Dimenze řádkového a sloupcového prostoru

Dána matice $\mathbf{A} = \begin{pmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & 2 & 1 \\ 1 & 1 & 2 & 0 \end{pmatrix} \in \mathbb{Z}_3^{3\times 4}$

Řádkový prostor:

$$\mathcal{R}(\mathbf{A}) = \left\{ \begin{array}{lll} (0,0,0,0)^T, & (\mathbf{1},\mathbf{2},\mathbf{0},\mathbf{1})^T, & (2,1,0,2)^T, \\ (\mathbf{2},\mathbf{0},\mathbf{2},\mathbf{1})^T, & (0,2,2,2)^T, & (\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{0})^T, \\ (1,0,1,2)^T, & (2,2,1,0)^T, & (0,1,1,1)^T \end{array} \right\} \subseteq \mathbb{Z}_3^4$$

Sloupcový prostor:

$$\mathcal{S}(\mathbf{A}) = \left\{ \begin{array}{lll} (0,0,0)^T, & (\mathbf{1},\mathbf{2},\mathbf{1})^T, & (2,1,2)^T, \\ (\mathbf{2},\mathbf{0},\mathbf{1})^T, & (\mathbf{0},\mathbf{2},\mathbf{2})^T, & (\mathbf{1},\mathbf{1},\mathbf{0})^T, \\ (1,0,2)^T, & (2,2,0)^T, & (0,1,1)^T \end{array} \right\} \subseteq \mathbb{Z}_3^3$$

$$\dim(\mathcal{R}(\mathbf{A})) = \dim(\mathcal{S}(\mathbf{A})) = \operatorname{rank}(\mathbf{A}) = 2$$