Addition to the 1st lecture

- affine hypersurface If F is a field then n-dim. affine space (over F) is just $A_n := F^n = \{\bar{a} = (a_1, a_2, \dots, a_n) \mid a_i \in F\}$. A hypersurface S in F^n ($n \in \mathbb{N} = \{1, 2, \dots\}$) is given by a (nonzero) polynomial $P \in F[x_1, x_2, \dots, x_n]$ and is the set

$$S = \{\bar{a} \in F^n \mid P(\bar{a}) = 0_F\}$$

of all solutions ~~of~~ to the equation $P = 0$, in F^n.

To explain projective first we have to introduce the projective space. Let F be a field and $n \in \mathbb{N}$. Then the n-dim. proj. space $\mathbb{P}_n$ is given as a set partition

$$\mathbb{P}_n := A^*_{n+1}/\sim,$$

where $A^\times_{n+1} := \{\bar{a} \in A_{n+1} \mid \bar{a} \neq \bar{0}_F = (0_F, 0_F, \dots, 0_F)\}$, by the following equivalence relation $\sim$: (2)

$\bar{a}, \bar{b} \in A^{\times}_{n+1}$ then $\bar{a} \sim \bar{b} \Leftrightarrow \lambda \cdot \bar{a} = \bar{b}$ for some $\lambda \in F^{\times}$. Thus the elements of $\mathbb{P}_n$ are the elements of A_{n+1} $\neq \bar{0}$, up to rescaling by a ~~...~~ non-zero element of F. We write the elements $\bar{a} \in \mathbb{P}_n$ as $\bar{a} = (a_0 : a_1 : a_2 : \dots : a_n)$, $a_i \in F$ and not all 0_F. For $\mathbb{P}_n$, $n \geq 0$, we have the ~~recursive~~ recurrent (?) structure

$$\mathbb{P}_0 = \{[(1_F)]_{\sim}\} = \{\{(a) \mid a \in F, a \neq 0_F\}\}$$ and for $n \geq 1$:

$$\mathbb{P}_n = A_n \dot{\cup} \mathbb{P}_{n-1}$$ where

"A_n" $= \{[(1_F, a_1, \dots, a_n)]_{\sim} \mid a_i \in F\}$ and "$\mathbb{P}_{n-1}$" $= \{[(0_F, a_1, \dots, a_n)]_{\sim} \mid a_i \in F$, not all are $0_F\}$.