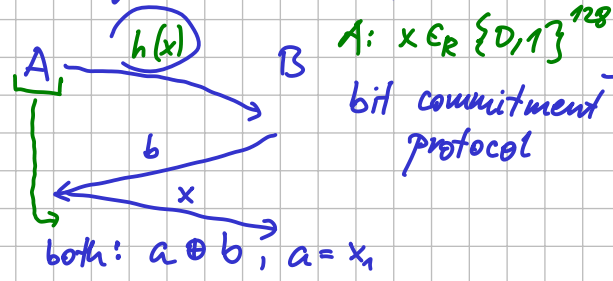


Tossing a coin over a phone call

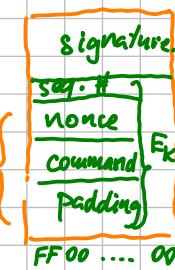


any fixed  $\oplus$  random bit  $\rightarrow$  random bit

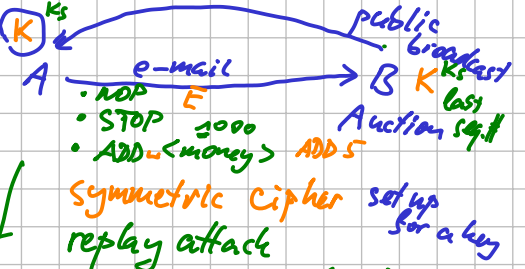
0	$\frac{1}{2}$	0 $\rightarrow$ 0
1	$\frac{1}{2}$	1 $\rightarrow$ 1
0	$\frac{1}{2}$	0 $\rightarrow$ 1
1	$\frac{1}{2}$	1 $\rightarrow$ 0
0	0	0 $\rightarrow$ 0
1	1	1 $\rightarrow$ 0

Auction Protocol

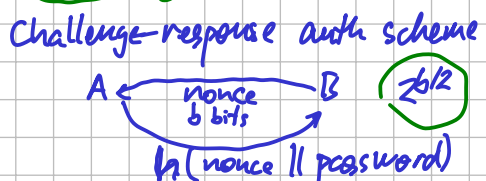
- secretly
- against whom
  - for how long



Message Authentication Code (MAC)  
 $h(K_s \parallel \text{encrypted part})$   
 Key Generation Function  
 $h(\dots) \rightarrow \text{key}$   
 Session ID (nonce)



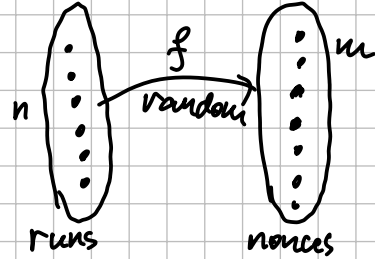
(Birthday Attacks)



How to measure strength?  
 b: security level (bits)  
 Attacker requires  $\geq 2^b$  operations to break our protocol.

- Kind of attacks
- known ciphertext  $\rightarrow$  recover plaintext
  - known plaintext  $\rightarrow$  recover key
  - chosen plaintext
  - distinguishing attack

23 people, 366 days  $\rightarrow \text{Pr}[2 \text{ share a birthday}] \geq 1/2$



$\text{Pr}_f[\text{injective}] = \frac{\# \text{inj. fncs}}{\# \text{all fncs}} = \frac{m \cdot (m-1) \cdot (m-2) \cdot \dots \cdot (m-n+1)}{m^n}$

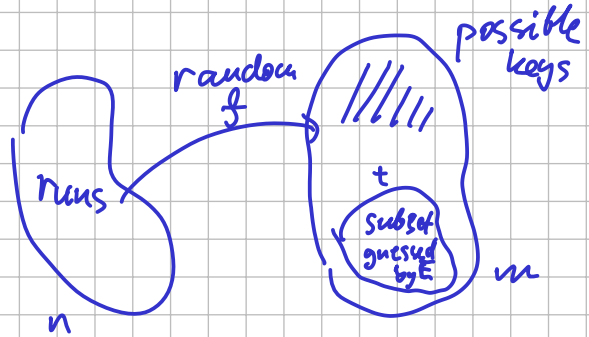
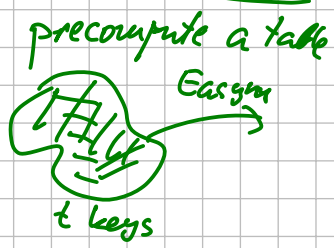
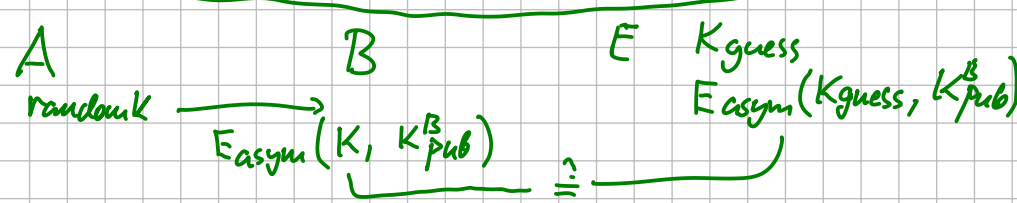
$1-x \approx e^{-x}$   
 $= \frac{1}{2}$

$e^{-\frac{n^2}{2m}} = \frac{1}{2}$   
 $-\frac{n^2}{2m} = \ln \frac{1}{2}$   
 $\frac{n^2}{m} = -2 \cdot \ln \frac{1}{2} \approx 1.38$   
 $n^2 \sim m$

$m^n$

$\approx 1 \cdot e^{-\frac{1}{m}} \cdot e^{-\frac{2}{m}} \cdot e^{-\frac{3}{m}} \cdot \dots \cdot e^{-\frac{n-1}{m}}$

$= e^{-\frac{1}{m} - \frac{2}{m} - \dots - \frac{n-1}{m}} = e^{-\frac{1+2+\dots+n-1}{m}} = e^{-\frac{n(n-1)}{2m}} \approx e^{-\frac{n^2}{2m}}$



$\text{Pr}_f[f \text{ avoids subset}] = \left(1 - \frac{t}{m}\right)^n \approx e^{-\frac{tn}{m}}$

const.  $\frac{1}{2}$   
 $tn \sim m$   
 e.g.,  $t := \sqrt{m}$   
 $n := \sqrt{m}$   
 Sec. level  $\leq \frac{\# \text{key bits}}{2}$

# One-time Pad (Vernam's Cipher)

message:  $x \in \{0,1\}^n$

key:  $K \in_R \{0,1\}^n$   
↑ uniformly at random

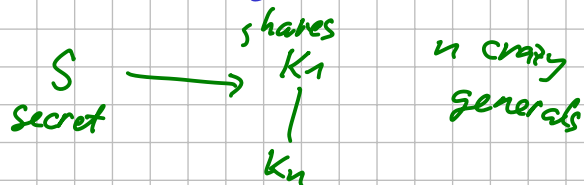
ciphertext:  $y = x \oplus K$   
 $[y_i = x_i \oplus k_i$  sequence of independent uniform random bit

decrypt:  $y \oplus K = (x \oplus K) \oplus K =$   
 $= x \oplus \underbrace{(K \oplus K)}_0 = x$

## Informational-theoretic security (perfect security)

① One-time Pad

② Secret sharing



t threshold  
if t generals meet → recover S  
< t → no inform. on S

↓  
Symmetric ciphers