) \[ \sum_{p \in P_i} x_p \leq 1 \quad \forall i \in I \quad \text{Trains} \]

(ii) \[ \sum_{q \in Q_j} y_q \leq 1 \quad \forall j \in J \quad \text{Configs} \]

(iii) \[ \sum_{a \in p \in P} x_p - \sum_{a \in q \in Q} y_q \leq 0 \quad \forall a \in A \quad \text{Coupling} \]

(iv) \[ x_p \geq 0 \quad \forall p \in P \quad \text{Integ.} \]

(v) \[ y_q \geq 0 \quad \forall q \in Q \quad \text{Integ.} \]
Lagrange Funktion des PCP

\[(\text{LD}) \quad \min_{\lambda \geq 0} \begin{bmatrix} \max_{Ax=1, \ x \in [0,1]^{|P|}} (u^T - \lambda^T C)x + \max_{By=1, \ y \in [0,1]^{|Q|}} (\lambda^T D)y \end{bmatrix} \]
Problem

Algorithm
- Subgradient
- Cutting Plane Model
- Update

Quadratic Subproblem

Primal Approximation

Inexact Bundle Method

\[ f(\lambda) := \min_{x \in X} c^T x + \lambda^T (b - Ax) \]

\[ \bar{f}_\mu(\lambda) = c^T x_\mu + \lambda^T (b - Ax_\mu) \]

\[ \hat{f}_k(\lambda) := \min_{\mu \in J_k} \bar{f}_\mu(\lambda) \]

\[ \lambda_{k+1} = \text{argmax} \hat{f}_k(\lambda) - \frac{u_k}{2} \| \lambda - \hat{\lambda}_k \|^2 \]

\[ \max \hat{f}_k(\lambda) - \frac{u_k}{2} \| \lambda - \hat{\lambda}_k \|^2 \iff \max v - \frac{u_k}{2} \| \lambda - \hat{\lambda}_k \|^2 \]

\[ \text{s.t. } v \leq \bar{f}_\mu(\lambda), \text{ for all } \mu \in J_k \]

\[ \Leftrightarrow \max \sum_{\mu \in J_k} \alpha_\mu \bar{f}_\mu(\hat{\lambda}) - \frac{1}{2u_k} \left\| \sum_{\mu \in J_k} \alpha_\mu (b - Ax_\mu) \right\|^2 \]

\[ \text{s.t. } \sum_{\mu \in J_k} \alpha_\mu = 1 \]

\[ 0 \leq \alpha_\mu \leq 1, \text{ for all } \mu \in J_k \]

\[ \| b - A\tilde{x}_k \| \to 0 \text{ } (k \to \infty) \]

\[ \tilde{x}_{k+1} = \sum_{\mu \in J_k} \alpha_\mu x_\mu \]
Problem

Algorithm
  ▶ Subgradient
  ▶ Cutting Plane Model
  ▶ Update
  ▶ Quadratic Subproblem

Primal Approximation

Inexact Bundle Method

Bundle Method

(Kiwi [1990], Helmberg [2000])

\[
f(\lambda) := \min_{x \in X} c^T x + \lambda^T (b - Ax)
\]

\[
\bar{f}_\mu(\lambda) = c^T x_\mu + \lambda^T (b - Ax_\mu)
\]

\[
\hat{f}_k(\lambda) := \min_{\mu \in J_k} \bar{f}_\mu(\lambda)
\]

\[
\lambda_{k+1} = \arg \max \hat{f}_k(\lambda) - \frac{u_k}{2} \|\lambda - \hat{\lambda}_k\|^2
\]

\[
\max \hat{f}_k(\lambda) - \frac{u_k}{2} \|\lambda - \hat{\lambda}_k\|^2 \iff \max \ \left\{ v - \frac{u_k}{2} \|\lambda - \hat{\lambda}_k\|^2 \right\}
\]

s.t. \( v \leq \bar{f}_\mu(\lambda) \), for all \( \mu \in J_k \)

\[
\sum_{\mu \in J_k} \alpha_\mu \bar{f}_\mu(\hat{\lambda}) - \frac{1}{2u_k} \left\| \sum_{\mu \in J_k} \alpha_\mu (b - Ax_\mu) \right\|^2
\]

s.t. \( \sum_{\mu \in J_k} \alpha_\mu = 1 \)

