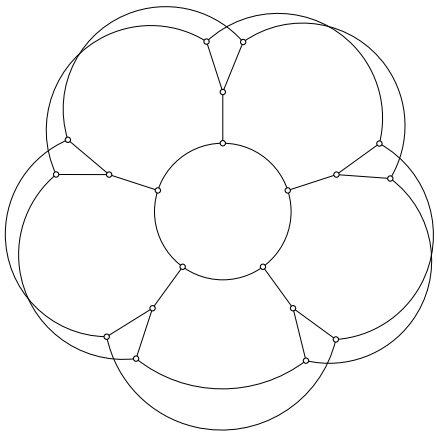
Flows and cycles in graphs – Exercise 2

1. Prove that every bridgeless graph has a NZF in every large-enough group. (Do not try to optimize the size – but also do not use the result about 8-NZF proved in class.)

2. Prove that the Flower snark (in the figure) is not 3-edge-colorable (so it is indeed a snark). We did it in class, but with some details skipped.



3. When proving that the Petersen graph does *not* have some property (in the previous set of exercises we discussed edge 3-coloring, resp. NZ $\mathbb{Z}_2^2$-flow) it is helpful, that the graph is extremely symmetric. Proving these symmetries is the topic of this exercise. First few ad-hoc definitions:

We say that graph G is *H-transitive*, if whenever H_1, H_2 are subgraphs of G, both isomorphic to H, there is an automorphism of G which maps H_1 to H_2.

We say that graph G is *ordered H-transitive*, if whenever H_1, H_2 are subgraphs of G, both isomorphic to H, and $f : H_1 \to H_2$ is an isomorphism, then there is an automorphism of G which extends f.

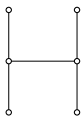
(a) *Kneser graph* $K(n, k)$ is a graph which has k-subsets of an n-set as vertices, and two vertices are adjacent iff the corresponding sets are disjoint. Show that the Petersen graph is isomorphic with $K(5, 2)$.

(b) The Petersen graph is K_1-transitive (or vertex-transitive).

(c) The Petersen graph is K_2-transitive (or edge-transitive).

(d) The Petersen graph is ordered K_2-transitive (or arc-transitive).

(e) The Petersen graph is ordered H-transitive where H is the following tree:



(f) The Petersen graph is M-transitive, where M is a matching with 5 edges.

4. Let G be a directed graph with edges colored by red, blue, yellow, and green. Let x, y be two vertices of G. We are looking for a path from x to y that may use blue edges in the forward direction, yellow in the backward direction, green in both directions and red in neither. Characterize graphs in which there exists such a path (and prove the characterization).