

Algorithmic game theory

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Regret minimization

Example

Example

No Regret Learning (review)

No single action significantly outperforms the dynamic.



1	0
0	1

Weather					Profit
Algorithm					3
Umbrella					3
Sunscreen					1

Polynomial weights algorithm

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Algorithm 0.4: POLYNOMIAL WEIGHTS ALGORITHM(X, T, η)

Input : A set of actions $X = \{1, \dots, N\}$, $T \in \mathbb{N}$, and $\eta \in (0, 1/2]$.

Output : A probability distribution p^t for every time step t .

$w_i^1 \leftarrow 1$ for every $i \in X$,

$p^1 \leftarrow (1/N, \dots, 1/N)$,

for $t = 2, \dots, T$

do
$$\begin{cases} w_i^t \leftarrow w_i^{t-1}(1 - \eta \ell_i^{t-1}), \\ W^t \leftarrow \sum_{i \in X} w_i^t, \\ p_i^t \leftarrow w_i^t / W^t \text{ for every } i \in X. \end{cases}$$

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Algorithm 0.8: NO-REGRET DYNAMICS(G, T, ε)

Input : A game $G = (P, A, C)$ of n players, $T \in \mathbb{N}$ and $\varepsilon > 0$.

Output : A prob. distribution p_i^t on A_i for each $i \in P$ and step t .

for every step $t = 1, \dots, T$

do $\left\{ \begin{array}{l} \text{Each player } i \in P \text{ independently chooses a mixed strategy } p_i^t \text{ using an algorithm with average regret at most } \varepsilon. \\ \text{Each player } i \in P \text{ receives a loss vector } \ell_i^t = (\ell_i^t(a_i))_{a_i \in A_i}, \\ \text{where } \ell_i^t(a_i) \leftarrow \mathbb{E}_{a_{-i}^t \sim \prod_{j \neq i} p_j^t} [C_i(a_i; a_{-i}^t)]. \end{array} \right.$



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Source: <https://img.etimg.com>

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Thank you for your attention.