Algorithmic game theory

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3rd lecture

October 21st 2021

Proof of the Minimax Theorem

The Minimax Theorem

The Minimax Theorem

- For every zero-sum game, worst-case optimal strategies for both players exist and can be efficiently computed. There is a number v such that, for any worst-case optimal strategies x^* and y^*, the strategy profile (x^*, y^*) is a Nash equilibrium and $\beta(x^*) = (x^*)^\top M y^* = \alpha(y^*) = v$.

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Figure: John von Neumann (1903–1957) and Oskar Morgenstern (1902–1977)

Sources: https://en.wikiquote.org and https://austriainusa.org

Duality of linear programming

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	Primal linear program	Dual linear program
Variables	$x_1, \ldots, x_m$	$y_1, \ldots, y_n$
Matrix	$A \in \mathbb{R}^{n \times m}$	$A^\top \in \mathbb{R}^{m \times n}$
Right-hand side	$b \in \mathbb{R}^n$	$c \in \mathbb{R}^m$
Objective function	$\max c^\top x$	$\min b^\top y$
Constraints	ith constraint has $\leq$	$y_i \geq 0$
	$\geq$	$y_i \leq 0$
	$=$	$y_i \in \mathbb{R}$
	$x_j \geq 0$	jth constraint has $\geq$
	$x_j \leq 0$	$\leq$
	$x_j \in \mathbb{R}$	$=$

Table: A recipe for making dual programs.

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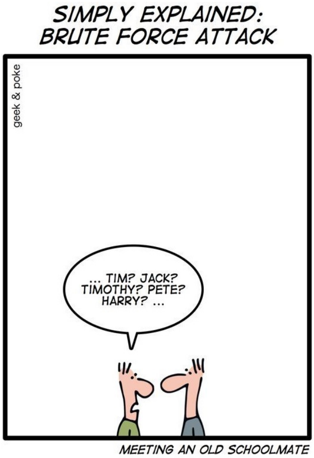

Source: https://pinterest.com

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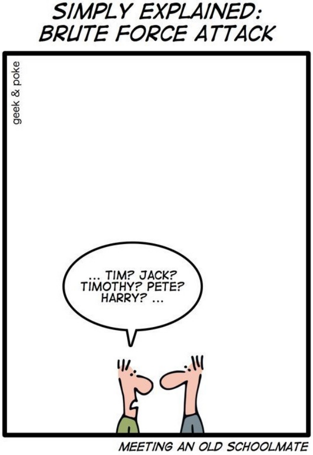

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- Later, we show the currently best known algorithm for this problem.

Examples of polytopes

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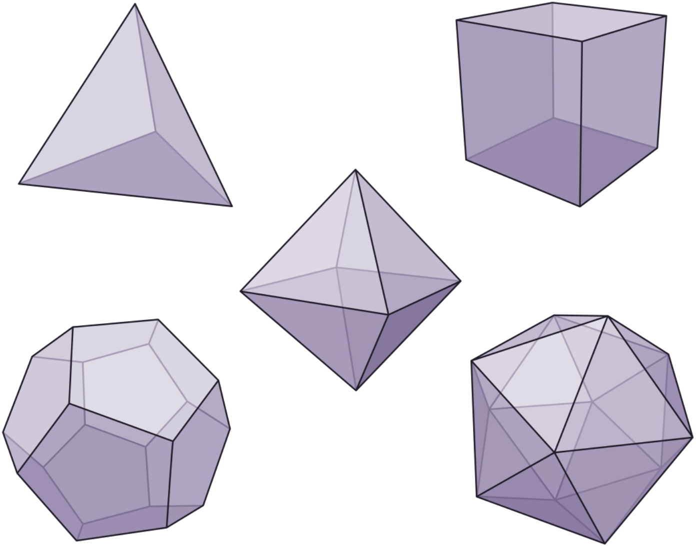

Best response polyhedra $\overline{P}$ and $\overline{Q}$ for the Battle of sexes

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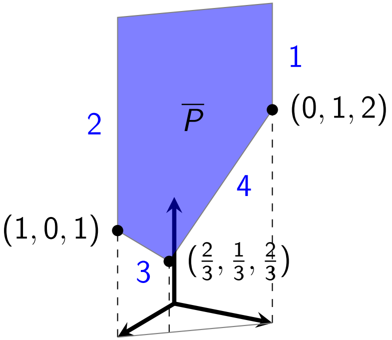

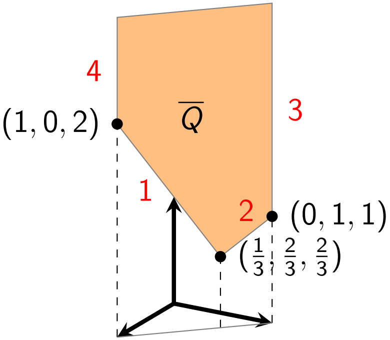

Best response polytopes P and Q for the Battle of sexes

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Thank you for your attention.