Algorithmic game theory

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5th lecture

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Nash equilibria in bimatrix games

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- Is there a chance to get an efficient algorithm?
- NASH = the problem of finding NE in bimatrix games.

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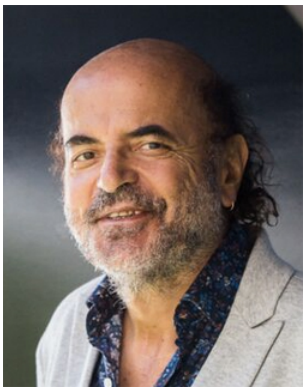

Figure: Christos Papadimitriou (born 1949).

Source: https://cs.columbia.edu

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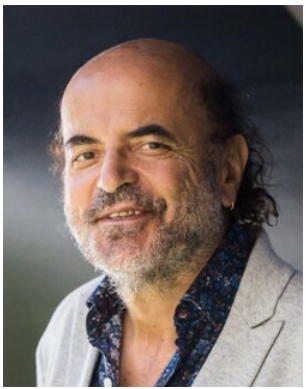

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- Abbreviation for "Polynomial Parity Arguments on Directed graphs".

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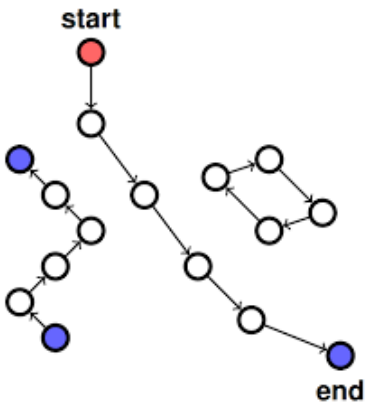

Source: R. Savani "Polymatrix Games" Tutorial at WINE 2015

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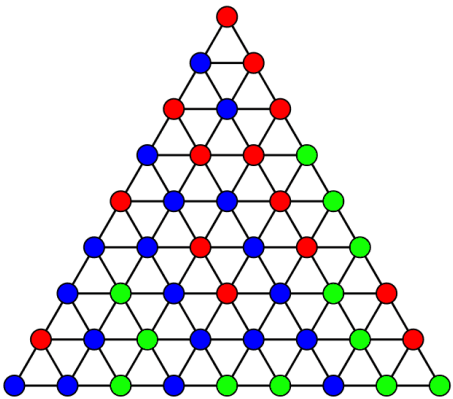

Source: https://lesswrong.com

Problems from PPAD: Ham sandwich theorem

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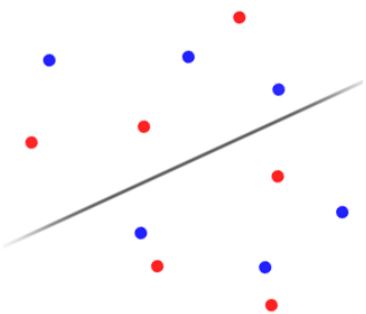
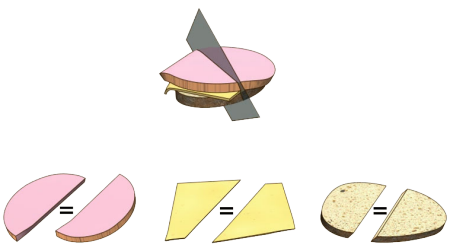
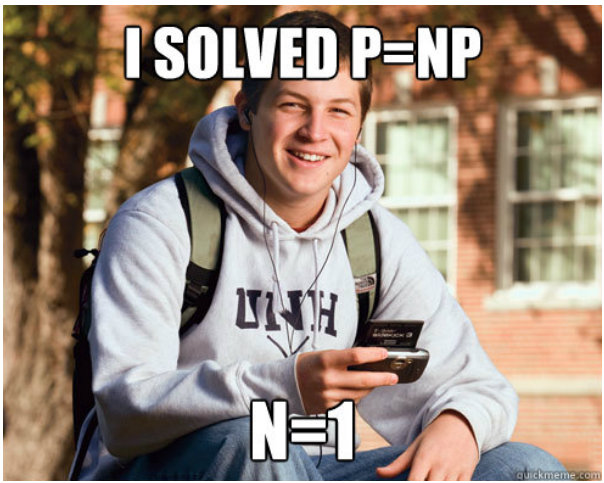




Sources: https://ejarzo.github.io and https://curiosamathematica.tumblr.com

Other notions of equilibria

- "P=NP" is one of the most important problems in computer science. The website https://www.win.tue.nl/~gwoegi/P-versus-NP.htm contains a collection of over 100 attempts to solve it.



Source: https://quickmeme.com

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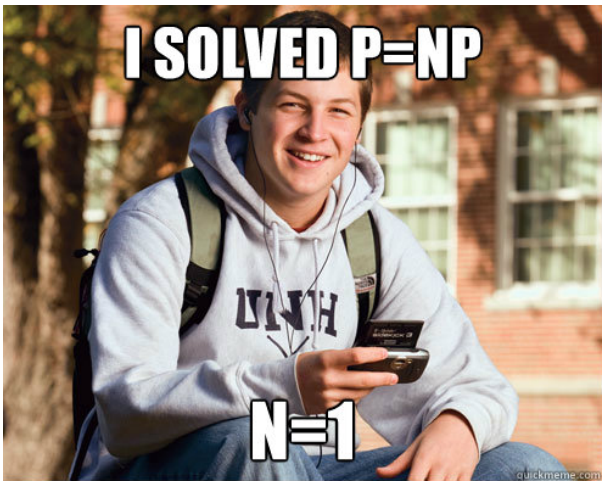

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Thank you for your attention.