Algorithmic game theory

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8th lecture

November 25th 2021

Regret minimization

Our notation

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- Given a comparison class $\mathcal{A}_X$ of agents A_i that select a single action i in all steps, we let $L_{min}^T = \min_{i \in X}\{L_{A_i}^T\}$ be the minimum cumulative loss of an agent from $\mathcal{A}_X$.

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- Our goal is to minimize the external regret $R_A^T = L_A^T - L_{min}^T$.
- We now restrict ourselves to losses from $\{0, 1\}$.

Example

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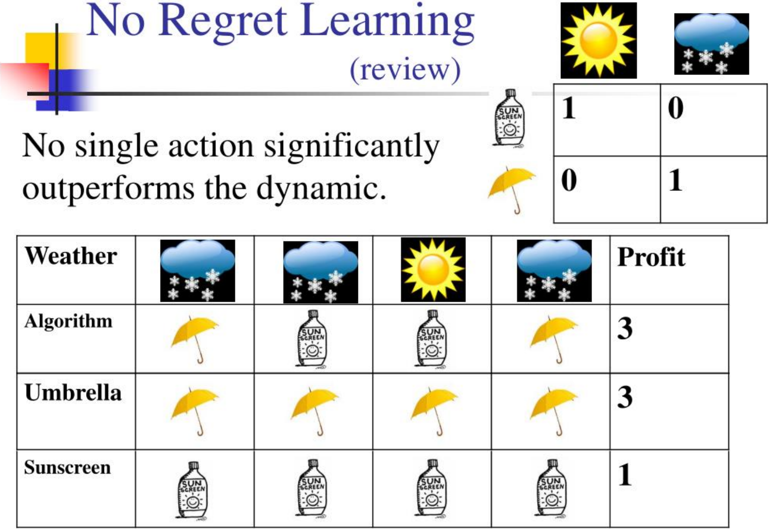
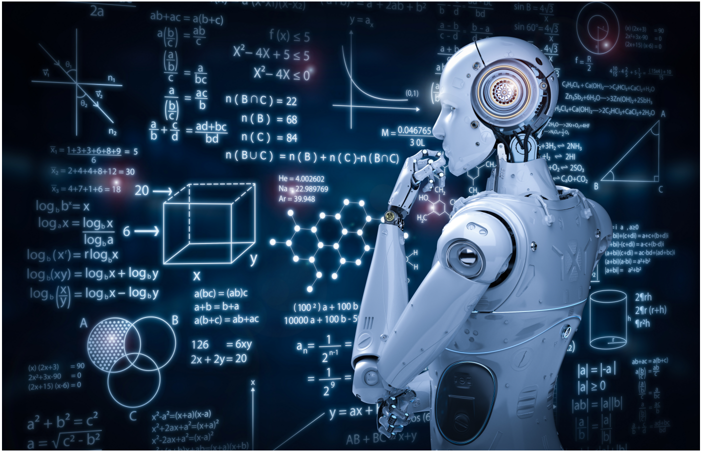


Source: No regret algorithms in games (Georgios Piliouras)

- Besides game theory, the "multiplicative weight update method" has many applications in various fields of science, for example in optimization, theoretical computer science, and machine learning.



Sources: https://clubitc.ro

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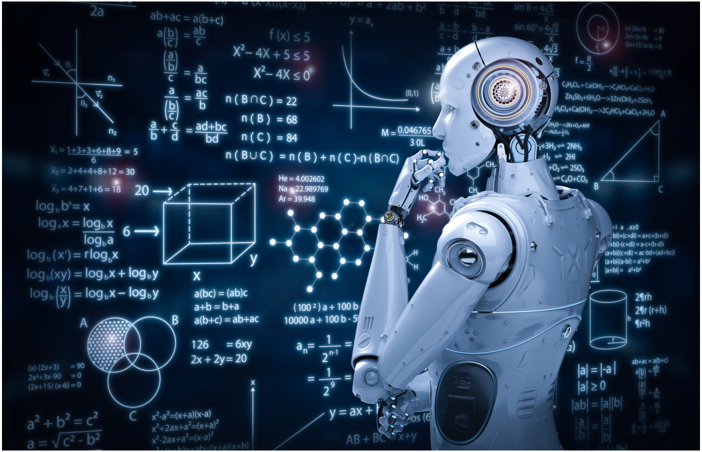

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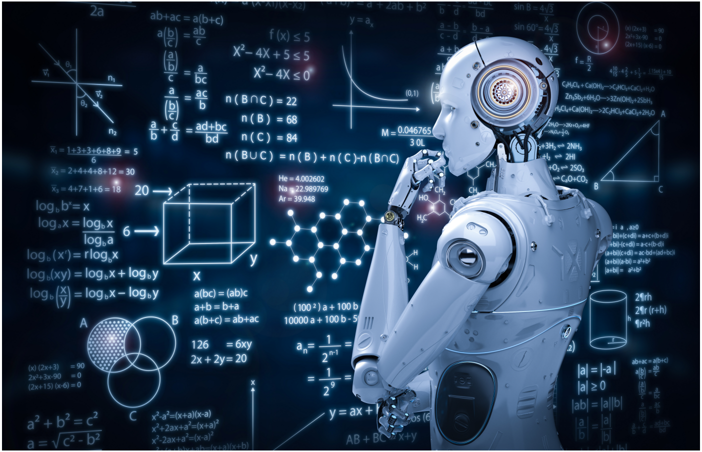

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Thank you for your attention.