Algorithmic game theory

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5th lecture

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Nash equilibria in bimatrix games

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- Is there a chance to get an efficient algorithm?
- NASH = the problem of finding NE in bimatrix games.
- Today, we discuss the computational complexity of NASH.

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- Without proof (but you can find it in the lecture notes).

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- Let PPAD be a complexity class consisting of problems that admit a polynomial-time reduction to END-OF-THE-LINE.

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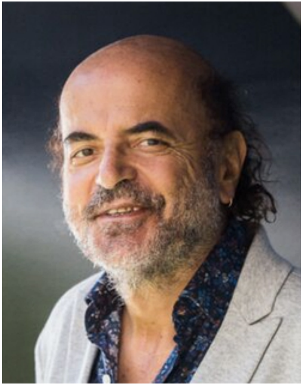

Figure: Christos Papadimitriou (born 1949).

Source: https://cs.columbia.edu

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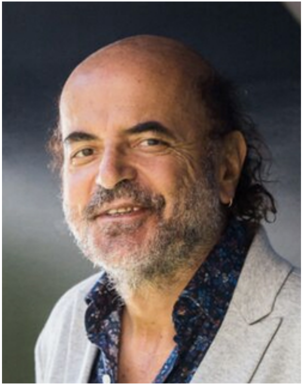

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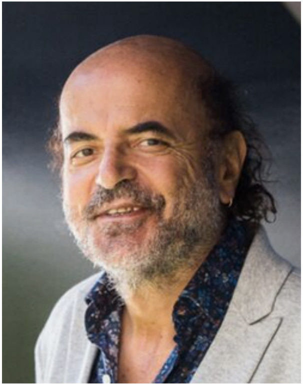

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- This complexity class contains a lot of well-known problems.

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- For an oriented graph G with max. indegree and outdegree 1 and a source in G, find a target in G. The graph is given by a polynomial-time computable function $f(v)$ that returns predecessor and successor of v.

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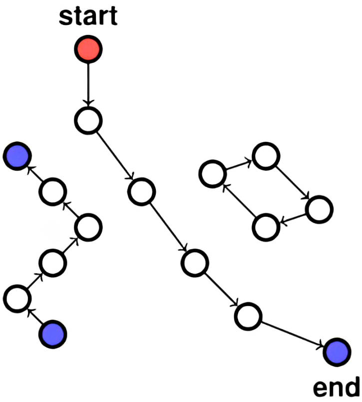

Source: R. Savani "Polymatrix Games" Tutorial at WINE 2015

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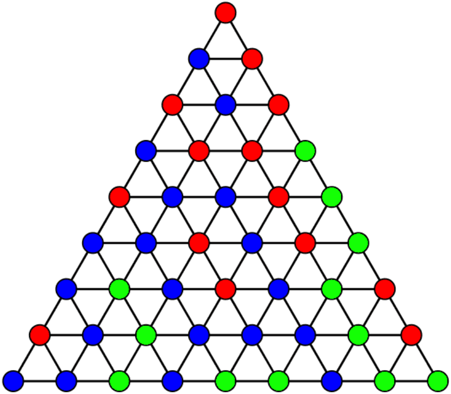

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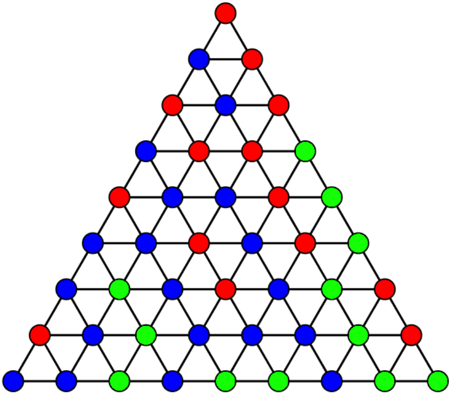

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- Discrete version of the Brouwer's fixed point theorem.

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- An approximate version of the following theorem is in PPAD: For each $d \in \mathbb{N}$, a non-empty compact convex set K in $\mathbb{R}^d$, and a continuous mapping $f : K \to K$, there exists $x_0 \in K$ such that $f(x_0) = x_0$.

Figure: L. E. J. Brouwer (1881–1966).