\( 0 \leq \alpha_\mu \leq 1 \), for all \( \mu \in J_k \)

\[
\|b - A\tilde{x}_k\| \rightarrow 0 \ (k \rightarrow \infty)
\]

\[
\tilde{x}_{k+1} = \sum_{\mu \in J_k} \alpha_\mu x_\mu
\]
Problem

Algorithm

- Subgradient
- Cutting Plane Model
- Update

Quadratic Subproblem

Bundle Method

(Kiwiel [1990], Helmberg [2000])

\[ f(\lambda) := \min_{x \in X} c^T x + \lambda^T (b - Ax) \]

\[ \bar{f}_\mu(\lambda) = c^T x_\mu + \lambda^T (b - Ax_\mu) \]

\[ \hat{f}_k(\lambda) := \min_{\mu \in J_k} \bar{f}_\mu(\lambda) \]

\[ \lambda_{k+1} = \arg\max \hat{f}_k(\lambda) - \frac{u_k}{2} \|\lambda - \hat{\lambda}_k\|^2 \]

\[ \max \hat{f}_k(\lambda) - \frac{u_k}{2} \|\lambda - \hat{\lambda}_k\|^2 \iff \max \left( v - \frac{u_k}{2} \|\lambda - \hat{\lambda}^k\|^2 \right) \]

\[ \text{s.t. } v \leq \bar{f}_\mu(\lambda), \text{ for all } \mu \in J_k \]

\[ \iff \max \sum_{\mu \in J_k} \alpha_\mu \bar{f}_\mu(\hat{\lambda}) - \frac{1}{2u_k} \left\| \sum_{\mu \in J_k} \alpha_\mu (b - Ax_\mu) \right\|^2 \]

\[ \text{s.t. } \sum_{\mu \in J_k} \alpha_\mu = 1 \]

\[ 0 \leq \alpha_\mu \leq 1, \quad \text{for all } \mu \in J_k \]

Primal $^2$-Approximation

Inexact Bundle Method

\[ \|b - A\tilde{x}_k\| \to 0 \ (k \to \infty) \]

\[ \tilde{x}_{k+1} = \sum_{\mu \in J_k} \alpha_\mu x_\mu \]
Bundle Method
(Kiwiæ [1990], Helmberg [2000])

Problem
Algorithm
  ▶ Subgradient
  ▶ Cutting Plane Model
  ▶ Update
Quadratic Subproblem

\[ f(\lambda) := \min_{x \in X} c^T x + \lambda^T (b - Ax) \]
\[ \bar{f}_\mu(\lambda) = c^T x_\mu + \lambda^T (b - Ax_\mu) \]
\[ \hat{f}_k(\lambda) := \min_{\mu \in J_k} \bar{f}_\mu(\lambda) \]
\[ \lambda_{k+1} = \arg\max \hat{f}_k(\lambda) - \frac{u_k}{2} \| \lambda - \hat{\lambda}_k \|^2 \]
\[ \max \hat{f}_k(\lambda) - \frac{u_k}{2} \| \lambda - \hat{\lambda}_k \|^2 \iff \max v - \frac{u_k}{2} \| \lambda - \hat{\lambda}_k \|^2 \]
\[ \text{s.t. } v \leq \bar{f}_\mu(\lambda), \text{ for all } \mu \in J_k \]

\[ \iff \max \sum_{\mu \in J_k} \alpha_\mu \bar{f}_\mu(\hat{\lambda}) - \frac{1}{2u_k} \left\| \sum_{\mu \in J_k} \alpha_\mu (b - Ax_\mu) \right\|^2 \]
\[ \text{s.t. } \sum_{\mu \in J_k} \alpha_\mu = 1 \]
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\[ \| b - A\tilde{x}_k \| \to 0 \ (k \to \infty) \]
\[ \tilde{x}_{k+1} = \sum_{\mu \in J_k} \alpha_\mu x_\mu \]

Primal Approximation
Inexact Bundle Method
Bundle Method

(Kiwiel [1990], Helmberg [2000])

