Tutorial 9, November 28, 2019

1. Explain each equality in the computation showing by the Moore–Osgood theorem 1 continuity at a of the uniform limit.
2. Show that the two definitions of the limit by $\mathcal{X}$ are equivalent.
3. Find some points $x \in \mathbb{R}$ where $(\cos(nx))$ does not converge.
4. (HW 3 pts.) Prove that $\sqrt{x^2 + \frac{1}{n^2}} \rightrightarrows |x|$ (on $\mathbb{R}$).
5. Explain why the Moore–Osgood theorem 2 generalizes Moore–Osgood theorem 1.
6. (HW 3 pts.) Is it true that
$$\lim_{n\to\infty} \int_0^1 f_n = \int_0^1 \lim_{n\to\infty} f_n \ ,$$
when $f_n(x) = nx(1-x)^n$?
7. Compute the limit
$$\lim_{n\to\infty} \int_0^{\pi/2} (\sin^{n+1} x - \sin^n x)\, dx$$
(and justify your computation).
8. (HW 3 pts.) Compute the limit
$$\lim_{n\to\infty} \int_0^1 (1 + x/n)^n \, dx$$
(and justify your computation).
9. Let us do this again in a general form. Let $f_n \to f$ on M but $f_n \not\rightrightarrows f$ on M. Prove that there is no inclusion-maximal set $A \subset M$ with $f_n \rightrightarrows f$ on A.