Source: https://arxiv.org/pdf/1612.06820.pdf

Problems from PPAD: Ham sandwich theorem

Source: https://www.seekpng.com/

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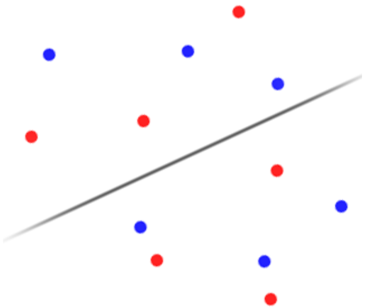

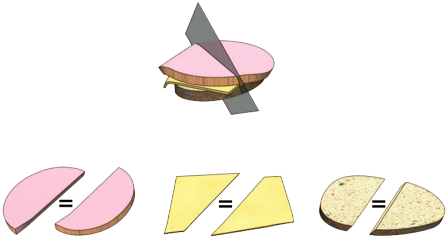

Sources: https://ejarzo.github.io and https://curiosamathematica.tumblr.com

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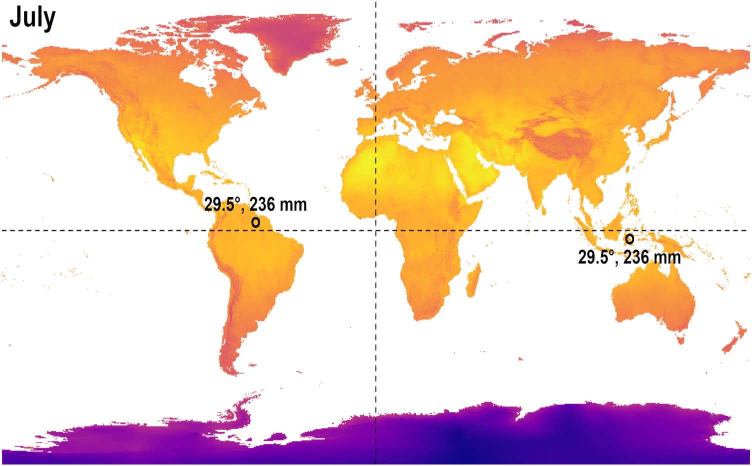

Source: https://scientificgems.wordpress.com/

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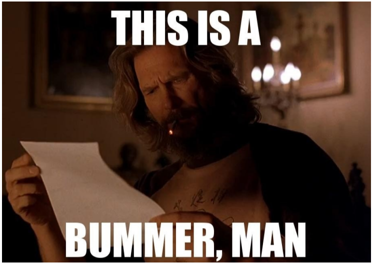

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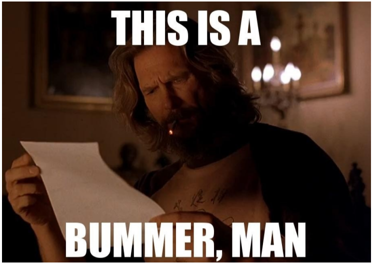

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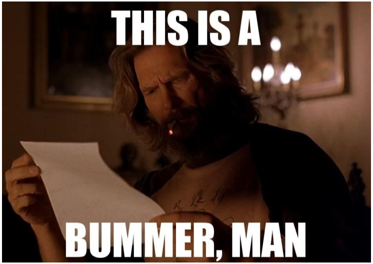

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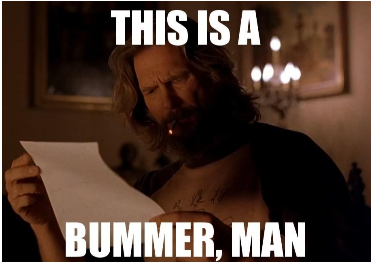

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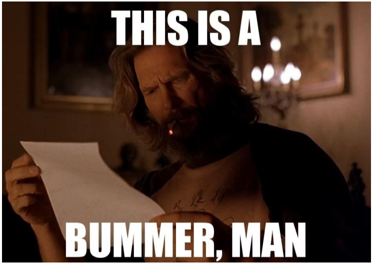

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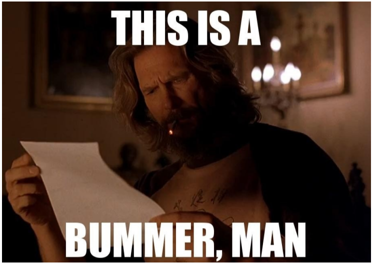

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- We introduce other solution concepts that possess some qualities of NE and yet are easier to compute.

Other notions of equilibria

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  - That is, no other strategy can improve the payoff by more than ε.

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  - We will see that his concept is also somehow computationally difficult.

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Theorem 2.37 (Lipton, Markakis, and Mehta, 2003)

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- I no longer present the proof (see the lecture notes).

