Tutorial 10, December 5, 2019

1. Prove that $\rightrightarrows$ of a series of functions is equivalent to the uniform Bolzano–Cauchy condition.

2. (HW 3 pts.) Prove that every broken line $f\colon [a,b] \to \mathbb{R}$ has on (a,b) a primitive (function) g, with arbitrary prescribed value $g(c) = d$ for any c in (a,b).

3. Is $\sum_{n=0}^{\infty} \frac{x^n}{n!} \rightrightarrows$ on $\mathbb{R}$?

4. Prove that the power series $\sum_{n\ge 0} n!x^n$ converges for no $x \ne 0$.

5. Let $p \in \mathbb{R}[x]$ be any polynomial. Determine the convergence radius of $\sum_{n\ge 0} p(n)x^n$.

6. Determine the convergence radii of
$$\sum_{n=0}^{\infty} \frac{4x^n}{3^n - 2n + 1} \quad \text{and} \quad \sum_{n=20}^{\infty} (5^n - 200n^2 + 7n - 2019)x^{3n}\ .$$

7. (HW 3 pts.) Determine the convergence radii of
$$\sum_{n=0}^{\infty} \binom{2n}{n} x^n \quad \text{and} \quad \sum_{n=0}^{\infty} (-1)^n x^{n^2}\ .$$

8. (HW 3 pts.) Yes or no: a nonzero function $f(x) = \sum_{n\ge 0} a_n x^n \colon \mathbb{R} \to \mathbb{R}$ that is given as a sum of a power series has, like a nonzero polynomial, only finitely many zeros (points $a \in \mathbb{R}$ with $f(a) = 0$).

9. If $\sum_{n\ge 0} a_n x^n$ and $\sum_{n\ge 0} b_n x^n$ are power series with positive convergence radii, what can be said about the convergence radius of
$$\sum_{n\ge 0} (a_n + b_n)x^n\ ?$$

10. If $\sum_{n\ge 0} a_n x^n$ and $\sum_{n\ge 0} b_n x^n$ are power series with positive convergence radii, what can be said about the convergence radius of
$$\sum_{n\ge 0} \left(\sum_{k=0}^{n} a_k b_{n-k} \right) x^n\ ?$$