- Problem
- Algorithm
  - Subgradient
  - Cutting Plane Model
  - Update
- Quadratic Subproblem

\[ f(\lambda) := \min_{x \in X} c^T x + \lambda^T (b - Ax) \]

\[ \overline{f}_{\mu}(\lambda) = c^T x_\mu + \lambda^T (b - Ax_\mu) \]

\[ \hat{f}_k(\lambda) := \min_{\mu \in J_k} \overline{f}_{\mu}(\lambda) \]

\[ \lambda_{k+1} = \arg\max \hat{f}_k(\lambda) - \frac{1}{2} \| \lambda - \hat{\lambda}_k \|^2 \]

\[ \max \hat{f}_k(\lambda) - \frac{1}{2} \| \lambda - \hat{\lambda}_k \|^2 \Leftrightarrow \max v - \frac{1}{2} \| \lambda - \hat{\lambda}_k \|^2 \]

s.t. \[ v \leq \overline{f}_{\mu}(\lambda), \text{ for all } \mu \in J_k \]

\[ \Leftrightarrow \max \sum_{\mu \in J_k} \alpha_\mu \overline{f}_{\mu}(\hat{\lambda}) - \frac{1}{2} u_k \left\| \sum_{\mu \in J_k} \alpha_\mu (b - Ax_\mu) \right\|^2 \]

s.t. \[ \sum_{\mu \in J_k} \alpha_\mu = 1 \]

\[ 0 \leq \alpha_\mu \leq 1, \text{ for all } \mu \in J_k \]

\[ \| b - A\tilde{x}_k \| \to 0 \text{ (} k \to \infty \text{)} \]

\[ \tilde{x}_{k+1} = \sum_{\mu \in J_k} \alpha_\mu x_\mu \]
Perturbation Branching

- Sequence of perturbed IP objectives $c_{j}^{i+1} := c_{j}^{i} - \alpha(x_{j}^{i})^2, \ \forall j, \ i=1,2,...$
- Fixing candidates in iteration $i$ $B_{i}^j := \{ j : x_{j}^{i} \geq 1 - \varepsilon \}$
- Potential function in iteration $i$ $v_{i}^j := c^{\top}x_{i} - w|B_{i}^j|$
- Go on while not integer and potential decreases, else
  - Perturb for $k_{\text{max}}$ additional iterations, if still not successful
    - Fix a single variable and reset objective every $k_{s}$ iterations
- Set of fixed variables (many) $B^* := B^{\text{argmin}} v_{i}^j$

Binary Search Branching

- Set of fixed variables (many) $B^* := \{j_1, \ldots, j_m\}, \ c_{j_1} \leq \ldots \leq c_{j_m}$$
- Sets $Q_{j}^k$ at perturbation branch $j$ $Q_{j}^k := \{ x : x_{j_1}=\ldots=x_{j_k}=1 \},$ $k=0,...,m$
- Branch on $Q_{j}^m$
  - Repeat perturbation branching to plunge
  - Backtrack to $Q_{j}^{\lfloor m/2 \rfloor}$ and set $m := \lfloor m/2 \rfloor$ to prune
(PRICE (x)) \( \exists \bar{p} \in P_i : \gamma_i < \sum_{a \in \bar{p}} (p_a - \lambda_a) \) 
\[ \eta_i := \max_{p \in P_i} \sum_{a \in p} (p_a - \lambda_a) - \gamma_i, \ \forall i \in I \implies \eta_i + \gamma_i \geq \sum_{a \in p} (p_a - \lambda_a) \ \forall i \in I, p \in P_i \]

(PRICE (y)) \( \exists \bar{q} \in Q_j : \pi_j < \sum_{a \in \bar{q}} \lambda_a \)
\[ \theta_j := \max_{q \in Q_j} \sum_{a \in \bar{q}} \lambda_a - \pi_j, \ \forall j \in J \implies \theta_j + \pi_j \geq \sum_{a \in q} \lambda_a \ \forall j \in J, q \in Q_j \]

(max\{\eta + \gamma, 0\}, max\{\theta + \pi, 0\}, \lambda) is feasible for (DLP)

\[ \beta(\gamma, \pi, \lambda) := \sum_{i \in I} \max\{\gamma_i + \eta_i, 0\} + \sum_{j \in J} \max\{\pi_j + \theta_j, 0\} \]

