

# Home assignment 3

## Probabilistic techniques 2

Submission deadline: June 16, 2023

1. Let  $X$  be a set of size  $n$  and let  $\mathcal{F} \subset 2^X$  be an up-set. Prove that  $f(m) = \frac{|\mathcal{F} \cap \binom{X}{m}|}{\binom{n}{m}}$  is increasing in  $m$ .
2. Consider the random hypergraph model  $\mathcal{H}_{n,M}^k$ , that is, a uniformly random  $k$ -uniform hypergraph of order  $n$  and size  $M$ . Show that if  $M = cn \log n$  and  $c = c(k)$  is a sufficiently small constant, then  $\mathcal{H}_{n,M}^k$  has isolated vertices w.h.p.
3. Using the Park-Pham theorem show that if  $M = Cn \log n$  and  $C = C(k)$  is a sufficiently large constant then  $\mathcal{H}_{n,M}^k$  contains w.h.p (when  $n$  is a multiple of  $k$ ) a perfect matching, i.e., a collection of  $n/k$  pairwise disjoint edges.
4. Show that if a graph  $F$  has more edges than vertices then it a.a.s. does not occur as a subgraph of the random  $d$ -regular graph.