Large digraphs of given degree and diameter and their properties

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Introduction

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Degree-diameter problem for graphs and digraphs: determination of the largest number $n(d,k)$ of vertices in a graph (digraph) of a given maximum degree d and diameter k

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Restricted classes of graphs and digraphs: vertex-transitive, Cayley, bipartite graphs (digraphs), graphs (digraphs) embaddable in a fixed surface...

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- Moore digraphs: $d = 1$ ......C_{k+1}, $k \geq 1$;

  $k = 1$.....K_{d+1} for $d \geq 1$.

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- There is no general upper bound on $vt(d,k)$ better than $M'(d,k) - 1$, for $d, k \geq 2$.

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- Gómez digraphs: a new family of large vertex transitive digraphs, giving the bound $n(d,k) \geq (d+[k-1/2])!/(d-[k+1/2])!$ for $k \geq 3$ and $d \geq \lceil \frac{k+1}{2} \rceil$.

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$$x_1 x_2 \dots x_k \to \begin{cases} x_2 x_3 x_4 \dots x_{k+1}, & x_{k+1} \neq x_1, x_2, \dots, x_k \\ x_2 x_3 x_4 \dots x_k x_1 \\ x_1 x_3 x_4 \dots x_k x_2 \\ x_1 x_2 x_4 \dots x_k x_3 \\ \dots \\ x_1 x_2 x_3 \dots x_k x_{k-1} \end{cases}$$

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- Order: $(d+1)_k = (d+1)!/(d-k+1)!$, diameter k, and d-regular.

Theorem

The automorphism group of the Faber-Moore-Chen digraphs is isomorphic to S_{d+1} acting on $V(\Gamma)$ in a natural way, that is, for any $\sigma \in S_{d+1}$, the assignment $x_1 \dots x_k \mapsto \sigma(x_1)\sigma(x_2)\dots\sigma(x_k)$ defines an automorphism of Γ.

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Theorem

The digraph $\Gamma(d,k)$ is a Cayley digraph if and only if

- *a) $d = k$*
- *b) $d = k + 1$ and any k,*
- *c) $d = q - 1$, where q is a prime power and $k = 2$.*
- *d) $d = q$, where q is a prime power and $k = 3$,*
- *e) $d = 10$ and $k = 4$,*
- *f) $d = 11$ and $k = 5$.*

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- We show that the Cayley digraphs arising from the Faber-Moore-Chen construction can be derived directly from the 'first principles': just from the definition of sharply k-transitive groups for $k \geq 2$, and not involving any theory of such groups and any prior knowledge of the Faber-Moore-Chen digraphs.

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- If this element g is unique, then G is sharply k-transitive.

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Proposition 1 Let G be a sharply 2-transitive permutation group on a finite set S with $|S| \geq 2$, so that $|G| = |S|(|S| - 1)$. Then, there exists a Cayley digraph Γ_1 of the group G of degree $|S| - 1$ and diameter 2.

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Proposition 2 Let G be a sharply 3-transitive permutation group on a finite set S with $|S| \geq 3$, so that $|G| = |S|(|S| - 1)(|S| - 2)$. Then, there exists a Cayley digraph Γ_2 of the group G of degree $|S| - 1$ and diameter 3.

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The digraph Γ_1 is isomorphic to the digraph $\Gamma(d,2)$, and the digraph Γ_2 is isomorphic to the digraph $\Gamma(d,3)$.

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- Order: tln^l and diameter at most $l(r+t) - 1$.

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Results

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Theorem

If the input digraph Γ *is* $r, r+1, r+2$*-alternately reachable and vertex-transitive, then the order of the automorphism group of the Comellas-Fiol digraph* $CF(\Gamma, l, 1)$ *is* $|Aut(CF(\Gamma, l, 1))| = l.|Aut(\Gamma)|^l$*. Consequently,* $Aut(CF(\Gamma, l, 1)) \cong [Aut(\Gamma)]^l \rtimes \mathbb{Z}_l$*.*

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- Computer experiments: the automorphism group H of the output digraph may be bigger than the group from our main result: $|H| > lt|G|^l$.
- **Example:** Consider the digraph Γ_1 with the vertex set $V(\Gamma_1) = Z_6$ and the dart set $(i, i+1), (i, i+2), i \in Z_6$. Let the automorphism group of the digraph Γ_1 and corresponding output digraph $CF(\Gamma_1, l, 2)$ for the parameters $l = t = 2$ be G_1 and H_1, respectively. With the help of a computer we have $|G_1| = 6$ and $|H_1| = 576 > 2.2.|G_1|^2 = 144$.

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- Open problem: Automorphism group of Gómez's digraphs.

Thank you for your attention.