Lemma (BS [2007]): \( \nu_{LP}(PCP) \leq \beta(\gamma, \pi, \lambda) \)
Solving the LP-Relaxation

scenario 570 trains

objective value

column generation iterations
HaKaFu, req32, 1140 requests, 30 mins time windows
<table>
<thead>
<tr>
<th>Article</th>
<th>Stations</th>
<th>Tracks</th>
<th>Trains</th>
<th>Modell/Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>Szpigel [1973]</td>
<td>6</td>
<td>5</td>
<td>10</td>
<td>Packing/Enumeration</td>
</tr>
<tr>
<td>Brännlund et al. [1998]</td>
<td>17</td>
<td>16</td>
<td>26</td>
<td>Packing/ Lagrange, BAB</td>
</tr>
<tr>
<td>Caprara et al. [2002]</td>
<td>74 (17)</td>
<td>73 (16)</td>
<td>54 (221)</td>
<td>Packing/ Lagrange, BAB</td>
</tr>
<tr>
<td>B. &amp; Schlechte [2007]</td>
<td>37</td>
<td>120</td>
<td>570</td>
<td>Config/PAB</td>
</tr>
<tr>
<td>Caprara et al. [2007]</td>
<td>102 (16)</td>
<td>103 (17)</td>
<td>16 (221)</td>
<td>Packing/PAB</td>
</tr>
<tr>
<td>Fischer et al. [2008]</td>
<td>656 (104)</td>
<td>1210 (193)</td>
<td>117 (251)</td>
<td>Packing/Bundle, IP Rounding</td>
</tr>
<tr>
<td>Lusby et al. [2008]</td>
<td>??</td>
<td>524</td>
<td>66 (31)</td>
<td>Packing/BAP</td>
</tr>
<tr>
<td>B. &amp; Schlechte [2010]</td>
<td>37</td>
<td>120</td>
<td>&gt;1.000</td>
<td>Config/Rapid Branching</td>
</tr>
</tbody>
</table>

▷ BAB: Branch-and-Bound
▷ PAB: Price-and-Branch
▷ BAP: Branch-and-Price
Discretization and Scheduling
Detailed railway infrastructure data given by simulation programs (Open Track)

- Signals
- Switches
- Tracks (with max. speed, acceleration, gradient)
- Stations and Platforms
Simplon micrograph: 1154 nodes and 1831 arcs, 223 signals etc.
Simulation tools provide exact running and blocking times

Basis for calculation of minimal headway times
Simulation of all possible routes with appropriate train types
- Generation of artificial nodes – “pseudo” stations

- No interactions between train routes

- Macro network definition is based on set of train routes
Interaction of Train Routes

- Generation of artificial nodes – „pseudo“ stations

- Diverging of train routes

- The same holds for converging routes
Interaction of Train Routes

- Generation of artificial nodes – pseudo stations

- Crossing of train routes

- Two pseudo stations were generated
Reduced Macrograph
(53 nodes and 87 track arcs for 28 train routes)
- Frequently many macroscopic station nodes are in the area of big stations
- Further aggregation is needed

\[
k = \begin{array}{|c|c|}
\hline
EC & 2 \\
R  & 4 \\
GV Auto & 2 \\
GV Rola & 2 \\
GV SIM & 4 \\
GV MTO & 6 \\
\hline
\end{array}
\]
- Planned times in macro network are possible in micro network
- Valid headways lead to valid block occupations (no conflicts)
  ⇒ feasible macro timetable can be transformed to feasible micro timetable
Micro

- 12 stations
- 1154 OpenTrack nodes
- 1831 OpenTrack edges
- 223 signals
- 8 track junctions
- 100 switches
- 6 train types
- 28 "routes"
- 230 "block segments"

Macro

- 18 macro nodes
- 40 tracks
- 6 Train types

Mathematische Optimierung
Scheduling Problems in Traffic and Transport
Cumulative Rounding Procedure

- Compute macroscopic running time with specific rounding procedure
- Consider again routes of trains (represented by standard trains)
- Example with \( \Delta = 6 \)

<table>
<thead>
<tr>
<th>Station</th>
<th>Dep/Pass</th>
<th>Rounded</th>
<th>Buffer</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>11</td>
<td>12 (2)</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>20</td>
<td>24 (4)</td>
<td>4</td>
</tr>
<tr>
<td>D</td>
<td>29</td>
<td>30 (5)</td>
<td>1</td>
</tr>
</tbody>
</table>

**Theorem:** If micro-running time \( d \geq \Delta \) for all tracks of the current train route, the cumulative rounding error (buffer) is always in \([0, \Delta)\).
Complex Traffic at the Simplon

- Slalom route
  - ROLA trains traverse the tunnel on the “wrong” side

- Crossing of trains
  - complex crossings of AUTO trains in Iselle

- Conflicting routes
  - complex routings in station area Domodossola and Brig

Dense Traffic at the Simplon

Scheduling Problems in Traffic and Transport
Estimation of the maximum theoretical corridor capacity

