Matematické struktury tutorial 8 on April 10, 2017: continuous maps and constructions of new topological spaces from old ones

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I went through the sections 3 and 4, pp. 101–105, of Chapter 5 "Topologie" of the lecture notes

- A. Pultr, *Matematické struktury*, 2005, 155 pp., available at `http://kam.mff.cuni.cz/~pultr/`

The most important thing mentioned is perhaps the notion of *product topology*. If (X_i, τ_i), $i \in J$, is any system of topological spaces, then the product topology (X, τ) on

$$X = \prod_{i \in J} X_i = \{(\alpha_i :\ i \in J) \mid \alpha_i \in X_i\}$$

is defined as generated by the subbase

$$\{\{(\alpha_i :\ i \in J) \mid \alpha_j \in U_j\} \mid j \in J, U_j \in \tau_j\}\ .$$

Equivalently, τ is given by the base

$$\{\{(\alpha_i :\ i \in J) \mid \alpha_{j_1} \in U_{j_1}, \dots, \alpha_{j_k} \in U_{j_k}\} \mid j_l \in J, U_{j_l} \in \tau_{j_l}, l = 1, 2, \dots, k\}$$

— as an exercise check that this set system on X satisfies the two conditions for a base of a topology. In plain speech, $U \subseteq X$ is open, belongs to τ, if and only if for every J-tuple $\alpha = (\alpha_i :\ i \in J) \in U$ we can find a *finite* (even though J may be infinite) set of coordinate indices $S \subseteq J$ and some open sets $U_s \in \tau_s$, $s \in S$, corresponding to them so that the J-tuples that for each index $s \in S$ have the corresponding coordinate in U_s (and have other coordinates arbitrary) contain α and all belong to U.

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