EXERCISES FOR TUTORIAL 9 OF MA 2, Dec 6, 2023

Review of the Riemann integral of real functions of one (real) variable.

1. Using your favorite definition of the Riemann integral compute $\int_0^1 x \, \mathrm{d}x$.
2. Using your favorite definition of the Riemann integral prove that if the function $f \colon [a, b] \to \mathbb{R}$ is unbounded then the integral $\int_a^b f$ does not exist.
3. Using your favorite definition of the Riemann integral prove that if the integral $I := \int_a^b f$ exists (and $a \leq b$), then we have the inequality

$$|I| \leq (b - a) \cdot \sup(\{|f(x)| \mid a \leq x \leq b\}) \ .$$

4. We have seen this already but still: give examples of Riemann integrable and nonnegative functions $f \colon [a, b] \to [0, +\infty)$ such that $\int_a^b f = 0$, but $f \neq 0$ (f is not constantly 0).
5. Give an example of a continuous and bounded function $f \colon (0, 1] \to \mathbb{R}$ that is not uniformly continuous.