Topological methods in combinatorics - tutorials

Class work 1 – Basics of general and algebraic topology

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1. Let $f\colon (X, \mathcal{O}_X) \to (Y, \mathcal{O}_Y)$ and $g\colon (Y, \mathcal{O}_Y) \to (Z, \mathcal{O}_Z)$ be a pair of continuous functions. Define $h\colon (X, \mathcal{O}_X) \to (Z, \mathcal{O}_Z)$ by setting $h(x) = g(f(x))$. Show that h defines a continuous function.

2. Decide and justify if the following functions are continuous

   (a) $id\colon x \in (\mathbb{R}, \mathcal{O}_{Euclid}) \mapsto x \in (\mathbb{R}, \mathcal{O}_{Discrete})$.

   (b) $id\colon x \in (\mathbb{R}, \mathcal{O}_{Discrete}) \mapsto x \in (\mathbb{R}, \mathcal{O}_{Euclid})$.

   (c) Let $p(x_0, \ldots, x_n)$ be a polynomial on variables $x_0, \ldots, x_n$ with real coefficients, define $eval_p\colon x \in (S^n, \mathcal{O}_{Euclid}) \mapsto p(x) \in (\mathbb{R}, \mathcal{O}_{Euclid})$.

3. $f\colon X \to Y$ is homeomorphism if and only if f is continuous, bijective and open/closed.

4. (Product topology) Let $(X, \mathcal{O}_X)$ and $(Y, \mathcal{O}_Y)$ be topological spaces. The cartesian product $X \times Y$ has the collection $\{U \times V\colon U \in \mathcal{O}_X, V \in \mathcal{O}_Y\}$ as basis. Decide if $(\mathbb{R}^2, \mathcal{O}_{Euclid})$ and $\mathbb{R}\times\mathbb{R}$ with the product topology are homeomorphic.

5. (Quotient topology) Let $(X, \mathcal{O}_X)$ topological space and $\simeq$ an equivalence relation on the elements of X. The topology on $X/\simeq$ on the equivalence classes is given by $q\colon X \to X/\simeq$, $U \subset X/\simeq$ is open if and only if $q^{-1}(U)$ is open. Show that $[0, 1]/\{0, 1\}$ and S^1 are homeomorphic.

6. Show that $\mathbb{R}$, $S^1 \setminus \{(0, 1)\}$ and $(0, 1)$ are homeomorphic.

7. Show that S^1 and $\partial([0, 1]^2)$ are homeomorphic.

Information about tutorials `https://kam.mff.cuni.cz/~dbulavka/teaching/ss2122/topomethods.html`