- Network accuracy of 6s
- Consider complete routing through stations
- Saturate by additional cargo trains

- Conflict free train schedules in simulation software (1s accuracy)
Aggregation-Test (Micro->Macro->Micro)

- Microscopic feasible 4h (8:00-12:00) reference plan in Open Track
- Reproducing this plan by an Optimization run
- Reimport to Open Track
Theoretical Capacities

- 180 trains for network small (without station routing and buffer times)
- 196 trains for network big with precise routing through stations (without buffer times)
- 175 trains for network big with precise routing through stations and buffer times
- No delays, no early coming
- Feasible train routing and block occupation
- Timetable is valid in micro-simulation
Network big with buffer times
Network big with buffer times

<table>
<thead>
<tr>
<th>Time discretization dt/s</th>
<th>6</th>
<th>10</th>
<th>30</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of trains</td>
<td>196</td>
<td>187</td>
<td>166</td>
<td>146</td>
</tr>
<tr>
<td>Cols in IP</td>
<td>504314</td>
<td>318303</td>
<td>114934</td>
<td>61966</td>
</tr>
<tr>
<td>Rows in IP</td>
<td>222096</td>
<td>142723</td>
<td>53311</td>
<td>29523</td>
</tr>
<tr>
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Hypergraph Scheduling
Scheduling Problems in Traffic and Transport (Visualization based on JavaView)
### Wagenstandanzeiger Gleis 11

<table>
<thead>
<tr>
<th>Zeit</th>
<th>Zug</th>
<th>Richtung</th>
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<td>Jan Kiepura</td>
</tr>
<tr>
<td>05:36</td>
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<td>Warszawa / Warschau</td>
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<td>06:21</td>
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<td>Zugleitung in Hamm</td>
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<td>IC 2236</td>
<td>Dienstag bis Donnerstag</td>
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<td>Montag und Freitag</td>
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<td>Bremen / Dortmund</td>
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<td>IC 2132</td>
<td>Ostfriesland</td>
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<tr>
<td>11:40</td>
<td>IC 2046</td>
<td>Bremen / Oldenburg / Oldenburg</td>
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<td>IC 2130</td>
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<td>16:45</td>
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<td>19:40</td>
<td>IC 2144</td>
<td>So Köln</td>
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<td>20:45</td>
<td>IC 2132</td>
<td>Bremen / Dortmund / Oldenburg</td>
</tr>
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</table>
Uniformity

(Blue: Uniform, ..., Red: Irregular)

Scheduling Problems in Traffic and Transport  (Visualization based on JavaView)
Uniformity

(Blue/Yellow: Uniform, ..., Red: Irregular, Fat: Maintenance)

Scheduling Problems in Traffic and Transport  (Visualization based on JavaView)
Modelling Uniformity Using Hyperarcs
Hyperassignment Problem

**Definition:** Let \( D=(V,A) \) be a directed hypergraph w. arc costs \( c_a \)

- \( H \subseteq A \) hyperassigment :\( \iff \delta^+(v) \cap H = \delta^-(v) \cap H = 1 \)
- Hyperassignment Problem :\( \iff \arg\min c(H), \ H \text{ hyperassignment} \)

\[
\begin{align*}
\min & \quad c^T x \\
x(\delta^+(v)) &= 1 \quad \forall v \in V \\
x(\delta^-(v)) &= 1 \quad \forall v \in V \\
x & \in \{0,1\}^A 
\end{align*}
\]

**Literature**

- Cambini, Gallo, Scutellà (1992): Minimum cost flows on hypergraphs; solves only the LP relaxation
- Jeroslow, Martin, Rarding, Wang (1992): Gainfree Leontief substitution flow problems; does not hold for the hyperassignment problem

**Theorem:** The HAP is NP-hard (even for simple cases).
Theorem: The LP/IP gap of HAP can be arbitrarily large.
**Theorem:** The LP/IP gap of HAP can be arbitrarily large.
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**Theorem:** The LP/IP gap of HAP can be arbitrarily large.

**Proposition:** The determinants of basis matrices of HAP can be arbitrarily large, even if all hyperarcs have head and tail size 2.

