Topological methods in combinatorics - tutorials

Class work – Tverberg's theorem, chessboard complexes, polyhedral complexes

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Definition 1. A *(geometric) polyhedral complex* is a collection of polyhedra $\mathsf{M} = \{M_1, \dots, M_k\}$ such that each M_i is a bounded polyhedron in $\mathbb{R}^d$ for some d and such that the following holds:

1. If $M \in \mathsf{M}$ and F is a face of M, then $F \in \mathsf{M}$.
2. If $M_1, M_2 \in \mathsf{M}$, then $M_1 \cap M_2$ is a face of both M_1 and M_2.

1. Let $v_1, \dots, v_{d+1}$ be vertices of a simplex in $\mathbb{R}_d$ and let B_i be a set of $r-1$ points lying very close to v_i. Prove that there is no partition of $B := B_1 \cup \dots \cup B_{d+1}$ into r disjoint parts whose convex hulls have a nonempty intersection.
2. A *chessboard complex* $\mathcal{C}_{m,n}$ is a simplicial complex whose vertex set is $[n]\times[m]$, and its simplices can be interpreted as placements of rooks on an $n \times m$ chessboard such that no rook threatens any other; that is, no two rooks share a row or a column.

   Which well-know topological space is homeomorphic to $\mathcal{C}_{3,4}$?
3. Is the following statement true? For each $d \geq 1$ every two simplicial subdivisions of the cube $[0, 1]^d$ (viewed as a polyhedral complex consisting of all faces of the cube), which do not use any new vertices, have the same number of d-simplices.

Information about tutorials `https://kam.mff.cuni.cz/~dbulavka/teaching/ss2122/topomethods.html`