Mathematical Programming and Polyhedral Combinatorics 2023/24
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The Capacitated Vehicle Routing Problem (CVRP)

We are **given**:

- A *depot*,
- A fleet of identical *vehicles* with capacity Q,
- n customers with *demands* $q_i \leq Q$, for $i = 1, \ldots, n$, to be delivered from the depot,
- Symmetric *travel cost* $c_{i,j}$ between points i and j for all pairs of points (i.e., the customers and the depot).

The **goal** is to determine routes of minimum total travel cost, subject to the following constraints:

- Each customer is serviced exactly once and she gets the desired demand,
- The total quantity delivered on each route does not exceed Q,
- Each route begins and ends at the depot.

Your task is

- To describe an integer linear programming (ILP) formulation of the problem,
- For each instance of the problem specified below, using your favorite software package for mathematical programming,
  - find an optimal solution of the LP relaxation derived from your ILP,
  - find an integral solution as good as possible (if doable),
  - provide the best known upper and lower bounds on optimal solution,
- Write a brief report.

Details

- For the ILP formulation of the problem, try to come up with your own formulation.
- If you don't have your favorite package for mathematical programming, I recommend Gurobi (they offer free academic licenses, https://www.gurobi.com/academia/academic-program-and-licenses/); as an introduction to this package, check https://www.gurobi.com/wp-content/plugins/hd_documentations/documentation/9.1/quickstart_linux.pdf .
- Alternatively, you can use the Neos Server that provides free access to many solvers, https://neos-server.org/neos/solvers/index.html .
- Concerning the instances, solve E-n13-k4, E-n22-k4, E-n23-k3, E-n30-k3, E-n31-k7, E-n33-k4, E-n51-k5, E-n76-k7, E-n76-k8, E-n76-k10, E-n76-k14, E-n101-k8, E-n101-k14, available at http://vrp.galgos.inf.puc-rio.br/index.php/en/ (set E, Christofides and Eilon, 1969); for explanation of the instance format see http://comopt.ifi.uni-heidelberg.de/software/TSPLIB95/tsp95.pdf .
- You are allowed to work in pairs but in this case, solve also instances from the set Rochat and Taillard (1995) (again at http://vrp.galgos.inf.puc-rio.br/index.php/en/).
- For the report, please
  - describe and explain your ILP,
  - provide details about your computer (processor, memory, OS), software package (if computed locally, not on the NEOS server),
  - prepare a table: for each instance a separate row, and in the columns specify the size of the instance (in number of customers), LP OPT, ILP OPT, time for obtaining LP OPT, time for obtaining ILP OPT, best known upper and lower bounds on optimum.

Other useful **links**

CVRP: https://en.wikipedia.org/wiki/Vehicle_routing_problem, http://dimacs.rutgers.edu/programs/challenge/vrp/cvrp/, https://eprints.lancs.ac.uk/id/eprint/72915/1/2015_mcf_cvrp_source.pdf

Software: https://www.pyomo.org/about, https://coin-or.github.io/pulp/