**Proposition:** HAP is APX-complete for hyperarc head and tail size 2 in general and for hyperarc head and tail cardinality 3 in the relevant cases.
### Computational Results

(CPLEX 12.1.0)

| # rows (2 \Q^I) | # columns (A|I) | nonzeros | LP-IP gap | root gap | root improvement | # clique cuts | # other cuts | root run time (sec.) |
|-----------------|---------------|----------|-----------|----------|------------------|--------------|--------------|---------------------|
| 534             | 52056         | 140081   | 11.16 %   | 6.81 %   | 4.90 %           | 160          | 14           | 8                   |
| 620             | 80477         | 236020   | 8.72 %    | 0.00 %   | 9.54 %           | 120          | 2            | 29                  |
| 812             | 102375        | 216566   | 0.38 %    | 0.18 %   | 0.20 %           | 24           | 16           | 40                  |
| 1128            | 267542        | 732134   | 4.59 %    | 0.26 %   | 4.55 %           | 263          | 0            | 160                 |
| 1310            | 363513        | 1006024  | 7.85 %    | 0.22 %   | 8.28 %           | 378          | 2            | 270                 |
| 1496            | 469932        | 1369224  | 18.70 %   | 1.86 %   | 20.71 %          | 809          | 0            | 971                 |
| 1696            | 618348        | 1787078  | 5.17 %    | 0.16 %   | 5.28 %           | 925          | 0            | 1705                |
| 1746            | 649525        | 1859898  | 7.52 %    | 4.88 %   | 2.86 %           | 563          | 0            | 1129                |
| 1798            | 647650        | 1822718  | 13.60 %   | 0.95 %   | 14.65 %          | 537          | 0            | 1099                |
| 1798            | 647650        | 1822718  | 13.35 %   | 0.62 %   | 14.69 %          | 604          | 0            | 873                 |
| 2006            | 855153        | 2491372  | 5.76 %    | 0.68 %   | 5.39 %           | 1025         | 0            | 2490                |
| 2260            | 1079535       | 3138752  | 9.89 %    | 2.03 %   | 8.73 %           | 954          | 0            | 5483                |
| 2502            | 1290750       | 3680124  | 7.06 %    | 0.76 %   | 6.79 %           | 801          | 0            | 4583                |
| 2620            | 1432355       | 4187296  | 9.05 %    | 1.15 %   | 8.68 %           | 1068         | 0            | 7910                |
| 2624            | 1439453       | 4087042  | 14.17 %   | 5.23 %   | 10.41 %          | 951          | 0            | (*) 14400           |

(*) = aborted
Partitioned Hypergraph and Configurations

Scheduling Problems in Traffic and Transport
**Theorem:** There is an extended formulation of HAP with $O(V^8)$ variables that implies all clique constraints.

\[
\begin{align*}
\min & \quad \mathbf{c}^T \mathbf{x} \\
x(\delta^+(v)) &= 1 \quad \forall v \in V \\
x(\delta^-(v)) &= 1 \quad \forall v \in V \\
x &\in \{0,1\}^A \\
y(C^+(a)) &= x_a \quad \forall a \in A \\
y(C^-(a)) &= x_a \quad \forall a \in A \\
y &\in \{0,1\}^C
\end{align*}
\]
Stochastic Scheduling
Cost of delays

- 72 €/minute average cost of gate delay over 15 minutes, cf. EUROCONTROL [2004]
- 840 – 1200 millions € annual costs caused by gate delays in Europe

Benefits of robust planning

- Cost savings
- Reputation
- Less operational changes

The Tail Assignment Problem – assign legs to aircraft in order to fulfill operational constraints such as preassignments, maintenance rules, airport curfews, and minimum connection times between legs, cf. Grönkvist [2005]
Delay Propagation

Scheduling Problems in Traffic and Transport
Delay Propagation Along Rotations

EDP (bad)  

EDP (good)
Goal: Decrease impact of delays

- Primary delays: genuine disruptions, unavoidable
- Propagated delays: consequences of aircraft routing, can be minimized

Rule-oriented planning

- Ad-hoc formulas for buffers
- These rules are costly and it is uncertain how efficient they are
- Calibrating these rules is a balancing act: supporting operational stability, while staying cost efficient

Goal-oriented planning

- Minimize occurrence of delay propagation on average
Delay distribution

- Delays are not homogeneously spread in the network
- Stochastic model must captures properties of individual airports and legs

Structure of the stochastic model

- Gate phase, representing time spent on the ground
- Flight phase, representing time spent en-route

