Flows and cycles in graphs – Exercises 6

1. Suppose that for a graph G exists a collection of cycles that covers every edge once or twice. Then there is another collection that covers every edge twice.

2. Suppose a graph G has a 4-edge cut but no smaller edge cuts. Let e_i $(i = 1, \dots, 4)$ be the edges of the 4-edge cut. Let G', G'' be graphs obtained by cutting each of the edges e_i "in the middle" and connecting the edges in each of the resulting components arbitrarily.

Formally: suppose $e_i = x_i y_i$ and in $G - \{e_1, \dots, e_4\}$ all vertices x_i are in one component, and all y_i's in the other. Choose a matching M_x on vertices $\{x_1, \dots, x_4\}$ and M_y on $\{y_1, \dots, y_4\}$. The graph $(G - \{e_1, \dots, e_4\}) \cup M_x \cup M_y$ consists of two components, we let them be G', G''. (They do depend on the choice of M_x, M_y.)

Question: are the graphs G', G'' bridgeless for some choice of M_x, M_y? Are the graphs G', G'' bridgeless for all choices of M_x, M_y?

3. In class we saw a proof that a minimal counterexample to the CDC conjecture does not have a 2-edge cut neither a 3-edge cut; the proofs used different approaches. Try to prove each of these results by "the other proof".