EXERCISES FOR TUTORIAL 8 OF MA 2, Nov 30, 2023

Problems are similar to the previous set but now they are for n variables, where $n = 1, 2, 3, \ldots$.

1. Compute complete Taylor expansion of the function $f(x, y) = \sqrt{x} + \sqrt{y} \colon (0, 1)^2 \to \mathbb{R}$, with the center in $(\frac{1}{2}, \frac{1}{2})$.
2. Compute complete Taylor expansion of the function $f(x_1, \ldots, x_n) = \exp(x_1 + \cdots + x_n) \colon \mathbb{R}^n \to \mathbb{R}$, with the center in $(0, \ldots, 0)$.
3. Using partial derivatives find (local and global) extremes of the function $f(x, y, z) = \frac{1}{1+x^2+y^2+z^2} \colon \mathbb{R}^3 \to \mathbb{R}$. Check your results by another argument.
4. Using Lagrange multipliers find (local and global) extremes of the function
$$f(x_1, \ldots, x_n) = x_1^2 + \cdots + x_n^2$$
on the set $M = \{(x_1, \ldots, x_n) \mid x_1 + \cdots + x_n = 1\}$. Explain your solution geometrically.
5. Do the same for the function $f(x_1, \ldots, x_n) = x_1 + \cdots + x_n$ and the set M equal to the sphere in $\mathbb{R}^n$ centered in $(0, \ldots, 0)$ and with radius n.