

Combinatorial Counting

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(Preliminary version)

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Chapter 1

Introduction

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Chapter 2

On Catalan numbers

Chapter 3

Pattern-avoiding permutations

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The Marcus–Tardos theorem

Chapter 5

Forbidding pattern of length 3

Chapter 6

Forbidding pattern of length 4

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