Annotation

- AVL-trees,
- red-black-trees,
- A-B-trees.

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- after `insert` and `delete` we perform the balance-renewing operations.
- For each vertex we define a value `balance` saying `depth_right - depth_left`, permitted values are `-1, 0` and `1`.

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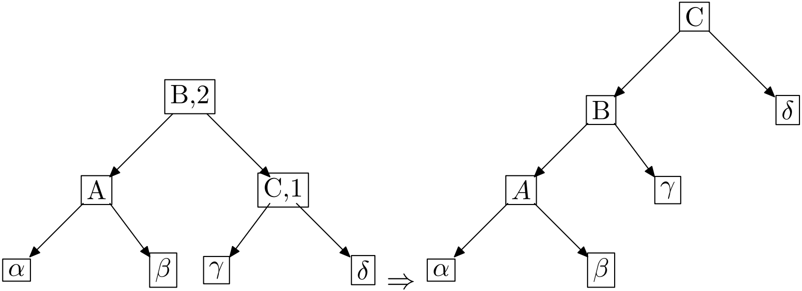
- We start solving on the bottom-most level with this balance.
- We explore two possibilities, the remaining 2 are symmetric.
- The tree may be falling "to the side" or "to the interior".
- In the former case we use a rotation, in the latter a double-rotation.

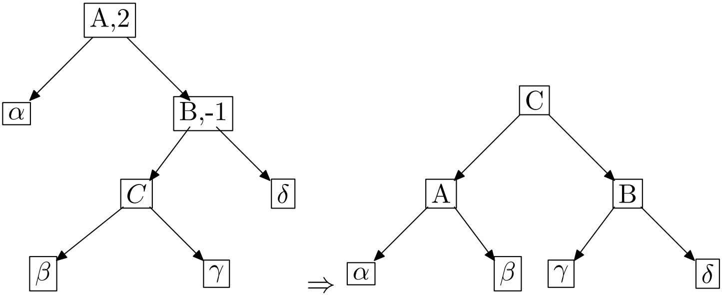
Rotation

Tree is falling "to the side".



Double-rotation

Tree is falling "to the interior".



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rotation, double-rotation, depths

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Shown on a blackboard

A-B-trees, *k*-ary tree canonical representation.