

## Exercise 8

February, 2006

1. Prove directly (i.e., without using Kolmogorov's zero-one law) the result of question 4 in the previous exercise: that percolation in  $\mathbb{Z}^2$  has a critical probability  $p_c$ .
2. What is the expectancy of the majority function on  $2n + 1$  inputs if each input has probability  $p$  of being 1?
3. (Beginning of Friedgut's proof of BKKKL) Let  $f : [0, 1]^n \rightarrow \{0, 1\}$  be monotone. Divide the cube into  $k^n$  subcubes, such that each subcube has an edge of length  $1/k$ , in the obvious way. Prove that  $f$  is nonconstant on at most  $nk^{n-1}$  subcubes.
4. Let  $f : \{0, 1\}^n \rightarrow \{0, 1\}$  be given. Define a probability space as follows: given an input vector  $x$ , we modify each bit of  $x$  with probability  $\epsilon$ . We then plug in the result to  $f$ . The function  $f_\epsilon(x)$  is the expectancy of the result. What is the Fourier expansion of  $f_\epsilon$ ?
5. Prove that  $T_{\epsilon_1} T_{\epsilon_2} = T_{\epsilon_1 \epsilon_2}$ .