$P(\pi)$ is sum of (edge) lengths of all nodes of a BST tree obtained by a permutation $\pi \in S_n$, where S_n is the set of all permutation of the set $\{1, \ldots, n\}$.

The average node depth of an average BST is the number

$$\frac{1}{n}\frac{1}{n!}\sum_{\pi \in S_n} P(\pi).$$

Denote

$$Q(n) = \frac{1}{n!}\sum_{\pi \in S_n} P(\pi).$$

There are
n choices of the 1st permutation element $\pi(0)$, let us denote it by j,
$\binom{n-1}{j-1}$ choices of the set $\{m \mid \pi(m) < j\} = A$,
$(j-1)!$ choices of mapping numbers $1, \ldots, j-1$ into A,
$(n-j)!$ choices of mapping numbers $j+1, \ldots, n$ into $\{2, \ldots, n\} - A$.

Note that

$$n\binom{n-1}{j-1}(j-1)!(n-j)! = n!.$$

It is

$$Q(n) = \frac{1}{n!}\sum_{\pi \in S_n} P(\pi) =$$

$$= \frac{1}{n!}\sum_{j=1}^{n}\binom{n-1}{j-1}\sum_{\lambda \in S_{j-1}}\sum_{\mu \in S_{n-j}}(P(\lambda) + j - 1 + P(\mu) + n - j) =$$

$$= \frac{1}{n}\sum_{j=1}^{n}\frac{1}{(j-1)!}\frac{1}{(n-j)!}\sum_{\lambda \in S_{j-1}}\sum_{\mu \in S_{n-j}}(P(\lambda) + P(\mu) + n - 1) =$$

$$= \frac{1}{n}\sum_{j=1}^{n}\frac{1}{(j-1)!}\frac{1}{(n-j)!}\sum_{\lambda \in S_{j-1}}\sum_{\mu \in S_{n-j}} P(\lambda) +$$

$$+ \frac{1}{n}\sum_{j=1}^{n}\frac{1}{(j-1)!}\frac{1}{(n-j)!}\sum_{\lambda \in S_{j-1}}\sum_{\mu \in S_{n-j}} P(\mu) +$$

$$+ \frac{1}{n}\sum_{j=1}^{n}\frac{1}{(j-1)!}\frac{1}{(n-j)!}\sum_{\lambda \in S_{j-1}}\sum_{\mu \in S_{n-j}}(n-1) =$$

$$= \frac{1}{n}\sum_{j=1}^{n}\frac{1}{(j-1)!}\sum_{\lambda\in S_{j-1}}P(\lambda) + \frac{1}{n}\sum_{j=1}^{n}\frac{1}{(n-j)!}\sum_{\mu\in S_{n-j}}P(\mu) + \frac{1}{n}\sum_{j=1}^{n}(n-1) =$$

$$= \frac{1}{n}\sum_{j=1}^{n}Q_{j-1}+\frac{1}{n}\sum_{j=1}^{n}Q_{n-j}+(n-1) = \frac{1}{n}\sum_{k=0}^{n-1}Q_k+\frac{1}{n}\sum_{k=0}^{n-1}Q_k+(n-1) = \frac{2}{n}\sum_{k=0}^{n-1}Q_k+(n-1).$$

Now

$$\sum_{k=1}^{n-1}k\log_2 k = \sum_{k=1}^{\lceil n/2\rceil-1}k\log_2 k + \sum_{k=\lceil n/2\rceil}^{n-1}k\log_2 k \leq$$

$$\leq \sum_{k=1}^{\lceil n/2\rceil-1}k(\log_2 n-1) + \sum_{k=\lceil n/2\rceil}^{n-1}k\log_2 n \leq$$

$$\leq \sum_{k=1}^{n-1}k\log_2 n - \sum_{k=1}^{\lceil n/2\rceil-1}k =$$

$$= \frac{n(n-1)}{2}\log_2 n - \frac{\lceil n/2\rceil(\lceil n/2\rceil-1)}{2} \leq$$

$$\leq \frac{1}{2}n^2\log_2 n - \frac{n^2}{8} + \left(\frac{n+1}{4} - \frac{n}{2}\log_2 n\right) \leq \frac{1}{2}n^2\log_2 n - \frac{n^2}{8}.$$

We are going to prove that $Q(n) \leq 4n\log_2 n$ for all $n > 0$.
It is $Q(1) = 0 = 4\log_2 1$, and by induction

$$Q(n) \leq \frac{2}{n}\sum_{k=1}^{n-1}4k\log_2 n + n \leq \frac{8}{n}\left(\frac{1}{2}n^2\log_2 n - \frac{n^2}{8}\right) + n =$$

$$= 4n\log_2 n - n + n = 4n\log_2 n.$$

Consequently, the average node depth of an average BST tree is at most $4n\log_2 n$.