REQUIREMENTS FOR THE MA 1 EXAM
AND MORE INFO (summer term 2022)
(M. Klazar)

Definitions of basic terms

1. definition of a function, injection, surjection, bijection (L. 1)
2. supremum and infimum of a set in a linear order (L. 1)
3. (at most) countable and uncountable sets (L. 1)
4. finite and infinite limits of sequences, subsequences (L. 2)
5. liminf and limsup of a sequence (L. 3)
6. series, partial sum of a series, sum of a series (L. 3)
7. geometric series and its sum, absolutely convergent series (L. 4)
8. limits of functions, one-sided limits of functions (L. 4 and 5)
9. the exponential function, logarithm, cosine and sine (L. 4)
10. continuity of a function at a point, one-sided continuity of a function at a point (L. 5)
11. asymptotic symbols O, o and $\sim$ (L. 5)
12. compact, open, closed sets (L. 6)
13. global, local and strict extremes of functions (L. 6)
14. derivative of a function, one-sided derivative of a function (L. 7)
15. standard definition of tangent lines (L. 7)
16. higher order derivatives (L. 8)
17. (strictly) convex and concave functions (L. 8)
18. points of inflection (L. 8)
19. vertical asymptotes and asymptotes at infinity (L. 8)

20. Taylor polynomial of a function, Taylor series of a function (L. 9)
21. primitive functions (L. 9)
22. uniform continuity (L. 9)
23. the (general) Newton integral of a function (L.10 and 11)
24. the Riemann integral (L. 12)
25. the Henstock–Kurzweil integral (L. 13)
26. the length of the graph of a function, area between two graphs, volume of a solid of revolution (L. 13)

<u>Theorems and propositions without proofs</u>

1. Existence of $\mathbb{R}$ (T. 8, L. 1) and the Fundamental Theorem of Algebra (T. 17, L. 1)
2. On subsequences (P. 6, L. 2) and Existence of monotone subsequence (P. 11, L. 2)
3. Geometric sequences (P. 5, L. 3) and Liminf and limsup exist (T. 10, L. 3)
4. On harmonic numbers (T. 4, L. 4) and Riemann's (T. 5, L. 4)
5. On $r(x)$ (P. 8, L. 5) and Limits of composite functions (T. 14, L. 5)
6. Heine's definition of continuity (T. 1, L. 6) and Blumberg's (T. 5, L. 6) and Counting continuous functions (T. 7, L. 6)
7. Derivatives of a composite functions (T. 15, L. 7) and Derivatives of an inverse functions (T. 16, L. 7)
8. l'Hospital's rule (T. 7, L. 8) f'', convexity and concavity (T. 12, L. 8)
9. Remainders of the Taylor polynomial (T. 6, L. 9) and Bell numbers B_n (P. 7, L. 9)
10. Riemann = Newton (C. 4, L. 10) and Integration by substitution (T. 13, L. 10)

11. (N) $\int_A^B f$ by parts (T. 4, L. 11) and $\int r(x)$ (T. 7, L. 11)
12. On restrictions (P. 5, L. 12) and Lebesgue's (T. 11, L. 12)
13. Riemann = Darboux (V. 4, L. 13) and HK. $\int$ and N. $\int$ (T. 7, L. 13) and FTC 2 (T. 15, L. 12) and Length of G_f (T. 13, L. 13) and the Integral criterion (C. 17, L. 13)

Theorems and propositions with proofs

1. $\sqrt{2} \notin \mathbb{Q}$ (T. 6, L. 1) and Cantor's (T. 14, L. 1)
2. Uniqueness of limit (P. 3, L. 2) and Bolzano–Weierstrass (T. 13, L. 2)
3. Limit and order (T. 6, L. 3) and Cauchy condition (T. 15, L. 2)
4. Necessary condition of convergence (P. 2, L. 4) and Harmonic series (P. 3, L. 4)
5. Heine's definition (T. 14, L. 4) and Arithmetic of limits of functions (T. 11, L. 5)
6. On intermediate values (T. 8, L. 6) and The min-max principle (T. 13, L. 6)
7. Necessary conditions for extreme (T. 4, L. 7) and The Leibniz formula (T. 13, L. 7)
8. Lagrange's (T. 2, L. 8) and Derivatives and monotonicity 1 (T. 4, L. 8)
9. The Taylor polynomial (T. 1, L. 9) and non-uniqueness of PF (T. 9, L. 9)
10. Monotonicity of the (N) $\int$ (P. 7, L. 10) and Derivatives are Darboux (T. 10, L. 10)
11. Bachet's identity (P. 10, L. 11) and calculate/prove in detail the primitive function $\int 1/(1+x^2) =?$ (it is not enough to just say that $(\dots)' = 1/(1+x^2)$, but it is necessary to justify also the derivative).

12. Unbounded functions are bad (P. 8, L. 12) and Baire's (T. 10, L. 12)
13. $\underline{\int} \le \overline{\int}$ (P. 3, L. 13) and FTC 1 (T. 16, L. 12) and Abel's summation (T. 19, L. 13)

Example of an exam test

1. (10 points) Calculate the volume of the solid of revolution

$$V(1,\, 2,\, \log x)\ .$$

   Justify your calculation.

2. (a) (2 points) Define the sum of the series.
   (b) (4 points) Yes or no: if $c \in \mathbb{R}$ and $\sum a_n$ is a series, then this series converges if and only if the series $\sum ca_n$ converges.
   (c) (4 points) $\frac{5}{9} + \frac{5}{27} + \frac{5}{81} + \frac{5}{243} + \cdots = ?$

   Justify your answers.

3. (a) (2 points) Write the proposition about Heine's definition of continuity.
   (b) (4 points) Yes or no: the function $f\colon \mathbb{R} \setminus \{0\} \to \mathbb{R}$, $f(x) = \frac{1}{x}\sin(1/x)$ can be continuously extended to 0.
   (c) (4 points) Yes or no: the function $f\colon \mathbb{R} \setminus \{0\} \to \mathbb{R}$, $f(x) = \frac{\sin x}{x}$ can be continuously extended to 0.

   Justify your answers.

4. (a) (4 points) Write Lagrange's mean value theorem.
   (b) (6 points) Prove it.

Remarks on the exam. Test for 90 minutes with 4 problems worth 10 points each: (1) computational (simplified graphing a function or computation of an integral), (2) definition, (3) theorem or proposition without proof and (4) theorem or proposition with proof, with additional questions (as in the above example). Rating: 0–19 points ... 4 (F), 20–26 points ... 3

(C), 27–33 points ... 2 (B) and 34–40 points ... 1 (A). When you gain points near the upper limit, improving your grade may be possible (if the examiner allows) by further oral examination. Specific exam questions are listed above. No tools are allowed for the exam (calculators, notes, mobile phones, friend on the phone, ...; exceptions for handicapped students are decided by the examiner). It is allowed to use a table of derivatives, e.g. within the scope of Thm. 17 in Lecture 7.