Phase durations are modelled by probability distribution

- $G_j$ is random variable for delay of gate phase of leg $j$
- $F_j$ is random variable for duration of flight phase of leg $j$
Robust Tail Assignment Problem

Mathematical model:

\[
\begin{align*}
\text{min} & \quad \sum_k \sum_{r \in R_k} d_r x_r^k \\
\sum_k \sum_{r: l \in r, r \in R_k} x_r &= 1 \quad \forall l \in L \\
\sum_k \sum_{p \in R_k} a_{bp} x_p^k &\leq r_b \quad \forall b \in B \\
\sum_{j \in R_k} x_{j}^k &= 1 \quad \forall k \\
x_r^k &\in \{0,1\} \quad \forall k, \forall r \in R_k
\end{align*}
\]

▷ Minimize non-robustness
▷ Cover all legs
▷ Fulfill side constraints
▷ One rotation for each aircraft
▷ Integrality

▷ Set partitioning problem with side constraints
▷ Problem has to be resolved daily for period of a few days
▷ Solved by Netline/Ops Tail xOPT (state-of-the-art column generation solver by Lufthansa Systems)
Column Generation

Scheduling Problems in Traffic and Transport

Start

Solve Tail Assignment Problem (LP)

Compute rotations

Compute prices

No

All fixed?

Yes

No

Conflict? Backtrack?

Yes

Fix rotations

Yes

No

Stop?

No

Yes

Stop
Column Generation

Start

Solve Tail Assignment Problem (LP)

Compute robust rotations

Compute prices

Stop?

All fixed?

No

Yes

Fix rotations

Conflict? Backtrack?

No

Yes

Yes

No

Stop
Robustness measure: total probability of delay propagation (PDP)

\[ d_r = \sum_{i \in r} P[PD_i^r > 0] \]

Resource constraint shortest path problem

\[
\min_{r \in R^k} \quad d_r - \sum_{i \in r} \pi_i + \sum_{b \in B} a_{br}\mu_b - \nu_k
\]

where \( PD_i^r \) is random variable of delay propagated to leg \( i \) in rotation \( r \) and \( \pi_i, \nu_k, \mu_b \) are dual variables corresponding to cover, aircraft, and side constraints

\[
\min_{r \in R^k} \sum_{i \in r} P[PD_i^r > 0] - \sum_{i \in r} \pi_i + \sum_{b \in B} a_{br}\mu_b - \nu_k
\]

To solve this problem one must compute \( PD_i^r \) along rotations.
Delay distribution $H_j$ of leg $j$

$\triangleright \quad H_j = G_j + F_j$

Delay propagation from leg $j$ to leg $k$ via buffer $b_{jk}$

$\triangleright \quad PD_k = \max( H_j - b_{jk} , 0)$

Delay distribution $H_k$ of next leg $k$

$\triangleright \quad H_k = PD_k + G_k + F_k$

and so on...
Convolution

- \( H = F + G \) and \( f, g \) and \( h \) are their probability density functions

\[
h(t) = \int_{0}^{t} f(x) g(t - x) dx
\]

Numerical convolution based on discretization

\[
\bar{h}_t = \sum_{i=1}^{t} \bar{f}_i (\bar{g}_{t-i} + \bar{g}_{t-i-1}) / 2
\]

where \( \bar{f}, \bar{g} \) are stepwise constant approximations of functions \( f, g \)

Alternative approaches

- Analytical convolution, cf. Fuhr [2007]
Path Search

Scheduling Problems in Traffic and Transport
Scheduling Problems in Traffic and Transport
Path Search

Scheduling Problems in Traffic and Transport
Path Search

Scheduling Problems in Traffic and Transport
Path Search
### Instance SC1: reference solution

- 100 legs, 16 aircraft, no preassignments, no maintenance
- Optimizer produces the same solution for each step size
- CPU time differs only in computation of the convolutions
- PDP values differ because of approximation error

<table>
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<tr>
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<th>step size [min]</th>
<th>CPU [s]</th>
<th>PDP</th>
<th>error [%]</th>
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<td>15.4</td>
<td>25.0586</td>
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<td>1.0</td>
<td>25.0672</td>
<td>0.15</td>
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### Instance SC1: optimized solution

- Different discretization step sizes may produce different solutions
- CPU time and PDP are not straightforward to compare

