Annotation

- Simple sorting algorithms,
- Units,
- Pointers.

Sorting – the motivation

- We have read the data.
- we want to process them in a monotone ordering.
- How to do that? Sort, process.
- Let us consider the data have been read into an array.

The problem of sorting – simple sorting algorithms

- BubbleSort,
- InsertSort,
- SelectSort,
- QuickSort.

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- Individual elements are "bubbling" in a correct direction.
- We iterate this process until no swap takes place.

Bubblesort in pseudocode

```
weswapped:=true;
while weswapped do
  begin
    ■ for i:=1 to length - 1 do
      begin
        weswapped:=false;
        ■ if numbers[i]>numbers[i+1] then
          begin swap(numbers[i],numbers[i+1]);
               weswapped:=true;
          end:
    end;
end;
```

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- Thus altogether $O(n^2)$.
- We may implement the algorithm when in odd iterations we bubble from left to right while in even iterations from right to left. This is called **Shakesort**. Its complexity is the same.

Insert- and Select-sort

Selectsort:

- Repeat until the array to sort is empty:
- Find a minimum in the array to sort and add it to the sorted array.

Insertsort:

- Repeat until the array to sort is empty:
- Take the first element of the array to sort and place it onto the correct position in the target array, i.e.: find the position where this element should be in the target array, add it there and the rest of the target array move one position further.

Complexity-analysis: We iterate the process ntimes. One iteration takes at most cn steps (for some constant c). Therefore altogether $O(n^2)$.

sorting using the recursion - the idea

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- Output the array B, output pivot p (as many times as it was in A), output C.

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- To improve this algorithm we want to find a median but we have to do it in linear time.



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- This method is specific by dividing the data in a fixed way, e.g., Quicksort.
- Technically we are designing recursive algorithms with complexity $T(n) = \sum_{i=1}^{k} T(n_i)$ where $\sum_{i=1}^{k} n_i = n$.

FIXME!!!

Here should be a quicksort implementation!

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- We may copy (click'n'paste) them into the other source files (bad idea)
- or we store them into a separate file that gets compiled separately.
- The latter approach is referred as the units.

Units – advantages and disadvantages

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- Source code gets spreaded into several files,
- it is not necessary to write the code more times when we want to share it in several projects.

Units

Units – syntax and semantic

- Instead of word program we start with keyword unit,
- again, we place the name of the unit. This time the name must correspond with the filename. Also the keyword unit is compulsory.
- A unit consists of an interface (what's visible from the outside)
- and of implementation (internal part where the interface is implemented).

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- prototype is the header of the function, i.e., the "first line".

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- definition of internal functions (not mentioned in interface).
- We finish the unit by keyword end. (followed by full-stop)

Units - example

```
unit sorting;
interface
    type po=array[0..9] of integer;
    procedure bubble(var arr:array of integer);
    procedure select(var a:po);
    procedure insert(var a:po);
    procedure quicksort(var arr:array of integer;number:integer);
    procedure output(a:array of integer);
```

Units – example (cont.)

```
implementation
      var inserted:integer;
      procedure bubble(var arr:array of integer);
      function extract_min(var a:po):integer;
      {This function will not be visible from
outside!}
      procedure select(var a:po):integer;
end.
```

Units – how to use them

- When using a unit, we announe it with a keyword uses followed by the name of the unit:
- Example: uses sorting;

Using the unit - example

```
program sort;
uses sorting;
var p:array [0..9] of integer;
    i:integer;
begin
    for i:=0 to 9 do
        read(p[i]);
    quicksort(p,1,10);
    output(p);
end.
```