Algorithmic game theory

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5th lecture

November 4th 2021

Nash equilibria in bimatrix games

Best response polyhedra

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- The best response polyhedron for player 1 in G is defined as

$$\overline{P} = \{(x, v) \in \mathbb{R}^m \times \mathbb{R} \colon x \geq \mathbf{0}, \mathbf{1}^\top x = 1, N^\top x \leq \mathbf{1} v\}.$$

Similarly, the best response polyhedron for player 2 in G is

$$\overline{Q} = \{(y, u) \in \mathbb{R}^n \times \mathbb{R} \colon y \geq \mathbf{0}, \mathbf{1}^\top y = 1, My \leq \mathbf{1} u\}.$$

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Best response polyhedra $\overline{P}$ and $\overline{Q}$ for the Battle of sexes

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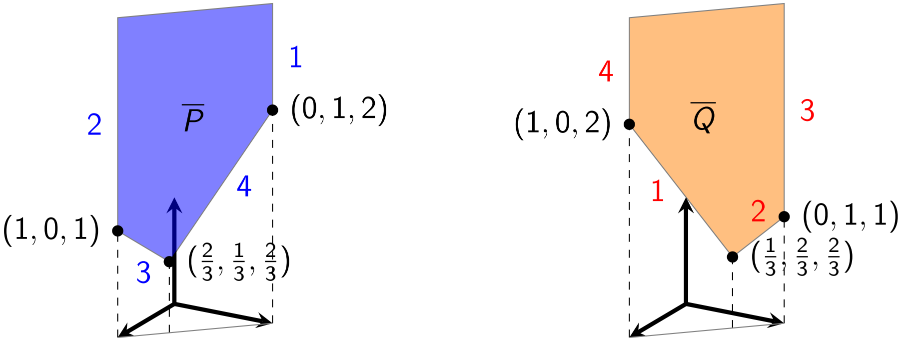

$$\overline{P} = \{(x_1, x_2, v) \in \mathbb{R}^2 \times \mathbb{R} \colon x_1, x_2 \geq \mathbf{0}, x_1 + x_2 = 1, x_1 \leq v, 2x_2 \leq v\}$$

$$\overline{Q} = \{(y_3, y_4, u) \in \mathbb{R}^2 \times \mathbb{R} \colon y_3, y_4 \geq \mathbf{0}, y_3 + y_4 = 1, 2y_3 \leq u, y_4 \leq u\}.$$

Best response polytopes P and Q for the Battle of sexes

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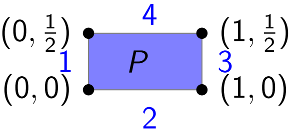

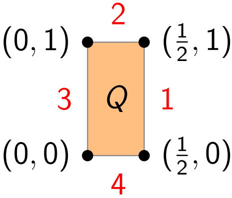

$$P = \{(x_1, x_2) \in \mathbb{R}^2 \colon x_1, x_2 \geq 0, x_1 \leq 1, 2x_2 \leq 1\}$$

$$Q = \{(y_3, y_4) \in \mathbb{R}^2 \colon y_3, y_4 \geq 0, 2y_3 \leq 1, y_4 \leq 1\}.$$

Lemke–Howson on the Battle of sexes ($k = 3$)

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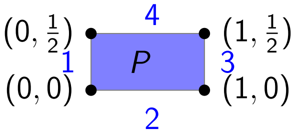

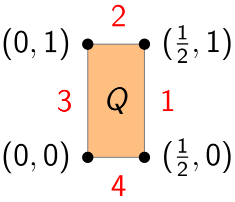

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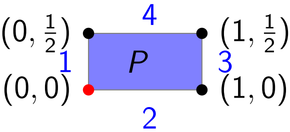

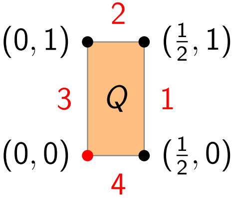

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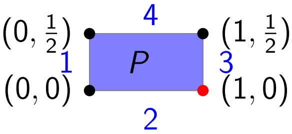

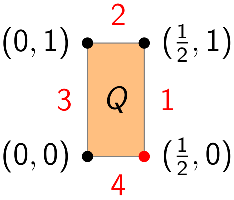

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Figure: Carlton E. Lemke (1920–2004) and J. T. Howson (?).

Source: https://oldurls.inf.ethz.ch

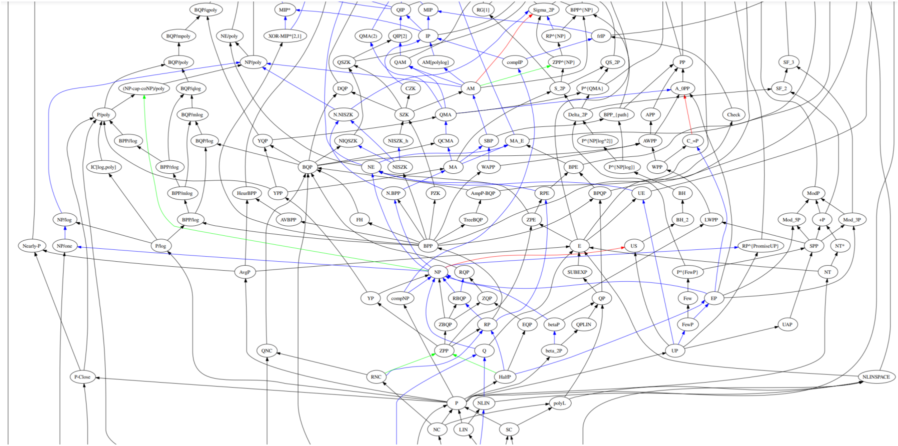
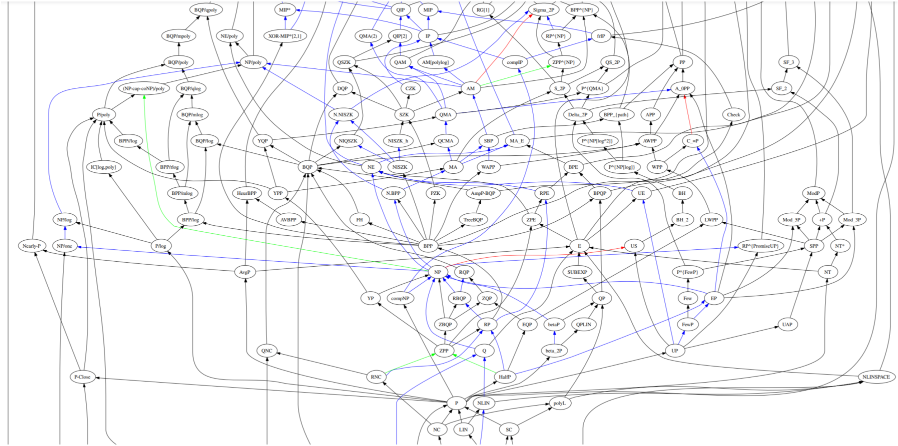


Figure: A view on the complexity classes classification.

Source: https://complexityzoo.uwaterloo.ca/Complexity_Zoo



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Thank you for your attention.