

Problem 1. For n sets $X_1, X_2, \dots, X_n$ we define the n -fold product $X_1 \times X_2 \times \dots \times X_n$ as the set

$$\prod_{i=1}^n X_i = \{(x_1, x_2, \dots, x_n) : x_i \in X_i \ \forall i \in \{1, 2, \dots, n\}\}$$

Let $A = \{1, 2\}$, $B = \{a, b\}$, $C = \{\mu, \varepsilon\}$, write down the set $A \times B \times C$.

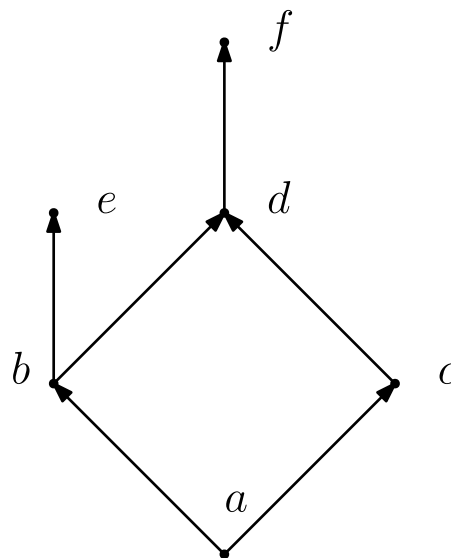
Problem 2. When is the following statement true?:

Let X, Y, Z be sets. If $X \times Y = Z \times Y$ then $X = Z$.

Prove correctness of your answer.

Problem 3. Consider the relations on people “is a brother of”, “is a sibling of”, “is a parent of”, “is married to”, “is a descendant of”. Which of the properties of reflexivity, symmetry, antisymmetry and transitivity do each of these relations have?

Problem 4. The picture below describes a relation R , where $(x, y) \in R$ means that there is a way to get from x to y using the arrows. Write down the relation R (as a set). Which of the properties of reflexivity, symmetry, anti-symmetry, and transitivity does the relation have?



Problem 5. Determine which of the following relations are reflexive, symmetric, transitive or antisymmetric.

1. $X = \{a, b, c\}$, $R = \{(a, a), (a, b), (b, b), (b, a), (c, c)\}$
2. $X = \{a, b, c\}$, $R = \{(a, a), (c, c)\}$
3. $(X, R) = (\mathbb{N}, \leq)$,

4. $X = \{1, 2, \dots, 10\}, R = \{(x, y) : \gcd(x, y) = 1\}$

5. X arbitrary, $R = \emptyset$

6. X arbitrary, $R = X \times X$

Problem 6. Let X be a set. Characterize all relations over X that are both an equivalence and an ordering.

Problem 7. Let R, S be two relations on a set X . Which of the following inherit the properties of reflexivity, symmetry, antisymmetry, transitivity? Prove your claims!

1. $R \cup S$

2. $R \cap S$

Composition of relations Let R be a relation from X to Y and let S be a relation from Y to Z . The composition $R \circ S$ is defined to be the relation

$$\{(x, z) \mid \exists y \in Y : (x, y) \in R, (y, z) \in S\}.$$

Problem 8. Let $A = \{1, 2, 3\}, R = \{(1, 2), (2, 3)\}, S = \{(2, 1), (3, 2)\}$.

1. Compute $S \circ R$ and $R \circ S$.

2. Decide which of $S \circ R, R \circ S$ are reflexive, symmetric, transitive on A .

Problem 9. Let R be a relation on a set X . Prove that R is transitive if and only if $R \circ R \subseteq R$

Problem 10. Let R, S be two equivalence relations on a set X . Prove that $R \circ S$ is an equivalence relation if and only if $R \circ S = S \circ R$