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Example of correlated equilibria: Game of Chicken

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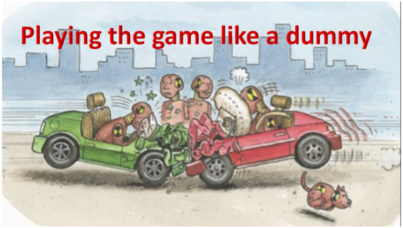
	Stop	Go
Stop	(0,0)	(-1,1)
Go	(1,-1)	(-10,-10)



Sources: https://peakd.com/

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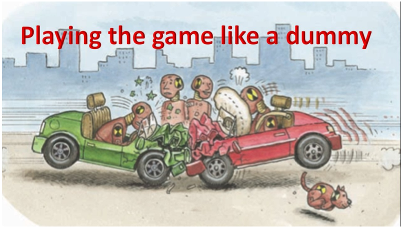

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- There are two pure NE with $(s_1(S), s_2(S)) = (1, 0)$ and $(s_1(S), s_2(S)) = (0, 1)$, and one mixed NE with $(s_1(S), s_2(S)) = (9/10, 9/10)$.

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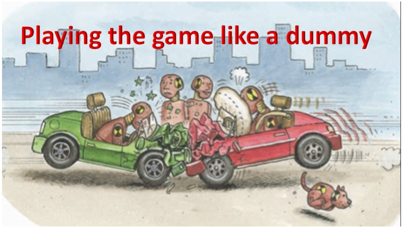

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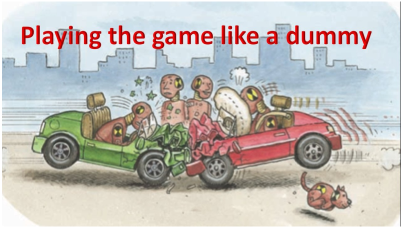

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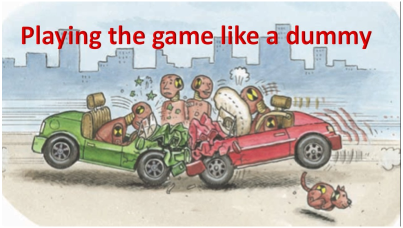

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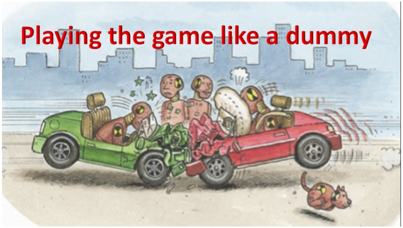

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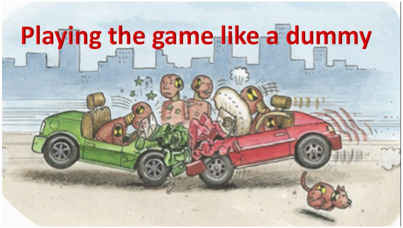

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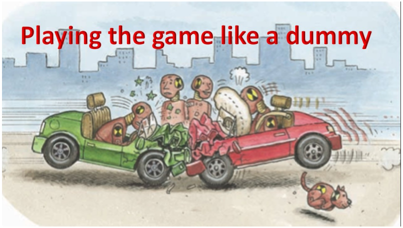

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  - By symmetry, driver 2 does not deviate as well.

Source: Students of MFF UK

Example of correlated equilibria: Battle of sexes

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	Football	Opera
Football	(2,1)	(0,0)
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Sources: https://media.istockphoto.com/

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  - By symmetry, the wife does not deviate as well.

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  - The objective function can be arbitrary as long as it is linear.

- The concept of correlated equilibria was introduced by Robert Aumann, who received a Nobel prize in economics for his work in game theory.

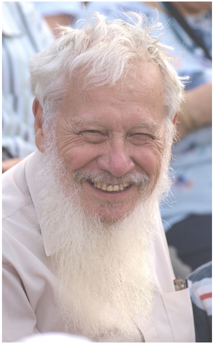

Figure: Robert Aumann (born 1930).

Sources: https://en.wikipedia.org and https://slideslive.com/38910863/strategic-information-theory

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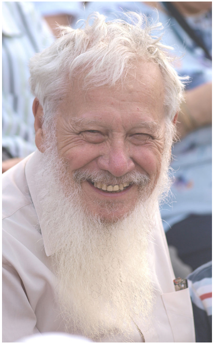

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Thank you for your attention.