<table>
<thead>
<tr>
<th>Step size [min]</th>
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<td>SC1 1</td>
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<td>SC1 2</td>
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<td>SC1 3</td>
<td>20.0651</td>
<td>29</td>
<td>19.7239</td>
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<tr>
<td>SC1 4</td>
<td>20.3353</td>
<td>31</td>
<td>19.7562</td>
</tr>
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Test Instances

Analyzed data

▷ approx. 350000 flights / 300 – 650 flights per day
▷ 28 months, 4 subfleets
▷ European airline with hub-and-spoke network

Test instances

▷ We optimize single day instances of one subfleet
▷ Data for 4 months, no maintenance rules and preassignments

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<tr>
<th></th>
<th>min</th>
<th>max</th>
<th>avg</th>
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</tr>
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<td>44</td>
<td>12</td>
</tr>
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<td>February</td>
<td>22</td>
<td>94</td>
<td>15</td>
</tr>
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<td>March</td>
<td>21</td>
<td>94</td>
<td>15</td>
</tr>
<tr>
<td>April</td>
<td>27</td>
<td>93</td>
<td>15</td>
</tr>
</tbody>
</table>
Probability of delay

- Depends on day time and departure airport

![Probability of delay graph]

Probability of departure delay during the day on various airports

Distribution of delay

- Independent of daytime and departure airport

![Distribution of delay graph]

Distribution of the length of gate primary delays on various airports

Gate phase

- Gate delay distribution $G_j$ of flight $j$

$$
\Pr[G_j = x] = \begin{cases} 
1 - p_j & x = 0 \\
p_j \ln(x, \mu, \sigma) & x > 0
\end{cases}
$$

where $\ln()$ is probability density function of Log-normal distribution with Power-law distributed tail and $p_j = c(t(j), a(j))$, $t(j)$ is departure time of flight $j$ and $a(j)$ is departure airport of flight $j$
Distribution of deviation from scheduled duration

- Depends on scheduled leg duration

\[
\Pr[F_j = x] = \text{Llg}(x + l_j, \alpha_j, \beta_j) \quad x \in \mathbb{R}
\]

where \(\text{Llg}()\) is probability density function of Log-logistic distribution and \(l_j\) is scheduled flight duration of leg \(j\).

Histogram of the flight duration and its representation by random variable. Left: scheduled flight duration 80 minutes, right: scheduled flight duration 45 minutes.
Parameters of the model:

- $p$ for every airport and day hour
- $\mu, \sigma$
- $\alpha, \beta$ for every flight length
- Parameters are estimated by automatic scripts in R and quality is proofed by Chi-Square test.

Model applied to South American airline data

Validation of various assumptions of the model

- Stability of parameters over time, ...
ORE

- Standard KPI method
- Bonus for ground buffer minutes
- Threshold value for maximal ground buffer time (15 minutes)

PDP

- Total probability of delay propagation

<table>
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<th>PDP</th>
<th>Savings</th>
</tr>
</thead>
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<td>EAD [min]</td>
<td>CPU [s] PDP</td>
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<td>26 414,51</td>
<td>28488</td>
<td>28 395,46</td>
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<td>February</td>
<td>22 540,48</td>
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<td>31 530,42</td>
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<td>March</td>
<td>21 516,69</td>
<td>30363</td>
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<tr>
<td>April</td>
<td>27 465,48</td>
<td>34453</td>
<td>42 449,16</td>
</tr>
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</table>

Scheduling Problems in Traffic and Transport
ORC vs. PDP on a single disruption scenario

- ORC outperforms PDP only in 21% of cases
- PDP saves on average 29 minutes of arrival delay
- For more disrupted days, PDP saves on average 62 minutes of arrival delay

Estimation of monetary savings by the cost model developed based on EUROCONTROL [2004]

Lufthansa Systems estimates annual saving of the method in the tail assignment to 300,000 € for short haul carrier with 30 aircraft

Application in other planning stages may increase the benefit
The 21st International Symposium on Mathematical Programming (ISMP) take place in Berlin, Germany from August 19 - 24 2012.

ISMP is a scientific meeting held every 3 years on behalf of the Mathematical Optimization Society. Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed diam nonumy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat.

GENERAL INFORMATIONS

WE MEET IN BERLIN
2012/08/19
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consequat. Duis autem vel eum iuure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi.

ANOTHER HEADLINE
2012/08/19
Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed diam nonumy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo.
Thank you for your attention

PD Dr. habil. Ralf Borndörfer

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