Definition. *A square matrix* Q *is* orthogonal *if* $Q^TQ = I$.

Exercise 1: Let $u \in \mathbb{R}^n$ be a vector, prove that a matrix uu^T has rank 1.
Exercise 2: Let $u \in \mathbb{R}^n$ be a vector, prove that a matrix $I - uu^T$ is orthogonal.
Exercise 3: Let $u, v, w \in \mathbb{R}^3$ be an orthonormal basis. Let $A = uu^T + vv^T + ww^T$.

1. find Au, Av, Aw
2. let $P = (u, v, w)$ (matrix with u, v, w as colomns), find AP
3. prove that $A = I(!)$.

Exercise 4: Prove that an othonormal matrix $A \in \mathbb{R}^{2\times 2}$ is only of the form

$$A = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \text{ or } A = \begin{pmatrix} \cos\varphi & \sin\varphi \\ \sin\varphi & -\cos\varphi \end{pmatrix}$$

Exercise 5: Show that $\forall a \in \mathbb{R}$ the matrix

$$A = \frac{1}{1+2a^2} \begin{pmatrix} 1 & -2a & 2a^2 \\ 2a & 1-2a^2 & -2a \\ 2a^2 & 2a & 1 \end{pmatrix}$$

is orthonormal.
Exercise 6: Let

$$\begin{pmatrix} a+b & b-a \\ a-b & b+a \end{pmatrix}$$

be a matrix with $a, b \in \mathbb{R}$. Give the conditions on a and b under which A is orthogonal.