Exercise 1: Th efollowing matrices represent a linear transformations in the plane $\mathbb{R}^2$. Determine eigenvalues and the associated eigenvectors, and interpret these in geometric terms.

a) $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

b) $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$

c) $\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$

d) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

e) $\frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

Exercise 2: Determine eigenvalues and the corresponding eigenvectors for the following matrix over the field $\mathbb{C}$:

a) $\begin{pmatrix} 2 & 6 \\ 6 & -3 \end{pmatrix}$

b) $\begin{pmatrix} 0 & 1 \\ -2 & 2 \end{pmatrix}$

c) $\begin{pmatrix} 1 & 5 \\ 2 & 4 \end{pmatrix}$

d) $\begin{pmatrix} 5 & 10 \\ 4 & -1 \end{pmatrix}$

Exercise 3: Determine eigenvalues and the corresponding eigenvectors for the following matrix over the field $\mathbb{Z}_5$. Decide whether this matrix is diagonalizable:

$$\begin{pmatrix} 1 & 0 & 0 \\ 2 & 3 & 3 \\ 1 & 1 & 0 \end{pmatrix}$$

Exercise 4: Determine eigenvalues of the matrix

$$\begin{pmatrix} 3 & 2 & 0 & 1 & -2 \\ 0 & 2 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 3 \\ 0 & 5 & 0 & 1 & 0 \\ 4 & 8 & 0 & 7 & -3 \end{pmatrix}.$$

Exercise 5: The matrix

$$\begin{pmatrix} 10 & 0 & 7 & -7 \\ 4 & 5 & 2 & -2 \\ 16 & 4 & 15 & -8 \\ 30 & 4 & 26 & -19 \end{pmatrix}$$

has three eigenvalues $3, -4$ and 5. Determine the remaining eigenvalue.